Large corrections to asymptotic $F_{\eta_c\gamma}$ and $F_{\eta_b\gamma}$ in the light-cone perturbative QCD

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The large-Q^2 behavior of η_c-γ and η_b-γ transition form factors $F_{\eta_c\gamma}(Q^2)$ and $F_{\eta_b\gamma}(Q^2)$ is analyzed in the framework of light-cone perturbative QCD with heavy quark (c and b) mass effects, parton transverse momentum dependence, and higher helicity components in the light-cone wave function are respected. It is pointed out that the quark mass effect brings significant modifications to the asymptotic predictions of the transition form factors in a rather broad energy region, and the modification for $F_{\eta_b\gamma}(Q^2)$ is much more severe than that for $F_{\eta_c\gamma}(Q^2)$ due to the b quark being heavier than the c quark. The parton transverse momentum and the higher helicity components also play a role in reducing the perturbative predictions. For the transition form factor $F_{\eta_c\gamma}(Q^2)$, they bring sizable corrections in the present experimentally accessible energy region ($Q^2 \leqslant 10$ GeV^2). For the transition form factor $F_{\eta_b\gamma}(Q^2)$, the corrections coming from these two factors are negligible since the b-quark mass is much larger than the parton's average transverse momentum. The coming e^+e^- collider (CERN LEP2) will provide the opportunity to examine these theoretical predictions.
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I. INTRODUCTION

Among a large number of exclusive processes, neutral meson production in two-photon collisions, $\gamma^*\gamma\to P(P$ being $\pi^0,\eta,\eta',\eta_c,\eta_b,\ldots)$, is the simplest one since two photons and one meson are involved in the initial and final states, respectively. Only one form factor, the meson-photon transition form factor ($F_{P\gamma}$), is necessary to describe this class of processes. Studying $F_{P\gamma}$ provides a rather simple and rigorous way to test QCD and the determination of the meson wave function (nonperturbative physics) [1]. Experimentally, many collaborations (TPC/Two-Gamma [2], CELLO [3], CLEO [4], and L3 [5], etc.) have measured the form factors $F_{\pi\gamma}(Q^2)$, $F_{\eta\gamma}(Q^2)$, and $F_{\eta'\gamma}(Q^2)$ in the Q^2 region up to 9, 20, and 30 GeV^2, respectively, where Q^2 is the virtuality of the virtual photon. Although with poor statistics, $c\bar{c}$ state (η_c,χ_{c0} and χ_{c2}) production has been observed [6]. In the CERN e^+e^- collider LEP2, the dominant process is $e^+e^-\to e^+e^-+X(\gamma\gamma\to X)$. Considering the higher energy (the center of mass energy will reach 100 GeV) and the higher luminosity [the cross section of this process grows as $(\ln s/m_e^2)^2$ with s being the invariant energy square of the incoming e^+e^- pair, whereas the annihilation cross section decrease as s^{-1}], LEP2 will be a good factory for the production of the heavy quarkonium ($c\bar{c}$ and $b\bar{b}$), and will greatly stimulate theoretical studies on these processes. At present, it seems a measurement of $F_{\eta_c\gamma}$ up to about 10 GeV^2 is possible [6]. Theoretically, there also have been many studies on these form factors [7–13]. In the large-Q^2 region, perturbative QCD can be employed as a powerful tool. The large-Q^2 behavior of the form factors $F_{\pi^0\gamma}$, $F_{\eta\gamma}$, and $F_{\eta'\gamma}$ has been studied in some detail by several authors [7–13]. Recently, the form factor $F_{\eta_c\gamma}$ has also been analyzed in covariant perturbative theory by adopting the Breit reference frame [14]. In this work, we present a theoretical study of $F_{\eta_c\gamma}$ and $F_{\eta_b\gamma}$ in the framework of light-cone perturbative QCD (LCPQCD). There are two differences between the LCPQCD calculations for the transition form factors of the heavy mesons (η_c,η_b) and those of the light mesons (π_0,η,η' etc.). First, the c- and b-quark masses cannot be neglected in the evaluation of the hard scattering amplitude, while the light quark masses may be readily neglected. Second, considering that the Wigner-Melosh rotation and c- and b-quark masses are large, one finds that there are contributions coming from the higher helicity components in the light-cone wave functions besides those which come from the ordinary helicity components. For the π, η, and η' mesons, the contributions from the higher helicity components can be neglected in the limit of the vanishing of the light quark masses.

