Semileptonic $B \to \rho$ and $B \to a_1$ transitions in a quark-meson model

A. Deandrea
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19, D-69120 Heidelberg, Germany

R. Gatto
Département de Physique Théorique, Université de Genève, 24 quai E. Ansermet, CH-1211 Genève 4, Switzerland

G. Nardulli and A. D. Polosa
Dipartimento di Fisica, Università di Bari and INFN Bari, via Amendola 173, I-70126 Bari, Italy

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We evaluate the form factors governing the exclusive decays $B \to \rho l \nu$, $B \to a_1 l \nu$, by using an effective quark-meson Lagrangian. The model is based on meson-quark interactions, and the computation of the mesonic transition amplitudes is performed by considering diagrams with heavy mesons attached to loops containing heavy and light constituent quarks. This approach was successfully employed to compute the Isgur-Wise form factors and other hadronic observables for negative and positive parity heavy mesons and is presently used for exclusive heavy-to-light weak transitions. We also evaluate a few strong coupling constants appearing in chiral effective Lagrangians for heavy and light mesons. [S0556-2821(99)04807-9]

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I. INTRODUCTION

The impressive experimental program for the study of B decays carried out in recent years has improved our knowledge of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and *CP* violations. In the next few years still more abundant data are to come, especially from the dedicated B factories Belle [1] and BaBar [2].

One of the most important goals in these investigations will be a more precise determination of the CKM matrix element V_{ub} and, to this end, both exclusive and inclusive $b \to u$ semileptonic transitions will be used. The two methods have their own uncertainties. Using the inclusive reaction implies the need to use perturbative QCD methods in the region near the end point of the lepton spectrum, where many resonances are present and perturbative methods are less reliable. This difficulty can be avoided by considering exclusive channels and summing them up or taking them separately; however, the use of the exclusive channels forces us to evaluate the hadronic matrix elements by nonperturbative methods that are either approximate or model dependent. Examples of these approximations are given by nonperturbative methods derived from quantum chromodynamics (QCD) first principles, i.e. lattice QCD and QCD sum rules. The drawback of these methods is the difficulty in improving the precision and in evaluating reliably the theoretical error, which follows from the nature of the approximations, i.e. the quenched approximation in lattice QCD and a truncated operator product expansion in QCD sum rules. Although less fundamental, other approaches can be reliably used to give estimates of the hadronic matrix elements that appear in exclusive $b \to u$ transitions and we refer here to constituent quark models. At the present stage of our understanding of hadronic interactions from first principles, they offer in our opinion a viable alternative, and the model dependence, which is obvious in this approach, can be used to estimate the overall theoretical uncertainty. In this paper we shall follow this route and use a constituent quark meson (CQM) model, introduced and defined in [3] (hereafter referred to as I) to compute two semileptonic exclusive heavy to light decays, viz. B to the light vector mesons ρ and a_1. The first decay has been recently observed by the CLEO Collaboration [4] (see also [5]) which has measured the branching ratio of the semileptonic decay $B \to \rho l \nu$:

$$\mathcal{B}(B^0 \to \rho^- l^+ \nu) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}. \tag{1}$$

This decay will be used as a test of the model of Ref. [3], since we do not introduce here any new parameter. On the other hand, $B \to a_1 l \nu$ is a prediction of this model yet to be tested by experiment.

The CQM model developed in I tries to conjugate the simplicity of an effective Lagrangian encoding the symmetries of the problem together with some dynamical information coming from the observed facts of confinement and chiral symmetry breaking. In spite of the simple way in which the dynamics is implemented, the results found in I are encouraging. We discussed there the following issues: leptonic constants for heavy mesons, Isgur-Wise form factors for heavy mesons of both negative and positive parity, strong coupling constants of heavy mesons and pions, radiative decays of D^*, B^*; the comparison with data was satisfactory whenever it was available.

