Induced parity breaking term in arbitrary odd dimensions at finite temperature

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We calculate the exact parity-odd part of the effective action (Γ_{odd}^{2d+1}) for massive Dirac fermions in $2d+1$ dimensions at finite temperature, for a certain class of gauge field configurations. We consider first Abelian external gauge fields, and then we deal with the case of a non-Abelian gauge group containing an Abelian U(1) subgroup. For both cases, it is possible to show that the result depends on topological invariants of the gauge field configurations, and that the gauge transformation properties of Γ_{odd}^{2d+1} depend only on those invariants and on the winding number of the gauge transformation. [S0556-2821(99)04506-3]

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I. INTRODUCTION

The issue of parity breaking at finite temperature in three-dimensional gauge theories with massive fermions posed a puzzle concerning the induced effective action: perturbative calculations indicated that it was simply a Chern-Simons (CS) term times a coefficient that was a smoothly varying function of the temperature but this was in contradiction with gauge invariance [1–3].

A crucial advance was made in [4] by studying a $D=1$ solvable model for which the *exact* effective action was gauge invariant although perturbative expansions produced gauge-noninvariant results. Subsequently, it was shown that the same phenomenon also takes place in $2+1$ dimensions. This was proved through nonperturbative calculations of the effective action in the Abelian case [5] and of its explicit, exact, temperature-dependent parity breaking part both in the Abelian and non-Abelian cases [6], for particular gauge backgrounds.

These results were discussed in connection with reduction of CS terms by a symmetry [7,8] and also confirmed by several alternative calculations [9–17].

It is the purpose of the present work to extend the results in [6] to the case of *arbitrary* odd dimensions, $D=2d+1$. Indeed, of the many interesting properties enjoyed by odd dimensional quantum field theories, not the least important is the possibility of equipping a gauge field with a Chern-Simons action. This parity breaking object is invariant under gauge transformations connected to the identity, but not necessarily so for ''large'' ones. Demanding invariance of the partition function under large gauge transformations has important consequences, particularly for the cases of nontrivial spacetimes (even in the Abelian case) or when the gauge group is non-Abelian.

As in the $D=3$ case, for arbitrary $D=2d+1$ dimensions the CS action arises as the result of integrating out fermionic degrees of freedom at zero temperature. At finite T, temperature-dependent parity breaking terms are also induced by integrating fermionic degrees of freedom, in such a form that their zero-T limit coincides, as we shall see, with the CS action [18,19].

The clue in the approach of [6] to the three-dimensional case was to choose a particular gauge background in which the temperature dependence in the parity breaking part of the effective action can be factored out, leaving all the spatial information encoded in the form of the two-dimensional chiral anomaly. The main point in the present paper is to show that the same holds in $D=2d+1$ dimensions. Namely, for particular gauge field backgrounds, the temperature dependence is isolated in a factor that can be related to the Polyakov loop and the spatial components of the gauge configuration give rise to a factor which is nothing but the chiral anomaly, now in $2d$ dimensions. This is done both in the Abelian case (Sec. II) and in the non-Abelian one (Sec. III) for a particular choice of the gauge background which is, however, sufficiently general as to allow to infer qualitative properties in the general case. We give a summary and discussion of our results in Sec. IV.

II. ABELIAN CASE

Let us start by stressing that in the imaginary time formalism of finite-temperature quantum field theory, the effective action for $D=2d+1$ dimensional Dirac fermions with mass M can be a nonextensive quantity whose temperature-dependent, parity-odd part, will be called here $\Gamma_{odd}^{2d+1}(A,M)$. It is defined as

$$\Gamma_{odd}^{2d+1}(A,M)=\frac{1}{2}[\Gamma^{2d+1}(A,M)-\Gamma^{2d+1}(A,-M)], \quad (1)$$

where

$$\exp[-\Gamma^{2d+1}(A,M)]=\int \mathcal{D}\psi\,\mathcal{D}\bar{\psi}\exp[-S_F(A,M)], \quad (2)$$

and the Euclidean action $S_F(A,M)$ is given by

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$$S_F(A,M)=\int_0^\beta d\tau\int d^{2d}x\,\bar{\psi}(\not{\partial}+ie\not{A}+M)\psi. \tag{3}$$

The Euclidean γ matrices in $2d+1$ dimensions are denoted as $\gamma_0,\gamma_1,\ldots,\gamma_{2d}$. From the point of view of the $2d$-dimensional theories (with coordinates x) that will arise below, γ_0 will act as a γ_5 chirality matrix.