II. LIGHT-CONE FORMALISM AND LIGHT-CONE WAVE FUNCTION

The light-cone (LC) formalism [15] provides a convenient framework for the relativistic description of hadrons in terms

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of quark and gluon degrees of freedom, and the application of perturbative QCD to exclusive processes has mainly been developed in this formalism (light-cone perturbative QCD) [16]. In this formalism, the quantization is chosen at a particular light-cone time $\tau = t + z$. Thereby, several characters arise in this formalism. (i) The hadronic wave function which describes the hadronic composite state at a particular τ is expressed in terms of a series of light-cone wave functions in Fock-state bases, for example,

$$|\pi\rangle = \sum_{q\bar{q}} |q\bar{q}\rangle \psi_{q\bar{q}/\pi} + \sum_{q\bar{q}g} |q\bar{q}g\rangle \psi_{q\bar{q}g/\pi} + \cdots, \qquad (1)$$

where $\psi_{q\bar{q}/\pi}$, $\psi_{q\bar{q}g/\pi}$, ..., are the light-cone wave functions. (ii) The temporal evolution of the state is generated by the light-cone Hamiltonian $H_{\rm LC} = P^- = P^0 - P^3$. (iii) The vacuum is very simple. The zero-particle state is the only one which has zero total P^+, since all quanta must have positive light-cone momentum k_i^+ and $P^+ = \Sigma_i k_i^+$. The zero-particle state cannot mix with the other states which contain a certain number of particles but with zero total P^+. Hence the vacuum state in the light-cone Fock bases [Eq. (1)] is an exact eigenstate of the full Hamiltonian $H_{\rm LC}$, and all bare quanta in the hadronic Fock state are parts of the hadron. This point differs from that in the equal-t perturbative theory in which the quantization is performed at a given time t. In the equal-t quantization, it is possible to make up a zero-momentum state which contains some particles, since the momentum of each particle may be positive or negative, and the momentum of a composite state is the sum of the momentum of each participant particle. Thus the zero-particle state may mix with some zero-momentum states which contain particles to build up the ground state, which makes the vacuum complex. (iv) The contributions coming from higher Fock states are suppressed by $1/Q^n$, therefore one may consider only the valence Fock state in the large-Q^2 region. Light-cone perturbative QCD is very convenient for light-cone dominated processes. For the detail calculation rules we refer to the literature [16,17].

The essential feature of light-cone PQCD applying to exclusive processes is that the amplitudes for these processes can be written as a convolution of the hadron light-cone wave function (LCWF) [or quark distribution amplitude (DA)] for each hadron involved in the process with a hard-scattering amplitude T_H. Both LCWF and T_H are the basic blocks for the LCPQCD calculation. T_H can be calculated from perturbative theory. Up to now, it has been very difficult to give the LCWF from the first principles of QCD. So one usually constructs some phenomenological models for the wave function. One widely used recipe is to connect the LCWF with the instant-form wave function. The momentum space wave functions can be connected by demanding the off-shell energies in the two formailsms to be equal (Brodsky-Huang-Lapage recipe [17]). Furthermore, for heavy quarkonia, the wave functions can be determined by applying nonrelatistic approximation and potential model. The spin structure of the light-cone wave function can be connected correctly with that of the instant-form wave function by considering the Wigner-Melosh rotation [18,19]. As the Wigner-Melosh rotation is respected, the light-cone wave function of the lowest valence Fock state of $\eta_c(\eta_b)$ can be expressed as [19]

$$|\psi_{q\bar{q}}^{\eta_c(\eta_b)}\rangle = \psi(x,\mathbf{k}_\perp,\uparrow,\downarrow)|\uparrow\downarrow\rangle + \psi(x,\mathbf{k}_\perp,\downarrow,\uparrow)|\downarrow\uparrow\rangle + \psi(x,\mathbf{k}_\perp,\uparrow,\uparrow)|\uparrow\uparrow\rangle + \psi(x,\mathbf{k}_\perp,\downarrow,\downarrow)|\downarrow\downarrow\rangle, \qquad (2)$$

where

$$\psi(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)\,\psi(x,k_\perp). \qquad (3)$$

Here $\psi(x,k_\perp)$ is the momentum space wave function in the light-cone formalism. The coefficients $C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)$ which result from the considering of the Wigner-Melosh rotation turn out to be [19]

$$\begin{aligned}
C_0^F(x,\mathbf{k}_\perp,\uparrow,\downarrow) &= \frac{m}{[2(m^2+\mathbf{k}_\perp^2)]^{1/2}},\\
C_0^F(x,\mathbf{k}_\perp,\downarrow,\uparrow) &= -\frac{m}{[2(m^2+\mathbf{k}_\perp^2)]^{1/2}},\\
C_0^F(x,\mathbf{k}_\perp,\uparrow,\uparrow) &= -\frac{(k_1 - ik_2)}{[2(m^2+\mathbf{k}_\perp^2)]^{1/2}},\\
C_0^F(x,\mathbf{k}_\perp,\downarrow,\downarrow) &= -\frac{(k_1 + ik_2)}{[2(m^2+\mathbf{k}_\perp^2)]^{1/2}}.
\end{aligned} \qquad (4)$$

where m is the c- (b-) quark mass for $\eta_c(\eta_b)$, and $\mathbf{k}_\perp$ is the quark transverse momentum. C_0^F satisfies the relation

$$\sum_{\lambda_1,\lambda_2} C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = 1. \qquad (5)$$

