Effective field theory for a three-brane universe

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A general effective field theory formalism is presented which describes the low-energy dynamics of a 3-brane universe. In this scenario an arbitrary four-dimensional particle theory, such as the standard model, is constrained to live on the world volume of a (3+1)-dimensional hypersurface, or "3-brane," which in turn fluctuates in a higher-dimensional, gravitating spacetime. The inclusion of chiral fermions on the 3-brane is given careful treatment. The power counting needed to renormalize quantum amplitudes of the effective theory is also discussed. The effective theory has a finite domain of validity, restricting it to processes at low enough energies that the internal structure of the 3-brane cannot be resolved. [S0556-2821(99)00308-2]

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I. INTRODUCTION

How many dimensions do we live in? Macroscopically, we see and feel three spatial dimensions using electromagnetism. Furthermore, Newton's $1/r^2$ law of gravity follows from general relativistic principles in 3+1 dimensions. Microscopically, calculations based on (3+1)-dimensional spacetime are in excellent accord with our most sensitive experimental tests of the standard model (SM). Yet, it is well known that extra spatial dimensions are possible if they are compactified at sufficiently small radii. To resolve a compact dimension, it must be probed by quanta with wavelengths smaller than its radius. Presently, our sharpest SM probes have wavelengths as short as $\sim 10^{-16}$ cm. Apparently this provides the upper bound on the radii of any higher dimensions occurring in nature. But this conclusion is based on the assumption that *all* particle species move in the same number of dimensions as the SM. This assumption is implicit in the standard Kaluza-Klein approach to higher dimensions, which, until recently, played a central role in string theory.

Can non-SM particles see extra dimensions that the SM does not? At present, we know of only one non-SM state, the graviton, but others might exist if they are weakly coupled or massive enough to have escaped detection thus far. Let us suppose there are extra spatial dimensions in which gravity can propagate¹ but in which the SM cannot. We can think of the SM as being "stuck" at some definite position in the extra dimensions. That is, if the full "bulk" spacetime is really d-dimensional, d>4, then we are considering the SM to be confined to a (3+1)-dimensional hypersurface. Since the bulk spacetime contains gravity and is therefore dynamical, the hypersurface cannot be rigid, but must also be dynamical. We can borrow some string-theory parlance and call this dynamical hypersurface a "3-brane."² The basic idea that the standard particles are confined to a 3-brane in higher dimensions goes back at least to Ref. [1]. Very recently, it has been discussed in Refs. [2] and [3], as a possible means of addressing the unnatural hierarchy between the weak and Planck scales. These papers give an up-to-date analysis of some of the theoretical and phenomenological possibilities. Related ideas involving 3-brane universes and/or relatively low compactification scales appear in Refs. [4-14].

Let us now impose the constraint that Newton's $1/r^2$ Law, or more generally, four-dimensional general relativity, is experimentally verified at macroscopic distances. Therefore by these distances, gravity must also be confined to 3+1 dimensions, the most obvious way being by compactifying the extra dimensions. But now the compactification radii are not constrained to be smaller than 10^{-16} cm because SM particles are not direct probes of the higher dimensions. To resolve the higher dimensions we must use gravity which has only been tested down to a distance of a centimeter. The radii of the extra dimensions need only be smaller than this for us to not yet have observed them. An exciting possibility is that future short-distance tests of gravity may do so [2,15].

Even if the compactification radii are too small to be seen directly in the forseeable future, they can lead to interesting indirect effects. One intriguing possibility is provided by the work initiated in Ref. [7] in the context of M theory, literally a case of parallel universes. We can effectively have two 3-branes, separated in a (4+1)-dimensional bulk spacetime. A supersymmetric and gauge extension of the SM lives on one 3-brane, while a strongly-interacting supersymmetric *hidden sector* lives on the other. The hidden sector dynamics can trigger spontaneous supersymmetry breaking on its 3-brane, which is transmitted by bulk modes to the SM 3-brane, where it shows up as soft supersymmetry breaking. (For an interesting study of this phenomenon in a simple setting see Ref. [16].)

If we do live on a 3-brane, what is it made of? It has been understood for some time that a quantum field theory can contain topological defects of various types and dimensionality, which can have low-energy particle-like modes trapped on them. It therefore seems plausible that our 3-brane is a (3+1)-dimensional defect in a higher dimensional field theory, and the SM particles are some of the light modes trapped on the defect. This is the scenario advocated in Ref.

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¹Indeed since gravity is so intimately tied to spacetime, it would be hard to conceive of gravity *not* being present in all the extra dimensions.

²In string theory, "3-brane" has a more restricted usage.

[2]. While it is known how to build theories of this type where scalars and fermions live on the defect, it is still problematic to obtain low-energy four-dimensional vector (gauge) fields, although some new ideas are pursued in Refs. [8,2]. It is therefore fair to say that while the 3-brane scenario has reasonable support within quantum field theory, we are still some way from a realistic model.

The situation improves if we consider superstring theory, the direction taken in Refs. [3,13]. Intrinsic to the theory are Dirichlet-branes (D-branes), defects of varying dimensionality on which open strings can end. See Ref. [17] for a review. The short open string modes trapped on D-branes can include light gauge fields, fermions and scalars, all the basic ingredients for realistic theories. It is possible to construct a variety of sandwiches of D-branes and strings (reveiwed in Ref. [18]), which reduce at low energies to interesting particle theories effectively living in four dimensions, weakly coupled to gravity and other modes which propagate in higher dimensions.³ However, a realistic SM sector has not yet been engineered in this way.

The purpose of the present paper is to show how one can construct realistic *effective field theories* to study the lowenergy consequences of the 3-brane scenario, in a systematic, economical and elegant way. The only degrees of freedom that appear are those that matter at low energies, the purely high-energy degrees of freedom are considered to be integrated out. Even if a fundamental description were known, the most efficient way of pursuing the low-energy dynamics would be to *match* the fundamental theory to such an effective field theory, and then use the latter for calculations and insight.

