Charm and bottom baryon decays in the Bethe-Salpeter approach: Heavy to heavy semileptonic transitions

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Charm and bottom baryons and mesons are studied within the framework of a relativistically covariant 3D reduction of the Bethe-Salpeter equation. We carry out an analysis of semileptonic decays of heavy hadrons within this framework using explicit oscillator-type wave functions where we calculate Isgur-Wise functions, decay rates and asymmetry parameters. Within this model we also study the effect of interactions between the light quarks inside the heavy baryon and how they affect the values of the computed heavy baryon observables. We also elaborate on the role of relativistic effects in the calculation of the heavy baryon Isgur-Wise function. [S0556-2821(99)01007-3]

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I. INTRODUCTION

There has been remarkable progress in the experimental study of hadrons containing a single heavy quark [1]. The experimental progress calls for the development of theoretical approaches that allow one to study bound systems of a heavy quark and light quarks or antiquarks. This would enable one to analyze different weak decay processes (leptonic, semileptonic and nonleptonic) of heavy baryons and heavy mesons on an equal footing. All information about heavy hadron decays is contained in a set of reduced form factors which are governed by the dynamics of their light constituents. Since the momentum dependence of these reduced form factors cannot yet be determined from first principles in QCD one has to turn to QCD-inspired model studies of these quantities. Such models should take into account the full content of the symmetries of the underlying strong interaction Lagrangian as e.g. the leading order spin-flavor symmetry of the heavy quark effective theory (HQET) Lagrangian [2].

Any model devised for the quantitative description of heavy hadron weak transitions should include relativistic effects. First, the average momenta of light quarks inside the hadron are of the order of the constituent quark mass. As a result, there are large relativistic effects in the dynamics of heavy meson decays (see, e.g. [3,4]). Moreover, a general model approach should also be applicable to the description of those decays of heavy hadrons which are accompanied by a large momentum (energy) transfer. Thus, one needs a genuine relativistic treatment of the problem under study. There exist various relativistic approaches which enable one to study heavy hadron weak transitions: QCD sum rules [2,5,6], QCD on the lattice [7–9], relativistic quark models [10–29] including the approaches based on the use of the Bethe-Salpeter (BS) equations [16–19,22–24,26–28].

The BS formalism provides a systematic field-theoretical basis for the treatment of bound-state to bound-state transitions in which the interaction operators between the constituents can, in principle, be constructed from the underlying Lagrangian of the theory. The presence of confining interactions in the bound system precludes a straightforward use of perturbation theory for the calculation of the BS kernel. One must necessarily make a nonperturbative model ansatz for the kernel. In particular, utilizing the skeleton expansion of the interquark kernel together with a plausible approximation for the long-range behavior of the gluon propagator responsible for the confinement of quarks one can obtain a coupled set of Schwinger-Dyson and Bethe-Salpeter equations in Euclidean space. This approach can be solved to compute hadronic observables such as masses, decay constants and the q^2-behavior of various form factors [22,23]. The merit of this explicitly covariant approach is that it takes into account the full content of global QCD symmetries from the very beginning.

In the present paper the treatment of the confining interactions is based on the widely used instantaneous approximation for the BS interaction kernel in the c.m. frame of the hadron. This approach, though fully relativistic, shares most of the simplicity and transparency of the nonrelativistic model approaches. One may even calculate the corrections due to the noninstantaneous character of the kernel using the quasipotential method [30–32]. Explicit forms of instanta-

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neous $q\bar{q}$ and $3q$ kernels are beginning to emerge from lattice simulations [33] and from QCD-based calculations in the continuum limit [34,35]. In order to test the kernels it is important to carry out a systematic quantitative analysis of both the heavy meson and heavy baryon transitions in the instantaneous picture. Although the effect of the global QCD symmetries can be consistently embedded in the three-dimensional (3D) approach [24], we do not deal with a such an extension of the model at the present stage. We restrict ourselves to the standard relativistic constituent quark model where the (momentum independent) mass of the constituent quark is the input parameter of the theory, rather than that it emerges through the solution of the quark Schwinger-Dyson equation.

Most of the recent studies on the BS approach have focused on systematic investigations of heavy meson weak transitions [16–19]. The calculations in [18,19] are done in the heavy quark limit, while $1/m_Q$ effects are studied in Refs. [16,17]. In the baryon sector there has been less activity in the context of the BS approach. The reason for this lies in the complexity of the three-body problem both technically and conceptually. Some authors have circumvented the difficulties of the three-body problem by invoking the quark-diquark picture for the heavy baryons which effectively reduces the three-quark system to a two-body bound-state problem [20,25]. Summarily one may say that there is ample room left for systematic studies of heavy baryons as genuine three-body bound states within the quantum field-theoretical BS approach.

In most of the BS studies of bound state transition amplitudes the so-called two-tier scheme is used [26–28] when the instantaneous approximation for the BS kernel is employed. In the two-tier scheme one connects 3D and 4D hadron wave functions according to the following sequence of steps. First, one reduces the BS equation in the instantaneous approximation to a 3D equation for the equal-time wave functions and then one solves the BS equation. Further, in order to be able to apply the Mandelstam formalism for the calculation of matrix elements, one has to ''reconstruct'' the 4D BS wave function from the equal-time wave function through the BS equation. The 4D wave function is then substituted in the resulting expression for the hadronic matrix elements. The two-tier scheme is well suited for the solution of the two-particle bound state problem. For the three-particle bound state problem, however, problems arise due to the *disconnectedness* of the three-particle interaction kernel and the choice of the form of the instantaneous interaction in these disconnected terms. As a result the final 4D wave function has a rather unusual structure, containing the square root of the Dirac δ-function. In our approach such ill-defined structures do not appear.

The aim of the present paper is to calculate the heavy baryon observables in the covariant instantaneous approximation for the pair-wise kernel of the BS equation. To this end we develop a framework where the abovementioned problem related to the ill-definedness of the BS baryon wave function is avoided. This is achieved by abandoning the two-tier scheme and expressing the matrix elements directly in terms of the equal-time wave functions following the ideas of the covariant quasipotential approach [36]. Within our approach we also calculate the characteristics of heavy meson decays, using the same set of parameters as in baryon sector.

As a first approximation to the full complexity of the spin-spin interactions we work in the well-known spectator picture [10,27,28,37,38] which provides a well-established setting to include the dynamical effects of relativity. We shall start, however, from the complete bound-state amplitude and outline the approximations which finally lead to the spectator picture. In brief, our approximation consists in expanding the Lorentz-spinor factors in the BS equation and matrix elements in powers of $|\vec{p}|/m$ and in retaining only the leading-order term in this expansion (here $|\vec{p}|$ denotes the magnitude of the relative three-momentum of the quark in the c.m. frame of the baryon and m stands for its constituent mass). It is obvious that this approximation differs somewhat from the ''static'' approximation of Ref. [37] which consists in setting all four components of the quark relative four-momenta equal to zero. We mention, though, that the two approaches lead to identical results in the analysis of the spin structure of hadron transition matrix elements. It should be emphasized that the spectator quark model has been extremely succesful in the description of heavy meson and heavy baryon weak transitions [10,37,38]. A comprehensive analysis of semileptonic and nonleptonic decay data has been carried out in this model in terms of a few fit parameters related to the overlap integrals of the radial part of meson and baryon 4D BS wave functions. To our knowledge the overlap integrals appearing in the spectator model have not been calculated yet except the preliminary calculations carried out within the so-called Lagrangian spectator model [14,15]. One of the aims of our paper consists in establishing a clear and unambiguous connection of the BS approach in the baryon sector to the spectator quark model. Such an approach will provide a tool for the microscopic calculation of the spectator model parameters as well as for a study of new effects beyond the spectator approximation in the heavy hadron weak transitions.

We attempt to remain close to the conventional nonrelativistic treatment of the bound state problem in terms of 3D equal-time wave functions, the advantage being that these have a clear physical interpretation. The merit of such an approach as ours lies in added transparency, and in the possibility of controlling the magnitude of new relativistic effects. This can be achieved by restricting the zeroth components of the individual quark momenta in the baryon equal-time wave functions to their mass shell values as has been done in most of the studies [16–19] in the meson sector. The expressions obtained for the matrix elements then have a very simple form and can be readily interpreted in terms of quantum mechanical overlap integrals of 3D wave functions.

In the present paper we employ the BS framework for heavy-to-heavy transitions both in the heavy meson and heavy baryon sectors. Using harmonic oscillator wave functions, we calculate heavy meson and heavy baryon IW functions, their decay rates and asymmetry parameters. Within the oscillator model for heavy baryons we also study an alternative approach where the interactions between the two light quarks in the heavy baryons are switched off. Also, we

study relativistic effects in the heavy baryon IW function.

The layout of our paper is as follows: In Sec. II we present the BS formalism for baryons where we discuss in some detail how the instantaneous approximation can be adapted to the description of heavy baryons. We discuss problems related to the disconnectedness of the three-particle BS kernel. In Sec. III we construct the matrix element of the weak current in terms of heavy baryon equal-time wave functions. In Sec. IV we present the calculation of meson and baryon observables in the harmonic oscillator potential model. Section V contains our conclusions.

