Reply to ''Comment on 'Brans-Dicke wormholes in the Jordan and Einstein frames' ''

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(Received 27 May 1998; published 16 March 1999)

This reply to the Comment by Bloomfield clarifies that the Brans-Dicke wormhole condition is $(C+1)^2>\lambda^2$ rather than $(C+1)>\lambda$, as suggested in our earlier paper. Various cases depending on the signs of $(C+1)$ and λ arise. The radial tidal accelerations are indeed finite, as correctly pointed out by Bloomfield. [S0556-2821(98)01424-6]

PACS number(s): 04.20.Gz, 04.50.+h, 04.62.+v

The preceding Comment by Bloomfield [1] points out the need to expand or amend a few points in our paper [2] which we do here. First, the wormhole range $-\frac{3}{2}<\omega<-\frac{4}{3}$ looks similar to a computational error because of a misleading inequality (17) in Ref. [2]. Actually, the wormhole condition

$$C(1-\omega C/2)>0 \tag{1}$$

is satisfied under a weaker condition:

$$(C+1)^2>\lambda^2. \tag{2}$$

Thus, $C+1$ and λ can have any sign provided that Eq. (2) is satisfied. Bloomfield [1] considers the case $(C+1)>0$ and $\lambda>0$ and rightly concludes that wormholes exist only for $\omega<-2$ in the weak field approximation. However, there are also other cases, as we will see below.

Note that $\rho<0$ and the Brans-Dicke (BD) scalar field ϕ plays the role of exotic matter [3,4]. Therefore, the scalar mass m_s would be nonpositive definite, and hence we generally allow for both signs. It has been shown by Scheel, Shapiro, and Teukolsky [5] that the scalar mass increases to zero at the end of a spherical collapse. Bloomfield [1] pointed out the question of various masses and these are useful for our purposes. For $C(\omega)=-(\omega+2)^{-1}$, $\phi_\infty=1$, $c=1, G=(4+2\omega)/(3+2\omega)$, the three types of masses are [5,6]

$$\text{Keplerian mass}\quad m_k=2B/\lambda=GM=M(4+2\omega)/(3+2\omega), \tag{3}$$

$$\text{scalar mass}\quad m_s=-(Cm_k)/2=M/(3+2\omega), \tag{4}$$

$$\text{tensor mass}\quad m_t=B(C+2)/\lambda=M. \tag{5}$$

The tensor mass is unobservable in the Jordan frame through ordinary test particle motion in the presence of a BD scalar field. Hence, we may regard m_t as only an undetermined integration constant. Let us rewrite the wormhole throat radii as

$$r_0^{\pm}=\lambda(\omega+2)m_s((C+1)/\lambda\pm\sqrt{[\{(C+1)/\lambda\}^2-1]}) \tag{6}$$

The transferability to Einstein frame requires that $\omega>-\frac{3}{2}$ which implies $\omega>-2$. Then, under Eq. (2), the following cases are possible: (i) $m_s>0, C+1>0, \lambda>0, B=\lambda(\omega+2)m_s>0$ such that $r_0^+>B$, $\Omega=1-\nu<0$, $\nu=(C+1)/\lambda$; (ii) $m_s<0, C+1<0, \lambda<0, B>0$ such that $r_0^+>B$, $\Omega<0$; (iii) $m_s>0, C+1>0, \lambda<0, B=-B'$, $B'>0, r_0^-$ $>B'$, $\Omega>2$; (iv) $m_s<0, C+1<0, \lambda>0, B=-B'$, $B'>0$, $r_0^->B'$, $\Omega>2$. Bloomfield's choice corresponds to case (i) [and (iii)]. For cases (ii) and (iv), $C+1<0$ which, together with Eq. (1) and $\lambda^2>0$, implies $-\frac{3}{2}<\omega<-\frac{4}{3}$. Hence, there is no theoretical reason to eliminate the weak field approximation from wormhole investigations.

Secondly, we now see that $r_0^+r_0^-=B^2$ can be interpreted both as $r_0^-<B<r_0^+$ [1] and $r_0^+<B<r_0^-$. There is no need to exclude r_0^- from our consideration.

Thirdly, the limits; the R radii of the throat are

$$R_0^+=r_0^+(1+B/r_0^+)^{1+\nu}(1-B/r_0^+)^{1-\nu},$$

$$R_0^-=r_0^-(1+B'/r_0^-)^{1+\nu}(1-B'/r_0^-)^{1-\nu}.$$

Bloomfield has correctly evaluated the limits of R_0^+, ρ_0^+, and ϕ_0^+ in his Eq. (13) [1]. Indeed, it may be verified that the same results are also obtained for the other set when $r_0^-\to B'$, $\nu\to 1$. The statement in this context in Ref. [2] needs to be amended. For more details about the subtleties of the limiting processes in the BD theory, see our recent work [7]. As to the traversability condition, Bloomfield [1] is again right. The radial tidal acceleration is indeed finite contrary to what was stated in Ref. [2].

We now have the following conclusion: For cases (ii) and (iv), which allow transferability to the Einstein frame, the wormhole range is $-\frac{3}{2}<\omega<-\frac{4}{3}$. The most interesting result is that such wormholes are physically traversable due to the finiteness of tidal forces at the throat. For cases (i) and (iii), wormholes are again traversable but the theory can not be transferred to the physically important Einstein frame since $\omega<-2$. In the Einstein frame, several energy conditions are not violated [2,8] and hence there is no question of wormholes there. Bloomfield [1] also reaches the same conclusion on a different argument and there is no contradiction here.

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