Free differential algebras and generic 2D dilatonic (super)gravities

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The field equations for both generic bosonic and generic locally supersymmetric 2D dilatonic gravity theories in the absence of matter are written as free differential algebras. This constitutes a generalization of the gauge theoretic formulation. Moreover, it is shown that the condition of free differential algebra can be used to obtain the equations in the locally supersymmetric case. Using this formulation, the general solution of the field equations is found in the language of differential forms. The relation with the ordinary formulation and the coupling to supersymmetric conformal matter are also studied. [S0556-2821(99)02408-X]

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I. INTRODUCTION

Low-dimensional models of gravity play an important role in present-day theoretical physics due to the complexity of four-dimensional gravity. They are used to investigate the consequences of general covariance or local supersymmetry in a simpler setting. In two dimensions, although pure gravity is trivial, there are some models that include an additional field which have some properties analogous to the ones arising in four-dimensional general relativity, a fact that may be used to try to answer some fundamental problems also present in the four-dimensional case. For instance, black hole formation and evaporation can be studied using dilatonic type models of two-dimensional gravity such as the string-inspired dilatonic gravity [also known as the Callan-Giddings-Harvey-Strominger (CGHS) model [1]].

The CGHS model is not the only one that has been studied. Other interesting models are the Jackiw-Teitelboim model [2] and the more realistic, although less simple, spherically symmetric reduction of four-dimensional gravity. In fact, by using appropriate conformal redefinitions of the metric, it is always possible to write the action of a generic first-order model of 2D dilaton gravity conformally coupled to matter in the form [3]

$$S=\int d^2x\sqrt{-g}\left[R\eta+V(\eta)-\frac{1}{2}\partial^\mu f\partial_\mu f\right], \tag{1.1}$$

where f is a matter field (only one is considered for simplicity), and $V(\eta)$ is an arbitrary potential term. The CGHS model is obtained when $V=4\lambda^2$ (for constant λ), the Jackiw-Teitelboim model corresponds to $V=4\lambda^2\eta$ and for the spherically symmetric reduction of four-dimensional gravity $V\propto 1/\sqrt{\eta}$. The case of the model with an exponential potential ($V=4\lambda^2 e^{\beta\eta}$, for constant β) has also been considered [4].

The study of the consequences of general covariance in two-dimensional theories has been extended to the locally supersymmetric case. This gives information not only about supergravity itself, but also about the consequences its presence may have in the bosonic part of the theory, such as positivity of the energy [5,6]. The generalization of Eq. (1.1) to the locally supersymmetric case was given in [5] using the superfield formulation of [7], which shows that, as in the bosonic case, field redefinitions can be found that cast the action in a standard form which is the superfield generalization of Eq. (1.1). A further generalization also in the superfield context that relaxes the usual torsion constraints has been given in [8]. However, this paper is mainly concerned with an alternative formulation based on free differential algebras, as will be explained in what follows.

The pure dilaton gravity part (i.e. in the absence of matter) of the CGHS and Jackiw-Teitelboim models can be given a gauge theoretic interpretation, in which the starting points are the two-dimensional extended Poincaré and de Sitter algebras, respectively. Given a Lie algebra $\mathcal{G}$ with commutators

$$[T_a,T_b]=C^c_{ab}T_c, \tag{1.2}$$

where $a=1,\ldots,r=\dim\mathcal{G}$, the gauge field

$$A=A^aT_a \tag{1.3}$$

and a scalar L with values in the dual of the above algebra, i.e.,

$$L=\eta_aT^a,\quad T^a(T_b)=\delta^a_b \tag{1.4}$$

are introduced. Then a gauge invariant action for these fields is given by

$$I=\int_{\mathcal{M}}L(F(A))=\int_{\mathcal{M}}\eta_aF^a, \tag{1.5}$$

where $\mathcal{M}$ is the two-dimensional spacetime and $F(A)$ is the curvature of the connection A, $F(A)=dA+A\wedge A$. Indeed, under a gauge transformation of the connection A the curvature transforms under the adjoint representation of the superalgebra (1.2). The action (1.5) is then invariant under the gauge transformations of the connection A provided that L transforms under the coadjoint representation of the algebra. Note that the Lie algebra does not have to admit a nondegenerate invariant inner product but, if it does, Eq. (1.5) may be replaced by the integral of $\langle L,F\rangle$, $\langle\,\rangle$ being the inner product symbol, where now L is an *algebra*-valued quantity. The field equations of the theory are

$$F(A)=0, \quad \left(dA^a=-\frac{1}{2}C^a_{bc}A^b\wedge A^c\right), \quad d\eta_a+C^c_{ab}A^b\eta_c=0. \tag{1.6}$$

It is clear from these that the scalar field L is a Lagrange multiplier that imposes the zero curvature condition for the connection A. Suitable choices for Lie algebras lead to theories with action (1.5) that can be interpreted as two-dimensional gravities.

The CGHS model has been constructed out of two different Lie algebras. In [9], the Poincaré algebra with generators M (Lorentz generator) and $\{P_a\,;\,a=1,2\}$ (translations), and commutators

$$[M,P_a]=\epsilon^b{}_aP_b, \quad [P_a,P_b]=0, \tag{1.7}$$

was used. The Levi-Cività symbol ϵ^{ab} is defined here by $\epsilon^{01}=1$, and the indices are raised and lowered using the Minkowski metric η_{ab}; the spacetime signature has been chosen to be $(-,+)$. The same definition applies to the corresponding Levi-Cività symbol with space-time indices $\epsilon^{\mu\nu}$, where ϵ_{01} is therefore equal to -1. The formulation based on Eq. (1.7) had some problems that were naturally solved in [10] by starting from a central extension of this algebra instead. This implies introducing a central generator I. The non-vanishing commutators of the new algebra are

$$[M,P_a]=\epsilon^b{}_aP_b, \quad [P_a,P_b]=-\epsilon_{ab}I. \tag{1.8}$$

It was shown in [11] that the Jackiw-Teitelboim model may be formulated as a theory based on the de Sitter algebra with generators $\{M,P_a\,;\,a=1,2\}$ and commutators

$$[M,P_a]=\epsilon^b{}_aP_b, \quad [P_a,P_b]=-\Lambda\epsilon_{ab}M, \tag{1.9}$$

where Λ is a constant. Supersymmetric extensions of these algebras have been studied [12,13] and they lead, by using the Z_2-graded version of the procedure just explained, to supergravity theories. In particular it is known that the algebra (1.7) admits a (p,q) supersymmetric extension, the algebra (1.8) admits two different $N=1$ supersymmetric extensions and the algebra (1.9) admits a unique (1,1) extension.