The fermionic fields in Eq. (2) obey antiperiodic boundary conditions in the timelike direction:

$$\psi(\beta,x)=-\psi(0,x),\quad \bar{\psi}(\beta,x)=-\bar{\psi}(0,x)\quad \forall x. \tag{4}$$

The gauge field, instead, is periodic in the same direction:

$$A_\mu(\beta,x)=A_\mu(0,x),\quad \forall x. \tag{5}$$

The effective action $\Gamma^{2d+1}(A,M)$ is, as usual, written in terms of a fermionic determinant:

$$\Gamma^{2d+1}(A,M)=-\log\det(\not{\partial}+ie\not{A}+M). \tag{6}$$

In order to get an exact result we choose a particular gauge field background which corresponds to a vanishing electric field and a time-independent magnetic field,

$$A_0=A_0(\tau), \tag{7}$$

$$A_j=A_j(x)\quad (j=1,2,\ldots,2d), \tag{8}$$

or any equivalent configuration obtained from this by a gauge transformation.

Using the same arguments as in Ref. [6] we can always perform a (nonanomalous) gauge transformation of the fermionic fields in the functional integral defining the fermionic determinant in Eq. (2), so that as the zero component of the gauge field becomes a constant that will be called $\tilde{A}_0$,

$$\tilde{A}_0=\frac{1}{\beta}\int_0^\beta d\tau A_0(\tau). \tag{9}$$

After redefining the fermionic fields according to this prescription, we get

$$S_F(A_j,\tilde{A}_0,M)=\int_0^\beta d\tau\int d^{2d}x\,\bar{\psi}\times[\not{\partial}+ie(\gamma_jA_j+\gamma_0\tilde{A}_0)+M]\psi, \tag{10}$$

where there is now no explicit τ dependence in the background. Then, if one performs a Fourier transformation on the time variable for ψ and $\bar{\psi}$,

$$\psi(\tau,x)=\frac{1}{\sqrt{\beta}}\sum_{n=-\infty}^{+\infty}e^{i\omega_n\tau}\psi_n(x),$$

$$\bar{\psi}(\tau,x)=\frac{1}{\sqrt{\beta}}\sum_{n=-\infty}^{+\infty}e^{-i\omega_n\tau}\bar{\psi}_n(x), \tag{11}$$

where $\omega_n=(2n+1)\pi/\beta$ is the usual Matsubara frequency for fermions, the Euclidean action becomes an infinite series of decoupled $2d$-dimensional actions, one for each Matsubara mode:

$$S_F(A_j,\tilde{A}_0,M)=\sum_{n=-\infty}^{+\infty}\int d^{2d}x\,\bar{\psi}_n(x)\times[\not{d}+M+i\gamma_0(\omega_n+e\tilde{A}_0)]\psi_n(x). \tag{12}$$

Here, $\not{d}$ is the $2d$ Euclidean Dirac operator corresponding to the spatial coordinates and the spatial components of the gauge field:

$$\not{d}=\gamma_j(\partial_j+ieA_j). \tag{13}$$

As the Matsubara modes introduced in Eq. (11) are decoupled, the $2d+1$ determinant arising from fermion integration becomes an infinite product of the determinants of $2d$ Euclidean Dirac operators:

$$\det(\not{\partial}+ie\not{A}+M)_{2d+1}=\prod_{n=-\infty}^{n=+\infty}\det[\not{d}+M+i\gamma_0(\omega_n+e\tilde{A}_0)]_{2d}. \tag{14}$$