II. BS APPROACH TO BARYONS AS A BOUND STATE SYSTEM OF THREE QUARKS

In this section we shall derive the general BS equation with an instantaneous kernel for the equal-time baryon wave functions. By taking the limit $|\vec{p}|/m\to 0$ in the spinor part we show that the solutions of the baryonic BS equation reduce to the well-known spectator model wave functions. We also derive the normalization condition for the equal-time baryon wave function. The aim of this section is to provide equal-time baryon wave functions which can be employed in the calculation of heavy baryon weak transition matrix elements. A second aim is to establish the connection with the quark spectator model wave functions.

A. Bound-state equation

Let $\Psi_{\alpha\beta\gamma}(p_1p_2p_3)$ denote the 4D BS wave function of the baryon. We shall express the individual quark momenta p_i through the total four-momentum of the baryon (P) and the relative Jacobi momenta (q_1,q_{23}) according to (the m_i stand for the constituent quark masses)

$$p_1=\frac{m_1}{m_1+m_2+m_3}P-\frac{1}{3}q_1$$

$$p_2=\frac{m_2}{m_1+m_2+m_3}P+\frac{1}{3}\frac{m_2}{m_2+m_3}q_1-\frac{1}{2\sqrt{3}}q_{23} \tag{1}$$

$$p_3=\frac{m_3}{m_1+m_2+m_3}P+\frac{1}{3}\frac{m_3}{m_2+m_3}q_1+\frac{1}{2\sqrt{3}}q_{23}.$$

The BS equation for the baryon wave function reads

$$(S^{(1)}(p_1))^{-1}\otimes(S^{(2)}(p_2))^{-1}\otimes(S^{(3)}(p_3))^{-1}\Psi(p_1p_2p_3)$$
$$=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}V(p_1p_2p_3;k_1k_2k_3)\Psi(k_1k_2k_3) \tag{2}$$

where $S^{(i)}(p_i)=i(m_i-\not{p}_i)^{-1}$ denotes the propagator of the i-th quark with momentum p_i. Assuming pair-wise interactions between quarks, the kernel $V(p_1p_2p_3;k_1k_2k_3)$ in Eq. (2) can be written in the following form:

$$V=(2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{3}p_i-\sum_{i=1}^{3}k_i\right)\sum_{cycl(ijn)}(2\pi)^4$$
$$\times\delta^{(4)}(p_i-k_i)(S^{(i)}(p_i))^{-1}\otimes V^{(jn)}(P_{jn};q_{jn},l_{jn}). \tag{3}$$

Here $V^{(jn)}(P_{jn};q_{jn},l_{jn})$ is the two-body potential of the (jn)th pair and l_i, l_{jn} denote the relative momenta for the system of quarks with individual momenta k_1, k_2, k_3, defined similar to Eq. (1), and $P_{jn}=p_j+p_n=k_j+k_n$.

Next we discuss the choice of the form of the instantaneous kernel in the three-particle BS equation. For the mesonic two-particle case there exists a well-established prescription how to obtain the instantaneous kernel. One constrains the zeroth components of the relative four-momenta of quarks in the c.m. frame of the meson by the condition $q^0=l^0=0$ which leads to an instantaneous kernel that depends only on the three-momenta of the quarks.

This procedure cannot be directly generalized to the disconnected three-particle kernel [Eq. (3)] due to the singular character of the δ-functions $\delta^{(4)}(p_i-k_i)$ corresponding to the four-momentum conservation of the spectator quark. In the literature one finds different prescriptions for the definition of the baryonic instantaneous kernel [26–28,39]. The definition of the instantaneous kernel given in Ref. [39], though natural in view of its nonrelativistic counterparts, does not possess a natural connection to its relativistic counterpart given by Eq. (3). There does not exist a simple prescription to smoothly extrapolate from kernel (3) to the instantaneous limit given in Ref. [39]. For this reason we adopt an alternative definition which was also used in Refs. [26–28]. According to this definition, only pair-wise interaction kernels $V^{(jn)}$ undergo a (well-defined) modification in the instantaneous limit. In the c.m. frame of the baryon the prescription is analogous to that for the two-particle case. If the instantaneous kernels are assumed to be local, the prescription reads

$$V^{(jn)}(P_{jn};q_{jn},l_{jn})\to\sum_{\Gamma}O_\Gamma^{(j)}\otimes O_\Gamma^{(n)}V_\Gamma^{(jn)}\left(-\frac{\vec{q}_{jn}-\vec{l}_{jn}}{2\sqrt{3}}\right), \tag{4}$$

where the matrices O_Γ describe the spin structure of the potential (scalar, vector...). In an arbitrary reference frame the three-vectors are replaced by the covariant expressions $\vec{q}_{jn}\to q_{jn}^T,\vec{l}_{jn}\to l_{jn}^T$, where $p_\mu^T=p_\mu-v_\mu(v\cdot p)$ and $p^\parallel=(v\cdot p)$ etc. (here v_μ stands for the four-velocity of the baryon). These substitutions define the transformation rule of the baryon wave function from the rest frame to an arbitrary frame, providing explicit Lorentz-covariance of the formalism.

Having chosen the form of the instantaneous interaction, we turn to the derivation of the bound-state equation where we shall follow the proposals of Refs. [26–28]. We define the equal-time bound-state wave function according to the conventional prescription [26]. In the c.m. frame of the baryon this definition reads

$$\tilde{\Psi}_{\alpha\beta\gamma}(\vec{p}_1\vec{p}_2\vec{p}_3)=\int_{-\infty}^{\infty}\prod_{i=1}^{3}\frac{dp_i^0}{2\pi i}\times\delta\left(M_B-\sum_{r=1}^{3}p_r^0\right)\Psi_{\alpha\beta\gamma}(p_1p_2p_3) \tag{5}$$

where M_B denotes the baryon mass. As mentioned before the covariant generalization of the above expression is straightforward: $\vec{p}_i$ is replaced by p_i^T and p_i^0 by $p_i^{\parallel}$.

Next, one substitutes the instantaneous kernel defined by Eqs. (3) and (4) into the BS equation (2) and integrates over relative energy variables. The integral over q_{jn}^0 can be easily done with the help of the Cauchy's theorem. The remaining integral over q_i^0, however, can not be evaluated by Cauchy's theorem since both the propagators and the wave function depend on q_i^0. In order to proceed one replaces $1/3q_i^0$ in the propagators by its mass-shell value [26]

$$1/3q_i^0\to\mu_iM_B-w_i,\quad \mu_i=\left(\sum_{r=1}^{3}m_r\right)^{-1}m_i. \tag{6}$$

After integrating over the relative energy variables with the use of the substitution (6) one can rewrite the BS equation (2) to obtain

$$\tilde{\Psi}(\vec{p}_1\vec{p}_2\vec{p}_3)=\sum_{i=1}^{3}\left(\frac{\Lambda_+^{(j)}(\vec{p}_j)\otimes\Lambda_+^{(n)}(\vec{p}_n)}{w_j+w_n+w_i-M_B-i0}+\frac{\Lambda_-^{(j)}(\vec{p}_j)\otimes\Lambda_-^{(n)}(\vec{p}_n)}{w_j+w_n-w_i+M_B-i0}\right)\hat{I}^{(i)}\tilde{\Psi} \tag{7}$$

where

$$\Lambda_\pm^{(i)}(\vec{p}_i)=\frac{w_i\pm\hat{h}_i(\vec{p}_i)}{2w_i},\quad \hat{h}_i(\vec{p}_i)=\gamma_0^{(i)}m_i+\gamma_0^{(i)}\vec{\gamma}^{(i)}\vec{p}_i,$$
$$w_i=(m_i^2+\vec{p}_i^{\,2})^{1/2} \tag{8}$$

and

$$\hat{I}^{(i)}\tilde{\Psi}=\frac{1}{(2\sqrt{3})^3}i\int\frac{d^3\vec{l}_{jn}}{(2\pi)^3}\sum_\Gamma(\gamma_0^{(j)}O_\Gamma^{(j)})\otimes(\gamma_0^{(n)}O_\Gamma^{(n)})\times V_\Gamma^{(jn)}\left(-\frac{\vec{q}_{jn}-\vec{l}_{jn}}{2\sqrt{3}}\right)\tilde{\Psi}(P;\vec{q}_i,\vec{l}_{jn}). \tag{9}$$

Equation (7) gives a complete set of 3D equations for the equal-time baryon wave function components, without the need to use a Gordon on-mass-shell expansion as employed in Refs. [26–28]. The components of the equal-time baryon wave function are defined as $\tilde{\Psi}^{\sigma_1\sigma_2\sigma_3}=\Lambda_{\sigma_1}^{(1)}\Lambda_{\sigma_2}^{(2)}\Lambda_{\sigma_3}^{(3)}\tilde{\Psi}$, with $\sigma_1,\sigma_2,\sigma_3=+,-$. Note that Eq. (7) differs from the corresponding equation obtained from the instantaneous kernel used in Ref. [39]. Namely, adopting the kernel given in Ref. [39], it is easy to demonstrate that only the components $\tilde{\Psi}^{+++}$ and $\tilde{\Psi}^{---}$ are nonzero whereas the ''mixed'' components present in Eq. (7) vanish identically.