Roughly speaking, the low-energy domain restricts us to processes which cannot resolve the stucture of the 3-brane. In many ways this approach is analogous to the chiral Lagrangian approach to pion dynamics, where the pion is treated as a point-particle whose internal quark-glue structure is outside the low-energy domain of validity. We will simply assume that there is some high-energy physics, perhaps string theory or an exotic field theory, which gives rise to a 3-brane moving inside a bulk *d*-dimensional spacetime. It is assumed that the SM, or some extension of it (which for convenience we will continue to call the "SM"), is constrained to propagate within the 3-brane world volume, and that gravity, and perhaps some other degrees of freedom, are free to propagate in the bulk. We then construct the most general consistent effective field theory describing the couplings between bulk gravity, the 3-brane fluctuations, and the SM particles. By this means we can outpace the fundamental theorists who must first tackle how a realistic 3-brane arises.

The organization of the paper is as follows. Section II sets up some basic notation. Section III describes the couplings of the purely bosonic degrees of freedom. The result of this section may appear rather obvious to anyone familiar with the literature on D-branes, and no great originality is claimed. Section IV develops the formalism needed to include chiral fermions, such as quarks and leptons, on the 3-brane. Section V explains how to gauge-fix reparametrization invariance of the 3-brane description. Section VI discusses the power counting needed to implement renormalization in the effective field theory program. Section VII discusses the sense in which the effective field theory is really a ''gauged chiral Lagrangian'' corresponding to the spontaneous breaking of higher-dimensional spacetime symmetries by the 3-brane ground state. In particular, our treatment is an adaptation of Volkov's general formalism for treating spontaneous breaking of spacetime symmetries [19]. Section VIII provides the conclusions.

The present formalism is not explicitly supersymmetric, but it is hoped that the extension to supersymmetry can be accomplished by methods similar to those of Sec. IV. It is expected that this will tie in closely with earlier work on low-energy effective theories describing the spontaneous (partial) breaking of (higher) supersymmetry. See for example Refs. [20] and [21]. Reference [21] gives a more complete list of references on this topic.

The present paper (in particular Sec. VI) assumes an aquaintance with the methodology of effective field theory. Reference [22] provides a good introduction to the basic concepts and techniques, in the relatively simple context of pion physics. Reference [23] describes how to interpret general relativity as a quantum effective field theory.

II. PRELIMINARIES

A. Fields, coordinates, and related notation

We are interested in four-dimensional SM fields living on the world-volume of a 3-brane, which in turn is free to move in a gravitating bulk spacetime of dimension, d>4. The fields we consider will be the minimal set needed to realize this scenario. (Adding non-minimal fields poses no extra problem.) For simplicity we take the 3-brane world-volume topology to be \mathbf{R}^4 , and the bulk topology to be either \mathbf{R}^d or $\mathbf{R}^4 \times \mathbf{T}^{d-4}$, where \mathbf{T}^k denotes a *k*-torus.

The coordinates of the bulk spacetime will be denoted X^M . The bulk coordinate indices are capital letters from the middle of the Roman alphabet, $M, N, \ldots = 0, \ldots, d-1$. We reserve the lower-case letters from the middle of the Roman alphabet to refer to just the last d-4 of these indices, $m, n, \ldots = 4, \ldots, d-1$. If one is only interested in bosonic fields (Sec. III), the components of the bulk metric, $G_{MN}(X)$, can be considered as the fundamental gravitational degrees of freedom. Otherwise, the gravitational degrees of freedom in the bulk are the components of the *d*-bein, $E^A_M(X)$ (that is, the *d*-dimensional vielbein). The *local Lorentz* indices in the bulk are capital letters from the beginning of the Roman alphabet, $A, B, \ldots = 0, \ldots, d-1$. The *d*-bein is related to the bulk metric, $G_{MN}(X)$, by

$$E_{M}^{A}(X) \eta_{AB} E_{N}^{B}(X) = G_{MN}(X),$$

$$E_{M}^{A}(X) G^{MN}(X) E_{N}^{B}(X) = \eta^{AB},$$
 (1)

where η_{AB} is the *d*-dimensional Minkowski metric. It is useful to subdivide the local Lorentz indices into two subsets:

³The way effective 3-branes arise in the scenario of Ref. [7], mentioned above, is more subtle. See Ref. [11] for a discussion.

the first four denoted by letters from the beginning of the Greek alphabet, $\alpha, \beta, \ldots = 0, \ldots 3$, while the remaining indices are denoted by lower-case letters from the beginning of the Roman alphabet, $a, b, \ldots = 4, \ldots, d-1$.

The coordinates intrinsic to the 3-brane will be denoted x^{μ} . The 3-brane coordinate indices are chosen from the middle of the Greek alphabet, $\mu, \nu, \ldots = 0, \ldots, 3$. The *bulk* coordinates describing the position occupied by a point *x* on the 3-brane, are denoted $Y^M(x)$. They are dynamical fields.

The last fields required are those of the SM, which are all functions of x, since they live only on the 3-brane. They come in three types: scalar fields, vector gauge fields, and left-handed Weyl spinors, denoted $\phi(x), A_{\mu}(x), \psi_L(x)$ respectively. Any right-handed spinor fields can be made left-handed by charge conjugation in the usual manner. Spinor and internal indices are suppressed because it is entirely straightforward to replace them whenever desired.