The plan of the present paper is as follows. The model is briefly reviewed in Sec. II. In Sec. III we compute the direct contribution to the form factors for the $B \to \rho l \nu$, $B \to a_1 l \nu$ decays, i.e. the contribution arising from diagrams where the weak current directly interacts with the quarks belonging to heavy and light mesons. In Sec. IV we compute the strong coupling constants of ρ and a_1 to heavy mesons: these couplings are relevant for the calculation of the polar diagrams, i.e. the diagrams where the weak current couples to B and ρ (or a_1) via an intermediate particle. These contributions (called polar contributions) are described in Sec. V. In Sec.

VI we present our results and compare them with other approaches and with available data. In Sec. VII we draw our conclusions and in the Appendix we collect formulas and integrals used in the calculations.

II. CQM MODEL

We begin with a short summary of the CQM model; for a more detailed treatment see I. The model is an effective field theory containing a quark-meson Lagrangian:

$$\mathcal{L}=\mathcal{L}_{ll}+\mathcal{L}_{hl}\,. \tag{2}$$

The first term involves only the light degrees of freedom (a constituent quark model for light quarks and mesons was originally suggested by Manohar and Georgi [6]). To the fields considered in I, i.e. the light quark field χ and the pseudo-scalar $SU(3)$ octet of mesons π, we add the vector meson and axial vector meson octets ρ_μ and a_μ. Considering only the kinetic part of the light quarks and mesons as well as the quark-meson interactions at the lowest order, we have, for $\mathcal{L}_{ll}$,

$$\mathcal{L}_{ll}=\frac{f_\pi^2}{8}\partial_\mu\Sigma^\dagger\partial^\mu\Sigma+\frac{1}{2g_V^2}\mathrm{tr}[\mathcal{F}(\rho)_{\mu\nu}\mathcal{F}(\rho)^{\mu\nu}]$$
$$+\frac{1}{2g_A^2}\mathrm{tr}[\mathcal{F}(a)_{\mu\nu}\mathcal{F}(a)^{\mu\nu}]+\overline{\chi}(iD^\mu\gamma_\mu-m)\chi$$
$$+\overline{\chi}(h_\pi\mathcal{A}^\mu\gamma_\mu\gamma_5-ih_\rho\rho^\mu\gamma_\mu-ih_a a^\mu\gamma_\mu\gamma_5)\chi. \tag{3}$$

Let us discuss the various terms in this equation. The first three terms refer to pions, light vector and axial vector respectively. We have $\xi=\exp(i\pi/f_\pi)$, $\Sigma=\xi^2$, $f_\pi=132$ MeV; the ρ and a_1 field strengths are given by

$$\mathcal{F}(x)_{\mu\nu}=\partial_\mu x_\nu-\partial_\nu x_\mu+[x_\mu,x_\nu], \tag{4}$$

where, consistently with the notations of [7] (see also [8]), we write

$$\rho_\mu=i\frac{g_V}{\sqrt{2}}\hat{\rho}_\mu\,,\qquad g_V=\frac{m_\rho}{f_\pi}\simeq 5.8. \tag{5}$$

By analogy we also write ($m_a\simeq 1.26$ GeV is axial vector meson mass)

$$a_\mu=i\frac{g_A}{\sqrt{2}}\hat{a}_\mu\,,\qquad g_A=\frac{m_a}{f_\pi}\simeq 9.5. \tag{6}$$

Here $\hat{\rho},\hat{a}$ are Hermitian 3×3 matrices of the negative and positive parity light vector mesons. The fourth term in Eq. (3) contains the light quarks, with $D_\mu=\partial_\mu-i\mathcal{V}_\mu$ and

$$\mathcal{V}^\mu=\frac{1}{2}(\xi^\dagger\partial^\mu\xi+\xi\partial^\mu\xi^\dagger). \tag{7}$$

Therefore it gives both the kinetic term of the light quark and its coupling to an even number of pions. For m, in I we took the value $m=300$ MeV (for non-strange quarks). The last three terms describe further interactions between light quarks and light mesons. The coupling of the light quark to an odd number of pions is mediated by

$$\mathcal{A}^\mu=\frac{i}{2}(\xi^\dagger\partial^\mu\xi-\xi\partial^\mu\xi^\dagger). \tag{8}$$