The above construction can be reinterpreted in the more general setting of free differential algebras (FDA's). A free differential algebra [14] generated by the differential forms $G^{n_i}_i$, where n_i is the degree of the form, is a mapping $G^{n_i}_i\mapsto dG^{n_i}_i$ defined by

$$dG^{n_i}_i=\sum_r \sum_{\substack{n_{j_1}+\cdots+n_{j_r}=n_i+1\\ n_{j_1}\geq 1\cdots n_{j_r}\geq 1}} \alpha_i^{j_1\cdots j_r}G^{n_{j_1}}_{j_1}\wedge\cdots\wedge G^{n_{j_r}}_{j_r}, \tag{1.10}$$

where $\alpha_i^{j_1\cdots j_r}$ are in general functions of the zero-forms ($G^{n_i}_i$ such that $n_i=0$), in such a way that by virtue of Leibniz's rule and the expressions for $dG^{n_i}_i$ themselves, $d(dG^{n_i}_i)$ is identically zero (i.e., vanishes without using any algebraic relation between the forms $G^{n_i}_i$). Analogously, it is possible to define Z_2-graded free differential algebras for which Eq. (1.10) remains the same, the only difference being that now $G^{n_i}_i\wedge G^{n_j}_j=(-1)^{n_in_j}(-1)^{\alpha_i\alpha_j}G^{n_j}_j\wedge G^{n_i}_i$, where the extra factor $(-1)^{\alpha_i\alpha_j}$ (not present in the bosonic case) takes into account the Z_2 parities of $G^{n_i}_i$ and $G^{n_j}_j$ (α_i and α_j, respectively). The use of free differential algebras in supergravity dates from 1981, and since then, they have appeared in many examples [15].

As shown before, associated to every Lie algebra it is possible to construct a FDA by demanding that the curvatures in Eq. (1.6) vanish, and the same can be said of superalgebras and Z_2-graded FDA's. Adding the Euler-Lagrange (E-L) equations for the Lagrange multipliers, a larger FDA is obtained. From this point of view, the equations of CGHS and Jackiw-Teitelboim models can be written as a FDA that has a subalgebra that corresponds to a finite-dimensional Lie algebra. For instance, Eqs. (1.6) in the Jackiw-Teitelboim case are (the exterior product symbol $\wedge$ will be omitted from now on)

$$\begin{gathered} de^a+\epsilon^a{}_b\omega e^b=0,\\ d\omega-\frac{\Lambda}{2}\epsilon_{ab}e^ae^b=0,\\ d\eta+\eta_a\epsilon^a{}_be^b=0,\\ d\eta_a-\eta_b\epsilon^b{}_a\omega+\eta\Lambda\epsilon_{ab}e^b=0, \end{gathered} \tag{1.11}$$

where $A=e^ap_a+\omega M$, and $e^a=e^a_\mu dx^\mu$, $\omega=\omega_\mu dx^\mu$ give the zweibein and spin connection, respectively.

One of the points of the article is to show that the family of models (1.1) (and its generalization to the locally supersymmetric case) can be given a free differential algebraic formulation that does not correspond in general to a finite-dimensional Lie (super)algebra. However, the models still have an interpretation in terms of symmetries and as the dual of an *infinite* dimensional Lie (super)algebra. Moreover, this approach also provides an alternative method to obtain the locally supersymmetric generalization of Eq. (1.1). It is convenient to note here that an approach to generic 2D dilatonic gravities different from that of FDA's is provided by the Poisson σ models of [16].

The other main point to be presented here is the following. Once the field equations of both dilatonic gravity and dilatonic supergravity are written as free differential algebras containing one-forms and zero-forms, it is possible to find their general solution in the differential form language [for the solution of Eq. (1.1) in a specific gauge see, for instance, [17]). The general solution in the locally supersymmetric case does not exist in the literature, to the author's knowledge, although the action is known. The search for the general solution without fixing the gauge may be motivated by the fact that in the bosonic case its knowledge has been used to prove that the corresponding models in the presence of conformal matter can be related by a canonical transformation to a theory of free fields with the constraints of a certain string theory [18]. It might happen that something similar is possible in the context of supergravity theories and superstrings.

The organization of the paper is as follows. Section II is devoted to the bosonic case. There, the FDA corresponding to a generic dilatonic gravity model will be given, and its group theoretical meaning will be explained. The section ends with the derivation of the general solution using the FDA structure of the equations. In Sec. III, the locally supersymmetric case is studied. This will include the generalization of the bosonic FDA of the previous section, its relation with the ordinary formulation, the general solution (which in this case has some peculiarities), and the coupling of the models to conformal matter by using Noether's method. Finally, there is a section that contains the conclusions and outlook.

II. THE GENERIC BOSONIC CASE

The FDA given by

$$
\begin{aligned}
&de^a+\epsilon^a{}_b\omega e^b=0,\\
&d\omega-\frac{V'}{2}e^ae^b\epsilon_{ab}=0,\\
&d\eta+\eta_a\epsilon^a{}_be^b=0,\\
&d\eta_b+V\epsilon_{ab}e^a-\epsilon^a{}_b\omega\eta_a=0,
\end{aligned}
\tag{2.1}
$$

obviously does not have a subalgebra that can be derived from a finite-dimensional Lie algebra when $V'(\eta)\neq$ const, although it generalizes Eq. (1.11) ($V=\Lambda\eta$ there).

