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One-loop corrections for a Schwarzschild black hole via 2D dilaton gravity

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We study quantum corrections for the Schwarzshild black hole by considering it as a vacuum solution of a two-dimensional (2D) dilaton gravity theory obtained by spherical reduction of 4D gravity coupled with matter. We find perturbatively the vacuum solution for the standard one-loop effective action in the case of null-dust matter and in the case of minimally coupled scalar field. The corresponding state is in both cases a 2D Hartle-Hawking vacuum, and we evaluate the corresponding quantum corrections for the thermodynamic parameters of the black hole. We also find that the standard effective action does not allow boundary conditions corresponding to a 4D Hartle-Hawking vacuum state. [S0556-2821(99)01604-5]

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I. INTRODUCTION

One of the most interesting problems in quantum gravity is the Hawking radiation of black holes [1]. As we do not yet have a complete quantum theory of gravity, the full description of this phenomenon is still missing. Note that string theory has given a microscopic explanation of this process [2]. However, a complete formalism for calculating large-radius back reaction effects does not exist. These effects are described by the effective action, and in the absence of a complete formalism for calculating the effective action, one has to resort to various approximations.

One way is to quantize matter fields in the fixed black hole background. It is then possible to calculate the quantum corrections to the classical metrics, the spectrum of the radiation, temperature, etc. The back reaction of the radiation to the metric is calculated by defining the appropriate expected value of the energy-momentum tensor of the matter field [3–5] and solving the corresponding "one-loop" equation

$$R_{\mu\nu}-g_{\mu\nu}R/2=\langle T_{\mu\nu}\rangle. \quad (1.1)$$

This approach brought a fairly clear qualitative picture of the process. A better approach is to integrate the gravitational and matter fields in the functional integral and obtain a one-loop effective action, which would allow a background-independent approach. This, unfortunately, cannot be done in four spacetime dimensions (4D) because of the nonrenormalizability of gravity. However, in two spacetime dimensions (2D), gravity is renormalizable, and this procedure can be done. Therefore if one considers the spherically symmetric general relativity with matter as a 2D field theory, then it is possible to calculate the corresponding one-loop effective action. It is plausible to assume that such an action would be a good approximation for a full theory for large radius. Recently several papers have appeared on how to calculate this effective action [6–9]. All these papers gave similar results, modulo ambiguities in coefficients of certain counterterms.

Given this action, one can now start to investigate the one-loop back-reaction effects. The simplest thing is to investigate the static vacuum solutions. This approach has been already started in [10] where the quantum corrections to the Reissner-Nordstrom black hole and the corresponding thermodynamic properties were calculated. In that paper the authors have used only the Polyakov-Liouville term in the effective action, while it is known that the large radius null-dust action also contains additional local terms [11]. It is very well known fact [12,13] that the local terms in the effective action can influence the form of the solution. This is one reason why we consider the null-dust model. Another reason is that the null dust model can serve as a preparatory study for the more complicated, and more realistic model which is spherically symmetric reduction of general relativity with minimally coupled scalar field (SSG). The one-loop effective action for SSG model is qualitatively different from the null-dust action. There is a new nonlocal term due to the coupling between the dilaton and the matter field. Its influence on the form of the one-loop solution could be important, so that we calculate the correction to the Schwarzschild black-hole solution and the corresponding corrections to the thermodynamic parameters. As we mentioned, there exists some ambiguity in the literature about the $R\Phi$ coefficient in the effective action for SSG, and therefore we investigate how its value affects the physical parameters of the solution.

The third motivation is to compare the properties of this solution to those obtained in 4D via Eq. (1.1) [5], in order to see how good is the 2D effective action approach. The plan of the paper is the following. In Sec. II we briefly review the spherically symmetric one-loop effective action and transform it to a local form by using two auxiliary fields which mimic trace anomaly in our case. As a warm-up exercise we consider first a simpler model of null-dust matter in Sec. III:

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its equations of motion, perturbative solutions, corrections to the radius, temperature, and entropy of the black hole. This section is very similar to Ref. [10], but we only consider an uncharged black hole. In Sec. IV the same is done for the spherically symmetric model, and we present the results for an arbitrary $R\Phi$ coefficient. In Sec. V we present our conclusions. Appendix A contains all relevant formulas given for the action with arbitrary $R\Phi$ coefficient. The alternative calculation of entropy using the conical singularity method is given in Appendix B.

II. ONE-LOOP EFFECTIVE ACTION

Spherically symmetric reduction of the Einstein-Hilbert action in 4D gives the following 2D dilaton gravity action:

$$\Gamma_0=\frac{1}{4G}\int d^2x\sqrt{-g}e^{-2\Phi}[R+2(\nabla\Phi)^2+2e^{2\Phi}], \quad (2.1)$$

where $G,\Phi,g_{\mu\nu}$ are the Newton constant, dilaton, and two-dimensional metric, respectively. The 4D line element is given by

$$ds^2_{(4)}=g_{\mu\nu}dx^\mu dx^\nu+e^{-2\Phi}d\Omega^2 \quad (2.2)$$

so that $r=r_0e^{-\Phi}$ can be identified as the spatial radius in appropriate gauge (r_0 is an arbitrary length constant, which is needed for dimensional reasons).

If one couples minimally N scalar 2D fields f_i to this action, one gets the null-dust model, with the action

$$\Gamma_0=\frac{1}{4G}\int d^2x\sqrt{-g}\times\left[e^{-2\Phi}[R+2(\nabla\Phi)^2+2e^{2\Phi}]-2G\sum_{i=1}^{N}(\nabla f_i)^2\right], \quad (2.3)$$

where the number of scalar fields N is introduced in order to obtain the semiclassical approximation from the large-N limit. If one adds N scalar fields which are minimally coupled to gravity in 4D and afterwards performs the spherically symmetric reduction the action becomes

$$\Gamma_0=\frac{1}{4G}\int d^2x\sqrt{-g}\times\left[e^{-2\Phi}[R+2(\nabla\Phi)^2+2e^{2\Phi}]-2G\sum_{i=1}^{N}e^{-2\Phi}(\nabla f_i)^2\right]. \quad (2.4)$$

The null-dust model differs from SSG by the absence of coupling between scalar field and dilaton. It can be thought of as a large radius approximation to SSG, since the rescaling $f_i\to f_i/r$ in Eq. (2.4) will give Eq. (2.3) plus terms of order $1/r$.