In the limit $|\vec{p}|/m\to0$ only the $\tilde{\Psi}^{+++}$-component of the baryon wave function survives. In this limit Eq. (7) reads

$$\left(\sum_{i=1}^{3}w_i-M_B\right)\tilde{\Psi}^{+++}=\Lambda_+^{(1)}\Lambda_+^{(2)}\Lambda_+^{(3)}\sum_{i=1}^{3}\hat{I}^{(i)}\tilde{\Psi}^{+++}. \tag{10}$$

Note that the istantaneous kernels of Refs. [39] and [26–28] yield the same equation (10) if the baryon wave function is restricted to the subspace of $(+++)$-components. Consequently, the difference of our approach to the prescriptions from Refs. [39] and [26–28] reveals itself in the way which the $(+++)$-component couples to the negative-frequency components.

B. Wave function

In the limit $|\vec{p}|/m\to0$ the projectors $\Lambda_\pm^{(i)}$ in Eq. (8) simplify to $\Lambda_\pm^{(i)}\to\frac{1}{2}(1+\gamma_0)^{(i)}$ in the c.m. frame of the baryon, or to $\Lambda_\pm^{(i)}\to\frac{1}{2}(1+\not{v})^{(i)}$ in the general frame. In the $|\vec{p}|/m\to0$ limit Eq. (10) is solved by the following ansatz for the wave function:

$$\tilde{\Psi}_{\alpha\beta\gamma}(p_1^Tp_2^Tp_3^T)=\theta_{\alpha\beta\gamma}(v)\phi(p_1^Tp_2^Tp_3^T) \tag{11}$$

where $\theta_{\alpha\beta\gamma}(v)$ obeys the following matrix equation for $(i,j,n)=\text{cycl}\ (1,2,3)$

$$\frac{1}{2}(1+\not{v})_{\alpha_1\beta_1}\frac{1}{2}(1+\not{v})_{\alpha_2\beta_2}\frac{1}{2}(1+\not{v})_{\alpha_3\beta_3}\times\delta_{\beta_i\delta_i}(\not{v}O_\Gamma)_{\beta_j\delta_j}(\not{v}O_\Gamma)_{\beta_n\delta_n}\theta_{\delta_1\delta_2\delta_3}=c_\Gamma\theta_{\alpha_1\alpha_2\alpha_3}. \tag{12}$$

The coefficients c_Γ are the eigenvalues of the matrix equation (12). Note that we have written $(\gamma_0^{(j)}O_\Gamma)\otimes(\gamma_0^{(n)}O_\Gamma)$ in its covariant form $(\not{v}^{(j)}O_\Gamma)\otimes(\not{v}^{(n)}O_\Gamma)$. Note also that in the limit $|\vec{p}|/m\to0$ only the scalar and zeroth component of vector interactions survive: $O_\Gamma=1,\not{v}$.

In the c.m. frame the radial wave function can be seen to satisfy the following equation:

$$\left(\sum_{r=1}^{3}w_r-M_B\right)\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=-\frac{1}{(2\sqrt{3})^3}\sum_{i=1}^{3}\int\frac{d^3\vec{l}_{jn}}{(2\pi)^3}u^{(jn)}\times\left(-\frac{\vec{q}_{jn}-\vec{l}_{jn}}{2\sqrt{3}}\right)\phi(P;\vec{q}_i\vec{l}_{jn}) \tag{13}$$

where $u^{(jn)}=-i\Sigma_\Gamma c_\Gamma V_\Gamma^{(jn)}$.

The usual ground state baryon spin wave functions can be seen to satisfy Eq. (12) with an eigenvalue $c_\Gamma=1$. Adding the flavor degree of freedom and putting in the appropriate spin-flavor symmetries one has (see [27,37,38])

$$J^P=1/2^+\ \theta_{ABC}(v)=[(\not{v}+1)\gamma^5 C]_{\beta\gamma}u_\alpha(v)B_{a[bc]}$$
$$+\text{cycl}\ \ (a\alpha,b\beta,c\gamma)$$
$$J^P=3/2^+\ \theta_{ABC}(v)=[(\not{v}+1)\gamma_\nu C]_{\beta\gamma}u^\nu_\alpha(v)B_{\{abc\}}$$
$$+\text{cycl}\ \ (a\alpha,b\beta,c\gamma) \tag{14}$$

where $B_{a[bc]}$ and $B_{\{abc\}}$ stand for the flavor wave functions with a mixed and full symmetry. The index pairs $A=(a\alpha)$, $B=(b,\beta)$, $C=(c\gamma)$ collect the isospin and Dirac indices. Details of the construction of flavor wave functions can be found in Ref. [40].

As a next step one has to specify the pair-wise interaction kernels $u^{(jn)}$. In the present paper we shall assume that the pair-wise interactions are of the harmonic oscillator type. One has

$$u^{(jn)}(\vec{q}-\vec{l})=\int d^3\vec{r}e^{-i(\vec{q}-\vec{l})\vec{r}}\left(\frac{1}{2}\mu_{jn}^2\omega_0^2\vec{r}^2+u_0^{jn}\right),$$
$$\mu_{jn}=\frac{m_j m_n}{m_j+m_n}. \tag{15}$$

Choosing a nonrelativistic form for the quark kinetic energy we obtain oscillator wave functions after substituting Eq. (15) into Eq. (13). We present these functions in the c.m. frame of the baryon $(\vec{p}_1+\vec{p}_2+\vec{p}_3=0)$. We distinguish between the following cases:

(1) Light baryons containing two quarks of equal mass: $m_2=m_3=m, m_1\neq m$

$$\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=C\exp\left[-\frac{1}{4m\omega_0}\left(\frac{(m_1+m)^{1/2}(m_1+2m)^{1/2}}{m}\times(\vec{p}_2+\vec{p}_3)^2+\frac{(m_1+m)^{1/2}}{\sqrt{2}(2m_1+m)^{1/2}}\times(\vec{p}_2-\vec{p}_3)^2\right)\right]. \tag{16}$$

(2) Heavy-light baryons: $m_1\to\infty$

$$\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=C\exp\left[-\frac{1}{2(m_2+m_3)\omega_0}\times\left((\vec{p}_2+\vec{p}_3)^2+\frac{(m_3\vec{p}_2-m_2\vec{p}_3)^2}{\sqrt{2}m_2m_3}\right)\right]. \tag{17}$$

(3) Model of noninteracting light quarks: $u^{(23)}=0$, $m_1\to\infty$. The wave function of such a system is remarkably simple since it factorizes in the variables $\vec{p}_2$ and $\vec{p}_3$

$$\phi(\vec{p}_1\vec{p}_2\vec{p}_3)=C\exp\left[-\frac{\vec{p}_2^{\,2}}{2m_2\omega_0}\right]\exp\left[-\frac{\vec{p}_3^{\,2}}{2m_3\omega_0}\right]. \tag{18}$$

The constant C in Eqs. (16)–(18) can be determined from the normalization condition for the wave function (see below). The three cases (16), (17) and (18) cover all cases of interest inasmuch we shall always assume $m_u=m_d$. As has been discussed before, moving frame wave functions are obtained by the substitution $\vec{p}^{\,2}\to-(p^T)^2$.

C. Normalization condition

In this section we derive the general normalization condition for the equal-time BS baryon wave function as given in Eq. (7). As usual, we start from the 4D BS equation for the six-particle Green's function G with the instantaneous kernel defined by Eqs. (3) and (4). Using Eq. (6), the BS equation can be reduced to the following 3D equation for the two-time Green's function $\tilde{G}$

$$\tilde{G}=-i\tilde{g}_0\Pi\Gamma_0+i\tilde{g}_0\sum_{i=1}^{3}\hat{U}^{(i)}(M_B)\tilde{G}$$
$$=-i\tilde{g}_0\Pi\Gamma_0+i\tilde{g}_0\hat{U}(M_B)\tilde{G} \tag{19}$$

where $\Gamma_0=\gamma_0^{(1)}\otimes\gamma_0^{(2)}\otimes\gamma_0^{(3)}$ and

$$\Pi=\Lambda_+^{(1)}\Lambda_+^{(2)}\Lambda_+^{(3)}+\Lambda_-^{(1)}\Lambda_-^{(2)}\Lambda_-^{(3)},$$
$$\tilde{g}_0=[M_B-\hat{h}_1(\vec{p}_1)-\hat{h}_2(\vec{p}_2)-\hat{h}_3(\vec{p}_3)]^{-1}$$
$$\hat{U}^{(i)}(M_B)\tilde{G}=\tilde{g}_0^{-1}\left(\frac{\Lambda_+^{(j)}\Lambda_+^{(n)}}{w_j+w_n+w_i-M_B-i0}+\frac{\Lambda_-^{(j)}\Lambda_-^{(n)}}{w_j+w_n-w_i+M_B-i0}\right)\hat{I}^{(i)}\tilde{G} \tag{20}$$

with the operator $\hat{I}^{(i)}$ given by Eq. (9). From Eq. (19) it immediately follows that

$$\tilde{G}\Gamma_0\Pi[\tilde{g}_0^{-1}-i\hat{U}(M_B)]\tilde{G}\Gamma_0\Pi\Gamma_0=-i\tilde{G}\Gamma_0\Pi\Gamma_0. \tag{21}$$

Extracting the bound-state pole in the function $\tilde{G}$, one obtains

$$\langle\tilde{\bar{\Psi}}|\Gamma_0\Pi[\tilde{g}_0^{-1}-i\hat{U}(M_B)]|\tilde{\Psi}\rangle=-(P^2-M_B^2) \tag{22}$$

$$\left\langle\tilde{\bar{\Psi}}\middle|\Gamma_0\Pi\left[1-i\frac{\partial}{\partial M_B}\hat{U}(M_B)\right]\middle|\tilde{\Psi}\right\rangle=-2M_B \tag{23}$$

where $\tilde{\bar{\Psi}}$ denotes the conjugate wave function. Again the generalization of the above formulas to an arbitrary reference frame is straightforward.