Remarkably, the parity odd piece of Γ defined by Eqs. (1),(6) can then be factorized, for arbitrary odd space-time dimension D, following the procedure discussed in Ref. [6] for $D=3$. Indeed, for any given mode, one can factorize the $2d$ determinant into mass-even and mass-odd pieces through a chiral transformation as

$$\det[\not{d}+M+i\gamma_0(\omega_n+e\tilde{A}_0)]_{2d}=J_n[A,M]\det[\not{d}+\rho_n]_{2d}, \tag{15}$$

where

$$\rho_n=\sqrt{M^2+(\omega_n+e\tilde{A}_0)^2} \tag{16}$$

and $J_n[A,M]$ is the anomalous Jacobian of a chiral transformation in $2d$ dimensions,

$$\psi_n(x)=\exp\left(-i\frac{\phi_n}{2}\gamma_0\right)\psi'_n(x),$$

$$\bar{\psi}_n(x)=\bar{\psi}'_n(x)\exp\left(-i\frac{\phi_n}{2}\gamma_0\right), \tag{17}$$

with phase

$$\phi_n = \arctan\left(\frac{\omega_n + e\tilde{A}_0}{M}\right). \tag{18}$$

It is important to stress at this point the reason why this procedure can be pursued in more than three dimensions. The Jacobian for a constant chiral transformation is exactly known for *any* even dimension, and this is all we need to evaluate the parity-odd part of the effective action. In fact, from the definition (1),(6) we see that Γ_{odd} depends solely on the Jacobians J_n,

$$\Gamma_{odd} = -\sum_{n=-\infty}^{n=+\infty} \log J_n[A,M]. \tag{19}$$

Now, each Fujikawa Jacobian can be seen to give

$$J_n[A,M] = \exp\left(-i\phi_n \int d^{2d}x \mathcal{A}_{2d}[A]\right), \tag{20}$$

with $\mathcal{A}_{2d}[A]$ denoting the $2d$-dimensional chiral anomaly. Then

$$\Gamma_{odd}^{2d+1} = i\Phi \int d^2x \mathcal{A}_{2d}[A], \tag{21}$$

where

$$\Phi = \sum_{n=-\infty}^{n=+\infty} \phi_n. \tag{22}$$

Note that the phases ϕ_n contain at this stage *all* the dependence on the Matsubara frecuencies ω_n. Moreover, they are *independent* of the number of spacetime dimensions, and hence the sum over ϕ_n is the same as the one already calculated in Ref. [6] for the $D=3$ case,

$$\Phi = \arctan\left[\tanh\left(\frac{\beta M}{2}\right)\tan\left(\frac{1}{2}e\beta\tilde{A}_0\right)\right]. \tag{23}$$

Thus the parity-odd part of Γ finally reads

$$\Gamma_{odd}^{2d+1} = i \arctan\left[\tanh\left(\frac{\beta M}{2}\right)\tan\left(\frac{e}{2}\int_0^\beta d\tau A_0(\tau)\right)\right] \times \int d^{2d}x \mathcal{A}_{2d}[A]. \tag{24}$$

This is one of the main results in our paper. We have been able to compute the *exact* temperature-dependent parity-odd piece of the effective action for massive fermions in a gauge field background *in arbitrary dimensions*. Remarkably, the temperature-dependent factor is universal in the sense it does not depend on the number $D=2d+1$ of the space-time dimensions. The particular background we have chosen makes evident the role of Polyakov loop in the temperature-dependent factor of the effective action as will be discussed below. Concerning the dependence on the spatial components of the gauge field, it is just given by the $2d$ chiral anomaly. As we shall see below, all these features are also valid in the non-Abelian case.

The $D=2d+1=3$ case was discussed in detail in Ref. [6]. Let us then, as another example, write down here the explicit expressions for $D=2d+1=5$. In this case the well-known four-dimensional chiral anomaly is given by

$$\mathcal{A}_4 = -\frac{e^2}{16\pi^2} F_{ij}{}^*F_{ij}, \tag{25}$$

so that

$$\Gamma_{odd}^5 = -i \arctan\left[\tanh\left(\frac{\beta M}{2}\right)\tan\left(\frac{e}{2}\int_0^\beta d\tau A_0(\tau)\right)\right] \times \frac{e^2}{16\pi^2}\int d^4x F_{ij}{}^*F_{ij}. \tag{26}$$

Let us note that the anomaly factor in Eq. (26) can be nontrivial in the Abelian case according to the properties of the $2d$ manifold $\mathcal{M}$ on which $\mathcal{A}_4$ is integrated. For example, if $\mathcal{M}=S^2\times S^2, \int \mathcal{A}_4 d^4x = 2$, the smallest value it can take for spin manifolds (for nonspin manifolds it can take also odd values [21]).