The one-loop effective action for the model Eq. (2.3) has been found in Ref. [11]. The large-N, large-r one-loop part of the action is given by

$$\Gamma=\frac{1}{4G}\int d^2x\sqrt{-g}\times\left[e^{-2\Phi}[R+2(\nabla\Phi)^2+2e^{2\Phi}]-\frac{1}{2}\sum_i(\nabla f_i)^2\right]-\frac{N}{96\pi}\int d^2x\sqrt{-g}\left[R\frac{1}{\Box}R-(\nabla\Phi)^2+2R\Phi\right]. \quad (2.5)$$

Note that this action differs from the one used in Ref. [10] by the presence of two local terms, $(\nabla\Phi)^2$ and $2R\Phi$. The action (2.5) is very similar to the BPP model [13]. The action (2.5) can be rewritten as

$$\Gamma=\frac{1}{4G}\left(\int d^2x\sqrt{-g}\left[r^2R+2(\nabla r)^2+2-2G\sum_i(\nabla f_i)^2\right]-\kappa\int d^2x\sqrt{-g}\left[R\frac{1}{\Box}R-2R\log r-\frac{(\nabla r)^2}{r^2}\right]\right), \quad (2.6)$$

where $r=r_0e^{-\Phi}$ and $\kappa=NG/24\pi$. Since we are interested in the vacuum solutions, we will take $f_i=0$. The action (2.6) can be transformed into a local form by using an auxiliary scalar field ψ:

$$\Gamma=\frac{1}{4G}\left(\int d^2x\sqrt{-g}[r^2R+2(\nabla r)^2+2]-\kappa\int d^2x\sqrt{-g}\left[2R\psi+(\nabla\psi)^2-2R\log r-\frac{(\nabla r)^2}{r^2}\right]\right), \quad (2.7)$$

where ψ satisfies

$$\Box\psi=R.$$

In the SSG case the quantization of the matter fields gives the following one-loop correction to the effective action [6–9]:

$$\Gamma_1=-\frac{N}{96\pi}\int d^2x\sqrt{-g}\left(R\frac{1}{\Box}R-12R\frac{1}{\Box}(\nabla\Phi)^2+cR\Phi\right), \quad (2.8)$$

where c is a nonzero constant, whose numerical value depends on the quantization procedure used. The dimensional regularization method gives $c=12$ [7,8]. Dimensional regularization with a rescaled metric also gives $c=12$, but after returning to the original variables c changes to $c=14$ [9]. The ζ-function regularization gives $c=-4$ [6]. It will turn out that this ambiguity does not change any of our calculations essentially, so we will proceed with the value $c=12$ and summarize the results for the case of an arbitrary coefficient in Appendix A.

Note that one can also calculate the one-loop contributions due to all fields [9]. This calculation simplifies when a rescaled metric $\tilde{g}_{\mu\nu}=e^{\Phi}g_{\mu\nu}$ is quantized, so that in the large-N and large-r limit one obtains [9]

$$\tilde{\Gamma}_1=-\frac{N}{96\pi}\int d^2x\sqrt{-\tilde{g}}\times\left(\tilde{R}\frac{1}{\Box}\tilde{R}-12\tilde{R}\frac{1}{\Box}(\tilde{\nabla}\Phi)^2+12\tilde{R}\Phi+12\sum_i\tilde{R}\frac{1}{\Box}(\tilde{\nabla}f_i)^2\right). \tag{2.9}$$

When compared to Eq. (2.8), the last term in Eq. (2.9) is new, and it comes from the finite parts of the graviton-dilaton loops. Since we are interested in vacuum solutions, for which $f_i=0$, this difference will not be essential. However, when transforming back to the original metric, the $f_i=0$ limit of Eq. (2.9) will produce an additional term in Eq. (2.8), proportional to $\int\sqrt{-g}(\nabla\Phi)^2$, which is of the relevant order in $1/r$. Since it is not clear whether the quantization of the original metric will produce this term, in this paper we will consider only the quantum corrections given by Eq. (2.8), although one should keep in mind that additional counterterms are possible due to different quantization procedures.

The analysis of the action given by the correction Eq. (2.8) will simplify if it is written in the local form. Note that in this case we are dealing with two nonlocal terms, $R(1/\Box)R$ and $R(1/\Box)(\nabla\Phi)^2$. This implies that we have to introduce two auxiliary fields, ψ and χ. After a tedious calculation we get the following local form of the correction

$$\Gamma_1=-\frac{N}{96\pi}\int d^2x\sqrt{-g}[2R(\psi-6\chi)+(\nabla\psi)^2-12(\nabla\psi)(\nabla\chi)-12\psi(\nabla\Phi)^2+12R\Phi]. \tag{2.10}$$

The additional fields then satisfy the equations of motion

$$\Box\psi=R \tag{2.11}$$

and

$$\Box\chi=(\nabla\Phi)^2. \tag{2.12}$$

Note that it is now easy to obtain the expression for trace anomaly for the SSG case from Eqs. (2.10), (2.11), and (2.12). It is given by

$$T=\frac{1}{24\pi}[R-6(\nabla\Phi)^2+6\Box\Phi]. \tag{2.13}$$

In the case of the null-dust model T is given by

$$T=\frac{1}{24\pi}R, \tag{2.14}$$

if we take the Polyakov-Liouville term only, or

$$T=\frac{1}{24\pi}(R+\Box\Phi) \tag{2.15}$$

in the case (2.7).