Equation (23) gives the general normalization condition in the instantaneous approximation for the baryon equal-time wave function given by Eq. (7). Note that there is an important difference of the normalization condition (23) for the *three-particle* wave function as compared to its two-particle counterpart (see, e.g. [41]). In the latter case the left-hand side (LHS) of the normalization condition does not depend on the bound-state energy M_B if the static kernel is energy-independent (a commonly accepted approximation). On the other hand the normalization condition for the three-particle

wave function is nonlinear in M_B irrespective of the form of the potential. This energy dependence arises from the energy denominators in Eq. (20).

The normalization condition considerably simplifies in the spectator approximation. All energy-dependent terms with at least one projector $\Lambda_-^{(i)}$ drop out in this limit. With the help of Eq. (11), one concludes from Eq. (23) that

$$N_C!\,\overline{\theta}\theta\frac{1}{(6\sqrt{3})^3}\int\frac{d^3\vec{q}_1}{(2\pi)^3}\frac{d^3\vec{q}_{23}}{(2\pi)^3}\phi^2(\vec{q}_1\vec{q}_{23})=2M_B \quad (24)$$

where the factor $N_C!=3!$ arises from the sum over (implicit) color indices and

$$\overline{\theta}\theta=\overline{\theta}_{\alpha\beta\gamma}\theta_{\alpha\beta\gamma},\quad \overline{\theta}_{\alpha\beta\gamma}=\theta^{\star}_{\alpha'\beta'\gamma'}(\gamma_0)_{\alpha'\alpha}(\gamma_0)_{\beta'\beta}(\gamma_0)_{\gamma'\gamma}. \quad (25)$$

Using Eq. (24) and the explicit expressions for the oscillator wave functions, Eqs. (16)–(18) it is then a simple task to calculate the normalization factor C. One has:

(1) Light baryons containing two quarks of equal mass: $m_2=m_3=m, m_1\neq m$

$$C=2^{13/8}\pi^{3/2}\left(\frac{M_B}{N_C!\,\overline{\theta}\theta m^3\omega_0^3}\right)^{1/2}\left(\frac{m_1+m}{m_1}\right)^{3/4}\left(\frac{2m_1+4m}{2m_1+m}\right)^{3/8}. \quad (26)$$

(2) Heavy-light baryons: $m_1\to\infty$

$$C=2^{13/8}\pi^{3/2}\left(\frac{M_B}{N_C!\,\overline{\theta}\theta(m_2m_3)^{3/2}\omega_0^3}\right)^{1/2}. \quad (27)$$

(3) Two noninteracting light quarks

$$C=4\pi^{3/2}\left(\frac{M_B}{N_C!\,\overline{\theta}\theta(m_2m_3)^{3/2}\omega_0^3}\right)^{1/2} \quad (28)$$

III. MATRIX ELEMENTS OF HEAVY BARYON TRANSITIONS

Below we give the expression of the matrix element of the weak current $\overline{Q}'(0)W^\mu Q(0)$ with $W^\mu=\gamma^\mu(1+\gamma^5)$ between heavy baryon states. In the derivation we follow the ideas of the covariant quasipotential approach [36]. However, there is a difference between our approach and the commonly used quasipotential approaches related to the treatment of the negative-frequency components of the baryon wave function. As is well known for the case of spin-$\frac{1}{2}$ constituents, neither the free nor the full equal-time Green's function can be inverted. In the quasipotential method the free Green's function is modified such that it can be inverted. The requisite modifications are by no means unique and differ by how the negative-frequency components of the wave function are treated. In general this may lead to different results for heavy baryon transition matrix elements. In our approach there is no need for such a modification and one can retain the full content of the equal-time BS wave function frequency components in the matrix elements.

After these introductory remarks we turn to the derivation of the matrix elements in the BS approach within the spectator model approximation. We denote the heavy quark (c or b) in the baryon by the label $1(1')$, while labels 2 and 3 correspond to the light quark constituents. For the time being we keep the mass of the heavy quark finite. Let R^μ denote the Green's function for the $Qqq\to Q'qq$ transition induced by the weak current. In the lowest-order approximation R^μ reads

$$R^\mu=S^{(1')}(p_1)W^\mu S^{(1)}(k_1)\otimes(2\pi)^4\delta^{(4)}(p_2-k_2)S^{(2)}(p_2)\otimes(2\pi)^4\delta^{(4)}(p_3-k_3)S^{(3)}(p_3).$$

The two-time operator $\tilde{R}^\mu$ is defined by

$$\tilde{R}^\mu(v',v)=\int_{-\infty}^{\infty}\prod_{i=1}^{3}\frac{da_i}{2\pi i}\frac{db_i}{2\pi i}2\pi i\,\delta\left(M_B'-\sum_{r=1}^{3}a_r\right)\times 2\pi i\,\delta\left(M_B-\sum_{s=1}^{3}b_s\right)R^\mu(p_1p_2p_3;k_1k_2k_3)$$

$$a_i=(v'\cdot p_i),\quad b_i=(v\cdot k_i) \quad (29)$$

where $v(v')$ and $M_B(M_B')$ denote the velocity and the mass of the initial (final) baryon.

In the conventional relativistic impulse approximation one takes the lowest-order result (29) and neglects the interaction terms $-i\hat{U}$ in Eq. (22). Extracting the double pole in R^μ with the use of Eqs. (22) and (23) one obtains the following expression for the matrix element of the weak current

$$\langle P'|\overline{Q}'(0)W^\mu Q(0)|P\rangle=-\langle\tilde{\bar{\Psi}}_v'|\Gamma_0(v)\Pi(v')\tilde{g}_0^{-1}(v')\tilde{R}^\mu(v',v)\Gamma_0(v')\times\Pi(v)\tilde{g}_0^{-1}(v)|\tilde{\Psi}_v\rangle \quad (30)$$

where all internal integrations are three-dimensional. In the spectator approximation the general structure of the transition matrix element (30) can be seen to further simplify. The details of the derivation can be found in the Appendix. The final result reads

$$\langle P'|\overline{Q}'(0)W^\mu Q(0)|P\rangle=2(M_BM_B')^{1/2}(\overline{\theta}\theta)^{-1}\overline{\theta}(v')W^\mu\theta(v)f(v\cdot v') \quad (31)$$

with

$$f(v\cdot v')=\frac{N_C!\,\overline{\theta}\theta}{2(M_BM_B')^{1/2}}\frac{1}{(6\sqrt{3})^8}\times\int\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_{23}}{(2\pi)^4}\frac{d^4l_1}{(2\pi)^4}\frac{d^4l_{23}}{(2\pi)^4}(2\pi)^4\delta^{(4)}(\Delta_2)\times(2\pi)^4\delta^{(4)}(\Delta_3)F(\vec{q}_1\vec{q}_{23})\overline{S}_H^{(1')}(p_1'^0)\times\overline{S}_H^{(1)}(k_1'^0)F(\vec{l}_1\vec{l}_{23})\overline{S}_L^{(2)}(p_2')\overline{S}_L^{(3)}(p_3') \quad (32)$$

where $F(\vec{p}_1\vec{p}_{23})=(\overline{\Lambda}-w_2-w_3)\phi(\vec{p}_1\vec{p}_{23}), M_B=m_1+\overline{\Lambda}+O(m_1^{-1})$ and $M_B'=m_1'+\overline{\Lambda}+O(m_1'^{-1})$. The four-vectors p_i', k_i' are defined by Eq. (A3) in Appendix. The propagators in the spectator approximation are given by the following expressions:

$$\overline{S}_H(p^0)=\frac{1}{-\overline{\Lambda}+m_2+m_3-p^0-i0} \quad \text{for heavy quarks}$$

$$\overline{S}_L^{(i)}(p)=\frac{1}{w_i'-m_i-p^0-i0} \quad \text{for light quarks.} \tag{33}$$

For the light quarks $(i=2,3)$ one has

$$\Delta_i^0=m_i(v'^0-v^0)+p_i'^0v'^0+\vec{p}_i'\vec{v}'-k_i'^0v^0-\vec{k}_i'\vec{v}$$

$$\vec{\Delta}_i=m_i(\vec{v}'-\vec{v})+\vec{p}_i'+p_i'^0\vec{v}'+(v'^0+1)^{-1}(\vec{p}_i'\vec{v}')\vec{v}' -\vec{k}_i'+k_i'^0\vec{v}+(v^0+1)^{-1}(\vec{k}_i'\vec{v})\vec{v}. \tag{34}$$

The Lorentz structure of the current matrix element Eq. (31) is determined by the spectator model factor $\overline{\theta}(v')W^\mu\theta(v)$. It is well known that in the heavy quark limit baryonic ground-state to ground-state transitions are determined by three independent form factor functions $\zeta(\omega),\xi_1(\omega)$ and $\xi_2(\omega)$ which depend on the momentum transfer variable $\omega=v\cdot v'$ [42]. In the spectator model these three functions become related and are given in terms of a single universal form factor function $f(\omega)$ [10]. One has

$$\zeta(\omega)=\xi_1(\omega)=\xi_2(\omega)(\omega+1)=f(\omega)\frac{\omega+1}{2}, \quad f(1)=1. \tag{35}$$

This result coincides with the prediction of large-N_c QCD [43].