In arbitrary dimension $D=2d$, we quote the form of the Abelian chiral anomaly, which is simply given by

$$\mathcal{A}_{2d} = -\frac{(-e)^d}{(4\pi)^d}\frac{1}{d!}\epsilon_{\mu_1\mu_2\cdots\mu_{2d}}F_{\mu_1\mu_2}\cdots F_{\mu_{2d-1}\mu_{2d}} \tag{27}$$

(see, for instance, [20]).

Let us check that the $D=5$ expression in Eq. (26) has the proper $T=0$ limit; i.e., it reduces to the usual Chern-Simons term. This limit reads

$$\Gamma_{odd}^5 \to -i\frac{M}{|M|}\frac{e^3}{32\pi^2}\int_0^\beta d\tau A_0(\tau)\int d^4x F_{ij}{}^*F_{ij}. \tag{28}$$

This is exactly the form taken by the Chern-Simons term [with a coefficient that is half the value necessary for making $\exp(S_{CS})$ gauge invariant even under large gauge transformations]

$$S_{CS}^5 = \frac{ie^3}{48\pi^2}\int d\tau d^4x \epsilon_{\mu\nu\rho\sigma\lambda}A_\mu\partial_\nu A_\rho\partial_\sigma A_\lambda \tag{29}$$

when evaluated on the configurations restricted by Eqs. (7) and (8). It is interesting to note that in the $T\to 0$ limit $\exp(-\Gamma_{odd}^5)$ is nothing but the Polyakov loop with a coefficient that corresponds to a topological invariant for the $2d$-dimensional gauge theory.

Notice that, in order to take into account all contributions to the above-mentioned configurations, one must approach them from a general one. Some terms that naively vanish for these configurations are actually finite since the limit $A_0(x)$

→const is undetermined. The same kind of indetermination is found in the $D=3$ case; we explain here the correct procedure for that simplest example (see Ref. [6]), since complications arising in higher dimensions are unessential. We write S^3_{CS} in momentum space,

$$S^3_{CS}=-\frac{e^2}{4\pi}\int\frac{d^3p}{(2\pi)^3}\epsilon_{\mu\nu\lambda}A_\mu(-p)p_\nu A_\lambda(p), \quad (30)$$

and explicitly separate the $\mu=0$ index,

$$S^3_{CS}=-\frac{e^2}{4\pi}\int\frac{d^3p}{(2\pi)^3}[\epsilon_{jk}A_0(-p)p_jA_k(p)+\epsilon_{jk}A_j(-p)p_kA_0(p)-\epsilon_{jk}A_j(-p)p_0A_k(p)]. \quad (31)$$

It is immediately seen that the first two terms contribute by the same amount, and that this amount is finite because $A_k(p)$ has a pole in p_j; the last term vanishes because $A_k(p)$ is also proportional to $\delta(p_0)$ (see [9] for details).

In other words, a safe procedure in coordinate space is the following: we first write the integrand without spatial derivatives acting on A_0, by means of integrations by parts, and only then use the fact that $\partial_0A_j=0$. This gives twice the result of the naive restriction given by using $\partial_0A_j=\partial_iA_0=0$.

The same check can be done in the general $(2d+1)$-dimensional case. The correct evaluation of the Chern-Simons term gives then $d+1$ times the naive result. In particular, one gets full agreement between Eqs. (28) and (29).