B. The "vacuum" state

The effective field theory will describe the smallamplitude, long-wavelength fluctuations of the dynamical fields about the following state:

$$E_{M}^{A}(X) = \delta_{M}^{A}, \quad G_{MN}(X) = \eta_{MN};$$

$$Y^{M}(x) = \delta_{\mu}^{M} x^{\mu},$$

$$\phi(x) = v.$$
(2)

That is, we expand about a Minkowski bulk spacetime, with the 3-brane occupying the subspace spanned by the four X^{μ} -axes, and with the intrinsic 3-brane coordinates, x^{μ} , agreeing with the bulk coordinates, X^{μ} . We also allow some of the scalar fields on the 3-brane to be non-zero, but constant over the 3-brane.

III. THE BOSONIC EFFECTIVE FIELD THEORY

This section describes the construction of the effective field theory when fermionic fields are absent. The procedure for adding fermions is treated in the next section.

A. Effective theory of gravity in the bulk

In isolation, the bulk gravitational fields are described by an action

$$S_{bulk} = \int d^d X \det(E) \{ -\Lambda + 2M^{d-2}R + \cdots \}, \qquad (3)$$

where the *d*-dimensional Einstein-Hilbert action has been explicitly written, with *M* being the *d*-dimensional Planck mass and *R* the *d*-dimensional curvature scalar, Λ is a *d*-dimensional cosmological constant term, and the ellipsis is the series of higher dimensional geometric invariants with coefficients given by powers of 1/M, multiplied by order one (or less) dimensionless couplings. The effective field theory philosophy and technology for using this non-renormalizable action is essentially the same as for the usual d=4 case, which has been discussed in detail in Ref. [23].

If we are considering the *d*-dimensional spacetime to be of the form $\mathbf{R}^4 \times \mathbf{T}^{d-4}$, then gravity becomes effectively four-dimensional at distances larger than the radii of the (d-4)-torus, with an effective Planck constant, M_{Pl} , given by $M_{Pl}^2 = M^{d-2}V_{\mathbf{T}}$, where $V_{\mathbf{T}}$ is the volume of the (d-4)-torus. See Refs. [2,3] for further discussion.

B. The induced metric on the 3-brane

The distance between two infinitesimally separated points on the 3-brane, x and x+dx, is given by

$$ds^{2} = G_{MN}(Y(x))dY^{M}dY^{N}$$
$$= G_{MN}(Y(x))\frac{\partial Y^{M}}{\partial x^{\mu}}dx^{\mu}\frac{\partial Y^{N}}{\partial x^{\nu}}dx^{\nu}, \qquad (4)$$

from which we deduce that the induced metric on the 3-brane is

$$g_{\mu\nu}(x) = G_{MN}(Y(x))\partial_{\mu}Y^{M}\partial_{\nu}Y^{N}.$$
 (5)

Given Eq. (2), it is clear that $g_{\mu\nu}(x)$ will consist of small fluctuations about the four-dimensional Minkowski metric, $\eta_{\mu\nu}$.

C. Effective field theory associated to the 3-brane

Let us write the most general action for the bosonic fields associated to the 3-brane in the background of the bulk metric, $G_{MN}(X)$. The action must be invariant under general X-coordinate transformations as well as x-coordinate transformations. The requirement of the first of these invariances is clear since it is the "gauge invariance" for the *d*-dimensional general relativistic bulk gravity. The requirement of x-coordinate invariance follows because the x-space is completely unphysical, just providing a convenient means of parametrizing the 3-brane embedding, Y(x).

The book-keeping to enforce these two invariances is straightforward. We first have to determine how our fields transform under the two types of coordinate transformations. $G_{MN}(X)$ is an X-space 2-tensor and x-scalar. The action cannot depend directly on all of Y(x) because it makes reference to the origin of X-coordinate space, which is unphysical (the usual statement that coordinates are not themselves generally-covariant tensors), but the action can depend on $\partial_{\mu}Y^{M}$, which is an X-vector and x-vector. $\phi(x)$ is obviously a scalar of both spaces. $A_{\mu}(x)$ is an x-vector and an X-scalar.⁴ An important composite field is the induced 3-brane metric, $g_{\mu\nu}(x)$. From Eq. (5), it is an X-scalar and an

⁴This point deserves some explanation. The gauge field $A_{\mu}(x)$ describes the parallel transport between two infinitesmally separated points on the 3-brane, x and x + dx. The parallel transport is as usual given by $1 + iA_{\mu}(x)dx^{\mu}$. Under an x-coordinate transformation, dx^{μ} transforms covariantly, and so A_{μ} must be taken to transform as a contravariant vector. But under an X-coordinate transformation nothing happens to the parallel transport, and so A_{μ} must be a scalar.

x-tensor. Using $g_{\mu\nu}$ and ∂_{μ} we can construct covariant *x*-derivatives in the standard way, and apply them to the various tensors already discussed in order to generate further tensors. Invariants can then be formed by contracting *x*-tensor indices using $g_{\mu\nu}$ and its inverse, $g^{\mu\nu}$.

From these ingredients we can build the action,

$$S_{bosons}^{3\text{-}brane} = \int d^{4}x \sqrt{-g} \left\{ -f^{4} + \frac{g^{\mu\nu}}{2} D_{\mu}\phi D_{\nu}\phi - V(\phi) - \frac{g^{\mu\nu}g^{\rho\sigma}}{4} F_{\mu\rho}F_{\nu\sigma} + \dots \right\},$$
(6)

where $F_{\mu\nu}(x)$ is the usual gauge field strength, and the ellipsis includes higher-dimensional invariants one can build out of the fields and covariant derivatives, with dimensionful coefficients set by powers of 1/f or 1/M. (See Sec. VI.) Note that by locality, when G_{MN} or its derivatives appear in this action they must be evaluated on the 3-brane, at Y(x). The dominant terms, explicitly displayed in Eq. (6), depend on G_{MN} only through $g_{\mu\nu}$ and Eq. (5), but higher invariants [in the ellipsis of Eq. (6)] can certainly depend on G_{MN} in a more general manner.