In order to determine the reduced form factor function $f(\omega)$, it is sufficient to consider only one particular transition. For example, take the $\Lambda_b\to\Lambda_c$ transition. Using the known spectator model wave functions Eq. (14), one obtains

$$\overline{\theta}_{\Lambda_c}(v')W^\mu\theta_{\Lambda_b}(v)=\frac{1}{2}\overline{\theta}_{\Lambda_c}\theta_{\Lambda_b}(1+\omega)\overline{u}(v')W^\mu u(v). \tag{36}$$

Then from Eqs. (31) and (36) one immediately concludes that

$$\langle\Lambda_c|\overline{Q}'(0)W^\mu Q(0)|\Lambda_b\rangle =2(M_{\Lambda_c}M_{\Lambda_b})^{1/2}\overline{u}(v')\gamma^\mu(1+\gamma^5)u(v)\frac{\omega+1}{2}f(\omega). \tag{37}$$

From Eq. (37) it is seen that the universal function $f(\omega)$ in Eq. (35) coincides with the one given by Eq. (32). Using the normalization condition for the baryon wave function (24), one can check that normalization condition $f(1)=1$ is satisfied.

In order to proceed further in the calculation of the heavy baryon weak semileptonic transition matrix element given by Eqs. (31) and (32), we choose a particular reference frame where $v^\mu=(1,0,0,0)$ and $v'^\mu=(\omega,0,0,(\omega^2-1)^{1/2})$. After integrating over the variables l_1 and l_{23} the arguments of the initial wave function and w_1, w_2 become dependent on the relative energy variables q_1^0 and q_{23}^0. Cauchy's theorem can therefore not be used in the evaluation of the integrals over q_1^0 and q_{23}^0. As mentioned before this is similar to that happens in the mesonic case. Also this dependence gives rise to a spurious imaginary part in the function $f(\omega)$ at $\omega\neq1$. A simple way to remedy this difficulty is to fix the relative energies on mass shell [the same, at the poles of the denominator in the Eq. (32)] in the wave functions and in the quantities w_1, w_2 such that one has

$$\frac{1}{3}q_1^0=w_2'+w_3'-m_2-m_3, \quad \frac{1}{2\sqrt{3}}q_{23}^0=\frac{m_2w_3'-m_3w_2'}{m_2+m_3}. \tag{38}$$

The Cauchy integration over the energy denominators can then easily be performed. The factor $(\overline{\Lambda}-w_2-w_3)(\overline{\Lambda}-w_2'-w_3')$ in the numerator is cancelled upon integration and we are left with the simple result

$$f(\omega)=\frac{N_C!\,\overline{\theta}\theta}{2(M_BM_B')^{1/2}}\frac{1}{(6\sqrt{3})^3} \times\int\frac{d^3\vec{q}_1}{(2\pi)^3}\frac{d^3\vec{q}_{23}}{(2\pi)^3}\phi(\vec{q}_1\vec{q}_{23})\phi(\vec{l}_1\vec{l}_{23}) \tag{39}$$

where

$$l_1^3=3(\omega^2-1)^{1/2}(w_2'+w_3')+\omega q_1^3,$$

$$l_{23}^3=\omega q_{23}^3+(\omega^2-1)^{1/2}2\sqrt{3}\,\frac{m_2w_3'-m_3w_2'}{m_2+m_3}, \quad \vec{l}_1^{\perp}=\vec{q}_1^{\perp}. \tag{40}$$

The physical meaning of the result is transparent. Equation (39) corresponds to the quantum-mechanical overlap of two baryon wave functions. The initial wave function is evaluated in the rest frame of the initial baryon and the final wave function in the frame moving with the velocity v' along the third axis. The arguments of the final wave function are Lorentz boosted where the energies of the light quarks are fixed by their mass shell values. Obviously, the same result can be obtained from the general expression (30), e.g., in the rest frame of the final baryon. In this case the initial wave function in Eq. (39) is substituted by the final wave function and vice versa. Since in the heavy quark limit the wave functions do not depend on heavy flavor, one ends up with the same heavy baryon IW function in both frames.

The calculation of the heavy meson IW function proceeds along similar lines but will not be presented in this paper.

We only give the final result obtained with the same assumptions as for the case of baryons. For the reduced form factor function one obtains

$$\xi(\omega)=\int \frac{d^3\vec{q}}{(2\pi)^3}\phi_M(\vec{q})\phi_M(\vec{l}), \quad \vec{l}^{\perp}=\vec{q}^{\perp},$$

$$l^3=\omega q^3+(\omega^2-1)^{1/2}(m^2+\vec{q}^2)^{1/2} \tag{41}$$

where the ET meson wave function $\phi_M(\vec{q})$ is normalized to unity, i.e., one has the normalization $\xi(1)=1$.

It is interesting to note that in the approximation of noninteracting light quarks within the heavy baryon the meson and baryon IW functions become related if one assumes that the interaction potentials between the heavy quark and the light quark-antiquark are the same [47]. Let $\zeta(\omega)$ be the IW function describing the transition $\Lambda_b\to\Lambda_c$ and $\xi(\omega)$ the mesonic IW function. Using Eq. (39) one then obtains

$$\zeta(\omega)=\frac{1}{2}(\omega+1)f(\omega)=\frac{1}{2}(\omega+1)\xi^2(\omega). \tag{42}$$

The two light quarks in the heavy baryon move independently in the mean field produced by the heavy quark where the heavy quark is fixed in the center of mass of the heavy baryon. Such a physical picture is quite attractive since one can relate the heavy baryon form factors to the heavy meson form factors (to be more precise, to the would-be heavy meson form factors, in which the interquark interaction potential coincides, by definition, with the potential acting between the heavy and light quarks in the heavy baryons). Anyway, the model of noninteracting light quarks enables one to effectively reduce the calculation of heavy baryon observables to the two-body case and thus enormously simplifies the treatment of the problem under study.

The assumption of noninteracting light quarks has two aspects which one may refer to as ''kinematical'' and ''dynamical'' aspects. Let us elaborate on these two aspects. The kinematical aspect deals with the spins of the quarks and manifests itself in the relativistic factor $\frac{1}{2}(\omega+1)$ in e.g. Eq. (42). We would like to emphasize that the kinematical aspect of the noninteracting light quark model is already implicit in the spectator model wave functions which are derived from the equal-velocity assumption (all quarks being on mass shell and propagating freely). Not surprisingly, the overlap integral of the baryon wave functions contains the factor $\frac{1}{2}(\omega+1)$ explicitly [see e.g. Eq. (36)]. The physical origin of this factor can be seen by considering the transition amplitude in the crossed channel, corresponding to the production of the heavy baryon-heavy antibaryon pair by the virtual photon born in the e^+e^- annihilation process.

Let us first consider the physical picture where both light quarks are produced independently from the vacuum through the exchange of many soft gluons with the total quantum numbers $J^P=0^+$ [Fig. 1(a)]. The intrinsic parity of the $(\frac{1}{2}^+\frac{1}{2}^+)$ pair is negative and, consequently, $(LSJ)=(110)$ for this transition. Thus one has a threshold factor of $|\vec{p}|$ for each of the two P-wave quark-antiquark pairs, i.e. in total

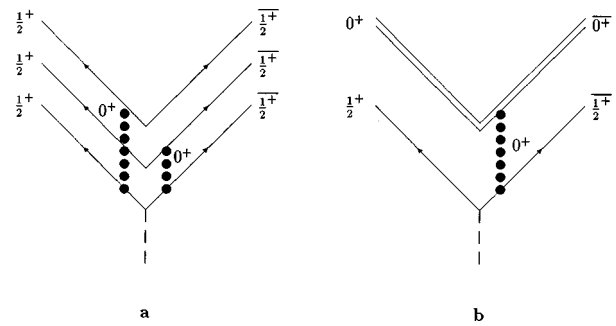

FIG. 1. Heavy baryon–heavy antibaryon pair production in the e^+e^- annihilation: (a) Light quarks are produced independently from the vacuum by soft gluon exchanges in the $J^P=0^+$ channel; (b) tightly bound light diquark is produced from the vacuum.

one has a $|\vec{p}|^2$ threshold factor where $|\vec{p}|$ is the magnitude of the c.m. relative three-momentum of the quark pair.

As opposed to this, let us consider the situation when two light quarks inside the heavy baryon are tightly bound in a diquark with the quantum numbers $J^P=0^+$ [Fig. 1(b)]. Then the amplitude for the transition $0^+\to 0^+ + \overline{0^+}$ is an S-wave transition without any threshold factor.