III. SPHERICALLY SYMMETRIC NULL-DUST MODEL

We will discuss first the simpler model of null-dust matter in order to prepare for the more complicated case of spherically symmetric scalar field. We will use the action in the form

$$\Gamma=\frac{1}{4G}\left(\int d^2x\sqrt{-g}[r^2R+2(\nabla r)^2+2]-\kappa\int d^2x\sqrt{-g}\left[2R\psi+(\nabla\psi)^2-2R\log r-\frac{(\nabla r)^2}{r^2}\right]\right). \tag{3.1}$$

The variation of this action with respect to the fields ψ, r, and $g^{\mu\nu}$ gives the equations of motion

$$\Box\psi=R, \tag{3.2}$$

$$rR-2\nabla^2r=-\kappa\left(\frac{R}{r}+\frac{(\nabla r)^2}{r^3}-\frac{\nabla^2r}{r^2}\right), \tag{3.3}$$

and

$$\begin{aligned}-2r\nabla_\mu\nabla_\nu r+g_{\mu\nu}(\nabla^2r^2-(\nabla r)^2-1)&=-\kappa T_{\mu\nu}\\&=\kappa\left(-2\nabla_\mu\nabla_\nu\psi+\nabla_\mu\psi\nabla_\nu\psi+g_{\mu\nu}\left(2R-\frac{1}{2}(\nabla\psi)^2\right)+2\frac{\nabla_\mu\nabla_\nu r}{r}-3\frac{\nabla_\mu r\nabla_\nu r}{r^2}+g_{\mu\nu}\left(-2\frac{\nabla^2r}{r}+\frac{5}{2}\frac{(\nabla r)^2}{r^2}\right)\right).\end{aligned} \tag{3.4}$$

From Eqs. (3.3) and (3.4) we can obtain the expression for the scalar curvature:

$$R=\frac{2-2(\nabla r)^2+\kappa/r^2}{r^2}. \tag{3.5}$$

This expression shows that the singularity of the curvature is at $r=0$, as in the classical case. The solution of Eq. (3.3) for the field r is $r=x^1$, so the field r really has the meaning of the radial coordinate, in accordance with Eq. (2.2). In Ref. [10], the quantum correction shifts the space-time singularity to the value $r_{cr}^2=2\kappa$; in our case it stays at $r=0$. This is a consequence of the fact that the quantum correction in Ref. [10] is given by the Polyakov-Liouville term only, while in our case there are two additional local terms.

The classical solution (corresponding to $\kappa=0$) of the equations of motion (3.3), (3.4) is

$$r=x^1,\quad ds^2=-f_0(r)dt^2+\frac{1}{f_0(r)}dr^2, \tag{3.6}$$

where $t=x^0$, $f_0=1-2GM/r=1-a/r$. We are interested in the quantum correction of this solution in the semiclassical approximation. This means that we are searching for the perturbative solution of Eqs. (3.2)–(3.4) by taking κ as a small parameter. By introducing the static ansatz as in Ref. [10],

$$ds^2 = -f(r)e^{2\phi(r)}dt^2 + \frac{1}{f(r)}dr^2, \qquad (3.7)$$

with $f(r) = 1 - a/r + \kappa[m(r)/r]$, we get, in the first order in κ:

$$\psi' = -\frac{f_0'}{f_0} + \frac{C}{f_0}, \qquad (3.8)$$

$$\phi' = \frac{\kappa}{2rf_0}\left(2f_0'' - \frac{f'^2_0}{f_0} + \frac{C^2}{f_0} - \frac{3f_0}{r^2}\right), \qquad (3.9)$$

$$m' = -2f_0'' + \frac{1}{2f_0}(f_0'^2 - C^2) - \frac{f_0'}{r} + \frac{5}{2}\frac{f_0}{r^2}. \qquad (3.10)$$

The integration constant C can be determined from the condition that the behavior of T_{00} at infinity is thermal. This boundary condition follows from the fact that the static solution we are constructing describes a black hole in thermal equilibrium with the Hawking radiation (the black hole emits as much energy as it absorbs, so that its mass does not change). Since the matter in this model behaves as a free 2D scalar field, we take that T_{00} at infinity (in the zeroth order in κ) has the temperature dependence of a 1D free bose-gas $(\pi/6)T_H^2$, where T_H is the classical Hawking temperature $4\pi T_H = 1/a$. This is consistent with the fact that for spherical null dust $\langle T_{uu}\rangle = \langle T_{vv}\rangle = 1/48\pi(4M)^2$ in the Hartle-Hawking vacuum, where u and v are the asymptotically flat Schwarzschild coordinates [14]. This gives $C^2 = 1/a^2$. By integrating Eqs. (3.9), (3.10) we obtain

$$m(r) = -\frac{1}{2a}\log\left(\frac{r}{l}\right) - \frac{r}{2a^2} - \frac{2}{r}, \qquad (3.11)$$

$$\phi(r) = F(r) - F(L),$$

$$F(r) = \kappa\left[\frac{1}{2a^2}\log\left(\frac{r}{l}\right) - \frac{1}{ar}\right], \qquad (3.12)$$

where L and l are the integration constants. We have assumed that our system is in a 1D spatial box of size $L(a \ll L)$, and that $e^{2\phi} = 1$ as $r \to L$.

The position of the horizon r_h can be found perturbatively from the condition $f(r_h) = 0$:

$$r_h = a - \kappa m(a) = a + \kappa\left[\frac{1}{2a}\log\left(\frac{a}{l}\right) + \frac{5}{2a}\right]. \qquad (3.13)$$

The Hawking temperature also changes due to the quantum corrections. In the case of the metric of the form (3.7), it is given by (see Appendix B)

$$4\pi T_H = e^{\phi}f'|_{r=r_h}, \qquad (3.14)$$

which, after insertion of ϕ and m, gives

$$4\pi T_H = \frac{1}{a}\left[1 - \kappa\left(\frac{5}{2a^2} + F(L)\right)\right]. \qquad (3.15)$$

The second term in Eq. (3.15) is the quantum correction of the temperature.

