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Bimaximal lepton flavor mixing matrix and neutrino oscillation

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Recently, many authors showed that if the solar and atmospheric neutrino data are both described by maximal mixing vacuum oscillations at the relevant mass scale, then there exists a unique bimaximal lepton mixing matrix for three neutrino flavors. We construct the lepton mass matrices from the symmetry principle so that maximal mixings for the atmospheric and the solar neutrino vacuum oscillations are naturally generated. Although the hierarchical patterns of the lepton sector are quite different from each other, we show how two different mass matrices suggested in this work can be generated in a unified way. We also give comments on possible future tests of the bimaximal lepton mixing matrix. [S0556-2821(99)50109-4]

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The recent atmospheric neutrino data from the Super-Kamiokande Collaboration [1] present convincing evidence for neutrino oscillation and hence a nonzero neutrino mass. The results indicate the maximal mixing between ν_μ and ν_τ with a mass squared difference $\delta m^2_{\rm atm}\simeq 5\times 10^{-3}$ eV^2. The long-standing solar neutrino deficit [2–4] can also be explained through matter enhanced neutrino oscillation [i.e., the Mikheyev-Smirnov-Wolfenstein (MSW) solution [5]] if $\delta m^2_{\rm solar}\simeq 6\times 10^{-6}$ eV^2 and $\sin^2 2\theta_{\rm solar}\simeq 7\times 10^{-3}$ (small angle case), or $\delta m^2_{\rm solar}\simeq 9\times 10^{-6}$ eV^2 and $\sin^2 2\theta_{\rm solar}\simeq 0.6$ (large angle case) and through long-distance vacuum neutrino oscillation called ''just-so'' oscillation [6] if $\delta m^2_{\rm solar}\simeq 10^{-10}$ eV^2 and $\sin^2 2\theta_{\rm solar}\simeq 1.0$. However, the recent data on the electron neutrino spectrum reported by Super-Kamiokande [3] seem to favor the ''just-so'' vacuum oscillation, even though the small angle MSW oscillation and the maximal mixing between the atmospheric ν_μ and ν_τ have been taken as a natural solution for the neutrino problems [7]. Moreover, as shown by Georgi and Glashow [8], solar neutrino oscillations may be nearly maximal if relic neutrinos comprise at least one percent of the critical mass density of the Universe. If this vacuum oscillation of the solar neutrino is confirmed in future experiments [3,9], the mixing angles in the lepton sector will turn out to be large in contrast with the quark sector in which all observed mixing angles among different families are quite small. This does not appear to be achieved in such a way as to unify quarks and leptons at the grand unified theory (GUT) scale. One can thus deduce that the origin of the lepton mass matrices would be different from the one in the quark sector [7]. Therefore, it is worthwhile to find any possible mechanism providing such neutrino mixing patterns. Gauge models such as the SO(10) grand unification model [10] and the left-right symmetric model [11] have been constructed so that the so called ''bimaximal'' neutrino mixing [12] for the solar and atmospheric vacuum oscillations are naturally accommodated. There have also been attempts to derive such a neutrino mixing from a lepton mass matrix ansatz [8,12–14].

Recently, Barger *et al.* [12] showed that if the solar and atmospheric neutrino data are both described by maximal mixing vacuum oscillations at the relevant mass scale, then there exists a unique mixing matrix for three neutrino flavors. Their solution necessarily conserves CP and automatically implies that there is no disappearance of atmospheric ν_e, consistent with indications from the Super-Kamiokande experiment. However, they did not construct the neutrino mass matrix from some simple symmetry principle, but inverted the process to obtain the neutrino mass matrix in the flavor basis from the mass eigenvalues and the bimaximal mixing matrix by using the fact that a Majorana mass matrix or a Hermitian Dirac mass matrix can be diagonalized by a single unitary matrix. From the phenomenological point of view, Georgi and Glashow [8] also suggested the neutrino mass matrix that is compatible with the ''bimaximal'' neutrino mixing, cosmological observation, and the nonexistence of neutrinoless double beta decay.

The purpose of this Rapid Communication is to construct the lepton mass matrices from the symmetry principle so that maximal mixings for the atmospheric and the solar neutrino vacuum oscillations are naturally generated. We note that the bimaximal lepton flavor mixing matrix $V_{\rm bi\text{-}max}$ can be constructed from the product of two unitary matrices:

$$(U^{\rm lepton}_{CKM})^\dagger \equiv V_{\rm bi\text{-}max} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1}$$

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$$
= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$$
\equiv U_\nu^\dagger \cdot U_l , \tag{2}
$$

where U_ν and U_l give the maximal mixing between the second and the third generations and between the first and the second generations, respectively. As will be shown later, the charged lepton mass matrix can be diagonalized by U_l, while the neutrino mass matrix can be diagonalized by U_ν. This is an outstanding feature of our lepton mass matrices. Although the hierarchical patterns of the lepton sector are quite different from each other, we will show how two different mass matrices suggested in this work can be generated in a unified way.