That Eq. (2.1) is indeed a FDA can be seen as follows. d^2 acting on e^a is proportional, by using the equation for $d\omega$, to a wedge product of three e^a forms, which vanishes in two dimensions. Applying d^2 to ω gives two terms: one of them is proportional to $e^ae_a=0$, and the other is of the form $V''d\eta e^ae^b\epsilon_{ab}$ and hence zero by virtue of the third equation. Next, d^2 acting on η can be seen to vanish from similar considerations. The only non-trivial identity to check is that of $d^2\eta_a$. But, using the first, third, and fourth equations,

$$
d^2\eta_b=-V'\eta_c\left(\epsilon^c{}_d\epsilon_{ab}+\frac{1}{2}\epsilon^c{}_b\epsilon_{ad}\right)e^ae^b,
\tag{2.2}
$$

which vanishes because in two dimensions the identity $\epsilon^c{}_{[d}\epsilon_{a]b}+\frac{1}{2}\epsilon^c{}_b\epsilon_{ad}-=0$ (d and a antisymmetrized with ''weight one'') holds.

Furthermore, Eqs. (2.1) are the Euler-Lagrange (E-L) equations of the Lagrangian two-form

$$
L=\eta_a(de^a+\epsilon^a{}_b\omega e^b)+\eta d\omega-\frac{V}{2}\epsilon_{ab}e^ae^b,
\tag{2.3}
$$

which is equivalent to the Lagrangian density (1.1) in the absence of matter. The equivalence is proved by solving for ω and η_a,

$$
\begin{aligned}
&\eta_a=-\epsilon^b{}_ae^\mu_b\partial_\mu\eta,\\
&\omega_\mu=e^{-1}\epsilon^{\rho\nu}\partial\rho e^a_\nu e_{a\mu},
\end{aligned}
\tag{2.4}
$$

where $e=\det(e^a_\mu)=\sqrt{-g}$ ($g_{\mu\nu}=e^a_\mu e^b_\nu\eta_{ab}$), and then substituting them into Eq. (2.3) to obtain an action that only depends on η and e^a_μ. In doing so, the relations $L=L_{\mu\nu}dx^\mu\wedge dx^\nu=\epsilon^{\mu\nu}L_{\mu\nu}d^2x\equiv\mathcal{L}d^2x$ and $\epsilon^{\mu\nu}\partial_\mu\omega_\nu=eR$ are used. Actually, substituting some E-L equations directly into the action does not necessarily mean that the equations obtained from the resulting action are the same as the ones obtained by making the substitution in the remaining E-L equations. However, this does happen in this case because the substituted fields (η_a,ω) are precisely the ones the equations of which are used (ω and η_a, respectively). This fact also guarantees that ω and η_a can be substituted into the symmetry transformation laws. On the other hand, Eqs. (2.1) can be obtained from Eq. (1.11) by letting Λ depend on η and demanding that the result is still a FDA (this procedure will be explained in more detail in the locally supersymmetric case).

The case $V=$const deserves a comment. It corresponds to the CGHS action, but it does not correspond to the *gauge formulation* of the CGHS model given in [10] because, by Eq. (1.8), this formulation contains a field (the one corresponding to the central generator I), which is not included in Eq. (2.1). If V is taken to be a constant in Eq. (2.1), the free differential algebra obtained is one that does have a subalgebra dual to a Lie algebra (the one considered in [9]), but that does not correspond to a Lagrangian of the form (1.5).

Although for a general V the FDA formulation is not the gauge theoretic formulation that corresponds to a finite-dimensional Lie algebra, it is still possible to work out the gauge symmetries of the FDA (2.1). A way to do that is to write $\delta e^a=dE^a+F^a$, $\delta\omega=d\Omega+G$, where E^a and Ω are the gauge parameters, and then fix both the *a priori* unknown one forms F^a, G, and the zero forms $\delta\eta$, $\delta\eta_a$ in such a way that the FDA is stable under the variations. The result is

$$
\begin{aligned}
&\delta e^a=dE^a-\epsilon^a{}_b\Omega e^b+\epsilon^a{}_bE^b\omega,\\
&\delta\omega=d\Omega+V'\epsilon_{ab}E^ae^b,\\
&\delta\eta=-\eta_a\epsilon^a{}_bE^b,\\
&\delta\eta_a=-V\epsilon_{ba}E^b+\epsilon^b{}_a\Omega\eta_b.
\end{aligned}
\tag{2.5}
$$

An interesting feature of these variations is that the action (2.3) is only quasi-invariant (i.e., invariant up to the differential of a one-form) when V is not proportional to η. In the case $V=\Lambda\eta$, these symmetries are the gauge transformations of the connections associated to the de Sitter algebra, as can be seen by computing the commutator of two such transformations. If this is done in general, say, for two E transformations, the following is obtained:

$$
\begin{aligned}
&[\delta_E,\delta_{E'}]e^a=-\epsilon^a{}_be^b(V'\epsilon_{cd}E^cE'^d),\\
&[\delta_E,\delta_{E'}]\omega=d(V'\epsilon_{ab}E^aE'^b)-V''\epsilon_{ab}E^aE'^b(d\eta+\eta_c\epsilon^c{}_de^d).
\end{aligned}
\tag{2.6}
$$

