(S)neutrino properties in *R*-parity-violating supersymmetry: *CP*-conserving phenomena

Yuval Grossman
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Howard E. Haber
Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064
and Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
(Received 12 November 1998; published 7 April 1999)

R-parity-violating supersymmetry (with a conserved baryon number *B*) provides a framework for particle physics with lepton-number- (*L*-) violating interactions. We examine in detail the structure of the most general *R*-parity-violating (*B*-conserving) model of low-energy supersymmetry. We analyze the mixing of Higgs bosons with sleptons and the mixing of charginos and neutralinos with charged leptons and neutrinos, respectively. Implications for neutrino and sneutrino masses and mixing and *CP*-conserving sneutrino phenomena are considered. *L*-violating low-energy supersymmetry can be probed at future colliders by studying the phenomenology of sneutrinos. Sneutrino-antisneutrino mass splittings and lifetime differences can provide new opportunities to probe lepton number violation at colliders. [S0556-2821(99)04709-8]

PACS number(s): 14.60.Pq, 12.60.Jv

I. INTRODUCTION

There is no fundamental principle that requires the theory of elementary particle interactions to conserve lepton number. In the standard model, lepton number conservation is a fortuitous accident that arises because one cannot write down renormalizable lepton-number-violating interactions that only involve the fields of the standard model [1]. In fact, there are some experimental hints for nonzero neutrino masses [2] that suggest that lepton number is not an exact symmetry.

In low-energy supersymmetric extensions of the standard model, lepton number conservation is not automatically respected by the most general set of renormalizable interactions. Nevertheless, experimental observations imply that lepton-number-violating effects, if they exist, must be rather small. If one wants to enforce lepton number conservation in the tree-level supersymmetric theory, it is sufficient to impose one extra discrete symmetry. In the minimal supersymmetric standard model (MSSM), a multiplicative symmetry called *R* parity is introduced, such that the *R* quantum number of a MSSM field of spin *S*, baryon number *B*, and lepton number *L* is given by $(-1)^{[3(B-L)+2S]}$. By introducing *B*-*L* conservation modulo 2, one eliminates all dimension-4 lepton-number- and baryon-number-violating interactions. Majorana neutrino masses can be generated in an *R*-parity-conserving extension of the MSSM involving new $\Delta L=2$ interactions through the supersymmetric seesaw mechanism [3,4].

In a recent paper [4] (for an independent study, see Ref. [5]), we studied the effect of such a $\Delta L=2$ interaction on sneutrino phenomena. In this case, the sneutrino ($\tilde{\nu}$) and antisneutrino ($\bar{\tilde{\nu}}$), which are eigenstates of lepton number, are no longer mass eigenstates. The mass eigenstates are therefore superpositions of $\tilde{\nu}$ and $\bar{\tilde{\nu}}$, and sneutrino mixing effects can lead to a phenomenology analogous to that of K-$\bar{K}$ and B-$\bar{B}$ mixing. The mass splitting between the two sneutrino mass eigenstates is related to the magnitude of lepton number violation, which is typically characterized by the size of neutrino masses.[1] As a result, the sneutrino mass splitting is expected generally to be very small. Yet it can be detected in many cases if one is able to observe the lepton number oscillation [4].

Neutrino masses can also be generated in *R*-parity-violating (RPV) models of low-energy supersymmetry [7–11]. However, all possible dimension-4 RPV interactions cannot be simultaneously present and unsuppressed; otherwise, the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose either *B* or *L* separately. For example, if *B* is conserved, but *L* is not, then the theory would violate *R* parity but preserve a $\mathbb{Z}_3$ baryon "triality."

In this paper we extend the analysis of Ref. [4] and study sneutrino phenomena in models without *R* parity (but with baryon triality). Such models exhibit $\Delta L=1$ violating interactions at the level of renormalizable operators. One can then generate $\Delta L=2$ violating interactions, which are responsible for generating neutrino masses. In general, one neutrino mass is generated at the tree level via mixing with the neutralinos, and the remaining neutrino masses are generated at one loop.

In Sec. II we introduce the most general RPV model with a conserved baryon number and establish our notation. In Sec. III we obtain the general form for the mass matrix in the neutral fermion sector (which governs the mixing of neutralinos and neutrinos) and in the neutral scalar sector (which governs the mixing of neutral Higgs bosons and sneutrinos). From these results, we obtain the tree-level masses of neutrinos and squared-mass splittings of the sneutrino-antisneutrino pairs. In Sec. IV we calculate the neutrino

[1]In some cases the sneutrino mass splitting may be enhanced by a factor as large as 10^3 compared to the neutrino mass [4,6].

masses and sneutrino-antisneutrino squared-mass splittings generated at one loop. The phenomenological implications of these results are addressed in Sec. V along with our summary and conclusions. An explicit computation of the scalar potential of the model is presented in Appendix A. For completeness, we present in Appendix B the general form for the mass matrix in the charged fermion sector (which governs the mixing of charginos and charged leptons) and in the charged scalar sector (which governs the mixing of charged Higgs bosons and charged sleptons). The relevant Feynman rules for the RPV model and the loop function needed for the one-loop computations of Sec. IV are given in Appendixes C and D.

II. *R*-PARITY VIOLATION FORMALISM

In *R*-parity-violating low-energy supersymmetry, there is no conserved quantum number that distinguishes the lepton supermultiplets $\hat{L}_m$ and the down-type Higgs supermultiplet $\hat{H}_D$. Here m is a generation label that runs from 1 to $n_g=3$. Each supermultiplet transforms as a $Y=-1$ weak doublet under the electroweak gauge group. It is therefore convenient to denote the four supermultiplets by one symbol $\hat{L}_\alpha$ $(\alpha=0,1,\ldots,n_g)$, with $\hat{L}_0\equiv\hat{H}_D$. We consider the most general low-energy supersymmetric model consisting of the MSSM fields that conserves a $\mathbb{Z}_3$ baryon triality. As remarked in Sec. I, such a theory possesses RPV interactions that violate lepton number.

The Lagrangian of the theory is fixed by the superpotential and the soft-supersymmetry-breaking terms (supersymmetry and gauge invariance fix the remaining dimension-4 terms). The theory we consider consists of the fields of the MSSM, i.e., the fields of the two-Higgs-doublet extension of the standard model plus their superpartners. The most general renormalizable superpotential respecting baryon triality is given by

$$W=\epsilon_{ij}[-\mu_\alpha\hat{L}^i_\alpha\hat{H}^j_U+\tfrac{1}{2}\lambda_{\alpha\beta m}\hat{L}^i_\alpha\hat{L}^j_\beta\hat{E}_m+\lambda'_{\alpha nm}\hat{L}^i_\alpha\hat{Q}^j_n\hat{D}_m -h_{nm}\hat{H}^i_U\hat{Q}^j_n\hat{U}_m], \tag{2.1}$$

where $\hat{H}_U$ is the up-type Higgs supermultiplet, the $\hat{Q}_n$ are doublet quark supermultiplets, $\hat{U}_m$ [$\hat{D}_m$] are singlet up-type [down-type] quark supermultiplets, and the $\hat{E}_m$ are the singlet charged lepton supermultiplets.[2] Without loss of generality, the coefficients $\lambda_{\alpha\beta m}$ are taken to be antisymmetric under the interchange of the indices α and β. Note that the μ term of the MSSM [which corresponds to μ_0 in Eq. (2.1)] is now extended to an (n_g+1)-component vector μ_α (while the latin indices n and m run from 1 to n_g). Then the trilinear terms in the superpotential proportional to λ and λ' contain lepton-number-violating generalizations of the down quark and charged lepton Yukawa matrices.

Next, we consider the most general set of (renormalizable) soft-supersymmetry-breaking terms. In addition to the usual soft-supersymmetry-breaking terms of the *R*-parity-conserving MSSM, one must also add new *A* and *B* terms corresponding to the RPV terms of the superpotential. In addition, new RPV scalar squared-mass terms also exist. As above, we can streamline the notation by extending the definitions of the coefficients of the *R*-parity-conserving soft-supersymmetry-breaking terms to allow for an index of type α which can run from 0 to n_g. Explicitly,

$$\begin{aligned}V_{\rm soft}=&(M^2_{\tilde{Q}})_{mn}\tilde{Q}^{i*}_m\tilde{Q}^i_n+(M^2_{\tilde{U}})_{mn}\tilde{U}^*_m\tilde{U}_n+(M^2_{\tilde{D}})_{mn}\tilde{D}^*_m\tilde{D}_n\\&+(M^2_{\tilde{L}})_{\alpha\beta}\tilde{L}^{i*}_\alpha\tilde{L}^i_\beta+(M^2_{\tilde{E}})_{mn}\tilde{E}^*_m\tilde{E}_n+m^2_U|H_U|^2-(\epsilon_{ij}b_\alpha\tilde{L}^i_\alpha H^j_U+\text{H.c.})\\&+\epsilon_{ij}[\tfrac{1}{2}a_{\alpha\beta m}\tilde{L}^i_\alpha\tilde{L}^j_\beta\tilde{E}_m+a'_{\alpha nm}\tilde{L}^i_\alpha\tilde{Q}^j_n\tilde{D}_m-(a_U)_{nm}H^i_U\tilde{Q}^j_n\tilde{U}_m+\text{H.c.}]\\&+\tfrac{1}{2}[M_3\tilde{g}\tilde{g}+M_2\tilde{W}^a\tilde{W}^a+M_1\tilde{B}\tilde{B}+\text{H.c.}].\end{aligned} \tag{2.2}$$

Note that the single *B* term of the MSSM is extended to an (n_g+1)-component vector b_α, the single squared-mass term for the down-type Higgs boson and the $n_g\times n_g$ lepton scalar squared-mass matrix are combined into an $(n_g+1)\times(n_g+1)$ matrix, and the matrix *A* parameters of the MSSM are extended in the obvious manner [analogous to the Yukawa coupling matrices in Eq. (2.1)]. In particular, $a_{\alpha\beta m}$ is antisymmetric under the interchange of α and β. It is sometimes convenient to follow the more conventional notation in the literature and define the *A* and *B* parameters as follows:

$$a_{\alpha\beta m}\equiv\lambda_{\alpha\beta m}(A_E)_{\alpha\beta m},\quad (a_U)_{nm}\equiv h_{nm}(A_U)_{nm},$$

$$a'_{\alpha nm}\equiv\lambda'_{\alpha nm}(A_D)_{\alpha nm},\quad b_\alpha\equiv\mu_\alpha B_\alpha, \tag{2.3}$$

where repeated indices are not summed over in the above equations. Finally, the Majorana gaugino masses M_i are unchanged from the MSSM.