In order to calculate the entropy of the quantum corrected black hole solution, we will use Wald technique [15]. In Refs. [16–18] it was shown that for the Lagrangian of the form $L = L(f_m, \nabla f_m, g_{\mu\nu}, R_{\mu\nu\rho\sigma})$ (f_m are the matter fields), the entropy is given by

$$S = -2\pi\epsilon_{\alpha\beta}\epsilon_{\chi\delta}\frac{\partial L}{\partial R_{\alpha\beta\chi\delta}}, \qquad (3.16)$$

evaluated at the horizon. Hence the evaluation of the entropy via Wald's method does not require the knowledge of the boundary terms (which might occur in the action), which are necessary if one evaluates the entropy from the Euclidean action [10]. For the lagrangian given by Eq. (3.1), we obtain

$$S = \frac{\pi a^2}{G} + \frac{\kappa\pi}{G}\left(3 + \log\frac{a}{l}\right). \qquad (3.17)$$

The first term in Eq. (3.17) is the Bekenstein-Hawking entropy, while the second term is the quantum correction. The same result is obtained when we define properly the boundary terms for the action Eq. (3.1), and use the conical singularity method and this calculation is presented in Appendix B.

Using Eqs. (3.16) and (3.18) and the first law of thermodynamics $TdS = dE$ [where the local temperature is given by $T = T_H/\sqrt{-g_{00}(L)}$], for the energy of the system we get the following expression:

$$E = M + \frac{\kappa}{4G^2M}\left(\frac{7}{4} + \frac{1}{2}\log\frac{L}{l}\right). \qquad (3.18)$$

The results which we have obtained are qualitatively similar to those of Ref. [10]. The difference in the numerical coefficients is due to the extra terms in the one-loop effective action. We also have a logarithmic correction to the entropy (term proportional to $\log M$). The main difference is that in our model the singularity of the curvature stays at the origin $r = 0$.

IV. SPHERICALLY SYMMETRIC SCALAR FIELD MODEL

We now examine the more realistic, and more complicated case of spherically symmetric scalar field. By adding the actions (2.4) and (2.8) and by setting $f_i = 0$ we get the one-loop effective action

$$\Gamma = \frac{1}{4G}\left(\int d^2x\sqrt{-g}[r^2R + 2(\nabla r)^2 + 2] - \kappa\int d^2x\sqrt{-g}\left[R\frac{1}{\Box}R - 12R\frac{1}{\Box}\frac{(\nabla r)^2}{r^2} - 12R\log r\right]\right), \qquad (4.1)$$

which is, in the local form

$$\Gamma=\frac{1}{4G}\bigg(\int d^2x\sqrt{-g}[r^2R+2(\nabla r)^2+2] -\kappa\int d^2x\sqrt{-g}\bigg[2R(\psi-6\chi)+(\nabla\psi)^2-12(\nabla\psi)(\nabla\chi) -12\frac{\psi(\nabla r)^2}{r^2}-12R\log r\bigg]\bigg). \tag{4.2}$$

Varying the action (4.2) with respect to r, $g_{\mu\nu}$,χ, and ψ we obtain the following equations of motion:

$$-rR+2\nabla^2 r=6\kappa\left(2\frac{\psi(\nabla r)^2}{r^3}-2\frac{\psi\nabla^2 r}{r^2}-2\frac{(\nabla\psi)(\nabla r)}{r^2}+\frac{R}{r}\right), \tag{4.3}$$

$$-2r\nabla_\mu\nabla_\nu r+g_{\mu\nu}(\nabla^2r^2-(\nabla r)^2-1) =\kappa\bigg[g_{\mu\nu}\bigg(2\Box\psi-12\Box\chi-12\frac{\nabla^2 r}{r}+(12+6\psi)\frac{(\nabla r)^2}{r^2} -\frac{1}{2}(\nabla\psi)^2+6(\nabla\psi)(\nabla\chi)\bigg)+\nabla_\mu\psi\nabla_\nu\psi -12\nabla_\mu\psi\nabla_\nu\chi-2\nabla_\mu\nabla_\nu\psi+12\nabla_\mu\nabla_\nu\chi +12\frac{\nabla_\mu\nabla_\nu r}{r}-12\frac{\nabla_\mu r\nabla_\nu r}{r^2}(\psi+1)\bigg], \tag{4.4}$$

$$\Box\psi=R, \tag{4.5}$$

$$\Box\chi=\frac{(\nabla r)^2}{r^2}. \tag{4.6}$$

The vacuum solution of the classical ($\kappa=0$) equations is the Schwarzschild black hole (3.6). We will, again, solve Eqs. (4.3)–(4.6) perturbatively in κ, starting with the static ansatz (3.7). Integration of the equations for ψ and χ in the zeroth order in κ gives

$$\psi=Cr+Ca\log\frac{r-a}{l}-\log\frac{r-a}{r}, \tag{4.7}$$

$$\chi'=\frac{2Dr^2-2r+a}{2r(r-a)}, \tag{4.8}$$

where C,D, and l are the integration constants.

The 00 and 11 components of Eq. (4.4), to the first order in κ, are

$$-rf'-f+1=\kappa\tau_0^0 =\kappa\bigg(f'\psi'-2R-6f'\chi'+\frac{1}{2}f\psi'^2-6f\psi'\chi' -6\frac{f\psi}{r^2}+6\frac{f'}{r}\bigg), \tag{4.9}$$

$$rf'+2\phi'fr+f-1=\kappa\tau_1^1 =\kappa\bigg(-f'\psi'+2R+6f'\chi'+\frac{1}{2}f\psi'^2 -6f\psi'\chi'-6(\psi+2)\frac{f}{r^2}-6\frac{f'}{r} -2f\psi''+12f\chi''\bigg). \tag{4.10}$$

From Eqs. (4.9) and (4.10) we easily obtain

$$m'=\tau_0^0,\quad \phi'=\frac{\kappa}{2rf}(\tau_1^1+\tau_0^0). \tag{4.11}$$

Using the expressions for ψ and χ' (4.7), (4.8), for m and ϕ we obtain the solutions

$$m(r)=\frac{11a}{4r^2}+\frac{-5+6aC}{2r}-\frac{1}{2}(C^2-12CD)r -\frac{5}{2a}\log\frac{r-a}{r}+\frac{6}{r}\log\frac{r-a}{r}-\frac{3a}{r^2}\log\frac{r-a}{r} -\frac{a(C^2-12CD)}{2}\log\frac{r-a}{l} -\frac{6aC}{r}\log\frac{r-a}{l}+\frac{3a^2C}{r^2}\log\frac{r-a}{l}, \tag{4.12}$$

and

$$\phi(r)=\kappa[F(r)-F(L)], \tag{4.13}$$

where $F(r)$ is given by

$$F(r)=\frac{3}{4r^2}+\frac{2+3aC}{ar}+\frac{1-6aC-a^2(C^2-12CD)}{2a(r-a)} +\frac{5}{2a^2}\log\frac{r-a}{r}-\frac{3}{r^2}\log\frac{r-a}{r} +\frac{(C^2-12CD)}{2}\log\frac{r-a}{l}+\frac{3aC}{r^2}\log\frac{r-a}{l}. \tag{4.14}$$

The constant L is chosen as a regularization parameter for large radius r. As the integration constant in $m(r)$ can be absorbed in the definition of the mass of the black hole [4,5], in Eq. (4.12) we have fixed it by using the previously introduced length scale l only.