The leading term of Eq. (6) is a "bare" tension for the 3-brane, the mass scale f being determined by the physics that gave rise to the 3-brane. This term can be renormalized at tree- and loop-level by the vacuum energy of the SM. The *renormalized* 3-brane tension dominates the interactions of bulk gravity with the 3-brane when it is close to its ground state. This term also contains kinetic energy for the *Y* fields as will be discussed in Sec. VI.

IV. FERMIONS ON THE 3-BRANE

A. The problem

Recall how spin-1/2 fermions are ordinarily introduced in a four-dimensional general relativistic context. (See Ref. [24] for more details.) The fermions, $\psi(x)$, are regarded as *x*-scalars, but as spinors of the local Lorentz group. The Lorentz generators in the spinor representation are as usual given by, $\sigma_{(\alpha\beta)} \equiv \frac{1}{4} \gamma_{[\alpha} \gamma_{\beta]}$, where the γ_{α} are Dirac matrices satisfying

$$\{\gamma_{\alpha},\gamma_{\beta}\}=2\,\eta_{\alpha\beta}.\tag{7}$$

The local Lorentz group can formally be thought of as an *internal SO*(3,1) gauge group. It gets related to spacetime through the vierbein, e^{α}_{μ} , an *x*-vector and local Lorentz vector which satisfies

$$e^{\alpha}_{\mu}(x) \eta_{\alpha\beta} e^{\beta}_{\nu}(x) = g_{\mu\nu}(x), \qquad (8)$$

$$e^{\alpha}_{\mu}(x)g^{\mu\nu}(x)e^{\beta}_{\nu}(x) = \eta^{\alpha\beta}.$$
(9)

A covariant derivative for ψ with respect to local Lorentz transformations can be constructed in terms of the vierbein

$$\omega_{\mu}^{\alpha\beta} = \frac{1}{2} g^{\rho\nu} e^{[\alpha}{}_{\rho} \partial_{[\mu} e^{\beta]}{}_{\nu]} + \frac{1}{4} g^{\rho\nu} g^{\tau\sigma} e^{[\alpha}{}_{\rho} e^{\beta]}{}_{\tau} \partial_{[\sigma} e^{\gamma}{}_{\nu]} e^{\delta}{}_{\mu} \eta_{\gamma\delta}.$$
(11)

An *x*-coordinate invariant and local Lorentz invariant action then follows:

$$S_{fermion} = \int d^4x \sqrt{-g} \{ \bar{\psi} i e^{\mu}_{\alpha} \gamma^{\alpha} D_{\mu} \psi + \cdots \}, \qquad (12)$$

where e^{μ}_{α} is the inverse of the vierbein, obtained from e^{α}_{μ} by using $g^{\mu\nu}$ to raise the *x*-coordinate index and $\eta_{\alpha\beta}$ to lower the local Lorentz index.

It therefore appears that incorporating fermions on the 3-brane will require deriving the vierbein induced from the bulk *d*-bein and the 3-brane embedding. Indeed this is generally the case, and, as will be seen below, the result is considerably more complicated than for the induced metric, Eq. (5). However, before embarking on this exercise it is enlightening to see why we cannot generally get away with a simpler approach.

We can consider the four-dimensional local Lorentz group at x on the 3-brane to be the subgroup of the d-dimensional local Lorentz group at Y(x) which acts nontrivially on the four-dimensional hyperplane tangent to the 3-brane (spanned by $\partial_{\mu}Y^{M}E_{M}^{A}$). It follows that we get fourdimensional local Lorentz invariance by demanding d-dimensional local Lorentz invariance. Just as in four dimensions, we can construct d-dimensional gauge fields for local Lorentz invariance, $\Omega_{M}^{AB}(X)$, in terms of the d-bein. The formula is exactly analogous to Eq. (11). From this we get an induced gauge field and covariant derivative on the 3-brane,

$$D_{\mu} \equiv \partial_{\mu} Y^{M} \Omega_{M}^{AB}(Y(x)),$$
$$D_{\mu} \equiv \partial_{\mu} + \frac{1}{2} \omega_{\mu}^{AB} \Sigma_{(AB)}, \qquad (13)$$

where $\Sigma_{(AB)} \equiv \frac{1}{4} \Gamma_{[A} \Gamma_{B]}$ are the *d*-dimensional Lorentz generators in spinor representation, and Γ_A are *d*-dimensional Dirac matrices satisfying,

$$\{\Gamma_A, \Gamma_B\} = 2 \eta_{AB}. \tag{14}$$

We can therefore write an *x*-coordinate invariant and *d*-dimensional local Lorentz invariant action,

$$S_{fermion} = \int d^4x \sqrt{-g} \{ g^{\mu\nu} \partial_{\nu} Y^M E^A_M \overline{\psi} i \Gamma_A D_{\mu} \psi + \dots \}.$$
(15)

Although Eq. (15) is a consistent means of introducing fermions onto the 3-brane, it is not the most general way, and in particular does not give rise to four-dimensional chiral fermions. The reason is that even if the ψ field appearing in Eq. (15) is in an irreducible (perhaps chiral) spinor representation of the *d*-dimensional local Lorentz group, it always corresponds to a *reducible* spinor representation of the four-dimensional Lorentz subgroup. This reducible representation

contains equal numbers of left- and right-handed Weyl doublets, as is familiar from dimensional reduction in Kaluza-Klein theory.

In superstring theory, fermions in the above reducible representation naturally arise in the *simplest* D-brane configurations. They are some of the massless modes of the open string that can attach to the D-brane. They are related by supersymmetry to massless vector fields on the D-brane, and so become gauginos of the (highly-supersymmetric) lowenergy gauge theory that lives on the D-brane.