In the equal-velocity approximation the magnitude of the c.m. relative three-momentum can be expressed in terms of the velocity transfer variable $\omega=(v\cdot v')=M_B^{-2}(p_1\cdot p_2)$ where p_1 and p_2 are the momenta of baryon and antibaryon produced in the e^+e^- annihilation. It is a simple task to derive $|\vec{p}|/m=|\vec{v}'|=(\frac{1}{2}(\omega-1))^{1/2}$. In the direct channel ω is replaced by $-\omega$ and thus the threshold factor $|\vec{p}|^2$ turns into $\frac{1}{2}(\omega+1)$ present in Eq. (36).

The dynamical aspect of the noninteracting light quark model consists in the assumption of the factorization of baryon radial wave function with regard to variables $\vec{p}_1$ and $\vec{p}_2$. This can be achieved by setting the interaction potential between the light quarks to zero. From a rigorous point of view, one should then also replace the interaction between the heavy and light quarks in the heavy baryon by some effective ''mean field'' interaction. Below we shall present the results of numerical calculations which demonstrate the effect of the noninteracting light quark approximation in heavy baryon observables.

IV. RESULTS

In this section we present our numerical results both for heavy meson and heavy baryon sectors. We use oscillator wave functions as given by Eqs. (16)–(18) for baryons and corresponding oscillator wave functions for mesons. Oscillator wave functions are known to provide a good basis of trial wave functions in the variational solution of the bound-state equation [32,50].

We would like to emphasize that in the present paper we have not attempted to obtain a precise description of meson and baryon data by the fine tuning of a large number of model parameters. Rather, we want to demonstrate that, in the framework considered in the present paper, one achieves a reasonably good description of experimental numbers both

TABLE I. Heavy meson leptonic decay constants.

Process	Quantity	Our	Lattice [7]
$D\to l\nu_l$	f_D (MeV)	226	200 ± 30
$B\to l\nu_l$	f_B (MeV)	134	180 ± 40

in the meson and baryon sector with only a few parameters. We have also checked that the dependence of our numerical results on these parameters is rather moderate.

In order to reduce the number of free parameters as much as possible we do not distinguish between the masses of the light quarks and set $m_u=m_d=m_s=m$. It is known that the effect of the m_s-m_u mass difference in the baryon wave functions is rather small and we neglect it in the present treatment.

First we turn to the calculation of the heavy meson leptonic decay constants defined by

$$iP_\mu f_P=-iN_C\int\frac{d^3\vec{p}}{(2\pi)^3}\mathrm{Tr}[\tilde{\chi}(P;\vec{p})\gamma_\mu\gamma^5] \tag{43}$$

where $\tilde{\chi}(P;\vec{p})$ denotes the equal-time meson wave function in the c.m. frame. In an arbitrary reference frame the heavy meson wave function is given by (see e.g. [38])

$$\tilde{\chi}(P;p^T)=c_M\gamma^5(1-\not{v})M_a^b\phi_M[-(p^T)^2]. \tag{44}$$

Here M_a^b denotes the meson flavor matrix and c_M is the normalization constant. Using the BS normalization condition and assuming the radial part of the meson wave function to be of the oscillator type $\phi_M\sim\exp[(p^T)^2/\Lambda^2]$, it is a straightforward task to obtain

$$f_P=\left(\frac{2N_C\Lambda^3}{\pi^{3/2}M_P}\right)^{1/2} \tag{45}$$

where M_P denotes the meson mass. As can be seen from Eq. (45) the calculated leptonic decay constant exibits the well-known $M_P^{-1/2}$ scaling behavior.

Next we turn to the calculation of the heavy meson Isgur-Wise (IW) function according to Eq. (41). The IW function depends only on the ratio m/Λ where m is the light quark mass. In order to fix this ratio we calculate the slope parameter of the heavy meson's IW wave function using again oscillator wave functions for the heavy meson. One obtains

TABLE II. Slope of the heavy meson IW function.

ρ^2	Approach
1.00 (input)	Our
$0.9^{+0.2+0.4}_{-0.3-0.2}$	Lattice [44]
0.84 ± 0.02	QCD sum rules [45]
0.70 ± 0.25	QCD sum rules [46]
$0.42-0.92$	Quark Confinement Model [29]
1.02	Quasipotential [17]

TABLE III. Experimental and theoretical values for the branching ratios (in %) and asymmetry parameters in the decay $B\to D(D^*)e\bar{\nu}$.

	Theory	Experiment
$Br(B\to D)$	$2.05\ \lvert V_{bc}/0.04\rvert^2$	1.6 ± 0.7 [1], 1.9 ± 0.5 [1]
$Br(B\to D^*)$	$5.35\ \lvert V_{bc}/0.04\rvert^2$	6.6 ± 2.2 [1], 4.4 ± 0.4 [1]
α_{pol}	1.71	$1.1\pm 0.4\pm 0.2$ [48]
α'	0.63	
A_{FB}	0.083	$0.20\pm 0.08\pm 0.06$ [49]
A_{FB}^T	0.20	

$$\rho^2=\frac{3}{4}+\frac{m^2}{\Lambda^2}. \tag{46}$$

We further use the calculated values of leptonic decay constants in Eq. (45) to provide absolute values for m and Λ. Most of the present theoretical investigations of the slope parameter converge around the value $\rho^2\approx 1$. With $\rho^2=1$ as input one obtains $\Lambda=2m$ from Eq. (46). Further, taking the value $m=250$ MeV for the constituent quark mass and, respectively, $\Lambda=500$ MeV for the wave function range parameter, we obtain a reasonable fit to the experimental leptonic decay constants as shown in Table I. For comparison, we have also listed the results of the recent lattice calculations of the same quantities in Table I. In Table II we give some recent results on the heavy meson IW function slope parameter. Note that the functional dependence of the heavy meson IW function in our approach is well approximated by the formula

$$\xi(\omega)=\left(\frac{2}{\omega+1}\right)^{2\rho^2} \tag{47}$$

in our approach where, as was mentioned above, we take $\rho^2=1$ as input.

With the above two parameter values we present our results of the calculation of the decay observables in B-meson semileptonic transitions in Table III. We give the branching ratios for the weak semileptonic decays $B\to D$, $B\to D^\star$ and values for the polarization-type observables α_{pol}, α', A_{FB} and A_{FB}^T [2,51] in these decays. As can be seen from Table III, the agreement of the calculated quantities with existing experimental data is satisfactory.

Next we present our results in the baryon sector. We use the same values for the two model parameters as in the meson sector. To begin with we discuss the model of noninteracting light quarks with the additional assumption that the interaction between the heavy and light quarks in the heavy baryon is the same as the interaction between the heavy quark and light antiquark in the heavy meson [47]. With this assumption the range parameter in the heavy baryon equal-time wave function defined by $\Lambda_B=2m\omega_0$ turns out to be the same as the parameter Λ in the meson wave function. Consequently, the heavy baryon IW function can be also parametrized by the ansatz (47) but with

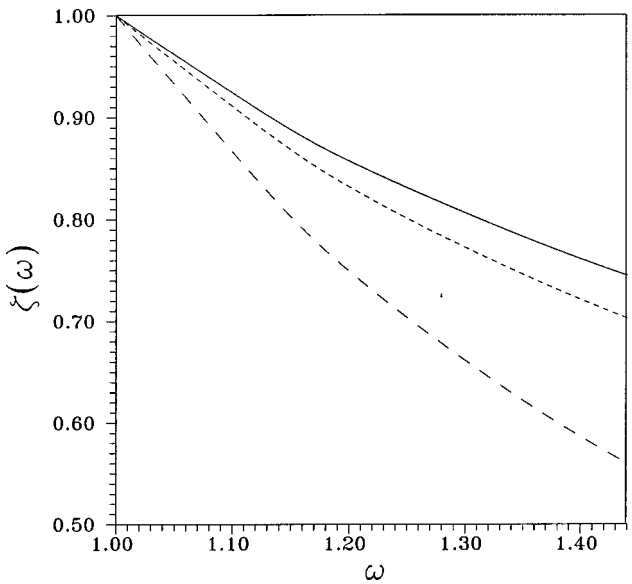

FIG. 2. The heavy baryon IW function $\zeta(\omega)$ in the noninteracting light quark model and in the full model for the different values of the wave function range parameter Λ_B: full model with $\Lambda_B = 500$ MeV (solid line); noninteracting light quark model with $\Lambda_B = 500$ MeV (long-dashed line); full model with $\Lambda_B = 355$ MeV (short-dashed line).

$$\rho_B^2 = 1 + \sum_{light} \left(\frac{m_{light}}{\Lambda} \right)^2 = 2\rho^2 - \frac{1}{2} \tag{48}$$

such that $\rho_B^2 = 1.5$ in the model of noninteracting light quarks.

When we present our results in the following on the functional dependence of the heavy baryon IW function $\zeta(\omega)$ we shall always compare the two cases where the interaction between the light quarks is switched either on or off. The results of our calculation are given in Fig. 2. As can be seen from Fig. 2 the IW function is substantially flatter (with the same set of model parameters $m/\Lambda = 0.5$) when the interactions in the light diquark are taken into account. At maximum recoil the difference between the values of the function $\zeta(\omega)$ calculated in the full (interacting) model and in the noninteracting light quark model amounts up to 30%. The function $\zeta(\omega)$ in the full model is well approximated by the functional dependence

$$\zeta(\omega) = \left(\frac{2}{\omega + 1} \right)^{a + b/\omega} \tag{49}$$

with $a = 1.23$ and $b = 0.4$. This corresponds to a slope parameter of $\rho_B^2 = 0.81$ which is much lower than the slope parameter $\rho_B^2 = 1.5$ in the noninteracting light quark model.