One character of the light-cone wave function [Eq. (2)] is that there are higher helicity $(\lambda_1+\lambda_2=\pm 1)$ components besides the ordinary helicity $(\lambda_1+\lambda_2=0)$ components, while the instant-form wave function has only the ordinary helicity components. This character has been employed in the studies of several processes: the proton ''spin puzzle'' [20], proton's structure, and the proton, neutron, and deuteron polarization asymmetries, etc. [21].

III. THE MESON-PHOTON TRANSITION FORM FACTORS $F_{\eta_c\gamma}$ AND $F_{\eta_b\gamma}$

In the following, we first analyze the η_c-γ transition form factor $F_{\eta_c\gamma}$. The analysis for $F_{\eta_b\gamma}$ can be obtained in a similar way. $F_{\eta_c\gamma}$ is extracted from the $\eta_c\gamma\gamma^*$ vertex,

$$\Gamma_\mu = -ie^2 F_{\eta_c\gamma}\epsilon_{\mu\nu\alpha\beta} p_{\eta_c}^\nu \epsilon^\alpha q^\beta, \qquad (6)$$

where p_{η_c} and q are the momenta of the η_c meson and the virtual photon respectively, and ϵ is the polarization vector of the on-shell photon. In the standard ''infinite-momentum'' frame [1], the momentum assignment can be written as

$$p_{\eta_c}=(p^+,p^-,p_\perp)=(1,m^2_{\eta_c},0_\perp),$$
$$q=(0,q^2_\perp-m^2_{\eta_c},q_\perp), \tag{7}$$
$$q'=(1,q^2_\perp,q_\perp),$$

where p^+ is arbitrary, and q' is the momentum of the final (on-shell) photon. For simplicity we choose $p^+=1$, and we have $q^2=-q^2_\perp=-Q^2$. Then $F_{\eta_c\gamma}$ is given by

$$F_{\eta_c\gamma}(Q^2)=\frac{\Gamma^+}{-ie(\epsilon_\perp\times q_\perp)}, \tag{8}$$

where $\epsilon=(0,0,\epsilon_\perp)$ and $\epsilon_\perp\cdot q_\perp=0$ are chosen.

The contribution coming from the ordinary helicity components $(\lambda_1+\lambda_2=0)$ turns out to be (see Fig. 1)

$$F^{(\lambda_1+\lambda_2=0)}_{\eta_c\gamma}(Q^2)=\frac{\sqrt{n_c}e_c^2}{i(\epsilon_\perp\times q_\perp)}\int_0^1[dx]\int_0^\infty\frac{d^2k_\perp}{16\pi^3}\frac{m_c}{\sqrt{m_c^2+k_\perp^2}}\psi(x,k_\perp)\times\left[\frac{\overline{v_\downarrow}(x_2,-k_\perp)}{\sqrt{x_2}}\not{\epsilon}\frac{u_\uparrow(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\times\frac{\overline{u_\uparrow}(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\gamma^+\frac{u_\uparrow(x_1,k_\perp)}{\sqrt{x_1}}\frac{1}{D}+(1\leftrightarrow2)\right], \tag{9}$$

where $[dx]=dx_1dx_2\delta(1-x_1-x_2),e_c$ is the c-quark charge in unit of e, and D is the ''energy-denominator,''

$$D=q^2_\perp-\frac{(q_\perp+k_\perp)^2+m_c^2}{x_1}-\frac{k_\perp^2+m_c^2}{x_2}=-\frac{(x_2q_\perp+k_\perp)^2+m_c^2}{x_1x_2}. \tag{10}$$

Different from the case of light mesons such as π,η, and η', the presence of the larger quark mass ($m_c\simeq1.5$ GeV) always prevents $1/D$ from the singularity point $D\to0$, i.e., the partons in the intermediate state are always far off energy shell. This means that even in the low Q^2 region, the LCPQCD calculation may be still available. By employing the LCPQCD calculation rules it can be found that Eq. (9) becomes [11,16]

$$F^{(\lambda_1+\lambda_2=0)}_{\eta_c\gamma}(Q^2)=2\sqrt{2}\sqrt{n_c}e_c^2\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}\frac{m_c}{\sqrt{m_c^2+k_\perp^2}}\psi(x,k_\perp)\times\left[\frac{q_\perp\cdot(x_2q_\perp+k_\perp)}{q^2_\perp[(x_2q_\perp+k_\perp)^2+m_c^2]}+(1\leftrightarrow2)\right]. \tag{11}$$