Clearly, in order to include two-component SM chiral fermions on the 3-brane we must adopt a different procedure. In fact we must explicitly determine an induced vierbein on the 3-brane, as mentioned above. Then we can take our action to be given by

$$S_{chiral-fermion}^{3-brane} = \int d^4x \sqrt{-g} \{ \bar{\psi}_L i e^{\mu}_{\alpha} \sigma^{\alpha} D_{\mu} \psi_L + y \phi \psi_L \psi_L + \text{H.c.} + \cdots \}, \qquad (16)$$

where the σ^{α} are the usual 2×2 chiral Dirac matrices for four-dimensional Minkowski space. The covariant derivative now contains the local Lorentz gauge fields as in Eqs. (10), (11) as well as gauge fields for internal gauge groups. Yukawa couplings to scalars are also included. The ellipsis contains higher dimension interactions that can be constructed with the help of the vierbein and covariant derivatives. Of course we have the usual requirement of cancellation of chiral gauge anomalies in order for our effective theory to make sense at the quantum level.

B. The induced vierbein

The vierbein can conveniently be thought of as a means of finding the components of the *x*-space differential, dx^{μ} , in (four-dimensional) local Lorentz coordinates, the result being just $e^{\alpha}_{\mu}dx^{\mu}$. Our strategy for obtaining the vierbein from the 3-brane embedding is as follows. At each point *x*, we will lift dx^{μ} to the corresponding infinitesimal *X*-space vector tangent to the 3-brane,

$$dY^M = \partial_\mu Y^M dx^\mu. \tag{17}$$

Then we will perform a local *d*-dimensional Lorentz transformation at Y(x), mapping the tangent hyperplane to lie in the $\alpha = 0, ..., 3$ directions. In particular, dY will be mapped to an infinitesimal vector with non-zero components only in the α directions, $e^{\alpha}_{\mu}dx^{\mu}$. The e^{α}_{μ} so obtained will be proven to form a vierbein. This approach has similarities with the constructions of Refs. [9,20,21]. For a different approach to chiral fermions on branes see Ref. [4].

The requisite Lorentz transformation, *R*, is determined as follows. Among the *d*-dimensional Lorentz generators (in the vector representation), $J^{(AB)}$, the subgroup generated by the $J^{(\alpha\beta)}$ and the $J^{(ab)}$ leaves invariant the subspace spanned by the α directions. We will drop these generators and consider *R* to be of the form

$$R(x) = \exp(i\theta_{\alpha a}(x)J^{(\alpha a)}).$$
(18)

The condition that the tangent hyperplane is mapped to lie in the α directions is equivalent to requiring the Lorentztransformed tangent vectors to be orthogonal to the *a* directions. That is,

$$R^a_B E^B_M(Y) \partial_\mu Y^M = 0, \text{ for all } a, \mu.$$
⁽¹⁹⁾

The *d*-bein has been used here to express $\partial_{\mu} Y$ in local Lorentz coordinates.

Equations (18) and (19) uniquely determine *R*, since they correspond to $4 \times (d-4)$ equations for $4 \times (d-4)$ unknowns, $\theta_{\alpha a}(x)$, and we are expanding about field values Eq. (2) for which there is a unique solution, $\theta_{\alpha a} = 0$. Equation (19) can be solved perturbatively to any desired order in the small fluctuations about Eq. (2). The precise algorithm for doing this is described in the Appendix.

The vierbein is then given by

$$e^{\alpha}_{\mu} \equiv R^{\alpha}_{A} E^{A}_{M}(Y) \partial_{\mu} Y^{M}.$$
⁽²⁰⁾

Let us prove that this indeed satisfies the properties of a vierbein, Eqs. (8), (9). Equation (20) implies

$$e^{\alpha}_{\mu}\eta_{\alpha\beta}e^{\beta}_{\nu} = R^{\alpha}_{A}E^{A}_{M}\partial_{\mu}Y^{M}\eta_{\alpha\beta}R^{\beta}_{B}E^{B}_{N}\partial_{\nu}Y^{N}.$$
 (21)

Now, by Eq. (19) we can replace the sums over $\alpha, \beta = 0, ..., 3$ on the right-hand side by sums from 0 to d-1, since the sums from 4 to d-1 add nothing. Therefore,

$$e^{\alpha}_{\mu}\eta_{\alpha\beta}e^{\beta}_{\nu} = R^{E}_{A}E^{A}_{M}\partial_{\mu}Y^{M}\eta_{EF}R^{F}_{B}E^{B}_{N}\partial_{\nu}Y^{N}.$$
 (22)

The fact that R is a d-dimensional Lorentz transformation implies that

$$R_A^E \eta_{EF} R_B^F = \eta^{AB}. \tag{23}$$

So Eq. (22) simplifies to

$$e^{\alpha}_{\mu}\eta_{\alpha\beta}e^{\beta}_{\nu} = E^{A}_{M}\eta_{AB}E^{B}_{N}\partial_{\mu}Y^{M}\partial_{\nu}Y^{N}$$
$$= G_{MN}\partial_{\mu}Y^{M}\partial_{\nu}Y^{N}$$
$$= g_{\mu\nu}, \qquad (24)$$

where the second equality follows from Eq. (1) and the third equality from Eq. (5). Thus Eq. (8) holds. Regarding Eq. (24) in matrix notation,

$$e^T \eta e = g, \qquad (25)$$

we can invert both sides, and then pre-multiply by e and post-multiply by e^{T} to get Eq. (9).

We have found an induced vierbein which we can use to construct the action for chiral fermions on the 3-brane, according to Eq. (16). The full action of our effective field theory is the sum of Eqs. (3), (6), (16).

V. GAUGE FIXING THE REPARAMETRIZATION INVARIANCE

Our formalism up to this point has been explicitly invariant under general *x*-coordinate transformations. This corresponds to a large reparametrization invariance in our description of the 3-brane. If $Y^M(x)$ describes a 3-brane configuration and if x'(x) is a general *x*-coordinate transformation, then $Y^M(x'(x))$ describes an identical 3-brane configuration.