Let us start with a general $S(3)_L \times S(3)_R$ symmetric mass matrix [15,16]:

$$
M_0 = C \begin{pmatrix} 1 & r & r \\ r & 1 & r \\ r & r & 1 \end{pmatrix}. \tag{3}
$$

By diagonalizing this matrix with the help of the unitary matrix

$$
U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{4}
$$

we obtain the eigenvalues

$$
C(1-r, \quad 1-r, \quad 1+2r).
$$

For $r=1$, only the third element becomes massive, which enables us to explain why the third generation quarks and charged leptons are much heavier than the others [17]. Thus we take $r=1$ for the charged lepton mass matrix. On the other hand, the neutrino data does not seem to support such a hierarchy. Moreover, if we regard the neutrinos as a part of hot dark matter, all three neutrinos may be almost degenerate in their masses [18,19]. This almost degenerate neutrino mass pattern can be achieved by taking r to be nearly zero [16]. Therefore, we choose, as the first step,[1]

$$
r=1 \quad \text{for charged lepton case,}
$$

$$
\text{and} \quad r=0 \quad \text{for neutrino case.}
$$

In order to generate the hierarchy of the charged lepton sector and the phenomenologically acceptable form of the mass matrix for the neutrino sector, we introduce the symmetry breaking terms so that the hierarchy or the mass difference between two generations can be accommodated, and the maximal mixing between those generations can be generated simultaneously. We will show that this can be achieved in the way that the $S(3)_L \times S(3)_R$ symmetry is broken down to $S(2)_L \times S(2)_R$. As is well known, Fritzsch and Xing [14,16] constructed lepton mixing matrices based on the so-called democratic mass matrix that reflects $S(3)_L \times S(3)_R$ symmetry. However, they do not provide the maximal mixings for the atmospheric and the solar neutrino oscillations. As will be shown later, in order to achieve the bimaximal mixing scenario we adopt different symmetry breaking schemes from those of Fritzsch and Xing. In Ref. [14], Fritzsch and Xing also commented on the bimaximal mixing scenario with completely different charged lepton and neutrino mass matrices from ours. Their nonzero off-diagonal entries are different from ours, which implies that $S(3)_L \times S(3)_R$ symmetry is broken down in different ways.

Now we consider the following 2×2 mass matrix, which provides the maximal mixing between two flavors [20],

$$
\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}. \tag{5}
$$

This form of mass matrix can be diagonalized by the unitary matrix

$$
U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \tag{6}
$$

and the eigenvalues are given as

$$
(\alpha+\beta, \quad \alpha-\beta).
$$

The matrix (5) can be easily generated by considering the so-called "democratic" 2×2 mass matrix, that reflects $S(2)_L \times S(2)_R$ symmetry, and by adding a symmetry breaking matrix, which has $S(2)$ symmetry under the interchange between the first and the second indices:

$$
M_2 = A \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + B \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} A+B & A-B \\ A-B & A+B \end{pmatrix}. \tag{7}
$$

[1] Actually, the case $r=0$ might not require the diagonalization of M_0 because that case already corresponds to the diagonal form before diagonalizing. However, we need the diagonalization as long as the value of r is not exactly zero but small enough to be negligible compared to the parameter C and even A, B.

With the help of Eq. (6), one can easily obtain the eigenvalues of M which are given as

$$(2A, \quad 2B).$$

Since we want to get the bimaximal mixing matrix while keeping the hierarchical charged lepton masses and degenerate neutrino masses, we add this symmetry breaking matrix M_2 to the previous hierarchical matrices M_0 appropriately. Then, we relate the parameters A and B (a) to the masses of the first and the second generations for the charged lepton sector, respectively, and (b) to the mass differences between two (the second and the third) generations for the neutrino sector.

At the end, we can obtain the realistic lepton mass matrices. That is, we add the above symmetry breaking M_2 matrix as the submatrix of M_0 in the (e,μ) basis for the charged lepton sector,

$$M_l=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C \end{pmatrix} \Rightarrow \begin{pmatrix} A+B & A-B & 0 \\ A-B & A+B & 0 \\ 0 & 0 & C \end{pmatrix}, \tag{8}$$

while we add it as the one in the (ν_μ,ν_τ) basis for the neutrino sector as follows:

$$M_\nu=\begin{pmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{pmatrix} \Rightarrow \begin{pmatrix} C & 0 & 0 \\ 0 & C+A+B & A-B \\ 0 & A-B & C+A+B \end{pmatrix}. \tag{9}$$

Then, one can see that these matrices M_l and M_ν can be diagonalized by U_l and U_ν, respectively, which in turn lead to the bimaximal lepton flavor mixing matrix $U_{CKM}^{\rm lepton}$, as given in Eqs. (1), (2) by combining U_ν with U_l.