In order to fix the values of the constants C and D, let us analyze the properties of the solution (4.12)–(4.14). Let us first calculate the determinant of the metric tensor, $g=e^{2\phi}$. In the first order in κ we get

$$g=1+2\phi=1+2\kappa[F(r)-F(L)],$$

which, in the vicinity of the point $r=a$ reduces to

$$g=\text{const}+\kappa\frac{1-6aC-a^2(C^2-12CD)}{a(r-a)}-\kappa\frac{1-6aC-a^2(C^2-12CD)}{a^2}\log(r-a).$$

Obviously g is divergent at $r=a$ unless

$$1-6aC-a^2(C^2-12CD)=0. \tag{4.15}$$

The position of the horizon is determined from the condition $f(r_h)=0$. If we assume that the correction is perturbative, $r_h=a+\kappa r_1$, we get

$$r_1=-m(a+\kappa r_1), \tag{4.16}$$

which gives

$$r_1=-\frac{1+12aC}{4a}+\frac{1}{2}a(C^2-12CD)+\frac{1}{2a}\log a-\frac{1-6aC-a^2(C^2-12CD)}{2a}\log(\kappa r_1)-3C\log l-\frac{a(C^2-12CD)}{2}\log l. \tag{4.17}$$

Note that the term proportional to $\log(\kappa r_1)$, which appears in this formula, is divergent in the limit $\kappa\to 0$, so one should take that the corresponding coefficient vanishes: $1-6aC-a^2(C^2-12CD)=0$, in order to stay in the region of the perturbative calculation.

The third argument also forces us to fix C and D in accordance with Eq. (4.15). Namely, if we calculate the Hawking temperature for the one-loop corrected geometry using Eq. (3.14), we get

$$4\pi T_H=\frac{1}{r_h}[1+\kappa m'(r_h)]e^{\phi(r_h)}. \tag{4.18}$$

Using the expressions for m, ϕ, and r_h,

$$4\pi T_H=\frac{1}{a}\left(1+\frac{1-6aC-a^2(C^2-12CD)}{ar_1}+\kappa\left(-\frac{1}{2a^2}+\frac{3C}{a}-(C^2-12CD)+\frac{C^2-12CD}{2}\log l-F(L)\right)\right). \tag{4.19}$$

Here, also, there is a nonperturbative term proportional to the factor $1-6aC-a^2(C^2-12CD)$. Therefore, Eq. (4.15) fixes one condition for C and D.

If we calculate the scalar curvature R in the first order in κ we obtain

$$R=\frac{2a}{r^3}+\frac{\kappa}{2ar^5}(a\{3a(12Cr-25)-36a^2C\log l+2r[5+(C^2-12CD)r^2]\}+2(18a^2-5r^2)\log r+2\{18a^2(aC-1)+[5+(C^2-12CD)a^2]r^2\}\times\log(r-a)). \tag{4.20}$$

From the last expression we see that the curvature singularities are $r=0$ and $r=a$. In order to remove the second singularity and keep the calculation perturbative, we get the second relation between C and D:

$$18aC-13+(C^2-12CD)a^2=0. \tag{4.21}$$

The conditions of applicability of perturbative calculation (4.15) and (4.21) give

$$C=\frac{1}{a},\quad D=\frac{1}{2a}. \tag{4.22}$$

We will now analyze the behavior of the stress-energy tensor for the SSG case. Using Eqs. (4.12)–(4.14) we get

$$T_{00}=\frac{1}{96\pi r^4}\left[-5a^2+4ar-12a^2Cr+18aCr^2+(C^2-12CD)r^4+12(a-r)^2\left(\log\frac{r-a}{r}-aC\log\frac{r-a}{l}\right)\right], \tag{4.23}$$

$$T_{11}=\frac{1}{96\pi r^2(a-r)^2}\left[11a^2-12ar-12a^2Cr+18aCr^2+(C^2-12CD)r^4+12(a-r)^2\left(\log\frac{r-a}{r}-aC\log\frac{r-a}{l}\right)\right]. \tag{4.24}$$

From Eqs. (4.23) and (4.24) the ingoing and outgoing fluxes are given by

$$T_{uu}=T_{vv}=\frac{1}{384\pi r^4}(6a^2-8ar-24Ca^2r+36aCr^2+2(C^2-12CD)r^4+24(a-r)^2[(1-aC)\log(r-a)-\log r+aC\log l]). \tag{4.25}$$

For the values of C and D given by Eq. (4.22), the conditions of the regularity of flux on the horizon calculated in the Kruscal coordinates (see the Appendix of [19])

$$T_{vv}<\infty,\quad (r-a)^{-1}T_{uv}<\infty,\quad (r-a)^{-2}T_{uu}<\infty \tag{4.26}$$

are fulfilled. It is interesting to note that we could take Eq. (4.26) as the regularity condition and obtain the same result (4.22) for C and D.

With these values of the constants our system is in the same 2D Hartle-Hawking vacuum state as in the null-dust case. This can be confirmed by performing a direct calculation of $\langle T_{uu}\rangle$ and $\langle T_{vv}\rangle$ [20]. From Eq. (4.25) we see that the system is in the thermodynamical equilibrium, but the emission and absorption fluxes at spatial infinity are negative, given by $-5/192a^2$. This property of SSG model can be understood from the fact that unlike the null-dust case, we have a strongly interacting gas in a 1D box, due to strong dilaton field at spatial infinity which couples to the 2D scalar field. Consequently the effective potential energy density is negative enough to make the total energy density negative, and hence the negative flux. As shown in Appendix A, the constants C and D which determine the flux at infinity, do not depend on the regularization-dependent coefficient c, so that negative flux is not a regularization artifact.