In our simplified approach the heavy baryon IW function depends only on the ratio m/Λ_B. Fixing m at $m = 250$ MeV we have evaluated the IW function for two different values of Λ_B. In Fig. 2 we present our results for the value $\Lambda_B = \Lambda/\sqrt{2} = 355$ MeV, corresponding to the popular one half rule for baryons (see e.g. [50]). According to this rule which is strongly supported by phenomenology, the interactions between quarks in the baryon are down by the factor $\frac{1}{2}$ as compared to the interactions between the quark and antiquark in the meson. From Fig. 2 one observes that even for such a substantial change of the value of the parameter Λ_B the IW function is only sligthly modified. In particular the difference between the noninteracting and full calculation persists. At maximum velocity transfer the IW functions calculated in the full model with these two values of Λ_B differ only by about 5%. The functional dependence of the IW function with $\Lambda_B = 355$ MeV can again be well approximated by the representation (49) but now with $a = 1.54$, $b = 0.4$; $\rho_B^2 = 0.97$.

TABLE IV. Exclusive decay rates of bottom baryons (in 10^{10} sec^{-1}) for $|V_{bc}| = 0.04$.

Process	[53]	[54]	[10]	[11]	Our (1)	Our (2)
$\Lambda_b^0 \to \Lambda_c^+$	5.9	5.1	5.14	5.39	6.52	6.09
$\Xi_b^0 \to \Xi_c^+$	7.2	5.3	5.21	5.27	6.83	6.42
$\Sigma_b^+ \to \Sigma_c^{++}$	4.3			2.23	1.90	1.65
$\Omega_b^- \to \Omega_c^0$	5.4	2.3	1.52	1.87	2.05	1.81
$\Sigma_b^+ \to \Sigma_c^{*++}$				4.56	4.17	3.75
$\Omega_b^- \to \Omega_c^{*0}$			3.41	4.01	4.55	4.13

To summarize our results on the functional dependence of the Isgur-Wise function one finds that the model of noninteracting light quarks can be clearly distinguished from the full model, at least within the spectator picture. There is no theoretical reason to prefer one to the other. The issue which of the two ansätze has to be preferred has to be settled by experiments. An attractive feature of our approach, apart from its simplicity, consists in the weak dependence of the predictions on the precise values of the model parameters which helps in distinguishing between various models.

In Table IV we present the calculated total widths of bottom baryon weak semileptonic transitions calculated in the full model with $\Lambda_B = 500$ MeV, column (1), and $\Lambda = 355$ MeV, column (2). For comparison we also list results from other model approaches. We see that the overall agreement of our results with results from other models discussed in the literature is reasonable. In the absence of experimental data, however, one can not fix the precise value of the wave function range parameter Λ_B owing to the weak dependence of the results on this parameter.

In Table V we give values for the asymmetry parameters in the Λ_b baryon semileptonic transitions [11,52]. Rows (1) and (2) are for $\Lambda_B = 500$ MeV and $\Lambda_B = 355$ MeV as in Table IV. For comparison, we again list the results obtained within different approaches. As can be seen from Table V

TABLE V. Asymmetry parameters for Λ_b decay.

	α	α'	α''	γ	α_P	γ_P
Our (1)	-0.78	-0.11	-0.55	0.54	0.41	-0.15
Our (2)	-0.78	-0.11	-0.55	0.54	0.41	-0.16
[11]	-0.76	-0.12	-0.53	0.56	0.39	-0.17
[13]	-0.74	-0.12	-0.46	0.61	0.33	-0.19

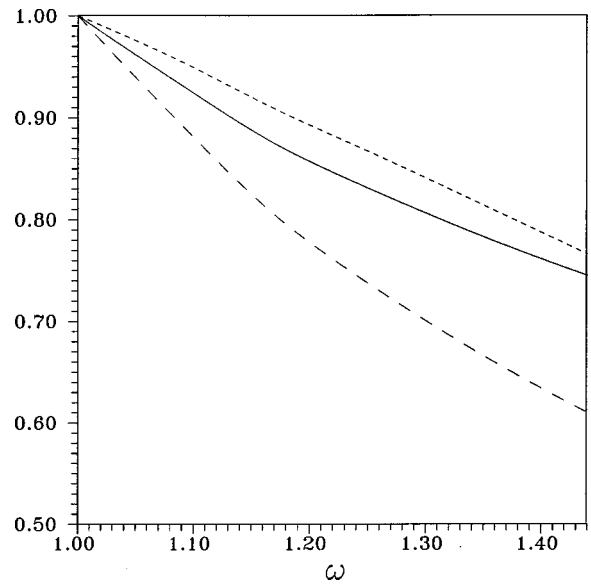

FIG. 3. Relativistic effect in the heavy baryon IW function in full model with $\Lambda_B = 500$ MeV: the function $\zeta(\omega)$; relativistic case (solid line); the universal form factor function $f(\omega)$; relativistic case (long-dashed line); the function $\zeta_{NR}(\omega)$; nonrelativistic case (short-dashed line).

the asymmetry parameters are rather insensitive to the particular model in which they are calculated.

Last but not least, we shall discuss the nonrelativistic limit of our approach where the crucial role of the factor $\frac{1}{2}(\omega+1)$ will become transparent. First, note that for $v^\mu = (1,\vec{0})\omega = v'^0 = \sqrt{1+\vec{v}'^2} \approx 1$ up to the terms of order of $\vec{v}'^2$. Thus this factor drops in the nonrelativistic limit. Further, from Eq. (40) one obtains

$$\vec{l}_1 = \vec{q}_1 + 6m\vec{v}', \quad \vec{l}_{23} = \vec{q}_{23} \tag{50}$$

in this limit which coincides with the formulas presented in Refs. [53,54]. From Eqs. (50) one immediately obtains the nonrelativistic (NR) result for the Isgur-Wise function

$$\zeta_{NR}(\omega) = f_{NR}(\omega) = \exp\left[-\frac{m^2}{\Lambda^2}(\omega^2-1)\right] \tag{51}$$

with $\rho^2_{B,NR} = 0.5$. In Fig. 3 we have plotted the functions $f(\omega)$ and $\zeta(\omega)$ in the full model in the relativistic case as well as the function $\zeta_{NR}(\omega)$ given by Eq. (51). It is interesting to note that although there exists a large relativistic dynamical effect since $\zeta_{NR}(\omega)$ and $f(\omega)$ are significantly different, the absence of the relativistic factor $\frac{1}{2}(\omega+1)$ in the nonrelativistic case compensates for this. In the heavy meson case where such a factor is absent the slope parameter ρ^2 of the IW function is grossly underestimated in the nonrelativistic treatment [3,4]. In our opinion our results unambigouosly indicate the importance of including relativistic effects when studying heavy baryon transitions.

V. SUMMARY AND OUTLOOK

We have presented a simple calculation of the heavy meson and heavy baryon semileptonic decay observables with the use of the field-theoretical BS approach where both heavy mesons and heavy baryons are treated on the same footing. While widely in use in the calculation of mesonic two-body bound-state observables, the BS approach has been known to encounter conceptual difficulties when applied to the baryonic three-body case. In the present paper we have explicitly demonstrated that the treatment of the three-body bound state systems can proceed along similar lines as in the two-body case when the constituents interact via instantaneous kernels.

Up to this point our investigation of the three-body problem has been restricted to the so-called ''spectator picture'' which provides a powerful tool for the study of heavy baryon weak interactions. In the spectator approximation the spin structure of the baryon wave functions and decay matrix elements is remarkably simple, and model-independent relations emerge between various decay amplitudes in this limit. This has been demonstrated by a systematic and comprehensive analysis of the heavy baryon weak nonleptonic decays, carried out in papers [37,38]. Further improvements on these ideas were given in Refs. [14,15], where the so-called Lagrangian spectator model has been proposed. The Lagrangian spectator model allows for the microscopic evaluation of the various ''overlap integrals'' (the reduced matrix elements, from the group-theoretical point of view), and thus allows one to compute a large number of experimental observables in heavy baryon decays: rates, asymmetry parameters, etc.

The spectator approximation is based upon a very simple and transparent physical picture: the internal motion of quarks (both heavy and light) inside the hadron is very slow; all quarks are assumed to be on mass shell and are assumed to have the same velocity, which coincides with the velocity of the hadron as a whole. All approximations which we have used in the treatment of the heavy baryon weak transitions are in accordance with the above physical picture, and can be deduced from it. Indeed, in the present paper we have demonstrated that the general BS approach to transition matrix elements can be reduced, step-by-step, to the spectator model treatment of the weak transition matrix elements with the use of the above approximations. We would like to emphasize that the BS approach enables one to evaluate the full matrix elements by expressing them in terms of the overlaps between equal-time BS wave functions. Note also, that even in the spectator limit our approach is *not* reduced to a nonrelativistic approach: only that part of motion which is related to the quark relative momenta is treated nonrelativistically, whereas the c.m. motion of hadrons is taken into account in a completely relativistic fashion. Thus, even at this stage our model is not reduced (and differs significantly) from the nonrelativistic baryon models which are used for the calculation of transition amplitudes [53,54]. Moreover, the necessity of the inclusion of the relativistic effects is readily seen even from the results of our calculations in the spectator picture.