Eigenvalues of the mass matrices M_l and M_ν are given as

$$M_l=(2A,\ 2B,\ C)$$

and

$$M_\nu=(C,\ C+2A,\ C+2B),$$

respectively. For the charged lepton sector, the parameters A, B, and C are determined by the following mass relations:

$$A=m_e/2, \qquad B=m_\mu/2, \qquad \text{and} \qquad C=m_\tau. \tag{10}$$

In order to solve A,B, and C for the neutrino sector, we first require two conditions,

$$\Delta m^2_{\rm solar}=10^{-10}\ \text{eV}^2 \quad \text{and} \quad \Delta m^2_{\rm atm}=2\times10^{-3}\ \text{eV}^2,$$

which can fit the available data quite well, where the mass differences $\Delta m^2_{ij}=m^2_{\nu_i}-m^2_{\nu_j}$ should be identified with, among the possibilities, $\Delta m^2_{\rm solar}=\Delta m^2_{12}$ and $\Delta m^2_{\rm atm}=\Delta m^2_{23}$. Thus we will consider henceforth only this case. In addition, if the neutrinos account for the hot dark matter of the universe, one has to require

$$\sum |m_{\nu_i}|\simeq 6\ \text{eV}.$$

Then the set of parameters (A,B,C) is given by

$$(A,B,C)\approx(10^{-10}, \quad 0.00025, \quad 2.0)\ (\text{eV}), \tag{11}$$

for which three light neutrinos are almost degenerate with masses around 2 eV.

Now, we check if the solution of three neutrino mass eigenvalues satisfies the constraints coming from the neutrinoless double β-decay, as well as other data from neutrino oscillation experiments. The neutrino mixing matrix Eq. (1) and neutrino mass eigenvalues lead to

$$\langle m_{\nu_e}\rangle \equiv \sum_{i=1}^{3} V^2_{ei} m_{\nu_i} \simeq 2.0\ \text{eV}.$$

However, that value of neutrino mass is not compatible with the current upper limit coming from the nonobservation of the neutrinoless double β-decay, which is given as [21]

$$\langle m_{\nu_e}\rangle \leqslant (0.5-1.5)\ \text{eV}. \tag{12}$$

In order to be satisfied with this constraint, $\Sigma|m_{\nu_i}|$ is allowed only up to 4.5 eV. If we take this value, the set of parameters (A,B,C) is determined to be

$$(A,B,C)\approx(10^{-10}, \quad 0.00035, \quad 1.5)\ (\text{eV}), \tag{13}$$

for which three light neutrinos are almost degenerate with masses around 1.5 eV. If we begin to increase the neutrino masses in order to make them dominant hot dark matter candidates, we cease to satisfy the $(\beta\beta)_{\nu0}$ constraint.

Further test of the bimaximal mixing scenario is provided with the long baseline experiments searching for $\nu_\mu\rightarrow\nu_\tau$ oscillation in the range of $\Delta m^2_{\mu\tau}\simeq10^{-3}$ eV^2 [22]. The MINOS [23] and K2K [24] sensitivities to Δm^2 at 90% C.L. can go down to $\Delta m^2=1.2\times10^{-3}$ eV^2 and 2.0×10^{-3} eV^2, respectively, while the Imaging of Cosmic and Rare Underground Signals (ICARUS) [25] sensitivity is achieved at $\Delta m^2=3.0\times10^{-3}$ eV^2. The bimaximal mixing scenario, in which $\sin^2 2\theta_{\mu\tau}$ is predicted to be 1 with $\Delta m^2_{\mu\tau}\simeq2\times10^{-3}$ eV^2, can be tested at the MINOS and K2K experiments searching for the $\nu_\mu\rightarrow\nu_\tau$ oscillations in the foreseeable future, but is beyond the sensitivity to Δm^2 at 90% C.L. being achieved at ICARUS. Future experiment on the $\nu_\mu\leftrightarrow\nu_\tau$ oscillation from the MINOS and K2K will exclude this scenario for charged lepton and neutrino mass matrices.

Finally, we comment on that the bimaximal neutrino mixing matrix Eq. (1) predicts zero for the V_{e3} element which makes $\nu_e\leftrightarrow\nu_\mu$ and $\nu_\mu\leftrightarrow\nu_\tau$ oscillations to be effectively a two-channel problem. This is supported by CHOOZ data [26] which give the mixing angle θ_{13} as less than 13° in most of the Super-Kamiokande allowed region. As one can see, the V_{e3} element becomes zero in the limit of $\theta_{13}=0$ [27]. However, note that a nonvanishing V_{e3} element is not completely excluded, but rather it can be larger in the region not covered by CHOOZ [28,29]. To justify this bimaximal mixing scenario, the precise determination of the V_{e3} element will probably be essential, which requires several oscillation channels to be probed at the same time. From the fact that the $\nu_\mu \rightarrow \nu_\tau$ disappearance channel is sensitive only to $V_{\mu 3}^2$ and the $\nu_\mu \rightarrow \nu_e$ appearance channel is sensitive to the product $V_{\mu 3}^2 V_{e3}^2$, one can determine the element V_{e3} by combining the regions to be probed in both channels. K2K [24] will be expected to perform this, but it does not, at present, seem to achieve sufficient sensitivity in the $\nu_\mu \rightarrow \nu_e$ appearance channel to probe the region of V_{e3}^2 allowed by Super-Kamiokande and CHOOZ [29].

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