Note that unless $V''=0$ the algebra only closes over the space of solutions. Another characteristic that is due to the departure from the gauge formulation is that, even when the field equations are taken into account,

$$[\delta_E,\delta_{E'}]=\delta_{\Omega=V'\epsilon_{ab}E^aE'^b}, \tag{2.7}$$

which means that, with the exception of the case $V=$const, the group of transformations that leave the FDA (2.1) invariant is intrinsically gauge (the commutator of two E rigid transformations gives a gauge Ω transformation). Alternatively, it may be said that Eq. (2.7) reflects the fact that the ''structure constants'' depend on the field η and are therefore not constant. This can be seen by computing the (vector space) dual of Eq. (2.1), for which the following elementary differential geometry relations can be used:

$$df(X)=X\cdot f,\quad \alpha(X,Y)=X\cdot\alpha(Y)-Y\cdot\alpha(X)-\alpha([X,Y]), \tag{2.8}$$

where α is a one-form, X,Y are vector fields, and f is a function. Now, since $e^a(P_b)=\delta^a_b$, $e^a(M)=0=\omega(P_a)$, and $\omega(M)=1$, Eq. (2.8) lead to

$$\begin{gathered}P_a(\eta)=-\eta_b\epsilon^b{}_a,\quad P_a(\eta_b)=-V\epsilon_{ab},\\ M(\eta)=0,\quad M(\eta_a)=\epsilon^b{}_a\eta_b,\\ [P_a,P_b]=-V'\epsilon_{ab}M,\quad [M,P_a]=\epsilon^b{}_aP_b.\end{gathered} \tag{2.9}$$

The first two lines of Eq. (2.9) mean that M and P_a are the vector fields that generate the transformation $\delta\eta$, $\delta\eta_a$ of Eq. (2.5), which in the case $V''=0$ is the coadjoint representation on the coalgebra of the corresponding Lie algebra. The last line can be viewed as an infinite-dimensional Lie algebra where one of its generators is $V'M$. The commutator of this generator gives, by virtue of Leibniz's rule and Eq. (2.9), new ones that are products of M and P_a by functions of η and η_a. These in turn produce new generators and so on. The end result is, in general, an algebra with an infinite number of generators. The Lie algebras that arise here should not be identified with the non-linear ones studied in [19].

An advantage of writing the field equations as in Eq. (2.1) is that the general solution can be easily obtained in the language of differential forms. The way to do it is to solve for η in terms of η_a *instead* of solving for η_a in terms of η [as it was done in Eq. (2.4)], so that the solution depends on the pair of free functions η_a. More explicitly, from the equations for $d\eta$ and $d\eta_a$ it is easy to deduce that

$$\frac{1}{2}\eta^a\eta_a+J=C, \tag{2.10}$$

where $J(\eta)$ is defined by $J'=V$ and the constant C is related to the Arnowitt-Deser-Misner (ADM) energy [20]. This expression gives implicitly η in terms of η_a. Next, the equation for $d\eta_a$ can be rewritten as

$$e^a=\frac{1}{V}(\eta^a\omega-\epsilon^{ba}d\eta_b). \tag{2.11}$$

If this equation is then substituted into those of de^a and $d\omega$, *one* equation involving ω and η_a is found:

$$d\left[\frac{\omega}{V}+\frac{1}{2}\left(\frac{1}{V[C-J]}+\frac{D}{C-J}\right)\epsilon^{ab}\eta_a d\eta_b\right]=0, \tag{2.12}$$

where D is another constant. Equations (2.12), (2.11), and (2.10) constitute the solution of Eq. (2.1), of the form $\eta=\eta(\eta_a,g)$, $\omega=\omega(\eta_a,g)$ and $e^a=e^a(\eta_b,g)$, where g is another free function which comes from integration of Eq. (2.12). Therefore, the number of free functions equals the number of gauge symmetries. Of course, this solution is by construction consistent with Eq. (2.4). The possibility $V=0$ may be avoided by considering only functions of η that are always different from zero except possibly when $\eta=0$ and excluding the points x for which $\eta(x)=0$ from the spacetime manifold (see, for instance, [21]). The presence of C-J in the denominator is related to the fact that the spacetime points where it vanishes can usually be interpreted as a black hole horizon.

III. GENERIC 2D SUPERGRAVITIES

Before considering the locally supersymmetric case, it is necessary to fix some conventions, which are the following. Spinors are taken to be real and two dimensional [they belong to the vector space of the reducible (1,1) spinorial representation of the Poincaré group in two dimensions]; correspondingly the gamma matrices are real. In terms of ϵ^{ab}, the matrix γ_3 is given by $\gamma_3\equiv\frac{1}{2}\epsilon_{ab}\gamma^a\gamma^b$. From this, it is possible to deduce some useful relations:

$$\begin{gathered}\epsilon^{\mu\nu}e_{b\mu}e_{c\nu}=e\epsilon_{bc},\quad \epsilon_{ab}e^a_\mu e^b_\nu=e\epsilon_{\mu\nu},\\ \gamma_3\gamma_3=1,\quad \gamma^a\gamma_3=\epsilon^a{}_b\gamma^b,\end{gathered} \tag{3.1}$$

and the Fierz reordering

$$\lambda\bar{\psi}=\frac{1}{2}\bar{\lambda}\gamma^b\psi\gamma_b-\frac{1}{2}\bar{\lambda}\psi+\frac{1}{2}\bar{\lambda}\gamma_3\psi\gamma_3 \tag{3.2}$$

for any two spinors λ, ψ. The following realization of the gamma matrices will be used (underlined indices are flat space indices):

$$\gamma^{\underline{0}}=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \gamma^{\underline{1}}=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \gamma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{3.3}$$