The total scalar potential is given by

$$V_{\rm scalar}=V_F+V_D+V_{\rm soft}. \tag{2.4}$$

[2]In our notation, $\epsilon_{12}=-\epsilon_{21}=1$. The notation for the superfields (extended to allow $\alpha=0$ as discussed above) follows that of Ref. [12]. For example, $(\tilde{e}^-_L)_m[(\tilde{e}^+_R)_m]$ are the scalar components of $\hat{L}^2_m[\hat{E}_m]$, etc.

In Appendix A, we present the complete expressions for V_F (which is derived from the superpotential [Eq. (2.1)]) and V_D. It is convenient to write out the contribution of the neutral scalar fields to the full scalar potential [Eq. (2.4)]:

$$V_{\text{neutral}}=(m_U^2+|\mu|^2)|h_U|^2+[(M_{\tilde{L}}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta^*]\tilde{\nu}_\alpha\tilde{\nu}_\beta^* -(b_\alpha\tilde{\nu}_\alpha h_U+b_\alpha^*\tilde{\nu}_\alpha^* h_U^*) +\tfrac{1}{8}(g^2+g'^2)[|h_U|^2-|\tilde{\nu}_\alpha|^2]^2, \tag{2.5}$$

where $h_U\equiv H_U^2$ is the neutral component of the up-type Higgs scalar doublet and $\tilde{\nu}_\alpha\equiv\tilde{L}_\alpha^1$. In Eq. (2.5), we have introduced the notation

$$|\mu|^2\equiv\sum_\alpha |\mu_\alpha|^2. \tag{2.6}$$

In minimizing the full scalar potential, we assume that only neutral scalar fields acquire vacuum expectation values: $\langle h_U\rangle\equiv v_u/\sqrt{2}$ and $\langle\tilde{\nu}_\alpha\rangle\equiv v_\alpha/\sqrt{2}$. From Eq. (2.5), the minimization conditions are

$$(m_U^2+|\mu|^2)v_u^*=b_\alpha v_\alpha-\tfrac{1}{8}(g^2+g'^2)(|v_u|^2-|v_d|^2)v_u^*, \tag{2.7}$$

$$[(M_{\tilde{L}}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta^*]v_\beta^*=b_\alpha v_u +\tfrac{1}{8}(g^2+g'^2)(|v_u|^2-|v_d|^2)v_\alpha^*, \tag{2.8}$$

where

$$|v_d|^2\equiv\sum_\alpha |v_\alpha|^2. \tag{2.9}$$

The normalization of the vacuum expectation values has been chosen such that

$$v\equiv(|v_u|^2+|v_d|^2)^{1/2}=\frac{2m_W}{g}=246\ \text{GeV}. \tag{2.10}$$

Up to this point, there is no preferred direction in the generalized generation space spanned by the $\hat{L}_\alpha$. It is convenient to choose a particular ''interaction'' basis such that $v_m=0$ $(m=1,\ldots,n_g)$, in which case $v_0=v_d$. In this basis, we denote $\hat{L}_0\equiv\hat{H}_D$. The down-type quark and lepton mass matrices in this basis arise from the Yukawa couplings to H_D, namely,[3]

$$(m_d)_{nm}=\frac{1}{\sqrt{2}}v_d\lambda'_{0nm}, \quad (m_l)_{nm}=\frac{1}{\sqrt{2}}v_d\lambda_{0nm}, \tag{2.11}$$

while the up-type quark mass matrices arise as in the MSSM:

$$(m_u)_{nm}=\frac{1}{\sqrt{2}}v_u h_{nm}. \tag{2.12}$$

In the literature, one often finds other basis choices. For example, the most common is one where $\mu_0=\mu$ and $\mu_m=0$ $(m=1,\ldots,n_g)$. Of course, the results for physical observables (which involve mass eigenstates) are independent of the basis choice.[4] In the calculations presented in this paper, when we need to fix a basis, we find the choice of $v_m=0$ to be the most convenient.

III. NEUTRINOS AND SNEUTRINOS AT THE TREE LEVEL

We begin by recalling the calculation of the tree-level neutrino mass that arises due to the *R*-parity violation. We then evaluate the corresponding sneutrino mass splitting. In all the subsequent analysis presented in this paper, we shall assume for simplicity that the parameters $(M_{\tilde{L}}^2)_{\alpha\beta}$, μ_α, b_α, the gaugino mass parameters M_i, and v_α are real. In particular, the ratio of vacuum expectation values,

$$\tan\beta\equiv\frac{v_u}{v_d}, \tag{3.1}$$

can be chosen to be positive by convention [with v_d defined by the positive square root of Eq. (2.9)]. That is, we neglect new supersymmetric sources of *CP* violation that can contribute to neutrino and sneutrino phenomena. We shall address the latter possibility in a subsequent paper [14].

A. Neutrino mass

The neutrino can become massive due to mixing with the neutralinos [7]. This is determined by the $(n_g+4)\times(n_g+4)$ mass matrix in a basis spanned by the two neutral gauginos $\tilde{B}$ and $\tilde{W}^3$, the Higgsinos $\tilde{h}_U$ and $\tilde{h}_D\equiv\nu_0$, and n_g generations of neutrinos ν_m. The tree-level fermion mass matrix, with rows and columns corresponding to $\{\tilde{B},\tilde{W}^3,\tilde{h}_U,\nu_\beta\ (\beta=0,1,\ldots,n_g)\}$, is given by [8,9]

[3]As shown in Appendix B, $(m_l)_{nm}$ is not precisely the charged lepton mass matrix, as a result of a small admixture of the charged Higgsino eigenstate due to RPV interactions.

[4]For a general discussion of basis-independent parametrizations of *R*-parity violation, see Refs. [13] and [11].

$$M^{(n)}=\begin{pmatrix} M_1 & 0 & m_Z s_W v_u/v & -m_Z s_W v_\beta/v \\ 0 & M_2 & -m_Z c_W v_u/v & m_Z c_W v_\beta/v \\ m_Z s_W v_u/v & -m_Z c_W v_u/v & 0 & \mu_\beta \\ -m_Z s_W v_\alpha/v & m_Z c_W v_\alpha/v & \mu_\alpha & 0_{\alpha\beta} \end{pmatrix}, \tag{3.2}$$

where $c_W\equiv\cos\theta_W$, $s_W\equiv\sin\theta_W$, v is defined in Eq. (2.10), and $0_{\alpha\beta}$ is the $(n_g+1)\times(n_g+1)$ zero matrix. In a basis-independent analysis, it is convenient to introduce

$$\cos\xi\equiv\frac{\sum_\alpha v_\alpha\mu_\alpha}{v_d\mu}, \tag{3.3}$$

where μ is defined in Eq. (2.6). Note that ξ measures the alignment of v_α and μ_α. It is easy to check that $M^{(n)}$ possesses n_g-1 zero eigenvalues. We shall identify the corresponding states with n_g-1 physical neutrinos of the standard model [8], while one neutrino acquires mass through mixing. We can evaluate this mass by computing the product of the five nonzero eigenvalues of $M^{(n)}$ [denoted below by $\det' M^{(n)}$][5]

$$\det' M^{(n)}=m_Z^2\mu^2 M_{\tilde\gamma}\cos^2\beta\sin^2\xi, \tag{3.4}$$

where $M_{\tilde\gamma}\equiv\cos^2\theta_W M_1+\sin^2\theta_W M_2$. We compare this result with the product of the four neutralino masses of the R-parity-conserving MSSM (obtained by computing the determinant of the upper 4×4 block of $M^{(n)}$ with μ_0, v_0 replaced by μ, v_d, respectively):

$$\det M_0^{(n)}=\mu(m_Z^2 M_{\tilde\gamma}\sin 2\beta-M_1M_2\mu). \tag{3.5}$$

To first order in the neutrino mass, the neutralino masses are unchanged by the R-parity-violating terms, and we end up with [9]

$$m_\nu=\frac{\det' M^{(n)}}{\det M_0^{(n)}}=\frac{m_Z^2\mu M_{\tilde\gamma}\cos^2\beta\sin^2\xi}{m_Z^2 M_{\tilde\gamma}\sin 2\beta-M_1M_2\mu}. \tag{3.6}$$

Thus $m_\nu\sim m_Z\cos^2\beta\sin^2\xi$, assuming that all the relevant masses are at the electroweak scale.

Note that a necessary and sufficient condition for $m_\nu\neq0$ (at the tree level) is $\sin\xi\neq0$, which implies that μ_α and v_α are not aligned.[6] This is generic in RPV models. In particular, the alignment of μ_α and v_α is not renormalization group invariant [9,10]. Thus exact alignment at the low-energy scale can only be implemented by a fine-tuning of the model parameters.

[5]To compute this quantity, calculate the characteristic polynomial $\det(\lambda I-M^{(n)})$ and examine the first nonzero coefficient of λ^n ($n=0,1,\ldots$). In the present case, $\det' M^{(n)}$ is given by the coefficient of λ^{n_g-1}.

[6]A necessary and sufficient condition for the alignment of μ_α and v_α is given in Eq. (3.11).

B. Sneutrino mass splitting

In RPV low-energy supersymmetry, the sneutrinos mix with the Higgs bosons. Under the assumption of *CP* conservation, we may separately consider the *CP*-even and *CP*-odd scalar sectors. For simplicity, consider first the case of one sneutrino generation. If R parity is conserved, the *CP*-even scalar sector consists of two Higgs scalars (h^0 and H^0, with $m_{h^0}<m_{H^0}$) and $\tilde\nu_+$, while the *CP*-odd scalar sector consists of the Higgs scalar A^0, the Goldstone boson (which is absorbed by the Z), and one sneutrino $\tilde\nu_-$. Moreover, the $\tilde\nu_\pm$ are mass degenerate, so that the standard practice is to define eigenstates of lepton number: $\tilde\nu\equiv(\tilde\nu_++i\tilde\nu_-)/\sqrt2$ and $\bar{\tilde\nu}\equiv\tilde\nu^*$. When R parity is violated, the sneutrinos in each *CP* sector mix with the corresponding Higgs scalars, and the mass degeneracy of $\tilde\nu_+$ and $\tilde\nu_-$ is broken. We expect the RPV interactions to be small; thus, we can evaluate the concomitant sneutrino mass splitting in perturbation theory. For $n_g>1$ generations of sneutrinos, one can consider nontrivial flavor mixing among sneutrinos (or antisneutrinos) in addition to n_g sneutrino-antisneutrino mass splittings.