Note that one can find states with positive flux at spatial infinity by choosing the values of constants C and D such that $C^2-12CD>0$. These will be also states of thermal equilibrium with temperature close to the classical Hawking temperature. However, the problem with such states is that the scalar curvature will diverge in the vicinity of the horizon, so that their physical interpretation is not clear.

The values for the position of the horizon and the temperature in the 2D Hartle-Hawking vacuum are respectively,

$$r_h=a+\kappa\left(-\frac{23}{4a}+\frac{1}{2a}\log\frac{a}{l}\right) \tag{4.27}$$

and

$$4\pi T_H=\frac{1}{a}\left[1+\kappa\left(\frac{15}{2a^2}+\frac{5}{2a^2}\log\frac{L}{l}\right)\right]. \tag{4.28}$$

The entropy can be found by using the Wald method. For the Lagrangian given by Eq. (4.2), from (3.17) we get

$$S=\frac{a^2\pi}{G}+\frac{\pi\kappa}{G}\left(-\frac{15}{2}+5\log\frac{a}{l}\right). \tag{4.29}$$

Note that the Euclidean method gives the same expression for the entropy as Eq. (4.29) (see the Appendix B for details). The first term in Eq. (4.29) is the Bekenstein-Hawking entropy, while the first term is the quantum correction. The quantum correction is of the same type as in the previous case. The energy of the system can be obtained from the second law of thermodynamics. This gives

$$E=M-\frac{5\kappa}{4MG^2}\left(\frac{7}{2}+\log\frac{L}{l}\right). \tag{4.30}$$

V. CONCLUSIONS

The 2D Hartle-Hawking boundary conditions which we have used for the SSG model give negative energy density at spatial infinity, which is in contrast with the 4D Hartle-Hawking vacuum state where the energy density is positive. If one tries to impose the 4D HH boundary conditions, one quickly sees that this is impossible in this model. The relation between the 4D energy-momentum tensor and the corresponding 2D energy-momentum tensor for spherically symmetric models is given by

$$T_{\mu\nu}=4\pi r^2 T^{(4)}_{\mu\nu}. \tag{5.1}$$

From Eqs. (4.23), (4.24), and (5.1) it can be seen that the behavior of $T_{\mu\nu}$ at spatial infinity is such that the corresponding $T^{(4)}_{\mu\nu}$ can not describe a 4D Hartle-Hawking (HH) vacuum since $T^{(4)}_{00}$ has the asymptotics $\sim$ const/r^2 instead of $\sim$ const. This is not surprising because the reflecting boundary conditions in 2D (gas in a 1D box) correspond to the reflection of only the s modes in 4D. For the 4D Hartle-Hawking vacuum one needs boundary conditions corresponding to a gas in a 3D box. Such boundary conditions can be implemented in a spherically symmetric situation, as explicitly demonstrated in Ref. [5]; however, the corresponding 2D effective action is clearly not the one given by Eq. (2.8). Including the extra local term coming from the action (2.9) does not improve the situation. Clearly a nontrivial modification is necessary, and further work should be done.

The failure of the action (2.8) (which was obtained by the standard perturbative calculation) to describe the most general situations of interest indicates that performing a spherical reduction first and then quantizing is not equivalent to quantizing first and then performing a spherical reduction. This is not surprising, given that in the first approach one neglects the quantum effects of the angular modes. One way to see the effect of the angular modes is to compare the results of 2D effective action approach to the approach based on Eq. (1.1). This requires further work, because the existing results in the latter approach [5] use boundary conditions corresponding to a 4D HH state. For example, the function $m(r)$ is given by

$$\begin{aligned}Km(r)=&\frac{r^3}{3a^3}+\frac{r^2}{a^2}+\frac{3r}{a}-\frac{13}{3}\\&+\left[22-120\left(\xi-\frac{1}{6}\right)\right]\frac{r-a}{r}\\&+\left[30-240\left(\xi-\frac{1}{6}\right)\right]\left(\frac{r-a}{r}\right)^2\\&-\left[11-120\left(\xi-\frac{1}{6}\right)\right]\left(\frac{r-a}{r}\right)^3+4\log\frac{r}{a}+C_0\end{aligned} \tag{5.2}$$

where $K=3840\pi$ and ξRf^2 is an additional term in the Lagrangian density of the scalar field. Its asymptotic behavior is $O(r^3)$, while the 2D HH vacuum requires the asymptotics $O(r)$.

In the 2D effective action approach one can compare the results for various models. Frolov, Israel and Solodukhin in Ref. [10] obtained for the 2D HH state

$$m(r) = -\frac{7a}{4r^2} + \frac{1}{2r} - \frac{r}{2a^2} - \frac{1}{2a}\log\frac{r}{l}. \tag{5.3}$$

Our result for the null-dust case is

$$m(r) = -\frac{2}{r} - \frac{r}{2a^2} - \frac{1}{2a}\log\frac{r}{l}. \tag{5.4}$$

In the SSG model case the 2D Hartle-Hawking state gives

$$m(r) = \frac{11a}{4r^2} + \frac{1}{2r} + \frac{5}{2}r + \frac{5}{2a}\log\frac{r}{l} - \frac{6}{r}\log\frac{r}{l} + \frac{3a}{r^2}\log\frac{r}{l}, \tag{5.5}$$

while the states with positive Hawking flux (for some C) have

$$m(r) = -\frac{11a}{4r^2} + \frac{-5+6aC}{2r} - \frac{1-6aC}{2a^2}r + \left(-\frac{3a}{r^2} + \frac{6}{r} - \frac{5}{2a}\right)\log\frac{r-a}{r} + \left(\frac{3a^2C}{r^2} - \frac{6aC}{r} - \frac{1-6aC}{2a}\right)\log\frac{r-a}{l}, \tag{5.6}$$

where Eq. (4.15) is taken. Note that the logarithmic divergence in Eq. (5.5) for $r\to a$ is superficial, as the corresponding coefficient goes to 0 in this limit.