As announced in the Introduction, the generalization to the locally supersymmetric case of Eq. (2.1) will be obtained from the FDA that leads to the supersymmetric Jackiw-Teitelboim model, which is the graded de Sitter–Lie superalgebra OSp(1,1|1) [12]. The field equations for the latter define the free differential algebra

$$
\begin{aligned}
&de^a+\epsilon^a{}_b\omega e^b+2i\bar{\psi}\gamma^a\psi=0,\\
&d\omega-2m^2\epsilon_{ab}e^ae^b+4im\bar{\psi}\gamma_3\psi=0,\\
&d\psi+\frac{1}{2}\omega\gamma_3\psi+me^a\gamma_a\psi=0,\\
&d\eta_a-\eta_b\epsilon^b{}_a\omega+4m^2e^b\epsilon_{ba}+im\bar{\chi}\gamma_a\psi=0,\\
&d\eta+\eta_a\epsilon^a{}_be^b+\frac{i}{2}\bar{\chi}\gamma_3\psi=0,\\
&d\chi+me^a\gamma_a\chi+\frac{1}{2}\omega\gamma_3\chi+4\eta_a\gamma^a\psi+8m\eta\gamma_3\psi=0,
\end{aligned}
\tag{3.4}
$$

from which the Lie superalgebra can be immediately recovered by duality ($A=e^aP_a+\omega M+\psi^\alpha Q_\alpha$, and Q_α are the supersymmetry generators). Next, the parameter m is taken to be a function of η. If this is done in the action, the E-L equations of which are Eqs. (3.4), a new term proportional to $\bar{\chi}e^a\gamma_a\psi$ has to appear in the equation for $d\omega$. The result is, however, not a *free* differential algebra because computing dd for each form and equating the result to zero would give extra algebraic relations between the forms. To convert the algebra into a free one, the terms proportional to m are substituted by terms of the same form but multiplied by unknown functions of η. On the other hand, the first equation of Eq. (3.4) should remain unaltered because it gives the usual torsion corresponding to ω. Then, imposing that $dde^a=0$ identically, makes it necessary to add a term proportional to $\epsilon^{ab}e_ae_b\chi$ in the equation for $d\phi$, which has to come from another one in the Lagrangian two-form proportional to $\epsilon^{ab}e_ae_b\bar{\chi}\chi$. This term introduces new elements in the equations of $d\eta_a$ and $d\omega$. The requirement $dd=0$, when applied to the other equations, fixes the arbitrary functions of η giving the result

$$
\begin{aligned}
&de^a+\epsilon^a{}_b\omega e^b+2i\bar{\psi}\gamma^a\psi=0,\\
&d\omega-2(uu')'\epsilon_{ab}e^ae^b+4iu'\bar{\psi}\gamma_3\psi\\
&\qquad+iu''\chi e^a\gamma_a\psi+\frac{i}{16}u'''\epsilon^{ab}e_ae_b\bar{\chi}\chi=0,\\
&d\psi+\frac{1}{2}\omega\gamma_3\psi+u'e^a\gamma_a\psi+\frac{1}{8}u''\epsilon^{ab}e_ae_b\chi=0,\\
&d\eta_a-\eta_b\epsilon^b{}_a\omega+4uu'e^b\epsilon_{ba}+iu'\bar{\chi}\gamma_a\psi-\frac{i}{8}u''\epsilon_{ba}e^b\bar{\chi}\chi=0,\\
&d\eta+\eta_a\epsilon^a{}_be^b+\frac{i}{2}\bar{\chi}\gamma_3\psi=0,\\
&d\chi+u'e^a\gamma_a\chi+\frac{1}{2}\omega\gamma_3\chi+4\eta_a\gamma^a\psi+8u\gamma_3\psi=0,
\end{aligned}
\tag{3.5}
$$

where $u(\eta)$ is an arbitrary function. In fact, these relations are the Euler-Lagrange equations derived from the Lagrangian two-form

$$
\begin{aligned}
L=&\,\eta_a(de^a+\epsilon^a{}_b\omega e^b+2i\bar{\psi}\gamma^a\psi)+\eta d\omega-2uu'\epsilon_{ab}e^ae^b\\
&+4iu\bar{\psi}\gamma_3\psi+iu'\bar{\chi}e^a\gamma_a\psi+i\bar{\chi}\left(d\psi+\frac{1}{2}\omega\gamma_3\psi\right)\\
&+\frac{iu''}{16}\epsilon^{ab}e_ae_b\bar{\chi}\chi.
\end{aligned}
\tag{3.6}
$$

Note that the anti–de Sitter case is recovered when $u=m\eta$. It is now simple to look for the local supersymmetry transformations under which this algebra is invariant (the Lagrangian two-form is then in general quasi-invariant, as it happened in the bosonic case). The comments made about the bosonic symmetries (2.5), as well as how to obtain them, also apply here. The transformation rule for ψ has to be of the form $\delta\psi=d\epsilon+\alpha$, where ϵ is the infinitesimal parameter of the transformation and α is a one-form that can be determined by substituting this expression into the third equation of Eq. (3.5), and demanding that the terms containing $d\epsilon$ cancel. Moreover, this condition may be used to obtain the form of the variation for the remaining one-forms. Having done that, it may be checked that the Lagrangian two-form is indeed quasi-invariant under the variation, except when $u=m\eta$, in which case it is strictly invariant. The following local supersymmetry transformations are obtained:

$$
\begin{aligned}
&\delta e^a=4i\bar{\psi}\gamma^a\epsilon,\\
&\delta\psi=d\epsilon+\frac{1}{2}\omega\gamma_3\epsilon+u'e^a\gamma_a\epsilon,\\
&\delta\omega=-8iu'\bar{\epsilon}\gamma_3\psi-iu''\bar{\epsilon}\gamma^a\chi e_a,\\
&\delta\eta=-\frac{i}{2}\bar{\chi}\gamma_3\epsilon,\\
&\delta\eta_a=-iu'\bar{\chi}\gamma_a\epsilon,\\
&\delta\chi=-8u\gamma_3\epsilon-4\eta_a\gamma^a\epsilon.
\end{aligned}
\tag{3.7}
$$

An immediate consequence of Eq. (3.7) is that the model with $u\propto\sqrt{\eta}$ is yet another supersymmetrization of the CGHS model, different from the ones considered in [12]. Specifically, the transformation rule for ψ is $\delta\psi=d\epsilon+(1/8)\omega\gamma_3\epsilon+c(1/\sqrt{\eta})e^a\gamma_a\epsilon$, where c is a constant, in contrast with the two cases of [12], for which either there is no $e^a\gamma_a\epsilon$ term or there is a term of the form $(1-\gamma_3)e^a\gamma_a\epsilon$ times a constant.