The *CP*-even and *CP*-odd scalar squared-mass matrices are most easily derived as follows. Insert $h_U=(1/\sqrt2)(v_u+ia_u)$ and $\tilde\nu_\alpha=(1/\sqrt2)(v_\alpha+ia_\alpha)$ into Eq. (2.5) and call the resulting expression $V_{\rm even}+V_{\rm odd}$. The *CP*-even squared-mass matrix is obtained from $V_{\rm even}$, which is identified by replacing the scalar fields in Eq. (2.5) by their corresponding real vacuum expectation values (or, equivalently, by setting $a_u=a_\alpha=0$ in $V_{\rm even}+V_{\rm odd}$). Then,

$$V_{\rm even}=\tfrac12 m_{uu}^2v_u^2+\tfrac12 m_{\alpha\beta}^2v_\alpha v_\beta-b_\alpha v_u v_\alpha+\tfrac{1}{32}(g^2+g'^2)(v_u^2-v_d^2)^2, \tag{3.7}$$

$$V_{\rm odd}=\tfrac12 m_{uu}^2a_u^2+\tfrac12 m_{\alpha\beta}^2a_\alpha a_\beta+b_\alpha a_u a_\alpha+\tfrac{1}{32}(g^2+g'^2)[(a_u^2-a_d^2)^2+2(a_u^2-a_d^2)(v_u^2-v_d^2)], \tag{3.8}$$

where $m_{uu}^2\equiv(m_U^2+\mu^2)$ and $m_{\alpha\beta}^2\equiv(M_{\tilde L}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta$. The minimization conditions $dV_{\rm even}/dv_p=0$ $(p=u,\alpha)$ yield Eqs. (2.7) and (2.8), with all parameters assumed to be real. In particular, it is convenient to rewrite Eq. (2.8). First, we introduce the generalized $(n_g+1)\times(n_g+1)$ sneutrino squared-mass matrix

$$(M_{\tilde\nu\tilde\nu^*}^2)_{\alpha\beta}\equiv(M_{\tilde L}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta-\tfrac18(g^2+g'^2)(v_u^2-v_d^2)\delta_{\alpha\beta}. \tag{3.9}$$

Then Eq. (2.8) assumes a very simple form

$$(M_{\tilde\nu\tilde\nu^*}^2)_{\alpha\beta}v_\beta=v_u b_\alpha. \tag{3.10}$$

As an aside, we note that Eq. (3.10) can be used to derive the necessary and sufficient condition for $\sin\xi=0$ (corresponding to the alignment of μ_α and v_α). If there exists some number c such that

$$(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}\mu_\beta=cb_\alpha, \tag{3.11}$$

then it follows that μ_α and v_α are aligned.[7] To prove that Eq. (3.11) implies the alignment of μ_α and v_α, simply insert Eq. (3.11) into Eq. (3.10) (thereby eliminating b_α), and note that $(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}$ must be nonsingular [otherwise, Eq. (3.10) would not yield a unique nontrivial solution for v_α].

Naively, one might think that if μ_α and v_α are aligned, so that all tree-level neutrino masses vanish, then one would also find degenerate sneutrino-antisneutrino pairs at the tree level. This is not generally true. Instead, the absence of degenerate sneutrino-antisneutrino pairs is controlled by the alignment of b_α and v_α. To see how this works, note that Eq. (3.10) implies that b_α and v_α are aligned if v_β is an eigenvector of $(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}$. One can then rotate to a basis in which $v_m=b_m=0$ (where $m=1,\ldots,n_g$); in this basis, the matrix elements $(M^2_{\tilde{\nu}\tilde{\nu}*})_{0m}=(M^2_{\tilde{\nu}\tilde{\nu}*})_{m0}=0$. It then follows (using the explicit forms for the scalar squared-mass matrices [Eqs. (3.13) and (3.15) given below]) that there is no mixing between the Higgs bosons and sneutrinos. Thus, although some RPV effects still remain in the theory, the *CP*-even and *CP*-odd sneutrino mass matrices are identical. Consequently, the conditions for the absence of tree-level neutrino masses (alignment of μ_α and v_α) and the absence of sneutrino-antisneutrino mass splitting at the tree level (alignment of b_α and v_α) are different.

Returning to the computation of the tree-level sneutrino-antisneutrino mass splittings, we must first calculate the *CP*-even and *CP*-odd scalar spectrum. The *CP*-even scalar squared-mass matrix is given by

$$(M^2_{\rm even})_{pq}=\frac{d^2V_{\rm even}}{dv_p dv_q}. \tag{3.12}$$

After using the minimization conditions of the potential, we obtain the following result for the *CP*-even squared-mass matrix:

$$M^2_{\rm even}=\begin{pmatrix} \frac{1}{4}(g^2+g'^2)v_u^2+b_\rho v_\rho/v_u & -\frac{1}{4}(g^2+g'^2)v_u v_\beta-b_\beta \\ -\frac{1}{4}(g^2+g'^2)v_u v_\alpha-b_\alpha & \frac{1}{4}(g^2+g'^2)v_\alpha v_\beta+(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}\end{pmatrix}, \tag{3.13}$$

where $(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}$ is constrained according to Eq. (3.10). The *CP*-odd scalar squared-mass matrix is determined from

$$(M^2_{\rm odd})_{pq}=\left.\frac{d^2V_{\rm odd}}{da_p da_q}\right|_{a_p=0}, \tag{3.14}$$

where $V_{\rm odd}$ is given by Eq. (3.8). The resulting *CP*-odd squared-mass matrix is then

$$M^2_{\rm odd}=\begin{pmatrix} b_\rho v_\rho/v_u & b_\beta \\ b_\alpha & (M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha\beta}\end{pmatrix}. \tag{3.15}$$

Note that the vector $(-v_u,v_\beta)$ is an eigenvector of $M^2_{\rm odd}$ with zero eigenvalue; this is the Goldstone boson that is absorbed by the Z. One can check that the following tree-level sum rule holds:

$$\operatorname{Tr} M^2_{\rm even}=m_Z^2+\operatorname{Tr} M^2_{\rm odd}. \tag{3.16}$$

This result is a generalization of the well-known tree-level sum rule for the *CP*-even Higgs masses of the MSSM [see Eq. (3.21)]. Equation (3.16) is more general in that it also includes contributions from the sneutrinos which mix with the neutral Higgs bosons in the presence of RPV interactions.

To complete the computation of the sneutrino-antisneutrino mass splitting, one must evaluate the nonzero eigenvalues of $M^2_{\rm even}$ and $M^2_{\rm odd}$, and identify which ones correspond to the sneutrino eigenstates. To do this, one must first identify the small parameters characteristic of the RPV interactions. We find that a judicious choice of basis significantly simplifies the analysis. Following the discussion at the end of Sec. II, we choose a basis such that $v_m=0$ (which implies that $v_d=v_0$).

To illustrate our method, we exhibit the calculation in the case of $n_g=1$ generation. In the basis where $v_1=0$, Eq. (3.10) implies that $(M^2_{\tilde{\nu}\tilde{\nu}*})_{\alpha 0}=b_\alpha\tan\beta$ $(\alpha=0,1)$. Then the squared-mass matrices, Eqs. (3.13) and (3.15), reduce to

[7]It is interesting to compare this result with the one obtained in Ref. [8], where it was shown that μ_α and v_α are aligned if two conditions hold: (i) $b_\alpha\propto\mu_\alpha$ and (ii) μ_α is an eigenvector of $(M^2_{\tilde{L}})_{\alpha\beta}$. From Eq. (3.11), we see that these two conditions are sufficient for alignment [since conditions (ii) and (iii) imply the existence of a constant c in Eq. (3.11)], but are not the most general.

$$M^2_{\rm even}=\begin{pmatrix} b_0\cot\beta+\frac{1}{4}(g^2+g'^2)v_u^2 & -b_0-\frac{1}{4}(g^2+g'^2)v_uv_d & -b_1 \\ -b_0-\frac{1}{4}(g^2+g'^2)v_uv_d & b_0\tan\beta+\frac{1}{4}(g^2+g'^2)v_d^2 & b_1\tan\beta \\ -b_1 & b_1\tan\beta & m^2_{\tilde\nu\tilde\nu*}\end{pmatrix} \qquad (3.17)$$

and

$$M^2_{\rm odd}=\begin{pmatrix} b_0\cot\beta & b_0 & b_1 \\ b_0 & b_0\tan\beta & b_1\tan\beta \\ b_1 & b_1\tan\beta & m^2_{\tilde\nu\tilde\nu*}\end{pmatrix}, \qquad (3.18)$$

where

$$m^2_{\tilde\nu\tilde\nu*}\equiv(M^2_{\tilde\nu\tilde\nu*})_{11}=(M^2_{\tilde L})_{11}+\mu_1^2-\tfrac{1}{8}(g^2+g'^2)(v_u^2-v_d^2). \qquad (3.19)$$

In the R-parity-conserving limit ($b_1=\mu_1=0$), one obtains the usual MSSM tree-level masses for the Higgs bosons and the sneutrinos.