Similarly, one can obtain the contribution coming from the higher helicity components,

$$F^{(\lambda_1+\lambda_2=\pm1)}_{\eta_c\gamma}(Q^2)=\frac{\sqrt{n_c}e_c^2}{i(\epsilon_\perp\times q_\perp)}\int_0^1[dx]\int_0^\infty\frac{d^2k_\perp}{16\pi^3}\frac{m_c}{\sqrt{m_c^2+k_\perp^2}}\psi(x,k_\perp)\times\left[\frac{\overline{v_\uparrow}(x_2,-k_\perp)}{\sqrt{x_2}}\not{\epsilon}\frac{u_\uparrow(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\times\frac{\overline{u_\uparrow}(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\gamma^+\frac{u_\uparrow(x_1,k_\perp)}{\sqrt{x_1}}\frac{1}{D}+(1\leftrightarrow2)\right]$$
$$=2\sqrt{2}\sqrt{n_c}e_c^2\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}\frac{m_c}{\sqrt{m_c^2+k_\perp^2}}\psi(x,k_\perp)\times\left[\frac{q_\perp\cdot k_\perp}{q^2_\perp[(x_2q_\perp+k_\perp)^2+m_c^2]}+(1\leftrightarrow2)\right]. \tag{12}$$

Once again, a nonzero quark mass m_c plays an important role in the calculation of $F^{(\pm1)}_{\eta_c\gamma}$, since in the $m_c\to0$ limit the matrix $\overline{v}_{\uparrow(\downarrow)}(x_2,-k_\perp)\not{\epsilon}u_{\uparrow(\downarrow)}(x_1,q_\perp+k_\perp)$ will go to zero. Therefore, for light mesons such as π, η, and η', neglecting the contributions coming from the higher helicity components should be a good approximation. Combining this matrix with the coefficients $C_0(x,k_\perp,\uparrow,\uparrow)$ and $C_0(x,k_\perp,\downarrow,\downarrow)$, one arrives at the second expression in Eq. (12). The full result is obtained by summing up the contributions from the ordinary helicity components [Eq. (11)] and those from the higher helicity components [Eq. (12)],

$$F_{\eta_c\gamma}(Q^2)=F^{(\lambda_1+\lambda_2=0)}_{\eta_c\gamma}(Q^2)+F^{(\lambda_1+\lambda_2=\pm1)}_{\eta_c\gamma}(Q^2). \tag{13}$$

Neglecting $k_\perp$ and m_c relative to $x_2q_\perp$ in Eqs. (11) and (12), and employing the asymptotic form distribution amplitude[1]

$$\phi(x)=\sqrt{3/2}f_{\eta_c}x_1x_2, \tag{14}$$

where f_{η_c} is the decay constant of the η_c meson, one can obtain the asymptotic prediction for the η_c-γ transition form factor

$$F_{\eta_c\gamma}(Q^2\to\infty)=\frac{8f_{\eta_c}}{3Q^2}. \tag{15}$$

Corrections to the asymptotic prediction [Eq. (15)] come from the c-quark mass, $k_\perp$ dependence, and the higher helicity components [see Eqs. (11), (12), and (13)]. All of these corrections are suppressed by the factor $1/Q^2$ in the large-Q^2

[1]Any meson distribution amplitude should evolve into the asymptotic form in the $Q^2\to\infty$ limit.

region. But in the present experimentally available energy region, these contributions may be important and should be taken into account.

In order to study the c-quark mass effect, one may first neglect the $k_\perp$ dependence in the hard-scattering amplitude of Eq. (11), then one can obtain

$$F_{\eta_c\gamma}(Q^2)=2\sqrt{2}\sqrt{n_c}e_c^2\int_0^1[dx]\phi(x)\times\left[\frac{x_2}{(x_2q_\perp)^2+m_c^2}+(1\leftrightarrow 2)\right], \qquad (16)$$

where $\phi(x)$ is the distribution amplitude of the η_c meson

$$\phi(x)=\int_0^\infty \frac{d^2k_\perp}{16\pi^3}\frac{m_c}{\sqrt{m_c^2+k_\perp^2}}\psi(x,k_\perp). \qquad (17)$$

The c-quark mass m_c which appears in the denominator of the right-hand side of Eq. (16) in the form of $x_2^2Q^2+m_c^2$ will dramatically modify the behavior of the hard-scattering amplitude in the end-point region. Thus Eq. (16) will approach to the asymptotic prediction [Eq. (15)] in a rather slow way, that is, the corrections coming from the c-quark mass effect are important in a rather broad energy region. The effects of the $k_\perp$ dependence and higher helicity components can be studied by comparing the results obtained from Eqs. (11), (12), (13), and (16). Also, it is interesting to note that the contribution coming from the higher helicity components is the same as that from the $k_\perp$ dependence in the ordinary helicity components. [The right-hand side of Eq. (12) is the same as the $k_\perp$ term on the right-hand side of Eq. (11).] These contributions may bring sizable corrections in the low and medium Q^2 regions.