As explained in the previous section, to connect this formulation with the ordinary one the standard procedure is to write all the forms and scalars on spacetime ($e^a=e^a_\mu dx^\mu$, $\omega=\omega_\mu dx^\mu$, $\psi=\psi_\mu dx^\mu$) and then to solve for η_a in the fourth equation of Eq. (3.5) to obtain

$$\eta_a=-\epsilon^b{}_ae^\mu_b\partial_\mu\eta-\frac{i}{2}\bar{\chi}\gamma_3\psi_\mu e^\mu_b\epsilon^b{}_a, \tag{3.8}$$

and use the first equation of Eq. (3.5) to determine ω_μ in terms of e^a_μ and ψ_μ as

$$\omega_\mu = e^{-1}\epsilon^{\rho\nu}\partial\rho e^a_\nu e_{a\mu} + 2ie^{-1}\epsilon^{\rho\nu}\overline{\psi}_\rho\gamma_\mu\psi_\nu$$
$$= \overline{\omega}_\mu + 2ie^{-1}\epsilon^{\rho\nu}\overline{\psi}_\rho\gamma_\mu\psi_\nu, \tag{3.9}$$

where $\overline{\omega}_\mu$ is the torsion-free spin connection. These two equations are then used to substitute η_a and ω both into the action and into the local supersymmetry transformations. The ordinary Lagrangian is obtained from the Lagrangian two-form as in the bosonic case, and is given by

$$\mathcal{L}_{sg} = eR\eta + 4euu' + 4iu\epsilon^{\mu\nu}\overline{\psi}_\mu\gamma_3\psi_\nu + iu'\epsilon^{\mu\nu}e^a_\mu\overline{\chi}\gamma_a\psi_\nu$$
$$+ i\epsilon^{\mu\nu}\overline{\chi}D_\mu\psi_\nu - \frac{ieu''}{8}\overline{\chi}\chi - 4\eta ie(D_\mu\overline{\psi}^\mu\gamma_\nu\psi^\nu$$
$$+ \overline{\psi}^\mu\gamma_\nu D_\mu\psi^\nu) + 2e\overline{\chi}\gamma_3\psi_\nu\overline{\psi}^\mu\gamma_\mu\psi^\nu, \tag{3.10}$$

where use is made of the fact that $\epsilon^{\mu\nu}\partial_\mu\overline{\omega}_\nu = eR$, and D_μ is the covariant derivative with respect to the spin connection $\overline{\omega}_\mu: D_\mu\psi_\nu = \partial_\mu\psi_\nu + \frac{1}{2}\overline{\omega}_\mu\gamma_3\psi_\nu$, and similarly for other spinors. This is the locally supersymmetric version of Eq. (1.1), which coincides, apart from conventions, with the superfield formulation of [5], once the appropriate field redefinitions are used to get $K(\Phi)=0$ and $J(\Phi)=0$ there. Note that, when $u''\neq 0$, it is possible to solve the algebraic equation for χ, which gives a Lagrangian density independent of χ. The local supersymmetry transformations are

$$\delta e^a_\mu = 4i\overline{\psi}_\mu\gamma^a\epsilon,$$
$$\delta\psi_\mu = D_\mu\epsilon + u'e^a_\mu\gamma_a\epsilon - i\overline{\psi}_\rho\gamma_\mu\psi_\nu\gamma^{\rho\nu}\epsilon,$$
$$\delta\eta = -\frac{i}{2}\overline{\chi}\gamma_3\epsilon, \tag{3.11}$$
$$\delta\chi = -8u\gamma_3\epsilon + 4\partial_\mu\eta\gamma^\mu\gamma_3\epsilon + 2i\overline{\chi}\gamma_3\psi_\mu\gamma^\mu\gamma_3\epsilon.$$

The general solution of the field equations written as a Z_2-graded free differential algebra can be found by using the same procedure as in the bosonic case, although there are a few differences. First, it is easy to check, by using the last three equations of Eq. (3.5), that

$$\frac{1}{2}\eta^a\eta_a + 2u^2 - \frac{i}{8}u'\overline{\chi}\chi = C, \tag{3.12}$$

[cf. Eq. (2.10)] where C is a constant. This expression defines implicitly η in terms of η^a and χ. The last equation in Eq. (3.5) can be used to obtain ψ in terms of e^a, ω, η^a, and χ. Multiplying it by $\eta_a\gamma^a + 2u\gamma_3$ and then using Eq. (3.12), the following equation is obtained:

$$(\eta_a\gamma^a + 2u\gamma_3)\nabla\chi + (8C + iu'\overline{\chi}\chi)\psi = 0, \tag{3.13}$$

where

$$\nabla\chi := d\chi + u'e^a\gamma_a\chi + \frac{1}{2}\omega\gamma_3\chi. \tag{3.14}$$

In some cases, this equation does not determine ψ because it is not possible to divide by the factor $8C + iu'\overline{\chi}\chi$. This happens when C is a Grassmann even number without an ordinary number part (i.e., when the body of C is zero), and when $C=0$. The first case will not be considered because finding the solution implies having to separate the components of C in terms of its components in a basis of the Grassmann algebra, and the use of non-commuting numbers is just a device motivated by the anticommuting character of the corresponding quantum operators and does not have a physical meaning by itself. However, the $C=0$ case still has to be considered, and it will be dealt with at the end of the section. In the other cases, Eq. (3.13) can be solved without using the Grassmann algebra structure, and the result can be substituted into the fourth equation of Eq. (3.5), which in turn gives e^a as an expression involving ω, η^a, and χ. Once the expressions for η, ψ, and e^a are known, they can be substituted into the first three equations of Eq. (3.5). They all give the same equation for ω [this is something that can be deduced from the integrability conditions of the last three equations of Eq. (3.5)], so it is possible to write the solution in terms of η^a and χ. The result is given by Eq. (3.12) plus