In both squared-mass matrices [Eqs. (3.17) and (3.18)], $b_1\ll m_Z^2$ is a small parameter that can be treated perturbatively. We may then compute the sneutrino mass splitting due to the small mixing with the Higgs bosons. Using second-order matrix perturbation theory to compute the eigenvalues, we find

$$m^2_{\tilde\nu_+}=m^2_{\tilde\nu\tilde\nu*}+\frac{b_1^2}{\cos^2\beta}\left[\frac{\sin^2(\beta-\alpha)}{(m^2_{\tilde\nu\tilde\nu*}-m^2_{H^0})}+\frac{\cos^2(\beta-\alpha)}{(m^2_{\tilde\nu\tilde\nu*}-m^2_{h^0})}\right],$$

$$m^2_{\tilde\nu_-}=m^2_{\tilde\nu\tilde\nu*}+\frac{b_1^2}{(m^2_{\tilde\nu\tilde\nu*}-m^2_{A^0})\cos^2\beta}. \qquad (3.20)$$

Above, we employ the standard notation for the MSSM Higgs sector observables [15]. Note that at leading order in b_1^2, it suffices to use the values for the Higgs parameters in the R-parity-conserving limit. In particular, the (tree-level) Higgs boson masses satisfy

$$m^2_{h^0}+m^2_{H^0}=m_Z^2+m^2_{A^0}, \qquad (3.21)$$

$$m^2_{h^0}m^2_{H^0}=m_Z^2m^2_{A^0}\cos^2 2\beta, \qquad (3.22)$$

while the (tree-level) CP-even Higgs mixing angle satisfies

$$\cos^2(\beta-\alpha)=\frac{m^2_{h^0}(m_Z^2-m^2_{h^0})}{m^2_{A^0}(m^2_{H^0}-m^2_{h^0})}. \qquad (3.23)$$

After some algebra, we end up with the following expression at leading order in b_1^2 for the sneutrino squared-mass splitting $\Delta m^2_{\tilde\nu}\equiv m^2_{\tilde\nu_+}-m^2_{\tilde\nu_-}$:

$$\Delta m^2_{\tilde\nu}=\frac{4b_1^2m_Z^2m^2_{\tilde\nu\tilde\nu*}\sin^2\beta}{(m^2_{\tilde\nu\tilde\nu*}-m^2_{H^0})(m^2_{\tilde\nu\tilde\nu*}-m^2_{h^0})(m^2_{\tilde\nu\tilde\nu*}-m^2_{A^0})}. \qquad (3.24)$$

We now extend the above results to more than one generation of sneutrinos. In a basis where $v_m=0$ ($m=1,\ldots,n_g$), the resulting CP-even and CP-odd squared mass matrices are obtained from Eqs. (3.17) and (3.18) by replacing b_1 with the n_g-dimensional vector b_m and $m^2_{\tilde\nu\tilde\nu*}$ by the $n_g\times n_g$ matrix $(M^2_{\tilde\nu\tilde\nu*})_{mn}$. In general, $(M^2_{\tilde\nu\tilde\nu*})_{mn}$ need not be flavor diagonal. In this case, the theory would predict sneutrino flavor mixing in addition to the sneutrino-antisneutrino mixing exhibited above. The relative strength of these effects depends on the relative size of the RPV and flavor-violating parameters of the model. To analyze the resulting sneutrino spectrum, we choose a basis in which $(M^2_{\tilde\nu\tilde\nu*})_{mn}$ is diagonal:

$$(M^2_{\tilde\nu\tilde\nu*})_{mn}=(m^2_{\tilde\nu\tilde\nu*})_m\delta_{mn}. \qquad (3.25)$$

In this basis, b_m is also suitably redefined. (We will continue to use the same symbols for these quantities in the new basis.) The CP-even and CP-odd sneutrino mass eigenstates will be denoted by $(\tilde\nu_+)_m$ and $(\tilde\nu_-)_m$, respectively.[8] It is a simple matter to extend the perturbative analysis of the scalar squared-mass matrices if the $(m^2_{\tilde\nu\tilde\nu*})_m$ are nondegenerate. We then find that $(\Delta m^2_{\tilde\nu})_m\equiv(m^2_{\tilde\nu_+})_m-(m^2_{\tilde\nu_-})_m$ is given by Eq. (3.24), with the replacement of b_1 and $m^2_{\tilde\nu\tilde\nu*}$ by b_m and $(m^2_{\tilde\nu\tilde\nu*})_m$, respectively. That is, while in general only one neutrino is massive, all the sneutrino-antisneutrino pairs are generically split in mass.[9] If we are prepared to allow for special choices of the parameters μ_α and b_α, then these results are modified. The one massive neutrino becomes massless if $\mu_m=0$ for *all* m (in the basis where $v_m=0$). In contrast, the number of sneutrino-antisneutrino pairs that remain degenerate in mass is equal to the number of the b_m that are zero. (Of course, all these tree-level results are modified by one-loop radiative corrections as discussed in Sec. IV.)

[8]The index m labels sneutrino generation, although one should keep in mind that in the presence of flavor violation, the sneutrino mass basis is not aligned with the corresponding mass bases relevant for the charged sleptons, charged leptons, or neutrinos.

[9]This is a very general tree-level result. Consider models with n_g generations of left-handed neutrinos in which some of the neutrino mass eigenstates remain massless. One finds that generically, *all* n_g sneutrino-antisneutrino pairs are split in mass. For example, in the three-generation seesaw model with one right-handed neutrino, only one neutrino is massive, while all three sneutrino-antisneutrino pairs are nondegenerate. (At the one-loop level, the nondegeneracy of the sneutrino-antisneutrino pairs will generate small masses for neutrinos that were massless at the tree level [16].)

If some of the $(m^2_{\tilde{\nu}\tilde{\nu}*})_m$ are degenerate, the analysis becomes significantly more complicated. We will not provide the corresponding analytic expressions (although they can be obtained using degenerate second-order perturbation theory). However, one can show that for two or more generations if $n_{\rm deg}$ of the $(m^2_{\tilde{\nu}\tilde{\nu}*})_m$ are equal (by definition, $n_{\rm deg}\geqslant 2$), and if $b_m\neq 0$ for all m, then only $n_g-n_{\rm deg}+2$ of the CP-even–CP-odd sneutrino pairs are split in mass. The remaining $n_{\rm deg}-2$ sneutrino pairs are exactly mass degenerate at the tree level. Additional cases can be considered if some of the b_m vanish.

IV. ONE-LOOP EFFECTS

In Sec. III, we showed that in the three-generation model for a generic choice of RPV parameters, mass for one neutrino flavor is generated at the tree level due to mixing with the neutralinos, while mass splittings of three generations of sneutrino-antisneutrino pairs at the tree level are a consequence of mixing with the Higgs bosons. Special choices of the RPV parameters can leave all neutrinos massless at the tree level and/or less than three sneutrino-antisneutrino pairs with nondegenerate tree-level masses.

Masses for the remaining massless neutrinos and mass splittings for the remaining degenerate sneutrino-antisneutrino pairs will be generated by one-loop effects. Moreover, in some cases, the radiative corrections to the tree-level generated masses and mass splittings can be significant (and may actually dominate the corresponding tree-level results). As a concrete example, consider a model in which RPV interactions are introduced only through the superpotential λ and λ' couplings [Eq. (2.1)]. In this case, μ_α, b_α, and v_α are all trivially aligned and no tree-level neutrino masses or sneutrino mass splittings are generated. In a realistic model, soft-supersymmetry-breaking RPV terms will be generated radiatively in such models, thereby introducing a small nonalignment among μ_α, v_α, and b_α. However, the resulting tree-level neutrino masses and sneutrino-antisneutrino mass splittings will be radiatively suppressed, in which case the tree-level and one-loop radiatively generated masses and mass splittings considered in this section would be of the same order of magnitude.

In this section, we compute the one-loop generated neutrino mass and sneutrino-antisneutrino mass splitting generated by the RPV interactions. However, there is another effect that arises at one loop from R-parity-conserving effects. Once a sneutrino-antisneutrino squared-mass splitting is established, its presence will contribute radiatively to neutrino masses through a one-loop diagram involving sneutrinos and neutralinos (with R-parity-conserving couplings). Similarly, a nonzero neutrino mass will generate a one-loop sneutrino-antisneutrino mass splitting. In Ref. [4], we considered these effects explicitly. The conclusion of this work was that

$$10^{-3}\lesssim\frac{\Delta m_{\tilde{\nu}}}{m_\nu}\lesssim 10^3. \tag{4.1}$$

This result is applicable to all models in which there is no unnatural cancellation between the tree-level and one-loop contributions to the neutrino mass or to the sneutrino-antisneutrino mass splitting.

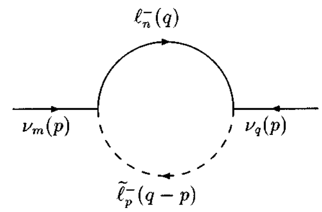

FIG. 1. One-loop contribution to the neutrino mass.

A. One-loop neutrino mass

At one loop, contributions to the neutrino mass are generated from diagrams involving a charged lepton-slepton loop (shown in Fig. 1) and an analogous down-type quark-squark loop [7]. We first consider the contribution of the charged lepton-slepton loop. We shall work in a specific basis, in which $v_m=0$ (i.e., $v_0=v_d$) and the charged lepton mass matrix is diagonal. In this basis, the distinction between charged sleptons and Higgs bosons is meaningful. Nevertheless, in a complete calculation, we should keep track of charged-slepton–Higgs-boson mixing and the charged-lepton–chargino mixing which determine the actual mass eigenstates that appear in the loop. For completeness, we write out in Appendix B the relevant mass matrices of the charged fermion and scalar sectors. In order to simplify the computation, we shall simply ignore all flavor mixing (this includes mixing between charged Higgs bosons and sleptons). However, we allow for mixing between the L-type and R-type charged sleptons separately in each generation, since this is necessary in order to obtain a nonvanishing effect.