We point out that the above analysis for $F_{\eta_c\gamma}$ is applicable to the form factor $F_{\eta_b\gamma}$ with the physics quantities corresponding to the c quark (e_c and m_c) and decay constant f_{η_c} being replaced by the ones corresponding to the b quark (e_b and m_b) and f_{η_b}, respectively. The differences resulting from the b quark being heavier than the c quark are as follows. First, the modification coming from b-quark mass effect becomes much more severe, i.e., the perturbative calculation with m_b effect being respected approaches the asymptotic prediction more slowly. Second, the corrections coming from the transverse momentum dependence and the higher helicity components of the light-cone wave function may become rather mild because the b-quark mass is much larger than the parton's average transverse momentum.

IV. NUMERICAL CALCULATIONS AND DISCUSSIONS

In principle, the wave functions of heavy quarkonia are known because of their nonrelativistic form and the successes of potential models—their form and parameters can be fixed by fitting the experimental data on their static properties (binding energy, radius, etc.) [22]. For simplicity, we employ a widely used model in the studies of exclusive processes with large momentum transfer, the Brodsky-Huang-Lepage (BHL) model [17], for the $\eta_c(\eta_b)$ meson light-cone wave function

$$\psi^{\rm BHL}(x,k_\perp)=A\exp\left[-\frac{k_\perp^2+m^2}{8\beta^2x(1-x)}\right]. \qquad (18)$$

This model has a maximun value at $x=1/2$ and is sharply peaked at $x=1/2$ for the η_c and η_b mesons, which is consistent with nonrelativistic dynamics and the potential model [for example, the distribution amplitudes of heavy quarkonia become $\delta(x-1/2)$ for the zero binding energy in the nonrelatistic limit].

The parameters A and β in Eq. (18) are determined by the following two constraints:

$$\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}\frac{m_q}{\sqrt{m_q^2+k_\perp^2}}\psi(x,k_\perp)=\frac{f_{\eta_q}}{2\sqrt{6}}, \qquad (19)$$

$$\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}|\psi(x,k_\perp)|^2=P_{q\bar{q}/\eta_q}, \qquad (20)$$

where $f_{\eta_q}(q=c,b)$ is the decay constant of the $\eta_c(\eta_b)$ meson corresponding to $f_\pi=131$ MeV, and $P_{q\bar{q}/\eta_q}$ is the probability of finding $|c\bar{c}\rangle(|b\bar{b}\rangle)$ Fock state in the $\eta_c(\eta_b)$ meson. Because of the lack of experimental information, one often evaluates f_{η_q} through various theoretical approaches. Employing the Van Royen–Weisskopf formula [23] for the decay constant[2]

$$f_M=\sqrt{\frac{12}{m_M}}|\psi_M(0)|, \qquad (21)$$

where m_M and $\psi_M(0)$ are the mass and the wave function at origin of the meson, respectively, one finds that the decay constant of the pseudoscalar meson is almost the same as that of the vector meson, i.e., $f_P=f_V$. Although the hyperfine splitting Hamiltonian may destroy this relation [24], the consideration of the difference coming from the mock meson spin structure may rescue it [25]. Hence, we adopt [25,26]

$$f_{\eta_c}\simeq f_{J/\psi}\simeq 420\ \text{MeV},\quad f_{\eta_b}\simeq f_\Upsilon\simeq 705\ \text{MeV}. \qquad (22)$$

As is well known, with the increasing of the constitute quark mass the valence Fock state occupies a bigger fraction in the hadron, and in the nonrelativistic limit the probability of finding the valence Fock state is going to approach unity. So one can expect $P_{q\bar{q}/\eta_q}=0.8$–1.0. Our calculation shows that

[2]The decay constants of the pseudoscalar and vector mesons are defined by $\langle 0|\bar{Q}\gamma^\mu\gamma_5Q'|M_P(\mathbf{K})\rangle=f_PK^\mu$ and $\langle 0|\bar{Q}\gamma^\mu Q'|M_V(\mathbf{K},\varepsilon)\rangle=f_Vm_V\varepsilon^\mu$, respectively, where ε is the polarization vector of the vector meson and K is the meson momentum.