Let us end this section by discussing the issue of gauge invariance under *large* gauge transformations, a question which, as explained in the Introduction, was put in doubt by perturbative calculations for $D=3$. For the particular Abelian background we are considering, such transformations Ω wind around the cyclic time direction, $\Omega(\beta,x)=\Omega(0,x)+(2\pi/e)k$, with $k\in Z$. The exact result we have obtained for the temperature-dependent, parity-odd effective action, Eq. (24), shows that large gauge transformations may change the temperature-dependent factor if its winding number is odd. Indeed, such a transformation, say, with a winding number $k=2p+1$, shifts the argument of the tangent in $(2p+1)\pi$. One has to keep track of this shift by shifting the branch used for the arctan definition. Now, if the integral of the anomaly is an even integer $n=2m$, the total change of the effective action is $2m(2p+1)\pi i$ and hence $\exp(-\Gamma_{odd})$ remains unchanged. In contrast, if $n=2m+1$, it changes its sign. However, as is well known, there is a mass- and temperature-independent parity anomaly contribution which we have not included in Eq. (24) [22–24] which precisely changes its sign so that the exponential of the complete effective action is indeed gauge invariant.

III. NON-ABELIAN CASE

The analysis performed in the previous section can be extended to certain class of non-Abelian background gauge fields. The model is defined by its Euclidean action

$$S_F=\int_0^\beta d\tau\int d^{2d}x\,\bar{\psi}(\not{D}+M)\psi, \quad (32)$$

where now

$$\not{D}_\mu=\partial_\mu+gA_\mu \quad (33)$$

and the anti-Hermitian gauge connection A_μ corresponds to the Lie algebra of some group G.

We consider configurations satisfying

$$A_0=|A_0|(\tau)\check{n}, \quad (34)$$

$$A_j=A_j(x), \quad [A_j,\check{n}]=0, \quad (j=1,\ldots,2d), \quad (35)$$

where $\check{n}$ is a fixed direction in the Lie algebra. Then, following exactly the same steps as those described in [6], one can arrive at a natural generalization of the result therein. We shall skip details and just present the final answer,

$$\Gamma_{odd}=\left(\frac{ig}{4\pi}\right)^d\frac{1}{d!}\,\mathrm{tr}\left\{\arctan\left[\tanh\left(\frac{\beta M}{2}\right)\times\tan\left(\frac{g}{2}\int_0^\beta d\tau A_0(\tau)\right)\right]\times\int d^{2d}x\,\epsilon_{j_1j_2\cdots j_{2d}}F_{j_1j_2}\cdots F_{j_{2d-1}j_{2d}}\right\}. \quad (36)$$

It is interesting at this point to consider in some detail a subset of configurations (34),(35) which generate a Γ^{2d+1}_{odd} with nice topological properties. Consider that G has an Abelian U(1) factor so that we can decompose A_μ as

$$A_\mu=iA^0_\mu+A^a_\mu\tau_a, \quad (37)$$

where A^0_μ is the component corresponding to the Abelian factor U(1), while A^a_μ denotes the ones for the non-Abelian subgroup that for definiteness we shall take to be SU(N). The matrices τ_a are the generators for SU(N), satisfying the relations

$$[\tau_a,\tau_b]=f_{abc}\tau_c\tau^\dagger_a=-\tau_a, \quad \mathrm{tr}(\tau_a\tau_b)=-\frac{\delta_{ab}}{2}. \quad (38)$$

We now fix the class of gauge configurations we consider to those verifying the conditions

$$A^0_0=A^0_0(\tau), \quad A^0_j=0,$$

$$A^a_0=0, \quad A^a_j=A^a_j(x). \quad (39)$$

The τ dependence of A_μ, present only through A_0^0, may be eliminated by an Abelian gauge transformation just as in the Abelian case. Then the fermionic action becomes

$$S_F(A_j^a,\tilde{A}_0^0,M)=\int_0^\beta d\tau\int d^{2d}x\bar{\psi}\times[\not\partial+g(\gamma_j A_j^a\tau_a+i\gamma_0\tilde{A}_0^0)+M]\psi. \tag{40}$$