It therefore suffices to consider the structure of a 2×2 (LR) block of the charged slepton squared-mass matrix corresponding to one generation. The corresponding charged slepton mass eigenstates are given by

$$\tilde{l}_i=V_{i1}\tilde{l}_L+V_{i2}\tilde{l}_R,\quad i=1,2, \tag{4.2}$$

where

$$V=\begin{pmatrix}\cos\phi_l & \sin\phi_l\\ -\sin\phi_l & \cos\phi_l\end{pmatrix}. \tag{4.3}$$

The mixing angle ϕ_l can be found by diagonalizing the charged slepton squared-mass matrix

$$M^2_{\rm slepton}=\begin{pmatrix}L^2+m_l^2 & Am_l\\ Am_l & R^2+m_l^2\end{pmatrix}, \tag{4.4}$$

where $L^2\equiv(M^2_{\tilde{L}})_{ll}+(T_3-e\sin^2\theta_W)m_Z^2\cos 2\beta$, $R^2\equiv(M^2_{\tilde{E}})_{ll}+(e\sin^2\theta_W)m_Z^2\cos 2\beta$, with $T_3=-1/2$ and $e=-1$ for the down-type charged sleptons, and $A\equiv(A_E)_{0ll}-\mu_0\tan\beta$. In terms of these parameters, the mixing angle is given by

$$\sin 2\phi_l=\frac{2Am_l}{\sqrt{(L^2-R^2)^2+4A^2m_l^2}}. \tag{4.5}$$

The two-point amplitude corresponding to Fig. 1 can be computed using the Feynman rules given in Appendix C. The result is given by

$$i\mathcal{M}_{qm}=\sum_{l,p}\sum_{i=1,2}\int\frac{d^4q}{(2\pi)^4}(-i\lambda_{qlp})C^{-1}P_LV_{i2}\frac{i(\not{q}+m_l)}{q^2-m_l^2}(i\lambda_{mpl})P_LV_{i1}\frac{i}{(q-p)^2-M_{p_i}^2}, \tag{4.6}$$

where m_l is the lepton mass, M_{p_i}, are the sleptons masses, and the V_{ij} are the slepton mixing matrix elements [Eq. (4.3)]. The charge conjugation matrix C appears according to the Feynman rules given in Appendix D of Ref. [17]. The integral above can be expressed in terms of the well-known one-loop integral B_0 (defined in Appendix D). The corresponding contributions to the one-loop neutrino mass matrix is obtained via $(m_\nu)_{qm}=-\mathcal{M}_{qm}(p^2=0)$. The end result is

$$\begin{aligned}(m_\nu)_{qm}^{(l)}&=\frac{1}{32\pi^2}\sum_{l,p}\lambda_{qlp}\lambda_{mpl}m_l\sin 2\phi_l[B_0(0,m_n^2,M_{p_1}^2)-B_0(0,m_n^2,M_{p_2}^2)]\\&\simeq\frac{1}{32\pi^2}\sum_{l,p}\lambda_{qlp}\lambda_{mpl}m_l\sin 2\phi_l\ln\left(\frac{M_{p_1}^2}{M_{p_2}^2}\right),\end{aligned} \tag{4.7}$$

where the superscript (l) indicates the contribution of Fig. 1. As expected, the divergences cancel and the final result is finite. In the last step, we simplified the resulting expression under the assumption that $m_l \ll M_{p_1},M_{p_2}$.

The quark-squark loop contribution to the one-loop neutrino mass may be similarly computed. Employing the same approximations as described above, the final result can be immediately obtained from Eq. (4.7) with the following adjustments: (i) multiply the result by a color factor of $N_c=3$, (ii) replace the Yukawa couplings λ with λ' and the lepton mass m_l by the corresponding down-type quark mass m_d, and (iii) replace the slepton mixing angle ϕ_l by the corresponding down-type squark mixing angle ϕ_d. Note that ϕ_d is computed using Eqs. (4.3) and (4.5), after replacing m_l, $e=-1$, $M_{\tilde{L}}^2$, $M_{\tilde{E}}^2$, and $(A_E)_{0ll}$ with m_d, $e=-1/3$, $M_{\tilde{Q}}^2$, $M_{\tilde{D}}^2$, and $(A_D)_{0dd}$, respectively. Here and below, $d[r]$ labels the generations of down-type quarks [squarks]. Then,

$$(m_\nu)_{qm}^{(d)}\simeq\frac{3}{32\pi^2}\sum_{d,r}\lambda'_{qdr}\lambda'_{mrd}m_d\sin 2\phi_d\ln\left(\frac{M_{r_1}^2}{M_{r_2}^2}\right). \tag{4.8}$$

The final result for the neutrino mass matrix is the sum of Eqs. (4.7) and (4.8). Clearly, for generic choices of the λ and λ' couplings, all neutrinos (including those neutrinos that were massless at the tree level) gain a one-loop generated mass.

B. One-loop sneutrino-antisneutrino mass splitting

We next consider the computation of the one-loop contributions to the sneutrino masses under some simplifying assumptions (which are sufficient to illustrate the general form of these corrections). Since the total R-parity-conserving contribution to the sneutrino and antisneutrino mass is equal and large (of order the supersymmetry breaking mass), it is sufficient to evaluate the one-loop corrections to the $\Delta L=2$ sneutrino squared masses. Flavor-nondiagonal contributions are significant only if sneutrinos of different flavors are mass degenerate. The one-loop generated mass splitting is relevant only when the tree-level contributions vanish or are highly suppressed. In the simplest case, for one generation of sneutrinos and without tree-level sneutrino-antisneutrino splitting, we get

$$(\Delta m_{\tilde{\nu}}^2)_n=2|\mathcal{M}_{nn}(p^2=m_{\tilde{\nu}}^2)|, \tag{4.9}$$

where $i\mathcal{M}_{nm}$ is the sum of all contributing one-loop Feynman diagrams computed below and $m_{\tilde{\nu}}$ is the R-parity-conserving tree-level sneutrino mass. In the more complicated case, where there are $n_{\rm deg}$ flavors of mass-degenerate sneutrinos, sneutrino-antisneutrino mass eigenstates are obtained by diagonalizing the $2n_{\rm deg}\times 2n_{\rm deg}$ sneutrino squared-mass matrix:

$$M_{\rm sneutrino}^2=\begin{pmatrix} m_{\tilde{\nu}}^2\delta_{mn} & \mathcal{M}_{mp}(p^2=m_{\tilde{\nu}}^2)\\ \mathcal{M}_{qn}^*(p^2=m_{\tilde{\nu}}^2) & m_{\tilde{\nu}}^2\delta_{qp}\end{pmatrix}, \tag{4.10}$$

where $m,n=1,\ldots,n_{\rm deg}$ and $p,q=n_{\rm deg}+1,\ldots,2n_{\rm deg}$. In the case that there are small mass splittings between sneutrinos of different flavors, we can treat such effects perturbatively by simply including such flavor nondegeneracies in the diagonal blocks above. Likewise, a small tree-level splitting of the sneutrino and antisneutrino can be accommodated perturbatively by an appropriate modification of the off-diagonal blocks above.

As discussed in Sec. IV A, we need only consider in detail the contribution of lepton and slepton loops. (In particular, we neglect flavor mixing, but allow for mixing between the L-type and R-type charged sleptons separately in each generation.) The corresponding contributions of the quark and squark loops are then easily obtained by appropriate substitution of parameters. The relevant graphs with an intermediate lepton and slepton loops are shown in Figs. 2 and 3, respectively.

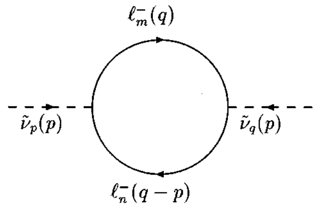

FIG. 2. Lepton pair loop contribution to the sneutrino-antisneutrino mass splitting.

Using the Feynman rules of Appendix C (including a minus sign for the fermion loop), the contribution of the lepton loop (Fig. 2) is given by

$$i\mathcal{M}_{pq}^{(f)} = -\sum_{m,n} \lambda_{pmn}\lambda_{qnm}\int \frac{d^4q}{(2\pi)^4} \times \frac{\mathrm{Tr}[(\not{q}+m_m)P_L(\not{p}+\not{q}+m_n)P_L]}{[q^2-m_m^2][(q+p)^2-m_n^2]} = \frac{-i}{8\pi^2}\sum_{m,n} \lambda_{pmn}\lambda_{qnm}m_m m_n B_0(p^2,m_m^2,m_n^2). \tag{4.11}$$

The contribution of the slepton loop (Fig. 3) contains two distinct pieces. In the absence of *LR* slepton mixing, we have *LL* and *RR* contributions in the loop proportional to the λ Yukawa couplings. When we turn on the *LR* slepton mixing, we find additional contributions proportional to the corresponding *A* terms. First, consider the contributions proportional to Yukawa couplings. For simplicity, we neglect the *LR* slepton mixing in this case. As before, we work in a basis where $v_m=0$ (i.e., $v_0=v_d$) and we choose a flavor basis corresponding to the one where the charged lepton mass matrices are diagonal. Then, the contribution of the slepton loop (Fig. 3), summing over $i=L,R$ type sleptons, is given by

$$i\mathcal{M}_{pq}^{(\lambda)} = \sum_{i,m,n} \lambda_{pmn}\lambda_{qnm}m_m m_n \int \frac{d^4q}{(2\pi)^4} \times \frac{1}{[q^2-M_{m_i}^2][(q+p)^2-M_{n_i}^2]} = \frac{i}{16\pi^2}\sum_{mn} \lambda_{pmn}\lambda_{qnm}m_m m_n[B_0(p^2,M_{m_R}^2,M_{n_R}^2) + B_0(p^2,M_{m_L}^2,M_{n_L}^2)], \tag{4.12}$$

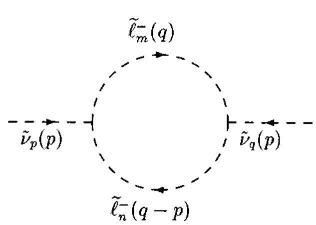

FIG. 3. Slepton pair loop contribution to the sneutrino-antisneutrino mass splitting.

where the m_n are *lepton* masses and M_{m_i} are slepton masses. It is easy to check that the divergences cancel from the sum $i\mathcal{M}_{pq}^{(f)}+i\mathcal{M}_{pq}^{(\lambda)}$, which results in a finite correction to the sneutrino mass. This serves as an important check of the calculation.

If *LR* slepton mixing is included, the above results are modified. The corrections to Eq. (4.12) in this case are easily obtained, but we shall omit their explicit form here. In addition, new slepton loop contributions arise that are proportional to the *A* parameters [defined in Eq. (2.2)]. We quote only the final result:

$$i\mathcal{M}_{pq}^{(A)} = \frac{i}{64\pi^2}\sum_{m,n} a_{pmn}a_{qnm}\sin 2\phi_m \sin 2\phi_n \times[B_0(p^2,M_{m_1}^2,M_{n_1}^2)+B_0(p^2,M_{m_2}^2,M_{n_2}^2) - B_0(p^2,M_{m_1}^2,M_{n_2}^2)-B_0(p^2,M_{m_2}^2,M_{n_1}^2)], \tag{4.13}$$

where ϕ_n is the slepton mixing angle of the nth generation and the corresponding slepton eigenstate masses are M_{n_1} and M_{n_2}. This result is manifestly finite. Note that this contribution vanishes when the *LR* mixing is absent.