$$\psi = -\frac{1}{8C}\left[1 - \frac{i}{8C}u'\overline{\chi}\chi\right](\eta_a\gamma^a + 2u\gamma_3)\nabla\chi,$$
$$e^a = \left[4uu' - \frac{i}{8}u''\overline{\chi}\chi + \frac{i}{4C}(u')^2u\overline{\chi}\chi\right]^{-1}$$
$$\times\left[\eta^a\omega\left(1 + \frac{i}{16C}u'\overline{\chi}\chi\right) - \epsilon^{ca}d\eta_c\right.$$
$$\left. + \frac{iu'}{8C}\epsilon^{ca}\overline{\chi}\gamma_c(\eta_b\gamma^b + 2u\gamma_3)d\chi\right], \tag{3.15}$$

$$d\left\{\left[\frac{4}{uu'} + \frac{i}{8}\frac{u''}{(uu')^2}\overline{\chi}\chi\right]\omega + \left[\left(\frac{2}{uu'[C-2u^2]} + \frac{D}{C-2u^2}\right)\right.\right.$$
$$\times\left(1 - \frac{iu'}{8[C-2u^2]}\overline{\chi}\chi\right) + \frac{i}{16u[C-2u^2]}$$
$$\left.\times\left(\frac{u''}{u(u')^2} - \frac{2}{C} + \frac{8u^2}{C^2}\right)\overline{\chi}\chi\right]\epsilon^{ab}\eta_a d\eta_b$$
$$\left. + i\left(\frac{1}{2Cu} - \frac{2u}{C^2}\right)\overline{\chi}\gamma_3 d\chi - \frac{i}{C^2}\eta_a\overline{\chi}\gamma^a d\chi\right\} = 0,$$

where D is another constant. Note that there are five arbitrary functions: η^a, χ and the one that comes from the integration of the last equation in Eq. (3.15). This number coincides with the number of gauge symmetries of the FDA. Of course, a gauge fixing greatly simplifies this expression but, as stated before, it may be important to control the gauge degrees of freedom.

A. The $C=0$ case

When $C=0$, Eq. (3.13) may, in principle, be solved for the components of ψ in a basis of the underlying Grassmann algebra, although it is not completely determined due to the fact that the body of $\bar{\chi}\chi$ is zero. This is not desirable from the point of view of the quantum theory, so it is convenient to restrict the space of solutions to those which can be expressed in terms of the fields themselves and not their components. This means that to solve Eq. (3.13) $\bar{\chi}\chi\psi$ has to vanish. Then it makes sense to try a solution of the form $\psi=\mu\chi$, where μ is a one-form to be determined. Indeed, it can be checked, following the same procedure as in the $C\neq 0$ case, that (D, D' are constants)

$$\psi=\frac{u'}{16u^2}e^a\eta_a\chi,$$

$$e^a=\left(4uu'-\frac{i}{8}u''\bar{\chi}\chi\right)^{-1}(\eta^a\omega-\epsilon^{ba}d\eta_b),$$

$$\eta^a\eta_a+4u^2-\frac{i}{4}u'\bar{\chi}\chi=0, \tag{3.16}$$

$$d\left\{\left[\frac{4}{uu'}+\frac{i}{8}\frac{u''}{(uu')^2}\bar{\chi}\chi\right]\omega-\left[\frac{i}{u^2}\left(\frac{1}{32}\frac{u''}{(uu')^2}+D'\right)\bar{\chi}\chi\right.\right.$$
$$\left.\left.+\left(1+\frac{i}{16}\frac{u'}{u^2}\bar{\chi}\chi\right)\left(\frac{1}{u^3u'}+\frac{D}{2u^2}\right)\right]\epsilon_{ab}\eta^a d\eta^b\right\}=0,$$

together with $\nabla\chi=0$ and $\eta_a\gamma^a\chi+2u\gamma_3\chi=0$, provides a solution of the FDA Eq. (3.5) when $C=0$.

However, Eq. (3.16) is not the only solution (this is not surprising, since it is expected that the solution includes two arbitrary functions to account for local supersymmetry). If ψ,e^a,ω,η is a solution, the set ψ',e^a,ω,η with $\psi'=\psi+\sigma\chi+\bar{\chi}\chi\zeta$ is also a solution provided σ is a closed one-form and and ζ obeys $\eta_a\gamma^a\zeta+2u\gamma_3\zeta=0$ and $\nabla\zeta=0$. The final result is, therefore, Eq. (3.16) except for the first equation, which takes the form

$$\psi=\frac{u'}{16u^2}e^a\eta_a\chi+\sigma\chi+\bar{\chi}\chi\zeta. \tag{3.17}$$

Both χ and ζ have to satisfy the same system of equations, with e^a,ω,η given in Eq. (3.16). Explicitly, writing ξ to denote either the spinor or the spinorial one-form,

$$\left(d+\frac{1}{2}\omega\gamma_3+u'e^a\gamma_a\right)\xi=0,$$
$$(\eta_a\gamma^a+2u\gamma_3)\xi=0. \tag{3.18}$$

The solution given in Eq. (3.16) is, as far as the bosonic fields are concerned, equal to the solution of the theory without fermions plus some corrections proportional to $\bar{\chi}\chi$. These corrections are irrelevant when solving Eq. (3.18), due to the presence of $\bar{\chi}\chi$ multiplying Eqs. (3.18) when $\xi=\zeta$ [see Eq. (3.17)] and to the presence of χ itself when $\xi=\chi$, because expressions involving the product of three χ spinors vanish. By virtue of the second and last equation in Eqs. (3.7), solving the equations is the same as, given a bosonic field configuration, looking for values of the supersymmetry parameter $\epsilon(x)$ such that supersymmetry is preserved. These particular values of $\epsilon(x)$ receive the name of Killing spinors. Not every configuration admits Killing spinors, but it turns out that those with $C=0$ do, and in fact these are the only configurations that are solutions of the field equations in the absence of matter and have Killing spinors [as can easily be seen by computing the square of the matrix in the second equation of Eqs. (3.18)]. To find the explicit expression of the solution of Eq. (3.18), it is convenient to define $\eta_{\neq}=\eta_{\underline{0}}+\eta_{\underline{1}}$, $\eta_{=}=\eta_{\underline{0}}-\eta_{\underline{1}}$ (underlined indices correspond to flat space indices) and write $\chi=\binom{\xi^L}{\xi^R}$ in the basis corresponding to Eq. (3.3). The solution may then be written

$$d(|\eta_{=}|^{-1/2}\xi^R)=0,\quad \xi^L=\frac{1}{2u}\eta_{\neq}\xi^R. \tag{3.19}$$