Fortunately, it is straightforward to eliminate this redundancy, leaving us with only the physical number of 3-brane degrees of freedom. This is done by imposing the gauge condition,

$$Y^{\mu}(x) - x^{\mu} = 0, \qquad (26)$$

while the d-4 fields, $Y^m(x)$, are physical and can fluctuate. For small fluctuations about Eq. (2), Eq. (26) can always be solved. Note that this is a complete gauge-fixing because if Y satisfies Eq. (26), then

$$Y^{\mu}(x'(x)) - x^{\mu} = 0$$
 if and only if $x'(x) = x$. (27)

In the quantum functional integral we need only integrate over Y(x) which satisfy Eq. (26). Furthermore, this gauge condition has a trivial ghost determinant, since if Y satisfies Eq. (26) and $x'(x) = x + \xi(x)$ is an *infinitesimal* coordinate transformation,

$$\frac{\delta}{\delta\xi^{\nu}(y)}\bigg|_{\xi=0} [Y^{\mu}(x'(x)) - x^{\mu}] = \partial_{\nu}Y^{\mu}(x)\,\delta^{4}(y-x)$$
$$= \delta^{\mu}_{\nu}\delta^{4}(y-x), \qquad (28)$$

which is field independent.

VI. POWER-COUNTING, CANONICAL FIELDS, AND RENORMALIZATION

The effective field theory construction described in the preceding sections admits and contains various types of nonrenormalizable interactions. We need to determine the power-counting dimension, and thereby the relevance, of these interactions, by writing our theory in terms of canonically normalized fields.

The gravitational fields can be decomposed as usual as [24]

$$E_{M}^{A}(X) = \delta_{M}^{A} + \frac{H_{M}^{A}(X)}{M^{d/2-1}},$$

$$G_{MN}(X) = \eta_{MN} + \frac{H_{MN} + H_{NM}}{M^{d/2-1}} + \frac{H_{ML}H_{N}^{L}}{M^{d-2}},$$
(29)

where $H_M^A(X)$ is the canonical graviton field, and its indices have been raised and lowered using the Minkowski metric. We must also canonically normalize the 3-brane coordinate fields, $Y^m(x)$ [where we are assuming that we have eliminated the reparametrization invariance according to Eq. (26)]. We see that a kinetic term quadratic in Y^m results from the expansion of the leading term of Eq. (6) in powers of (derivatives of) Y^m . A canonically normalized set of fields, $\mathcal{Z}^m(x)$, can be introduced by writing

$$Y^{m}(x) \equiv \frac{\mathcal{Z}^{m}(x)}{f^{2}}.$$
(30)

From Eqs. (5), (6), (26) we see that the dominant couplings of a bulk graviton to the SM (without exciting the \mathcal{Z}^m field) are given by

$$S_{graviton-SM} = \int d^4x T_{SM}^{\mu\nu}(x) \frac{H_{\mu\nu}(x, X^m = 0)}{M^{d/2 - 1}}, \quad (31)$$

where the SM stress tensor is defined in the usual fourdimensional way,

$$T_{SM}^{\mu\nu}(x) \equiv \frac{\delta S_{3\text{-}brane}}{\delta g_{\mu\nu}(x)} \bigg|_{g_{\mu\nu} = \eta_{\mu\nu}}.$$
(32)

This is a special case of our general expectation that a *d*-dimensional graviton, H_{MN} , should couple to the *d*-dimensional matter stress tensor, T^{MN} . When the matter is restricted to move on a 3-brane as in the case of the SM, the only non-vanishing components of T^{MN} are the $T^{\mu\nu}$, localized on the 3-brane.

It is a troublesome but necessary feature of the presence of fermions in the effective field theory [via Eq. (16)], that dependence on the gravitons and Y^m is implicit in the $\theta_{\alpha a}$ angles that determine *R* and the vierbein through Eqs. (18), (19), (20). To determine the interaction vertices in Eq. (16) we have to determine these angles from Eq. (19) perturbatively in powers of (derivatives of) *H* and *Y*, as described in the Appendix. Fortunately, for any process, computed to some fixed loop order, only vertices with a limited number of *H* and *Y* will contribute.

We now consider the structure of our effective field theory for the three possible cases, (i) $f \sim M$, (ii) $f \ll M$, (iii) $f \gg M$.

(i) $f \sim M$: For power-counting purposes we can take all higher-dimension interactions involving canonical fields to be of order a power of 1/M, given by dimensional analysis. In order to make sense of this non-renormalizable theory with an infinite number of possible terms in the ellipses of Eqs. (3), (6), (16) we must restrict its domain of validity to momenta and field fluctuations [away from Eq. (2)] much smaller than M. For processes outside this domain, we require a more fundamental description of quantum gravity and the physics that gave rise to the 3-brane. The effective field theory procedure in the domain of validity is to work to some fixed but arbitrary order in 1/M, say $\mathcal{O}(1/M^k)$, balanced by powers of fields and momenta for the process under consideration. We then throw away all terms in our effective Lagrangian of higher order, leaving only a finite number of interactions.

Now, if we only wish to do *classical* field theory, we can simply use the truncated effective Lagrangian. In the modern effective field theory view, this is precisely the sense in which ordinary classical general relativity, using only the Einstein-Hilbert action, is a valid approximation. However we can also do *quantum* effective field theory. In computing Feynman diagrams we will encounter local ultraviolet divergences which are formally of higher order than $1/M^k$. We can simply throw them away. The remaining divergences (finite in number) will correspond precisely to the (counter) terms we have retained in our effective Lagrangian, so renormalization can proceed in the usual way.