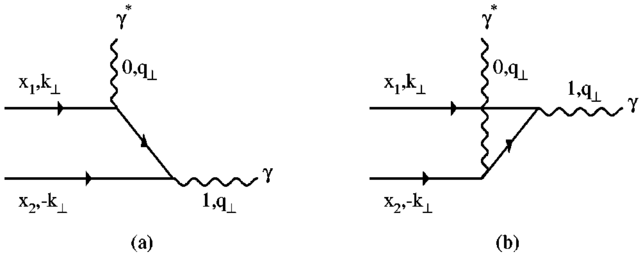

FIG. 1. The lowest order diagrams contributing to $F_{\eta_c\gamma}$ and $F_{\eta_b\gamma}$ in the light-cone perturbative QCD. The momenta are expressed in the light-cone variables $(+,\perp)$.

the prediction for the $\eta_c(\eta_b)$ transition form factor, $F_{\eta_c\gamma}(F_{\eta_b\gamma})$ are not sensitive to the value of $P_{c\bar{c}/\eta_c}(P_{b\bar{b}/\eta_b})$ [14]. So we may take

$$P_{c\bar{c}/\eta_c}=0.8, \quad P_{b\bar{b}/\eta_b}=1.0. \tag{23}$$

From the above constrains, one can fix the parameters in the wave functions

$$A=54.44\ \text{GeV}^{-1}, \quad \beta=0.994\ \text{GeV for } \eta_c, \tag{24}$$

$$A=4146\ \text{GeV}^{-1}, \quad \beta=1.507\ \text{GeV for } \eta_b. \tag{25}$$

The average transverse momenta of the quark in the mesons defined by $\langle k_\perp\rangle=\sqrt{\langle \mathbf{k}_\perp^2\rangle}$ with

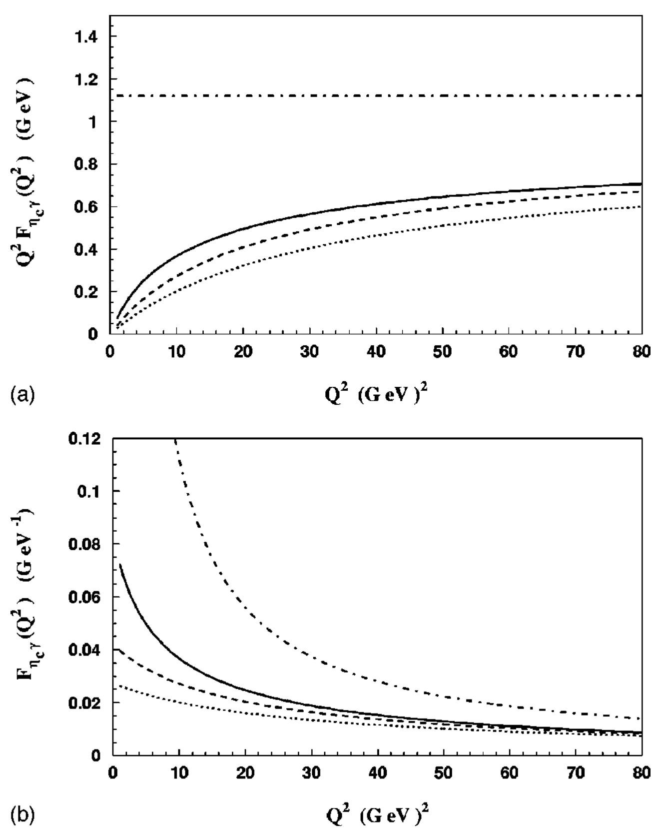

FIG. 2. (a) The η_c-γ transition form factor given in $Q^2F_{\eta_c\gamma}(Q^2)$. (b) The η_c-γ transition form factor given in $F_{\eta_c\gamma}(Q^2)$.

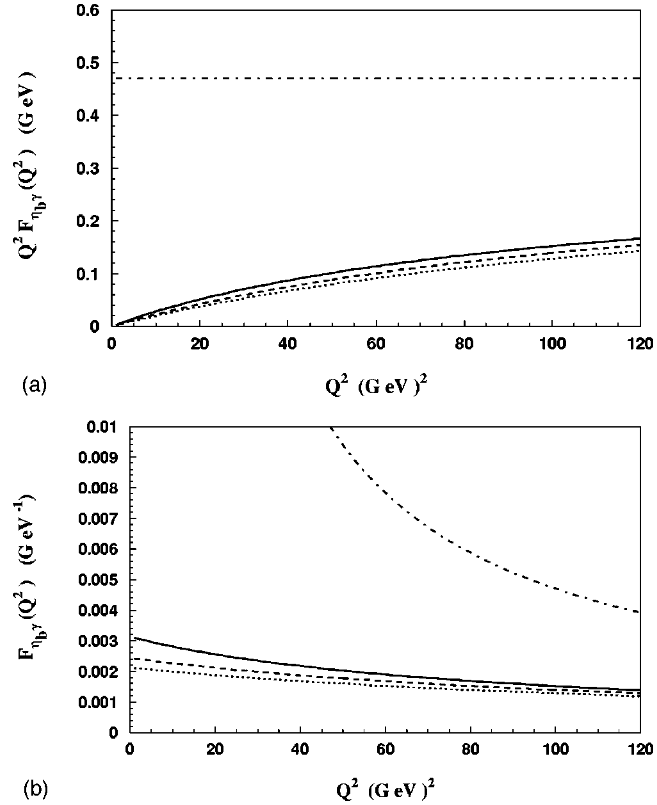

FIG. 3. (a) The η_b-γ transition form factor given in $Q^2F_{\eta_b\gamma}(Q^2)$. (b) The η_b-γ transition form factor given in $F_{\eta_b\gamma}(Q^2)$.