Moreover, consistently with a low energy theorem for pions, we put $h_\pi=1$. Concerning the interactions of vector particles, we put $h_\rho=\sqrt{2}m_\rho^2/g_V f_\rho$, $h_a=\sqrt{2}m_a^2/g_A f_a$, where f_ρ and f_a are the leptonic constants. For the ρ leptonic constant we use $f_\rho=0.152$ GeV^2, as given by ρ^0,ω decay into e^+e^-. For f_a a phenomenological determination using $\tau\to\nu_\tau\pi\pi\pi$ was obtained in [9], i.e. $f_a=0.25\pm0.02$ GeV^2, a result which agrees with the one found by QCD sum rules [10]. On the other hand from lattice QCD one obtains $f_a=0.30\pm0.03$ GeV^2 [11]. Since $1/f_a$ multiplies all the amplitudes involving the a_1 meson, the uncertainty in f_a will induce a normalization uncertainty in all the amplitudes involving the light axial-vector meson. We note that our choice for h_ρ and h_a implements the hypothesis of the vector and axial-vector meson dominance. Numerically we find

$$h_\rho\simeq h_a\simeq 0.95. \tag{9}$$

We also observe that our choice for the normalization of the light axial vector meson field, Eq. (6), is conventional since g_A disappears from the physical quantities (in [8] $g_A=g_V$ is assumed). We also differ from the phenomenological analyses of Ref. [8] since we do not assume the current algebra relations $m_a^2=2m_\rho^2$ and $f_a=f_\rho$ that seem to have substantial violations.

Let us now discuss $\mathcal{L}_{hl}$, i.e. the part of the Lagrangian that contains both light and heavy degrees of freedom, in particular the heavy quark (Q) and mesons ($Q\overline{q}$). According to heavy quark effective theory (HQET) [12], in the limit $m_Q\to\infty$, these mesons can be organized in spin-parity multiplets. We shall consider here the negative parity spin doublet (P,P^*) (e.g. B and B^*) and its extension to P-waves, i.e. the doublet containing the 0^+ and 1^+ degenerate states $(P_0,P_1^{*\prime})$. Incidentally, we note that HQET predicts another low-lying multiplet, comprising two degenerate states with 1^+ and 2^+ [13], which is of no interest here. In matrix notation these fields can be represented by two 4×4 Dirac matrices H and S, with one spinor index for the heavy quark and the other for the light degrees of freedom. An explicit matrix representation is, for negative parity states,

$$H=\frac{(1+\not{v})}{2}[P_\mu^*\gamma^\mu-P\gamma_5] \tag{10}$$

($\overline{H}=\gamma_0 H^\dagger\gamma_0$), whereas, for positive parity states,

$$S=\frac{1+\not{v}}{2}[P_{1\mu}^{*\prime}\gamma^\mu\gamma_5-P_0]. \tag{11}$$

In these equations v is the heavy meson velocity, $v^\mu P^*_\mu = v^\mu P^{*'}_{1\mu}=0$; $P^{*\mu}$, P, $P^{*'}_{1\mu}$ and P_0 are annihilation operators normalized as follows:

$$\langle 0|P|Q\bar{q}(0^-)\rangle=\sqrt{M_H} \tag{12}$$

$$\langle 0|P^{*\mu}|Q\bar{q}(1^-)\rangle=\epsilon^\mu\sqrt{M_H}, \tag{13}$$

with similar equations for the positive parity states ($M_H = M_P = M_{P*}$ is the common mass in the H multiplet). With these notations the heavy-light interaction Lagrangian is written as follows:

$$\mathcal{L}_{h\ell}=\bar{Q}_v iv\cdot\partial Q_v-(\bar{\chi}(\bar{H}+\bar{S})Q_v+\text{H.c.}) + \frac{1}{2G_3}\text{Tr}[(\bar{H}+\bar{S})(H-S)], \tag{14}$$