As it can be seen, the form of the corrections of $m(r)$ in all given cases is similar; they all diverge as $r\to 0$, which is to be expected. But it is interesting to note the large r behavior of $m(r)$: it goes as r in the 2D case while r^3 in the 4D case. This means that the metric diverges for large r in the 4D model [5], while in 2D models it is finite. This gives some restrictions on the applicability of 4D perturbative calculations. The 2D results which we noted above are all qualitatively similar. Unlike Ref. [10], we obtained that the Schwarzschild singularity stays at $r=0$ in the null dust and the SSG case. The usual 2D HH state has negative Hawking flux in the SSG case, which is the consequence of the interaction between the dilaton an the matter field. The positive Hawking flux states exist in the SSG model, but their physical interpretation is not clear due to curvature singularity at $r=a$. This is related to the pathological behavior of the flux for the freely falling observer at the horizon in this case. One may think that these states may be related to a 4D Unruh vacuum, but the relation $\langle T_{uu}\rangle = \langle T_{vv}\rangle$ does not allow this. The quantum corrections for the entropy have similar forms in various 2D models.

The SSG action was recently used in Ref. [21] to obtain the quantum correction to the Schwarzschild–de Sitter solution, which represents a nonasymptotically flat black hole. We succeeded to put this action in the local form introducing the two auxiliary fields, which, we believe, will be of importance in future investigations of this model. It would be interesting to see how our perturbative analysis extends to the case of nonvacuum solutions which are time dependent. In this case it would be useful to use the null-dust model as a zeroth-order approximation, since it is exactly solvable [22].

At the end of calculation our solutions contain two-dimensional constants l and L. This is the common case in the literature [10,4,5]: l is short-distance cutoff, of order of the Planck length, while L describes the infrared divergencies related to the radiated matter and is of order of the magnitude of space.

We obtained the entropy from the Wald approach in both cases which we analyzed. The same value for the entropy is obtained from the Euclidean method. The logarithmic form of the correction to the classical entropy is obtained, containing the cutoff parameters l and L. This also occurs in other models of quantum black holes [10,23,24]. Let us note that the calculation of entropy is more straightforward in 2D than in 4D. By using the black hole thermodynamics laws we determined the corrected energy of the black hole. Our results imply that $r_h \neq 2GM_{bh}$ in the quantum case. It would be interesting to calculate the Arnowitt-Deser-Misner (ADM) mass of the one-loop solution and compare it to the value obtained from the thermodynamics.

APPENDIX A

The general form of the action is

$$S = \frac{1}{4G}\left(\int d^2x\sqrt{-g}[r^2R + 2(\nabla r)^2 + 2] - \kappa\int d^2x\sqrt{-g}\left[2R(\psi-6\chi) + (\nabla\psi)^2 - 12(\nabla\psi)(\nabla\chi) - 12\frac{\psi(\nabla r)^2}{r^2} - 12\gamma R\log r\right]\right), \tag{A1}$$

where $\gamma = c/12$. In the previous analysis, $\gamma=1$.

The perturbative solution of the equations of motion is

$$m(r) = \frac{(36\gamma-25)a}{4r^2} + \frac{19-24\gamma+6aC}{2r} - \frac{1}{2}(C^2-12CD)r - \frac{5}{2a}\log\frac{r-a}{r} + \frac{6}{r}\log\frac{r-a}{r} - \frac{3a}{r^2}\log\frac{r-a}{r} - \frac{a(C^2-12CD)}{2}\log\frac{r-a}{l} - \frac{6aC}{r}\log\frac{r-a}{l} + \frac{3a^2C}{r^2}\log\frac{r-a}{l}, \tag{A2}$$

$$\phi(r) = \kappa[F(r) - F(L)], \tag{A3}$$

where $F(r)$ is given by

$$F(r)=\frac{3(4\gamma-3)}{4r^2}+\frac{2+3aC}{ar}+\frac{1-6aC-a^2(C^2-12CD)}{2a(r-a)}+\frac{5}{2a}\log\frac{r-a}{r}-\frac{3}{r^2}\log\frac{r-a}{r}+\frac{(C^2-12CD)}{2}\log\frac{r-a}{l}+\frac{3aC}{r^2}\log\frac{r-a}{l}. \quad \text{(A4)}$$

The values of the constants C and D do not depend on γ. Using Eqs. (A2)–(A4), it is straightforward to obtain the expressions for the temperature, entropy, etc.

APPENDIX B

In this appendix we will present the other derivation of the entropy of the corrected Schwarzschild solution. We will calculate entropy using the conical singularity method, developed in Refs. [10,24,25]. In order to define thermodynamical properties of the system, one considers the Euclidean theory. In that case, the free energy of the system F is proportional to the Euclidean action. We will consider the system at the arbitrary temperature $\overline{T}=(2\pi\overline{\beta})^{-1}$.

In Euclidean theory, $\tau=it$, the metric (3.7) takes the form

$$ds^2=f(r)e^{2\phi(r)}d\tau^2+\frac{1}{f(r)}dr^2, \quad \text{(B1)}$$

where $\tau\in[0,2\pi\overline{\beta}]$. If we define $\rho=\int[dr/\sqrt{f(r)}]$, the metric (B1) becomes

$$ds^2=g(\rho)d\tau^2+d\rho^2, \quad \text{(B2)}$$

where $g(\rho)=e^{2\phi}f$. Near the horizon $\rho\approx0$, Eq. (B2) may be written in the form

$$ds^2=\frac{\rho^2}{\beta_H^2}d\tau^2+d\rho^2, \quad \text{(B3)}$$

where $1/\beta_H=2\pi T_H=\frac{1}{2}e^{\phi}f'|_{r=r_h}$. From Eq. (B3) we see that for regular solution $\overline{\beta}=\beta_H$, meaning that the conical singularity in Eq. (B3) is absent. The metric (B2) can be conformally related to the metric of the cone C_α [10,25]:

$$ds^2=e^{\sigma}\left(dz^2+\frac{1}{\beta_H^2}z^2d\tau^2\right)=e^{\sigma}ds^2_{C_\alpha}. \quad \text{(B4)}$$