(ii) $f \leq M$: For power-counting purposes, Eq. (30) suggests that the strength of non-renormalizable interactions involving only canonical 3-brane fields should be taken of order a power of 1/f, the scale fixed by the physics which gave rise to the 3-brane. Equation (29) then suggests that interactions involving extra gravitons, H, are further suppressed by powers of 1/M. Now our effective field theory is valid for momenta and field fluctuations much smaller than f. For processes beyond this domain, we require a more fundamental description of the physics that gave rise to the 3-brane. However, this may not require a more fundamental description of quantum gravity, the present general relativistic description continuing to make sense for momenta all the way up to M.

The effective field theory procedure is now to do a double expansion. We must work to some fixed but arbitrary order in 1/f, balanced by powers of 3-brane field fluctuations and momenta for the process under consideration, and to some fixed order in 1/M, balanced by powers of the graviton field and momenta. For example, if f/M is small enough it may be a good approximation to work to some non-trivial order in 1/f, but to zeroth order in 1/M. In this approximation we are simply neglecting bulk gravity altogether, as we frequently do in SM applications, but we are retaining the 3-brane fluctations in the flat bulk spacetime. Once again, to any order in the double expansion, renormalization proceeds in the usual manner once the effective Lagrangian and ultraviolet divergences are truncated to the finite number that are within the order to which we are working.

(iii) $f \ge M$: This is the case of a "large" 3-brane tension. In this case it is quite unnatural to expect that the higherdimension interactions involving 3-brane fields are suppressed by powers of 1/f, even if the interactions contain no explicit gravitons. The reason is that gravity couples to everything and gravitational loops will dress all possible interactions. We can therefore expect that any higher-dimension interaction will naturally be of order powers of 1/M, unless protected by X-coordinate or x-coordinate invariance. That is, for power counting purposes, we should first write our effective Lagrangian in terms of X-coordinate and x-coordinate invariants as we have in Eqs. (3), (6), (16). Then the naive coefficients of the various higher-dimension invariants should be given by powers of M determined by dimensional analysis. The right powers of 1/f will then emerge when the effective Lagrangian is expanded in terms of the canonical fields.

While in principle any ultraviolet regulator can be used to regulate the Feynman diagrams of the effective theory, it is of course preferable to use a regularization that respects as many of the symmetries of the theory as possible. The simplest procedure appears to be dimensional regularization where one analytically continues the dimensionality of both the 3-brane as well as that of the bulk spacetime. As is always the case, this regularization does not respect chiral gauge invariance when fermions are present on the 3-brane, but this nuisance is no more severe than in ordinary fourdimensional field theories.

VII. THE FORMALISM OF SPONTANEOUS SYMMETRY BREAKING

The effective field theory developed above is a particular case of the chiral Lagrangian approach to spontaneous symmetry-breaking, and it is quite useful to understand the deep sense in which this is so. Let us recall the broad essence of this method. The chiral Lagrangian is a low-energy theory for the Nambu-Goldstone modes associated to spontaneously broken symmetries. If the full group of dynamical symmetries is G, and the vacuum spontaneously breaks this down to a subgroup H, then the Nambu-Goldstone modes transform under all of G, but the transformations outside of H are realized non-linearly. The chiral Lagrangian dynamics is tightly constrained to respect the full G symmetry. If there are other low-energy fields which are not Nambu-Goldstone modes, but which transform linearly under H, they are to be included in the chiral Lagrangian, and coupled to the Nambu-Goldstone modes so that the full G-invariance is respected. Although initially G is taken to be a *global* symmetry group, it can subsequently be weakly gauged in a straightforward manner at the level of the chiral Lagrangian. The beauty of this method is that it separates the question of what the low-energy consequences of spontaneous symmetry-breaking are from the (frequently more difficult) question of what the dynamical mechanism for the spontaneous symmetry-breaking is.

In general, there are two types of symmetry that can be spontaneously broken, the familiar case of internal symmetries and the less familiar case of spacetime symmetries. The general formalism for constructing the chiral Lagrangian in the former case was worked out in Ref. [25], while for the latter case the formalism was provided in Ref. [19]. In this paper, spacetime symmetry is spontaneously broken, and this symmetry is "weakly gauged" by gravity. Let us begin by turning off gravity, leaving d-dimensional Minkowski spacetime. Formally, we set $E_M^A = \delta_M^A$. From Eq. (2) we see that the 3-brane vacuum spontaneously breaks the d-dimensional Poincare symmetry by picking out a four-dimensional hyperplane to occupy. Specifically, this breaks the translations transverse to the 3-brane, generated by P_m , and the Lorentz transformations that change the orientation of the 3-brane, generated by $J^{\alpha a}$. The corresponding Nambu-Goldstone modes are the $Y^m(x)$ and $\theta_{\alpha a}(x)$ (see Sec. IV B) respectively. Using these modes, our effective theory, given by Eq. (6) plus Eq. (16), is invariant under the full d-dimensional Poincare symmetry, the four-dimensional Poincare subgroup being linearly realized and the remaining symmetry transformations being non-linearly realized. It is quite remarkable that the dynamics of this purely four-dimensional theory can respect d-dimensional Poincare invariance. The magic comes from the special couplings to the Nambu-Goto modes. In this non-gravitational limit our effective theory is essentially an adaptation of the general formalism of Ref. [19]