We would like to mention that the physical assumption of

''slow'' interquark motion for the light quarks (which lies at the heart of the whole spectator picture) cannot be rigorously justified from a theoretical point of view. According to common belief one has the value $|\vec{p}|/m \sim 1$ for light quarks inside the hadron. Since, the quark spin-spin interaction effects are proportional to powers of $|\vec{p}|/m$ they are *a priori* expected to give a sizeable contribution to the calculated baryon observables in contrast to the assumptions of the spectator model. Nevertheless, the present treatment of heavy hadron transitions is based on a relativistically consistent formulation of the spectator picture. It can be used as a stepping stone for the inclusion of $O(|\vec{p}|/m)$ spin effects at a later stage. We plan to address this problem in future publications.

In addition, we plan to apply the present BS approach and its possible modification with the inclusion of the spin effects to the more involved and interesting problems of heavy hadron physics, such as nonleptonic, one-pion and radiative decays of heavy baryons.

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APPENDIX A: MATRIX ELEMENTS IN THE SPECTATOR APPROXIMATION

In this appendix we present details of how to evaluate the heavy baryon weak transition matrix elements in the spectator approximation. In particular, we shall demonstrate that the spin structure of Eq. (30) considerably simplifies in the spectator picture. First, note that the factors Γ_0 and Π in Eq. (30) can be dropped since they are reduced to an identity operator when acting on the spectator model wave functions. Moreover, $\tilde{g}_0^{-1}$ is reduced to

$$\tilde{g}_0^{-1}(v') \to M_B - \sum_{i=1}^{3} (m_i^2 - p_i^2 + (p_i \cdot v')^2)^{1/2}. \tag{A1}$$

An analogous relation holds for $\tilde{g}_0^{-1}(v)$.

In order to further simplify Eq. (30) we perform a change of integration variables corresponding to Lorentz boosts which boost the initial and final baryon wave functions to the rest frame. One has

$$\begin{aligned}
&q_1^0 \to (q_1 \cdot v'), \quad \vec{q}_1 \to \vec{q}_1 - q_1^0 \vec{v}' + (v'^0+1)^{-1}(\vec{q}_1 \vec{v}')\vec{v}' \\
&l_1^0 \to (l_1 \cdot v), \quad \vec{l}_1 \to \vec{l}_1 - l_1^0 \vec{v} + (v^0+1)^{-1}(\vec{l}_1 \vec{v})\vec{v} \\
&q_{23}^0 \to (q_{23} \cdot v'), \quad \vec{q}_{23} \to \vec{q}_{23} - q_{23}^0 \vec{v}' \\
&\qquad + (v'^0+1)^{-1}(\vec{q}_{23}\vec{v}')\vec{v}' \\
&l_{23}^0 \to (l_{23} \cdot v), \quad \vec{l}_{23} \to \vec{l}_{23} - l_{23}^0 \vec{v} + (v^0+1)^{-1}(\vec{l}_{23}\vec{v})\vec{v}.
\end{aligned} \tag{A2}$$

Under this transformation the wave functions of the final and initial baryon are transformed to $C\overline{\theta}(v')\phi(\vec{q}_1\vec{q}_{23})$ and $C\theta(v)\phi(\vec{l}_1\vec{l}_{23})$, whereas $\tilde{g}_0^{-1}(v')$ and $\tilde{g}_0^{-1}(v)$ transform to $M_B' - \Sigma_{i=1}^3 w_i'$ and $M_B - \Sigma_{i=1}^3 w_i$, respectively, with

$$\begin{aligned}
&w_i' = (m_i^2 + \vec{p}_i'^2)^{1/2}, \quad w_i = (m_i'^2 + \vec{k}_i'^2)^{1/2} \\
&p_1' = -\frac{1}{3} q_1, \\
&p_2' = \frac{m_2}{3(m_2+m_3)} q_1 - \frac{1}{2\sqrt{3}} q_{23}, \\
&p_3' = \frac{m_3}{3(m_2+m_3)} q_1 + \frac{1}{2\sqrt{3}} q_{23}, \\
&k_1' = -\frac{1}{3} l_1, \quad k_2' = \frac{m_2}{3(m_2+m_3)} l_1 - \frac{1}{2\sqrt{3}} l_{23}, \\
&k_3' = \frac{m_3}{3(m_2+m_3)} l_1 + \frac{1}{2\sqrt{3}} l_{23}.
\end{aligned} \tag{A3}$$

After the change of integration variables one has

$$S^{(i')}(p_i) \to \frac{(\mu_i' M_B' + p_i'^0)\not{v}' + w_i' - (m_i' + w_i')^{-1}\vec{p}_i'^2 + (\vec{p}_i'\vec{v}')(\gamma_0 + (v'^0+1)^{-1}(\vec{\gamma}\vec{v}')) - (\vec{\gamma}\vec{p}_i')}{w_i'^2 - (\mu_i' M_B' + p_i'^0)^2 - i0}, \tag{A4}$$

$$S^{(i)}(k_i) \to \frac{(\mu_i M_B + k_i'^0)\not{v} + w_i - (m_i + w_i)^{-1}\vec{k}_i'^2 + (\vec{k}_i'\vec{v})(\gamma_0 + (v^0+1)^{-1}(\vec{\gamma}\vec{v})) - (\vec{\gamma}\vec{k}_i')}{w_i^2 - (\mu_i M_B + k_i'^0)^2 - i0}. \tag{A5}$$

In the spectator approximation one neglects the transformed momenta $\vec{p}_i'$,$\vec{k}_i'$ in the numerators of Eq. (A4). Further, we can set $\not{v}\to 1$ and $\not{v}'\to 1$ in the numerators since they act either on the final or initial spectator wave functions. As a result one obtains in the spectator approximation

$$S^{(i')}(p_i)\to\frac{1}{w_i'-\mu_i' M_B'-p_i'^0-i0},$$

$$S^{(i)}(k_i)\to\frac{1}{w_i-\mu_i M_B-k_i'^0-i0}. \tag{A6}$$

Further, in the heavy quark limit one neglects terms of order $(2m_1)^{-1}\vec{p}_1'^2$ and $(2m_1')^{-1}\vec{k}_1'^2$. One then obtains the following effective propagators

$$S^{(1')}(p_1)\to\frac{1}{-\bar{\Lambda}+m_2+m_3-p_1'^0-i0},$$

$$S^{(1)}(k_1)\to\frac{1}{-\bar{\Lambda}+m_2+m_3-k_1'^0-i0},$$

$$S^{(i)}(p_i)\to\frac{1}{w_i'-m_i-p_i'^0-i0},$$

$$S^{(i)}(k_i)\to\frac{1}{w_i-m_i-k_i'^0-i0},\quad i=2,3. \tag{A7}$$

The factors $\tilde{g}_0^{-1}$ turn into

$$\tilde{g}_0^{-1}(v')\to\bar{\Lambda}-w_2'-w_3',\quad \tilde{g}_0^{-1}(v)\to\bar{\Lambda}-w_2-w_3 \tag{A8}$$

where all reference to the heavy quark mass has disappeared, as it should indeed be. Further, substituting these expressions in Eq. (30), one immediately arrives at the Eqs. (31) and (32).

It is interesting to note that one can write down ''Feynman rules'' for the construction of the current matrix elements in the spectator approximation which are remarkably simple since the vertices and propagators have a trivial identity structure in the space of Lorentz indices. As an example we evaluate the current-induced transition between heavy baryons according to the diagram Fig. 4 where we have added the appropriate vertex and propagator structure. The spectator model Feynman rules for this transitions may be summarized as follows:

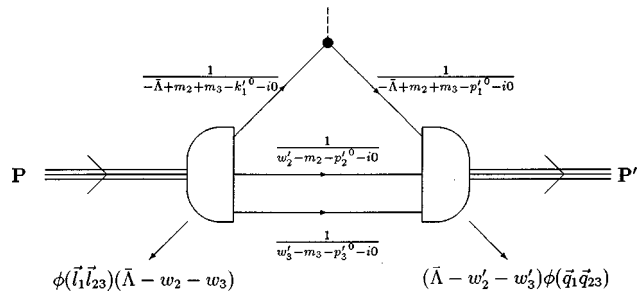

FIG. 4. ''Feynman rules'' in the spectator quark approximation.

(1) A factor $(\bar{\Lambda}-\Sigma_{light}w_{light}(\vec{p}))\phi(\vec{p}_1\vec{p}_2\vec{p}_3)$ for each heavy baryon vertex where $\phi(\vec{p}_1\vec{p}_2\vec{p}_3)$ denotes the radial part of the heavy baryon equal-time BS wave function.

(2) The heavy quark propagator:

$$\frac{1}{-\bar{\Lambda}+\sum_{light} m_{light}-p'^0-i0}. \tag{A9}$$

(3) The light quark propagator:

$$\frac{1}{w_i'-m_i'-p_i'^0-i0}. \tag{A10}$$

(4) Dirac δ-functions corresponding to the four-momentum conservation of light spectator quarks. All momenta are boosted to the rest frame of either the final or the initial baryon.

(5) Integration over all relative four-momenta.

(6) The spin-flavor structure of matrix elements is given by spectator model wave function scalar products.

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