Conformal factor σ and coordinate z can be found easily. The conical metrics in Eq. (B4) must be regularized at the tip of the cone. This is done using [25] the regular metrics

$$ds^2_{\tilde{C}_\alpha}=u(z,a,\alpha)dz^2+\frac{z^2}{\beta_H^2}d\tau^2, \quad \text{(B5)}$$

where a is the regularization parameter. The simplest choice for the function u is

$$u=\frac{z^2+a^2\alpha^2}{z^2+a^2}.$$

Instead of the manifold $M_\alpha(C_\alpha)$ we will consider the regularized manifold $\tilde{M}_\alpha(\tilde{C}_\alpha)$. On the regularized manifold our effective action (4.2) takes the form

$$\begin{aligned}\Gamma=&-\frac{1}{4G}\left(\int_{\tilde{M}_\alpha}d^2x\sqrt{g}[r^2R+2(\nabla r)^2+2]\right.\\&-\kappa\int d^2x\sqrt{g}\left[2R(\psi-6\chi)+(\nabla\psi)^2-12(\nabla\psi)(\nabla\chi)\right.\\&\left.\left.-12\frac{\psi(\nabla r)^2}{r^2}-12R\log r\right]\right)\\&-\frac{1}{2G}\left(\int_{\partial\tilde{M}_\alpha}r^2kds-\kappa\int_{\partial\tilde{M}_\alpha}(2\psi-12\chi-12\log r)kds\right).\end{aligned} \quad \text{(B6)}$$

In the action (B6) we added appropriate surface terms. k is the external curvature of the boundary of the manifold. After the conformal transformation (B4), from (B6) we get the action

$$\begin{aligned}\Gamma=&-\frac{1}{4G}\left(\int_{\tilde{C}_\alpha}d^2x\sqrt{\tilde{g}}[r^2\tilde{R}-r^2\tilde{\nabla}^2\sigma+2(\tilde{\nabla}r)^2+2]\right.\\&-\kappa\int_{\tilde{C}_a}d^2x\sqrt{\tilde{g}}[\tilde{R}\tilde{\psi}-12\tilde{R}\tilde{\chi}-2\sigma\tilde{R}+\sigma\tilde{\nabla}^2\sigma\\&+12(\tilde{\nabla}\sigma)(\tilde{\nabla}\chi)+12\tilde{\chi}\tilde{\nabla}^2\sigma\\&\left.-12\sigma\tilde{\nabla}^2\tilde{\chi}-12\tilde{R}\log r+12\log r\tilde{\nabla}^2\sigma]\right)\\&-\frac{1}{2G}\int r^2\left(\tilde{k}+\frac{1}{2}\tilde{n}^\mu\tilde{\partial}_\mu\sigma\right)+\frac{\kappa}{G}\int\left(\tilde{\psi}\tilde{k}-3\tilde{\psi}\tilde{n}^a\partial_a\tilde{\chi}\right.\\&+\frac{1}{4}\sigma\tilde{n}^\mu\tilde{\partial}_\mu\sigma+\frac{1}{4}\tilde{\psi}\tilde{n}^\mu\partial_\mu\tilde{\psi}+\left(\tilde{k}+\frac{1}{2}\tilde{n}^\mu\partial_\mu\sigma\right)\\&\left.\times(-\sigma-6\tilde{\chi}-6\log r)\right),\end{aligned} \quad \text{(B7)}$$

where $\tilde{R}=u'/zu^2$ is the curvature of the regularized cone $\tilde{C}_\alpha$, while $\tilde{\psi}=\psi-\sigma$ and $\tilde{\chi}=\chi$. $\tilde{k}=1/z\sqrt{u}$ is the external curvature of the boundary of $\tilde{C}_\alpha$. We now take the limit $\tilde{C}_\alpha\to C_\alpha$. After that it is easy to rewrite the effective action in terms of quantities defined on the manifold M_α/Σ and Σ, where Σ is the tip of the cone. Finally, we obtain

$$\Gamma = -\frac{1}{4G}\left(\int_{M_\alpha/\Sigma} d^2x\sqrt{g}[r^2\bar{R} + 2(\nabla r)^2 + 2] - \kappa\int_{M_\alpha/\Sigma} d^2x\sqrt{g}\left[2\bar{R}(\bar{\psi} - 6\bar{\chi}) + (\nabla\bar{\psi})^2 - 12(\nabla\bar{\psi})(\nabla\bar{\chi}) - 12\frac{\bar{\psi}(\nabla r)^2}{r^2} - 12\bar{R}\log r \right]\right) - \frac{\pi}{G}(1-\alpha)r^2(\Sigma) + \frac{2\pi\kappa}{G}(1-\alpha)(\bar{\psi}(\Sigma) - 6\bar{\chi}(\Sigma) - 6\log r(\Sigma)) - \frac{1}{2G}\left(\int_{\partial M_\alpha} r^2\bar{k}ds - \kappa\int_{\partial M_\alpha}(2\bar{\psi} - 12\bar{\chi} - 12\log r)\bar{k}ds \right) + O((1-\alpha)^2), \tag{B8}$$

where $\bar{R}$ is the regular part of curvature, $\bar{\psi}$ is the solution of the equation (4.5) in the case $\alpha=1$. Also, $\bar{\chi}=\chi$, while $\bar{k}=k|_{\alpha=1}$. Using the definition of entropy

$$S = \left(\alpha\frac{\partial}{\partial\alpha} - 1 \right)\Gamma|_{\alpha=1} \tag{B9}$$

from Eq. (B8) we easily get

$$S = \frac{\pi}{G}[r_h^2 - \kappa(2\psi_h - 12\chi_h - 12\log r_h)] \tag{B10}$$

up to the numerical coefficient. From Eqs. (4.7), (4.8), and (4.27) and (B10) we get the expressions for the entropy which are equal to Eq. (4.29).

A similar calculation can be done for the null dust case, with the same conclusion that the expression for the entropy is the same as Eq. (3.18).

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