In this way, the closed form σ of Eq. (3.17) gives a free function, and the first equation of Eq. (3.19), when applied to ζ, gives another one. Hence, the number of free functions is five as expected.

B. The coupling to conformal matter

When studying the physical consequences of 2D models, such as black hole formation and evaporation, it is necessary to add matter fields. For this reason it may be interesting to do it in the supersymmetric case, providing explicit expressions in components. The next thing to do is, therefore, to couple these locally supersymmetric dilatonic gravity models to conformal matter. Here, this will be done by using the Noether method, although the same result can be easily obtained using superfields. The starting point is the flat space, rigid (1,1) supersymmetry invariant matter Lagrangian

$$\mathcal{L}_m=-\frac{1}{2}\eta^{\mu\nu}\partial_\mu f\partial_\nu f+\frac{i}{4}\bar{\lambda}\gamma^\mu\partial_\mu\lambda, \tag{3.20}$$

where the first term corresponds in curved space to the usual conformal matter Lagrangian, and the spinorial term makes it supersymmetric for the rigid variation

$$\delta f=i\bar{\epsilon}\lambda,$$
$$\delta\lambda=-2\partial_\mu f\gamma^\mu\epsilon. \tag{3.21}$$

The curved space version of Eq. (3.20) [which is the one needed to couple it to Eq. (3.10)] is obviously not invariant under the variations (3.21) when they are written in curved space and the parameter ϵ is made spacetime dependent. There are terms in the variation that come from the variation of e_a^μ, and there are also terms proportional to $D_\mu\epsilon$ that would also appear even in flat space because the variation is now local. The latter can be cancelled by adding to the action terms that involve ψ_μ. They can be seen to be equal to $\Delta_1\mathcal{L}=ie\partial_\mu f\bar{\psi}_\nu\gamma^\mu\gamma^\nu\lambda$. Among the other terms, plus the new ones coming from the variation of $\Delta_1\mathcal{L}$, there are some that contain $D_\mu\lambda$. These can be cancelled by adding a new piece to the variation of λ: $\delta'\lambda=2i\overline{\lambda}\psi_\mu\gamma^\mu\epsilon$. The new variation contains terms involving $D_\mu\epsilon$, which means that the term $\Delta_2\mathcal{L}=-\frac{e}{4}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\psi_\mu\overline{\lambda}\lambda$ must be added to the action. The process stops here, because at this point the complete variation vanishes up to a total derivative. The resulting Lagrangian density is then

$$\mathcal{L}=\mathcal{L}_{sg}-\frac{1}{2}eg^{\mu\nu}\partial_\mu f\partial_\nu f+i\frac{e}{4}\overline{\lambda}\gamma^\mu D_\mu\lambda + ie\partial_\mu f\overline{\psi}_\nu\gamma^\mu\gamma^\nu\lambda-\frac{e}{4}\overline{\psi}_\nu\gamma^\mu\gamma^\nu\psi_\mu\overline{\lambda}\lambda, \tag{3.22}$$

and the local variation of the matter fields is given by

$$\begin{aligned}\delta f&=i\overline{\epsilon}\lambda,\\ \delta\lambda&=-2\partial_\mu f\gamma^\mu\epsilon+2i\overline{\lambda}\psi_\mu\gamma^\mu\epsilon.\end{aligned}\tag{3.23}$$

Due to the fact that the matter multiplet is the one corresponding to a conformally coupled matter field f, the coupling of matter to the locally supersymmetric models is exactly the same as that for the pure Poincaré case. If a coupling of the form $eh(\eta)g^{\mu\nu}\partial_\mu f\partial_\nu f$ was added [an interesting case, corresponding to a scalar field in four dimensions is $V\propto 1/\sqrt{\eta}$, $h(\eta)\propto\eta$], both the variation of the matter fields and the matter Lagrangian density itself would have to involve the field χ, because $\delta\eta=-(i/2)\overline{\chi}\gamma_3\epsilon$. However, it is not immediate how to apply the Noether method in this case, and superspace methods would presumably be more appropriate here.

IV. CONCLUSIONS AND OUTLOOK

This paper shows that a generic two-dimensional dilatonic gravity theory in the absence of matter can be expressed as a free differential algebra. This has several consequences. First, there is room for a symmetry interpretation which generalizes that of the gauge theoretic formulation. Second, it provides a method for obtaining the general solution in terms of differential forms in both the bosonic and the locally supersymmetric case. Third, it provides an alternative method to obtain the generic supergravity Lagrangians.

It is still to be investigated how the free functions appearing in the solutions obtained relate to the different gauge fixings. This will be important when the program of relating the dilatonic theories to free field theories is carried out for the locally supersymmetric case. In that context, having a general solution of the starting models in the absence of matter might help one to find the new canonical variables, or to prove that they exist. Another point to be analyzed is whether it is possible to couple the models to matter while maintaining the symmetries of the free differential algebras, as was done in [22] for the CGHS model. Finally, many aspects of the derivation presented in this paper do not apply to the case of non-trivial topology. For instance, it is not guaranteed that a generic dilatonic action can be cast in the form (1.1). On the other hand, finding the general solution relies on the integration of certain closed forms, which may then be non-exact. These issues deserve further study.

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