The total contribution of the lepton and slepton loops are given by the sum of Eqs. (4.11), (4.12), and (4.13):

$$i\mathcal{M}_{pq}^{(l)} = i\mathcal{M}_{pq}^{(f)}+i\mathcal{M}_{pq}^{(\lambda)}+i\mathcal{M}_{pq}^{(A)}. \tag{4.14}$$

Finally, one must add the contributions of the quark and squark loops. The results of this subsection can be used, with the substitutions described in Sec. IV A to derive the final expressions. Once again, we see that for generic choices of the λ, A, λ', and A' parameters, all sneutrino-antisneutrino pairs (including those pairs that were mass degenerate at the tree level) are split in mass by one-loop effects.

V. PHENOMENOLOGICAL CONSEQUENCES

The detection of a nonvanishing sneutrino-antisneutrino mass splitting would be a signal of lepton number violation. In particular, it serves as a probe of $\Delta L=2$ interactions, which also contributes to the generation of neutrino masses. Thus sneutrino phenomenology at colliders may provide access to physics that previously could only be probed by observables sensitive to neutrino masses.

Some proposals for detecting the sneutrino-antisneutrino mass splitting were presented in Ref. [4]. If this mass splitting is large (more then about 1 GeV), one may hope to be able to reconstruct the two masses in sneutrino pair production and measure their difference. In a RPV theory with *L* violation, resonant production of sneutrinos becomes possible [18] and the sneutrino mass splitting may be detected either directly [19] or by using tau-spin asymmetries [20]. If the mass splitting is much smaller than 1 GeV, sneutrino-antisneutrino oscillations can be used to measure $\Delta m_{\tilde{\nu}}$. In analogy with B-$\overline{B}$ mixing, a same-sign lepton signal will indicate that the two sneutrino mass eigenstates are not mass degenerate. In practice, one may only be able to measure the ratio $x_{\tilde{\nu}} \equiv \Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}}$. In order to be able to observe the oscillation, two conditions must be satisfied: (i) $x_{\tilde{\nu}}$ should not be much smaller than 1, and (ii) the branching ratio into a lepton-number-tagging mode should be significant.

The sneutrino-antisneutrino mass splitting is proportional to the RPV parameters b_m (for tree-level mass splitting) and λ, A, λ', and A' (for loop-induced mass splitting). Generally speaking, these parameters can be rather large, and the strongest bounds on them come from the limits on neutrino masses. In the following discussion, we will consider the possible values of the relevant parameters: (i) the ratio of the sneutrino-antisneutrino mass splitting to the neutrino mass ($r_\nu \equiv \Delta m_{\tilde{\nu}}/m_\nu$), (ii) the sneutrino width ($\Gamma_{\tilde{\nu}}$), and (iii) the branching ratio of the sneutrino into a lepton-number-tagging mode.

A. Order of magnitude of $\Delta m_{\tilde{\nu}}/m_\nu$

To determine the order of magnitude of $\Delta m_{\tilde{\nu}}/m_\nu$, we shall take all R-parity-conserving supersymmetric parameters to be of order m_Z. In the one-generation model, the neutrino acquires a mass of order $m_\nu \sim \mu_1^2 \cos^2\beta/m_Z$ via tree-level mixing, where we have used $\sin\xi = \mu_1/\mu$ in a basis where $v_1 = 0$. The tree-level mass splitting of the sneutrino-antisneutrino pair is obtained from Eq. (3.24), and we find $\Delta m_{\tilde{\nu}}^2 \sim b_1^2 \sin^2\beta/m_Z^2$. Using $\Delta m_{\tilde{\nu}}^2 = 2 m_{\tilde{\nu}\tilde{\nu}^*} \Delta m_{\tilde{\nu}}$, it follows that

$$r_\nu \equiv \frac{\Delta m_{\tilde{\nu}}}{m_\nu} \sim \frac{b_1^2 \tan^2\beta}{m_Z^2 \mu_1^2}. \tag{5.1}$$

To appreciate the implications of this result, we note that Eq. (3.10) in the $v_1 = 0$ basis yields

$$b_1 = [(M_{\tilde{L}}^2)_{10} + \mu_1 \mu_0] \cot\beta. \tag{5.2}$$

The natural case is the one where all terms in Eq. (5.2) are of the same order. Then $b_1 \sim \mathcal{O}(m_Z \mu_1 \cot\beta)$, and it follows that $r_\nu \sim \mathcal{O}(1)$. On the other hand, it is possible to have $r_\nu \gg 1$ if, e.g., $(M_{\tilde{L}}^2)_{10} \gg \mu_1 \mu_0$. The upper bound $r_\nu \lesssim 10^3$ [see Eq. (4.1)] still applies in the absence of unnatural cancellations between the tree-level and one-loop contributions to m_ν.

We do not discuss here any models that predict the relative size of the relevant RPV parameters. We only note that while we are not familiar with specific one-generation models that lead to $r_\nu \gg 1$, we are aware of models that lead to $r_\nu \sim 1$. One such example is a class of models based on horizontal symmetry [8].

In the three-generation model, there is at most one tree-level nonzero neutrino mass, while all sneutrino-antisneutrino pair masses may be split. This provides far greater freedom for the possible values of $(\Delta m_{\tilde{\nu}})_m \sim b_m^2 \sin^2\beta/m_Z^3$, since in many cases these are not constrained by the very small neutrino masses. In general, significant regions of parameter space exist in which $r_\nu \gg 1$ for at least $n_g - 1$ generations of neutrinos and sneutrinos.

Consider next the implications of the RPV one-loop corrections. These are proportional to different RPV parameters as compared to those that control the tree-level neutrino masses and sneutrino-antisneutrino mass splittings. Thus one may envision cases where the RPV one-loop results are either negligible, of the same order, or dominant with respect to the tree-level results. If the RPV one-loop results are negligible, then the discussion above applies. In particular, in the three-generation model with generic model parameters, one typically expects $r_\nu \sim \mathcal{O}(1)$ for one of the generations, while $r_\nu \gg 1$ for the other two generations. In contrast, if the RPV one-loop corrections are dominant, then the results of Sec. IV imply that $r_\nu \sim \mathcal{O}(1)$ for all three generations, for generic model parameters.

B. Sneutrino width and branching ratios

Besides their effect on the sneutrino-antisneutrino mixing, the RPV interactions also modify the sneutrino decays. This can happen in two ways. First, the presence of the λ and λ' coupling can directly mediate sneutrino decay to quark and/or lepton pairs. Second, the sneutrinos can decay through their mixing with the Higgs bosons (which would favor decay into the heaviest fermion or boson pairs that are kinematically allowed). These decays are relevant if the sneutrino is the lightest supersymmetric particle (LSP) or if the R-parity-conserving sneutrino decays are suppressed (e.g., if no two-body R-parity-conserving decays are kinematically allowed).

Consider two limiting cases. First, suppose that the RPV decays of the sneutrino are dominant (or that the sneutrino is the LSP). Then, in the absence of CP-violating effects, the sneutrino and antisneutrino decay into the same channels with the same rate. Moreover, the RPV sneutrino decays violate lepton number by one unit. Hence one cannot identify the decaying (anti)sneutrino state via a lepton tag, as in Ref. [4]. However, oscillation phenomena may still be observable if there is a significant difference in the CP-even and CP-odd sneutrino lifetimes. For example, if the RPV sneutrino decays via Higgs mixing dominate, then for sneutrino masses between $2m_W$ and $2m_t$ the dominant decay channels for the CP-even scalar would be W^+W^-, ZZ, and h^0h^0, while the CP-odd scalar would decay mainly into $b\bar{b}$. In this case, the ratio of sneutrino lifetimes would be of order m_Z^2/m_b^2. Adding up all channels, one finds a ratio of lifetimes of order 10^3. Moreover, the overall lifetimes are suppressed by small RPV parameters: so one can imagine cases where a LSP sneutrino would decay at colliders with a displaced vertex. Oscillation phenomena similar to that of the K-$\bar{K}$ system would then be observable for the sneutrino-antisneutrino system. Including all three generations of sneutrinos would lead to a very rich phenomenology that would provide a precision probe of the underlying lepton number violation of the theory.

Second, suppose that the R-parity-conserving decays of the sneutrino are dominant. Then the considerations of Ref. [4] apply. In particular, in most cases, there are leptonic final states in sneutrino decays that tag the initial sneutrino state. Thus the like-sign dilepton signal of Ref. [4] can be used to measure $x_{\tilde{\nu}} = \Delta m_{\tilde{\nu}}/\Gamma_{\tilde{\nu}}$. Since only values of $x_{\tilde{\nu}} \gtrsim 1$ are practically measurable, the most favorable case corresponds to very small $\Gamma_{\tilde{\nu}}$. In typical models of *R*-parity-conserving supersymmetry, the sneutrino decays into two-body final states with a width of order 1 GeV. This result can be suppressed somewhat by chargino-neutralino mixing angle and phase space effects, but the suppression factor is at most a factor of 10^4 in rate (assuming that the tagging mode is to be observable). If the LSP is the $\tilde{\tau}^{\pm}$, then supersymmetric models can be envisioned where two-body sneutrino decays are absent, and the three-body sneutrino decays $\tilde{\nu}_l \to \tilde{\tau}_R \nu_\tau l$ can serve as the tagging mode. In Ref. [4], we noted that a LSP $\tilde{\tau}_R$ is strongly disfavored by astrophysical bounds on the abundance of stable heavy charged particles [21]. In *R*-parity-violating supersymmetry, this is not an objection, since the LSP $\tilde{\tau}_R$ would decay through a RPV interaction. Three-body sneutrino decay widths can vary typically between 1 eV and 1 keV, depending on the supersymmetric parameters. Thus, in this case, the like-sign dilepton signature can also provide a precision probe of the underlying lepton number violation of the theory.