Note that the $\theta_{\alpha a}$ are not independent degrees of freedom

from the Y^m . This is a peculiarity of spacetime symmetry breaking and can be traced back to the fact that both translations and Lorentz transformations share the same conserved current, the *d*-dimensional energy-momentum tensor, whereas in the case of internal symmetries, each generator has its own conserved current. In particular, the fact that the effective theory is invariant under the full *d*-dimensional Poincare group implies that in addition to the usual fourdimensional energy-momentum tensor we have extra conserved currents, $T^{\mu m}(x)$,

$$\partial_{\mu}T^{\mu m} = 0. \tag{33}$$

Finally, we can turn gravity back on, thinking of it as weakly gauging the spontaneously broken d-dimensional Poincare group. We now become aware of another peculiarity of the spacetime symmetry breaking. Since the P_m are among the broken generators, in principle they do not provide good quantum numbers for labelling the bulk quanta, in the present case just the gravitons. Of course, far away from the 3-brane it does seem sensible to label bulk quanta by their *d*-dimensional momenta, but the interactions arising from our effective field theory between these quanta and the 3-brane quanta violate momentum conservation in the m $=4, \ldots, d-1$ directions. This is not hard to understand intuitively. When the bulk quanta are very soft compared to f, the scale setting the 3-brane tension, the 3-brane vacuum state appears approximately as a rigid wall, extending infinitely in the \vec{x} -directions. The bulk quanta can lose(gain) transverse momentum to(from) this infinitely massive wall. On closer inspection we see that the wall is not perfectly rigid, so that impacts from bulk quanta can create distortions in the wall, parametrized by $Y^{m}(x)$, which can propagate along the wall and can also excite the SM modes. More technically, for very soft momenta we can neglect the Y^m modes since they are derivatively coupled and therefore difficult to excite in the infrared. In this limit, the dominant coupling of the SM to bulk gravity is given by Eq. (31). The localization of the bulk graviton to $X^m = 0$ in coordinate space implies in momentum space that the SM model states couple equally to gravitons with any value of momentum transverse to the 3-brane, P_m . This breakdown of global P_m conservation remains true even when we take into account the presence of the Y^m modes, although we now have a local version of transverse momentum conservation, namely conservation of the momentum "current," Eq. (33). Note however that four-dimensional momenta are well-defined global charges which are conserved in all processes, since they correspond to unbroken symmetry generators.

VIII. CONCLUSIONS

This paper has described the minimal effective field theory formalism needed to explore the low-energy implications of a 3-brane universe. It appears relatively straightforward to generalize the present framework in several directions, for example, adding non-minimal bulk fields, making the formalism supersymmetric, considering more complicated spacetime topologies, or considering more than one brane embedded in the bulk spacetime.

The effective field theory formalism may help address the questions pursued in Ref. [16], regarding the transmission of supersymmetry breaking between branes in the scenario of Ref. [7]. Even the supersymmetry-breaking mechanism need not be explicitly described, since its consequences can be pursued via the chiral Lagangian approach to supersymmetry breaking initiated in Ref. [26].

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APPENDIX

Here we derive a procedure for determining the $\theta_{\alpha a}(x)$ that appear implicitly [via Eqs. (18), (19), (20)] in the fermion action, Eq. (16), in powers of the graviton and 3-brane fluctuations. In the vector representation of the *d*-dimensional Lorentz group, SO(d-1,1), we can write the generators, $J^{(AB)}, A < B$, in the explicit form

$$J_D^{(AB)C} = i \,\eta^{CE} (\,\delta_E^A \delta_D^B - \delta_D^A \delta_E^B). \tag{A1}$$

Let us define a set of small quantities in which we can perturb, by writing,

$$E_{M}^{B}(Y)\partial_{\mu}Y^{M} \equiv \delta_{\mu}^{B} - \epsilon_{\mu}^{B}.$$
 (A2)

Thus ϵ contains the small fluctuations around Eq. (2). We will do perturbation theory by formally expanding in powers of ϵ ,

$$\theta_{\alpha a} = \sum_{n=0}^{\infty} \theta_{\alpha a}^{(n)}$$
$$R \equiv e^{i\theta_{\alpha a}J^{(\alpha a)}} = \sum_{n=0}^{\infty} R^{(n)}.$$
(A3)

Substituting Eq. (A2) into Eq. (19) gives

$$R^a_{\mu} = R^a_B \epsilon^B_{\mu}. \tag{A4}$$

Now, to zeroth order in ϵ we have the obvious solution, $\theta_{\alpha a}^{(0)} = 0, R^{(0)} = I$. Higher order solutions can be obtained iteratively using Eq. (A4). Suppose that we have already determined $\theta^{(0)}, \ldots, \theta^{(n)}$. Then $\theta^{(n+1)}$ is determined as follows. The n+1 order term of Eq. (A4) reads

$$R^{(n+1)a}_{\mu} = R^{(n)a}_{B} \epsilon^{B}_{\mu}. \tag{A5}$$

Given the simple exponential series expansion for *R* in terms of θ , it is obvious that $R^{(n)}$ is a computable $\mathcal{O}(\epsilon^n)$ polynomial in $\theta^{(0)}, \ldots, \theta^{(n)}$. It follows that $\theta^{(n+1)}$ does not appear

on the right-hand side of Eq. (A5). The left-hand side has a simple linear dependence on $\theta^{(n+1)}$ which can be expressed by writing

$$R_{\mu}^{(n+1)a} = i \,\theta_{\gamma c}^{(n+1)} J_{\mu}^{(\gamma c)a} + R_{\mu|_{\theta^{(n+1)}=0}}^{(n+1)a}, \tag{A6}$$

where the second term of the right-hand side of Eq. (A6) is

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to be computed in terms of $\theta^{(0)}, \ldots, \theta^{(n)}$, with $\theta^{(n+1)}$ set to zero. By Eq. (A1), the first term of the right-hand side of Eq. (A6) is simply $\theta_{\mu c}^{(n+1)} \eta^{ac}$. Therefore subsituting Eq. (A6) into Eq. (A5) yields

$$\theta_{\mu c}^{(n+1)} = \eta_{ac} (R_B^{(n)a} \epsilon_{\mu}^B - R_{\mu|_{\theta^{(n+1)}=0}}^{(n+1)a}).$$
(A7)

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