$$\langle \mathbf{k}_\perp^2\rangle=\frac{1}{P_{q\bar{q}/\eta_q}}\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}|\mathbf{k}_\perp|^2\psi(x,k_\perp) \tag{26}$$

turn out to be 0.95 and 1.48 GeV for the η_c and η_b, respectively.

We present our numerical results for $F_{\eta_c\gamma}$ in Fig. 2. The dash-dotted line is the asymptotic prediction [Eq. (15)]. The solid curve is obtained by respecting the m_c effect but neglecting the corrections from the $k_\perp$ dependence and the higher helicity components [Eq. (16)]. The dashed curve is obtained by taking into account m_c effect and the $k_\perp$ dependence in the ordinary light cone wave function but neglecting the contributions form the higher helicity components [Eq. (11)]. Considering all of these corrections gives the dotted curve [Eq. (13)]. In the $Q^2\to\infty$ limit, all of these calculations approach the asymptotic prediction. But, because of the c quark being heavier, taking into account the quark mass effect significantly modifies the perturbative prediction in a rather broad energy region. At $Q^2\simeq 10\ \text{GeV}^2$, the result obtained by including the c-quark mass effect is only about 1/3 of the asymptotic prediction for $F_{\eta_c\gamma}$. At $Q^2\simeq 100\ \text{GeV}^2$ the ratio is about 70%. We point out that the asymptotic prediction [Eq. (15)] is essentially valid for $\log(Q^2/M_{\eta_c,\eta_b})\gg 1$, while the leading twist predictions from Eqs. (11)–(13) and (16) are valid for $Q^2/M_{\eta_c,\eta_b}\gg 1$. The wave function of heavy quarkonia in the medium energy region [Eq. (18)] being sharply peaked than their asymptotic form [Eq. (14)] also results in a Q^2-independent reduction in the prediction for $F_{\eta_c\gamma}$. Also it can be found from Fig. 2 that in the energy region of $Q^2 \leqslant 10\ \mathrm{GeV}^2$ where the the present experiments seem to be able to approach, the parton's transverse momentum and higher helicity components bring sizable corrections to the prediction of $F_{\eta_c\gamma}$.

The numerical results of $F_{\eta_b\gamma}$ are given in Fig. 3. The curve explanations are similar to those in Fig. 2. It can be found that the modification resulting from the b-quark mass effect is much more severe than that in the case of $F_{\eta_c\gamma}$ due to the b quark being heavier than the c quark. At $Q^2 \simeq 10\ \mathrm{GeV}^2$, the result obtained by including the b-quark mass effect is only about 10% of the asymptotic prediction for the $F_{\eta_b\gamma}$. At $Q^2 \simeq 100\ \mathrm{GeV}^2$ the ratio is about 30%. On the other hand, the corrections coming form the parton's transverse momentum and the higher helicity components are negligible in the calculation of $F_{\eta_b\gamma}$ since the b-quark mass m_b is much heavier than the parton's average transverse momentum in the η_b meson. One can expect that LEP2 may examine all of these theoretical predictions in the near future.

V. SUMMARY

In summary, the meson photon transition form factors $F_{P\gamma}(Q^2)$(P being $\pi^0, \eta, \eta', \eta_c, \eta_b, \ldots$) extracted from the two photon collision are the simplest exclusive processes which can provide a rather simple and rigorous way to the test of QCD and the determination of the meson wave function (nonperturbative physics). A lot of experimental collaborations such as TPC/Two-Gamma, CELLO, CLEO, L3, etc., have studied these processes. A measurement for the $F_{\eta_c\gamma}(Q^2)$ is very likely to be feasible in LEP2. In this work, we analyzed the η_c- and η_b-photon transition form factors in the light-cone perturbative theory with the quark mass effect, the parton's transverse momentum dependence, and the higher helicity components of the light cone wave function respected. It is pointed out that due to the c- (b-) quark being heavy, considering the quark mass effect brings significant modifications to the asymptotic predictions in a rather broad energy region. This effect is much more severe for $F_{\eta_b\gamma}$ than that for $F_{\eta_c\gamma}$ because of the b quark being heavier than the c quark. Also it is found that the parton's transverse momentum and higher helicity components bring sizable corrections to $F_{\eta_c\gamma}$ in the present experimentally accessible energy region ($Q^2 \leqslant 10 \sim 20\ \mathrm{GeV}^2$), while these corrections are negligible in the perturbative calculation of $F_{\eta_b\gamma}$. The coming e^+e^- collider LEP2 will provide the opportunity to examine all of these theoretical predictions.