C. Conclusions

R-parity-violating low-energy supersymmetry with baryon number conservation provides a framework for particle physics with lepton number violation. Recent experimental signals of neutrino masses and mixing may provide the first glimpse of the lepton-number-violating world. The search for neutrino masses and oscillations is a difficult one. Even if successful, such observations will provide few hints as to the nature of the underlying lepton number violation. In supersymmetric models that incorporate lepton number violation, the phenomenology of sneutrinos may provide additional insight to help us unravel the mystery of neutrino masses and mixing. Sneutrino flavor mixing and sneutrino-antisneutrino oscillations are analogous to neutrino flavor mixing and Majorana neutrino masses, respectively. Crucial observables at future colliders include the sneutrino-antisneutrino mass splitting, sneutrino oscillation phenomena, and possible long sneutrino and antisneutrino lifetimes. In this paper, we described *CP*-conserving sneutrino phenomenology that can probe the physics of lepton number violation. In a subsequent paper, we will address the implications of *CP* violation in the sneutrino system. The observation of such phenomena at future colliders would have a dramatic impact on the pursuit of physics beyond the standard model.

ACKNOWLEDGMENTS

We thank Yossi Nir for helpful discussions. Y. G. is supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00515, and H.E.H. is supported in part by the U.S. Department of Energy under Contract No. DE-FG03-92ER40689 and in part by Fermi National Accelerator Laboratory.

APPENDIX A: THE SCALAR POTENTIAL

In softly broken supersymmetric theories, the total scalar potential is given by Eq. (2.4), where V_F and V_D originate from the supersymmetry-preserving sector, while $V_{\rm soft}$ contains the soft-supersymmetry-breaking terms. V_F is obtained from the superpotential W by first replacing all chiral superfields by their leading scalar components and then computing

$$V_F = \sum_{\Phi} \left| \frac{dW}{d\Phi} \right|^2, \tag{A1}$$

where the sum is taken over all contributing scalar fields Φ. For the superpotential in Eq. (2.1) we obtain

$$\begin{aligned}
\frac{dW}{dD_m} &= \lambda'_{\alpha nm} L^i_\alpha Q^j_n \epsilon_{ij}, \\
\frac{dW}{dU_m} &= -h_{nm} H^i_U Q^j_n \epsilon_{ij}, \\
\frac{dW}{dQ^j_m} &= (\lambda'_{\alpha nm} L^i_\alpha D_m - h_{nm} H^i_U U_m)\epsilon_{ij}, \\
\frac{dW}{dE_m} &= \frac{1}{2}\lambda_{\alpha\beta m} L^i_\alpha L^j_\beta \epsilon_{ij}, \\
\frac{dW}{dL^i_\alpha} &= (\lambda_{\alpha\beta m} L^j_\beta E_m + \lambda'_{\alpha nm} Q^j_n D_m - \mu_\alpha H^j_U)\epsilon_{ij}, \\
\frac{dW}{dH^j_U} &= (h_{nm} Q^i_n U_m - \mu_\alpha L^i_\alpha)\epsilon_{ij}.
\end{aligned} \tag{A2}$$

Inserting these results into Eq. (A1), one ends up with

$$\begin{aligned}
V_F ={}& \lambda'_{\alpha nm}\lambda'^*_{\gamma km} L^i_\alpha Q^j_n (L^{i*}_\gamma Q^{j*}_k - L^{j*}_\gamma Q^{i*}_k) + h_{nm} h^*_{km} H^i_U Q^j_n (H^{i*}_U Q^{j*}_k - H^{j*}_U Q^{i*}_k) \\
&+ \lambda'_{\alpha nm}\lambda'^*_{\gamma nk} L^i_\alpha L^{i*}_\gamma D_m D^*_k + h_{nm} h^*_{nk} |H_U|^2 U_m U^*_k - (h_{nm}\lambda'^*_{\gamma nk} H^i_U L^{i*}_\gamma U_m D^*_k + \text{H.c.}) + \tfrac{1}{2}\lambda_{\alpha\beta m}\lambda^*_{\gamma\delta m} L^i_\alpha L^{i*}_\gamma L^j_\beta L^{j*}_\delta \\
&+ \lambda_{\alpha\beta m}\lambda^*_{\alpha\gamma k} L^i_\beta L^{i*}_\gamma E_m E^*_k + \lambda'_{\alpha nm}\lambda'^*_{\alpha pk} Q^i_n Q^{i*}_p D_m D^*_k + |\mu_\alpha|^2 |H_U|^2 + (\lambda_{\alpha\beta m}\lambda'^*_{\alpha pk} L^i_\beta Q^{i*}_p E_m D^*_k + \text{H.c.}) \\
&- (\mu_\alpha \lambda^*_{\alpha\gamma k} H^i_U L^{i*}_\gamma E^*_k + \text{H.c.}) - (\mu_\alpha \lambda'^*_{\alpha pk} H^i_U Q^{i*}_p D^*_k + \text{H.c.}) + \mu_\alpha \mu^*_\beta L^i_\alpha L^{i*}_\beta + h_{nm} h^*_{pq} U_m U^*_q Q^i_n Q^{i*}_p \\
&- (\mu_\alpha h^*_{pq} L^i_\alpha Q^{i*}_p U^*_q + \text{H.c.}).
\end{aligned} \tag{A3}$$

V_D is obtained from the following formula:

$$V_D=\tfrac{1}{2}[D^aD^a+(D')^2], \tag{A4}$$

where

$$D^a=\frac{1}{2}g\left[H_U^{i*}\sigma^a_{ij}H_U^j+\sum_m \tilde{Q}_m^{i*}\sigma^a_{ij}\tilde{Q}_m^j+\sum_\alpha \tilde{L}_\alpha^{i*}\sigma^a_{ij}\tilde{L}_\alpha^j\right],$$

$$D'=\frac{1}{2}g'\left[|H_U|^2-\sum_\alpha|\tilde{L}_\alpha|^2+2\sum_m|\tilde{E}_m|^2+\frac{1}{3}\sum_m|\tilde{Q}_m|^2-\frac{4}{3}\sum_m|\tilde{U}_m|^2+\frac{2}{3}\sum_m|\tilde{D}_m|^2\right]. \tag{A5}$$

Then,

$$\begin{aligned}V_D=&\frac{1}{8}g^2\Bigg\{\left(|H_U|^2-\sum_\alpha|\tilde{L}_\alpha|^2-\sum_m|\tilde{Q}_m|^2\right)^2-2\sum_{\alpha\neq\beta}|\epsilon_{ij}\tilde{L}^i_\alpha\tilde{L}^j_\beta|^2+4\sum_\alpha|H_U^{i*}\tilde{L}^i_\alpha|^2\\&-2\sum_{m\neq n}|\epsilon_{ij}\tilde{Q}^i_m\tilde{Q}^j_n|^2+4\sum_m|H_U^{i*}\tilde{Q}^i_m|^2-4\sum_{\alpha m}|\epsilon_{ij}\tilde{L}^i_\alpha\tilde{Q}^i_m|^2\Bigg\}\\&+\frac{1}{8}g'^2\left[|H_U|^2-\sum_\alpha|\tilde{L}_\alpha|^2+2\sum_m|\tilde{E}_m|^2+\frac{1}{3}\sum_m|\tilde{Q}_m|^2-\frac{4}{3}\sum_m|\tilde{U}_m|^2+\frac{2}{3}\sum_m|\tilde{D}_m|^2\right]^2.\end{aligned} \tag{A6}$$

Finally, the soft-supersymmetry-breaking contribution to the scalar potential has already been given in Eq. (2.2).

APPENDIX B: THE CHARGED FERMION AND SCALAR SECTORS

Using the same techniques discussed in Sec. III, one can evaluate the tree-level masses of charged fermions and scalars. For completeness, we include here the results for the general R-parity-violating, baryon-triality-preserving model exhibited in Sec. II. (For related results in a minimal RPV model in which μ_m is the only RPV parameter, see Ref. [22].)

First, we consider the sector of charged fermions. The charginos and charged leptons mix: so we must diagonalize a $(n_g+2)\times(n_g+2)$ matrix for n_g generations of leptons. Following the notation of Ref. [23], we assemble the two-component fermion fields as follows:

$$\psi^+=(-i\lambda^+,\psi^+_{H_U},\psi^+_{E_k}),$$

$$\psi^-=(-i\lambda^-,\psi^-_{L_\alpha}), \tag{B1}$$

where $-i\lambda^\pm$ are the two-component W-ino fields and the remaining fields are the fermionic components of the indicated scalar field. As before, $m=1,\ldots,n_g$ and $\alpha=0,1,\ldots,n_g$, with $L_0\equiv H_D$. The mass term in the Lagrangian then takes the form [8,9,24]

$$\mathcal{L}_{\rm mass}=-\frac{1}{2}(\psi^+\psi^-)\begin{pmatrix}0 & X^T\\ X & 0\end{pmatrix}\begin{pmatrix}\psi^+\\ \psi^-\end{pmatrix}, \tag{B2}$$

where[10]

$$X=\begin{pmatrix}M_2 & \frac{1}{\sqrt{2}}gv_u & 0_m\\ \frac{1}{\sqrt{2}}gv_\alpha & \mu_\alpha & (m_l)_{\alpha m}\end{pmatrix}. \tag{B3}$$

In Eq. (B3), 0_m is a row vector with n_g zeros and

$$(m_l)_{\alpha m}\equiv\frac{1}{\sqrt{2}}v_\rho\lambda_{\rho\alpha m}. \tag{B4}$$

Note that in the basis where $v_n=0$, the definition of $(m_l)_{nm}$ reduces to the one given in Eq. (2.11). The charged fermion masses are obtained by either diagonalizing $X^\dagger X$ (with unitary matrix V) or $XX^\dagger$ (with unitary matrix U^*), where the two unitary matrices are chosen such that U^*XV^{-1} is a diagonal matrix with the non-negative fermion masses along the diagonal. The following relation is noteworthy:

$$\mathrm{Tr}(X^\dagger X)=\mathrm{Tr}(XX^\dagger)=|M_2|^2+|\mu|^2+2m_W^2+\mathrm{Tr}(m_l^\dagger m_l), \tag{B5}$$

[10]The result given in Eq. (B3) corrects a minor error that appears in Refs. [8] and [9].