Now, as a result of the commutativity of $\tilde{A}_0$ with A_j, the same steps leading to the calculation of Γ_{odd}^{2d+1} may be performed here, with trivial modifications, except for the fact that the anomaly $\mathcal{A}$ will be the one corresponding to a $2d$-dimensional Abelian chiral rotation for a Dirac fermion in presence of a non-Abelian connection $A_j^a(x)$. This gauge field is to be regarded as an arbitrary SU(N) gauge field for the $2d$-dimensional sector of the theory. The anomaly is then of course the well-known "singlet" anomaly [20]

$$\mathcal{A}_{2d}=-\frac{(ig)^d}{(4\pi)^d}\frac{1}{d!}\epsilon_{j_1j_2\cdots j_{2d}}\mathrm{tr}[F_{j_1j_2}\cdots F_{j_{2d-1}j_{2d}}]. \tag{41}$$

Now, as the integral of $\mathcal{A}_{2d}$ is proportional to the Pontryagin index of the configuration

$$\int d^{2d}x\mathcal{A}_{2d}(x)=n, \tag{42}$$

we may write Γ_{odd}^{2d+1} as

$$\Gamma_{odd}^{2d+1}=i\arctan\left[\tanh\left(\frac{\beta M}{2}\right)\tan\left(\frac{g}{2}\int_0^\beta d\tau A_0(\tau)\right)\right]n. \tag{43}$$

Some remarks about this expression are in order. First, note that it is nontrivial only for $D=2d+1>3$ dimensions since for $2d=2$ the singlet anomaly vanishes. Depending on the gauge group and the $2d$ manifold over which the anomaly is integrated the Pontryagin index n can be a nontrivial integer. Second, it is an object which is sensitive to large gauge transformations in $2d+1$ spacetime, putting together the winding associated with the timelike direction τ (reflected in A_0^0), with the usual winding transformations in $2d$. However, the restrictions on the background gauge fields that we have imposed do not allow us to analyze general large gauge transformations although we expect that, as in the Abelian case, gauge invariance is respected.

IV. SUMMARY AND DISCUSSION

As a summary of our results we should like to stress the following points.

(i) The exact finite-temperature effective action induced by massive fermions in arbitrary odd dimensions has the proper behavior under gauge transformations. Although finite-temperature calculations to fixed perturbative order necessarily violate gauge invariance, when all orders are taken into account the invariance is restored.

(ii) Using a certain class of gauge field configurations, the temperature-dependent parity-odd part of the effective action (i.e., the relevant part to investigate possible gauge-invariant violations at finite temperature) can be calculated *exactly in arbitrary odd dimensions*. The result is a gauge-invariant action which is not just a Chern-Simons term with a temperature-dependent coefficient but which reduces, in the low-temperature regime, to this product and confirms, at $T=0$, that massive fermions induce a CS action.

(iii) An exact calculation was possible because γ_0 in $2d+1$ dimensions can be always taken as the chiral γ_5 matrix in $2d$ dimensions so that the temperature-dependent part of the effective action could be decoupled through a γ_0 rotation with constant phase. As is well known, the resulting chiral Fujikawa Jacobian can be exactly computed and yields to the $2d$-dimensional chiral anomaly. This gives another example of the connection between CS terms in odd dimensions and even-dimensional topological invariants connected to chiral anomalies [18,19].

(iv) Although our result is obtained for a particular class of gauge field backgrounds (vanishing electric field and time-independent magnetic field in the Abelian case), similar to those considered in the pioneering works at zero temperature [22,23], there is no doubt that the same gauge-invariant answer should be confirmed for general gauge field configurations, using, for example, a ζ-function regularization analysis.

(v) Remarkably, the temperature dependence of the parity-odd effective action is the same irrespective of the number of space-time dimensions. This could be attributed to the particular background we considered but the topological nature of the result suggests that a similar result should hold in general. This is also sustained by the fact that the dependence on the $2d$-dimensional components of the gauge field background occurs through the axial anomaly, also a quantity of topological nature.

We would like to end this work by noting that the results we derived in a finite-temperature quantum field theory language could also be interpreted in terms of a compactified Euclidean theory in an odd number of dimensions, where the curled coordinate is not necessarily the Euclidean time, but it may be a compact dimension of length $L=\beta$. If this interpretation is adopted, and one takes only the lowest Kaluza-Klein modes for the parity conserving part of the effective action, one then has a $2d$ reduced theory, where the odd part of the effective action we evaluated plays the role of a θ-vacuum term (we assume, of course, that there is also a Yang-Mills action for the gauge field).

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