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[1] S. J. Brodsky and G. P. Lepage, Phys. Rev. Lett. **43**, 545 (1979); Phys. Lett. **87B**, 359 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); **24**, 1808 (1981); S. J. Brodsky, SLAC-PUB-7604, hep-ph/9708345.

[2] TPC/Two-Gamma Collaboration, H. Aihara *et al.*, Phys. Rev. Lett. **64**, 172 (1990).

[3] CELLO Collaboration, H.-J. Behrend *et al.*, Z. Phys. C **49**, 401 (1991). Note that there is a factor of $4\pi\alpha$ for the definition of form factors between our analysis and this reference.

[4] CLEO Collaboration, V. Savinov *et al.*, hep-ex/9507005.

[5] L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. B **418**, 399 (1998).

[6] P. Aurenche *et al.*, hep-ph/9601317.

[7] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **24**, 1808 (1981).

[8] C.-R. Ji and F. Amiri, Phys. Rev. D **42**, 3764 (1990).

[9] S. Ong, Phys. Rev. D **52**, 3111 (1995).

[10] P. Kroll and M. Raulfs, Phys. Lett. B **387**, 848 (1996); R. Jakob, P. Kroll, and M. Raulfs, J. Phys. G **22**, 45 (1996).

[11] F. G. Cao, B. Q. Ma, and T. Huang, Phys. Rev. D **53**, 6582 (1996).

[12] L. V. Musatov and A. V. Radyushkin, Phys. Rev. D **56**, 2713 (1997); A. V. Radyushkin and R. T. Ruskov, Nucl. Phys. **B481**, 625 (1996); Phys. Lett. B **374**, 173 (1996).

[13] V. V. Anisovich, D. J. Melikhov, and V. A. Nikonov, Phys. Rev. D **55**, 2918 (1997).

[14] T. Feldmann and P. Kroll, Phys. Lett. B **413**, 410 (1997).

[15] See, e.g., J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970); J. B. Bjorken, J. B. Kogut, and D. E. Soper, *ibid.* **3**, 1328 (1971); S. J. Brodsky, R. Roskies, and R. Suaya, *ibid.* **8**, 4574 (1973).

[16] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); S. J. Brodsky and G. P. Lepage, *Perturbative Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989), p. 93.

[17] S. J. Brodsky, T. Huang, and G. P. Lepage, in *Particles and Fields-2*, Proceedings of the Banff Summer Institute, Banff, Alberta, Canada, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), p. 143; G. P. Lepage, S. J. Brodsky, T. Huang, and P. B. Mackenize, *ibid.*, p. 83; T. Huang, in *Proceedings of XXth International Conference on High Energy Physics*, Madison, Wisconsin, 1980, edited by L. Durand and L. G. Pondrom, AIP Conf. Proc. No. 69 (AIP, New York, 1981), p. 1000.

[18] E. Wigner, Ann. Math. **40**, 149 (1939); H. J. Melosh, Phys. Rev. D **9**, 1095 (1974).

[19] See, e.g., B. Q. Ma, Z. Phys. A **345**, 321 (1993); B.Q. Ma and T. Huang, J. Phys. G **21**, 765 (1995); F. G. Cao, J. Cao, B. Q. Ma, and T. Huang, Phys. Rev. D **55**, 7107 (1997).

[20] B. Q. Ma, J. Phys. G **17**, L53 (1991); B. Q. Ma and Q. R. Zhang, Z. Phys. C **58**, 479 (1993); S. J. Brodsky and F.

Schlumpf, Phys. Lett. B **329**, 111 (1994).
[21] B. Q. Ma, Phys. Lett. B **375**, 320 (1996); S. J. Brodsky and B. Q. Ma, *ibid.* **381**, 317 (1996).
[22] See, for example, M. R. Ahmady and R. R. Mendel, Phys. Rev. D **51**, 141 (1995); Y. B. Ding, X. H. Guo, and X. Q. Li, *ibid.* **54**, 1136 (1996); E. J. Eichten and C. Quigg, *ibid.* **52**, 1726 (1995).
[23] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50**, 617 (1967); **51**, 583(E) (1967).
[24] M. R. Ahmady and R. R. Mendel, Phys. Rev. D **51**, 141 (1995); Z. Phys. C **65**, 263 (1995).
[25] D. S. Hwang and G.-H. Kim, Z. Phys. C **76**, 107 (1997).
[26] H. W. Huang, J. H. Liu, J. Tang, and K. T. Chao, Phys. Rev. D **56**, 368 (1997).