where $|\mu|^2$ is defined in Eq. (2.6). Note that in the R-parity-conserving MSSM, Tr $M_\chi^2 \equiv |M_2|^2 + |\mu|^2 + 2m_W^2$ is the sum of the two chargino squared masses and m_l is the charged lepton mass matrix. In the presence of RPV interactions, Eq. (B5) remains valid despite the mixing between charginos and charged leptons. Of course, m_l no longer corresponds precisely to a mass matrix of physical states. For example, in the $v_m = 0$ basis,

$$X^\dagger X = \begin{pmatrix} |M_2|^2 + \frac{1}{2}g^2|v_d|^2 & \frac{1}{\sqrt{2}} g(M_2^* v_u + v_d^* \mu \cos\xi) & 0_m \\ \frac{1}{\sqrt{2}} g(M_2 v_u^* + v_d \mu^* \cos\xi) & |\mu|^2 + \frac{1}{2}g^2|v_u|^2 & \mu_n^*(m_l)_{nm} \\ 0_k & \mu_n(m_l^*)_{nk} & (m_l^\dagger m_l)_{km} \end{pmatrix}, \tag{B6}$$

where $\cos\xi$ is defined in Eq. (3.3). As expected, if $\mu_m \neq 0$ (but small), then the physical lepton eigenstates will have a small admixture of the charged Higgsino eigenstate. It is amusing to note that in the exact limit of $m_l = 0$, there are n_g massless fermions (i.e., the charged leptons), in spite of the mixing with the charged Higgsinos through the RPV terms.[11]

We next turn to the charged scalar sector. In this case, the charged sleptons mix with the charged Higgs boson and charged Goldstone boson (which is absorbed by the $W^\pm$). The resulting $(2n_g+2)\times(2n_g+2)$ squared-mass matrix can be obtained from the scalar potential given by Eqs. (A3), (A6), and (2.2). In the $\{H_U^1, \widetilde{L}_\beta^{2*}, \widetilde{E}_m\}$ basis, the charged scalar squared-mass matrix is given by

$$M_C^2 = \begin{pmatrix} m_{uu}^2 + D & b_\beta^* + D_\beta & \mu_\beta^*(m_l)_{\beta m} \\ b_\alpha + D_\alpha^* & m_{\alpha\beta}^2 + (m_l m_l^\dagger)_{\alpha\beta} + D_{\alpha\beta} & \frac{1}{\sqrt{2}}(a_{\rho\alpha m} v_\rho - \mu_\rho^* \lambda_{\rho\alpha m} v_u^*) \\ \mu_\alpha(m_l^*)_{\alpha k} & \frac{1}{\sqrt{2}}(a_{\rho\beta k}^* v_\rho^* - \mu_\rho \lambda_{\rho\beta k}^* v_u) & (M_{\widetilde{E}}^2)_{km} + (m_l^\dagger m_l)_{km} + D_{km} \end{pmatrix}, \tag{B7}$$

where the matrix m_l is defined in Eq. (B4) and

$$\begin{aligned} m_{uu}^2 &\equiv m_U^2 + |\mu|^2, \\ m_{\alpha\beta}^2 &\equiv (M_{\widetilde{L}}^2)_{\alpha\beta} + \mu_\alpha \mu_\beta^*, \\ D_{\alpha\beta} &\equiv \tfrac{1}{4}g^2 v_\alpha^* v_\beta + \tfrac{1}{8}(g^2 - g'^2)(|v_u|^2 - |v_d|^2)\delta_{\alpha\beta}, \\ D_{km} &\equiv \tfrac{1}{4}g'^2(|v_u|^2 - |v_d|^2)\delta_{km}, \\ D_\alpha &\equiv \tfrac{1}{4}g^2 v_\alpha v_u, \\ D &\equiv \tfrac{1}{8}(g^2 + g'^2)(|v_u|^2 - |v_d|^2) + \tfrac{1}{4}g^2|v_d|^2. \end{aligned} \tag{B8}$$

As a check of the calculation, we have verified that $(-v_u, v_\beta^*, 0)$ is an eigenvector of M_C^2 with zero eigenvalue, corresponding to the charged Goldstone boson that is absorbed by the $W^\pm$. The computation makes use of the minimization conditions of the potential [Eqs. (2.7) and (2.8)] and the antisymmetry of $\lambda_{\rho\beta k}$ and $a_{\rho\beta k}$ under the interchange of ρ and β.

A useful sum rule can be derived in the CP-conserving limit. We find

$$\begin{aligned} \mathrm{Tr}\, M_C^2 = m_W^2 + \mathrm{Tr}\, M_{\mathrm{odd}}^2 + \mathrm{Tr}\, M_{\widetilde{E}}^2 + 2\, \mathrm{Tr}(m_l^\dagger m_l) \\ - \tfrac{1}{4} n_g m_Z^2 \cos 2\beta. \end{aligned} \tag{B9}$$

This is the generalization of the well-known sum rule $m_{H^\pm}^2 = m_W^2 + m_A^2$ of the MSSM Higgs sector [15]. The charged sleptons are also contained in the above sum rule. As a check, consider the one-generation R-parity-conserving MSSM limit. Removing the Higgs sum rule contribution from Eq. (B9), the leftover pieces are

$$m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 - m_{\tilde{\nu}}^2 = 2m_e^2 + M_{\widetilde{E}}^2 - \tfrac{1}{4} m_Z^2 \cos 2\beta. \tag{B10}$$

The term in Eq. (B10) that is proportional to m_Z^2 is simply the D-term contribution to the combination of slepton squared masses specified above.

[11]It may seem from Eq. (B6) that the charged leptons are unmixed if $m_l = 0$. But one can shown that this is not the case by computing $XX^\dagger$. The mixing originates from $\mu_m \neq 0$ appearing in the matrix X [Eq. (B3)].

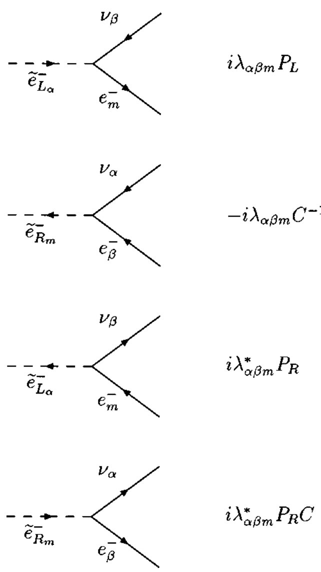

FIG. 4. Feynman rules for the scalar-fermion interactions.

APPENDIX C: FEYNMAN RULES

The fermion-scalar Yukawa couplings take the form

$$\mathcal{L}_{\text{Yukawa}} = -\frac{1}{2}\left(\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\right)\psi_i\psi_j + \text{H.c.}, \tag{C1}$$

where superfields are replaced by their scalar components after taking the second derivative of the superpotential W [given in Eq. (2.1)] and the ψ_i are two-component fermion fields. Converting to four-component Feynman rules (see, e.g., the Appendixes of Ref. [17]) and defining $P_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)$, we obtain the Feynman rules listed in Fig. 4. The charge conjugation matrix C appears in fermion-number-violating vertices.

The Feynman rules for the cubic scalar interactions can be obtained from the scalar potential [Eqs. (A3), (A6), and (2.2)] by putting $\tilde{L}^1_\alpha \to \tilde{L}^1_\alpha + v_\alpha/\sqrt{2}$. The Feynman rules for the interaction of the sneutrinos with slepton pairs are given in Fig. 5, where $(m_l)_{\gamma m}$ is defined in Eq. (B4). In Sec. IV, we have applied the rules of Fig. 5 to the $\tilde{\nu}_p\tilde{e}_m\tilde{e}_n$ couplings $(p,m,n=1,\ldots,n_g)$ in this basis where $v_m=0$ and $(m_l)_{nm}$ is diagonal. In this basis, the terms in Fig. 5 proportional to gauge couplings do not contribute.

APPENDIX D: THE B_0 FUNCTION

The B_0 function is defined as follows:

$$\frac{i}{16\pi^2}B_0(p^2,M^2,m^2) = \int \frac{d^nq}{(2\pi)^n}\,\frac{1}{(q^2-m^2)[(q-p)^2-M^2]}. \tag{D1}$$

One can express B_0 as a one-dimensional integral:

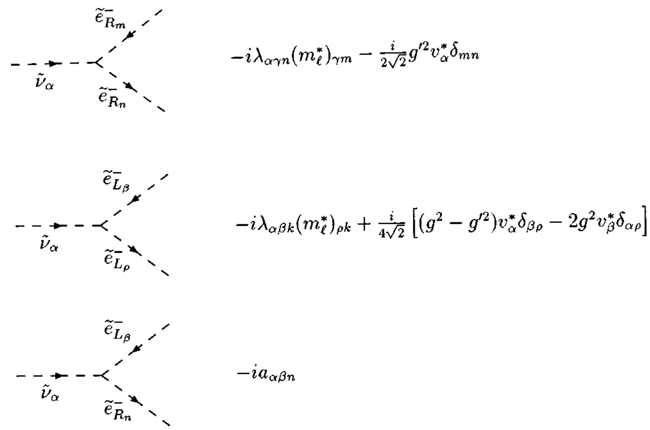

FIG. 5. Feynman rules for the interactions of the sneutrinos and charged sleptons.

$$B_0(p^2,M^2,m^2) = \Delta - \int_0^1 dx\,\ln\left(\frac{m^2x + M^2(1-x) - p^2x(1-x)}{\mu^2}\right), \tag{D2}$$

where

$$\Delta \equiv (4\pi)^\epsilon \quad \Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + \ln(4\pi) + \mathcal{O}(\epsilon), \quad \epsilon = 2 - \frac{n}{2}. \tag{D3}$$

Two limiting cases are useful for the calculations performed in Sec. IV. In the $p^2 \to 0$ limit,

$$B_0(0,M_1^2,m^2) - B_0(0,M_2^2,m^2) = \frac{M_2^2}{m^2-M_2^2}\ln\left(\frac{m^2}{M_2^2}\right) - \frac{M_1^2}{m^2-M_1^2}\ln\left(\frac{m^2}{M_1^2}\right). \tag{D4}$$

If we furthermore take the $m \to 0$ limit, we obtain

$$B_0(0,M_1^2,0) - B_0(0,M_2^2,0) = \ln\left(\frac{M_1^2}{M_2^2}\right). \tag{D5}$$

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