where Q_v is the effective heavy quark field of HQET and we have assumed that the fields H and S have the same coupling to the quarks, which is a dynamical assumption based on a simplicity criterion (in I we justify it on the basis of a four quark Nambu–Jona-Lasinio interaction by partial bosonization [14]). After renormalization of the heavy fields H and S [3] one obtains the kinetic part of the heavy meson Lagrangian in a form that is standard for heavy meson effective chiral theories [7]:

$$\mathcal{L}_{hl}=\text{Tr }\bar{\hat{H}}(iv\cdot\partial-\Delta_H)\hat{H}+\text{Tr }\bar{\hat{S}}(iv\cdot\partial-\Delta_S)\hat{S}. \tag{15}$$

Here Δ_H and Δ_S are the mass difference between the meson and the heavy quark at the lowest order; typical values considered in I are $\Delta_H=0.3-0.5$ GeV. Δ_H and Δ_S are related: for example, for $\Delta_H=0.4$ GeV one obtains the value $\Delta_S=0.590$ GeV [3]. These values correspond to a value for the S-multiplet mass $m=2165\pm 50$ MeV; these states, called in the literature (for the charmed case) $D_0, D_1^{*'}$, have not been observed yet, since they are expected to be rather broad. $\hat{H}$ and $\hat{S}$ are the renormalized fields and are given in terms of the bare fields H,S by

$$\hat{H}=\frac{H}{\sqrt{Z_H}} \tag{16}$$

$$\hat{S}=\frac{S}{\sqrt{Z_S}}. \tag{17}$$

Z_H, Z_S are renormalization constants that have been computed in [3] with the results (the integral I_3 can be found in the Appendix)

$$Z_H^{-1}=\left[(\Delta_H+m)\frac{\partial I_3(\Delta_H)}{\partial\Delta_H}+I_3(\Delta_H)\right] \tag{18}$$

$$Z_S^{-1}=\left[(\Delta_S-m)\frac{\partial I_3(\Delta_S)}{\partial\Delta_S}+I_3(\Delta_S)\right], \tag{19}$$

where m is the constituent light quark mass.

Let us finally discuss the way to compute the quark-meson loops arising from the previous Lagrangian. As we have seen, the CQM model describes the interactions in terms of effective vertices between a light quark, a heavy quark and a heavy meson [Eq. (14)]. We describe the heavy quarks and heavy mesons consistently with HQET; for example the heavy quark propagator is given by

$$\frac{i}{v\cdot k+\Delta}, \tag{20}$$

where Δ is the difference between the heavy meson and heavy quark mass and k is the residual momentum arising from the interaction with the light degrees of freedom.

The light quark momentum is equal to the integrated loop momentum. It is therefore natural to assume an ultraviolet cut-off on the loop momentum of the order of the scale at which the chiral symmetry is broken, i.e. $\Lambda\simeq 1$ GeV (in I we assumed the value $\Lambda=1.25$ GeV). Since the residual momentum of the heavy quark does not exceed few units of Λ_{QCD} in the effective theory, imposing such a cut-off does not cut any significant range of ultraviolet frequencies. We also observe that the value of the ultraviolet cut-off Λ is independent of the heavy quark mass, since it does not appear in the effective Lagrangian.

Concerning the infrared behavior, the model is not confining and thus its range of validity cannot be extended below energies of the order of Λ_{QCD}. In order to drop the unknown confinement part of the quark interaction one introduces an infrared cut-off μ. These parameters appear in the regularized amplitudes; as discussed in [3] (see also [14]) we have chosen a proper time regularization; the regularized form for the light quark propagator (including integration over momenta) is

$$\int d^4k_E\frac{1}{k_E^2+m^2}\to\int d^4k_E\int_{1/\Lambda^2}^{1/\mu^2}ds\, e^{-s(k_E^2+m^2)}, \tag{21}$$

where μ and Λ are the infrared and ultraviolet cut-offs. For μ in I we assumed the value $\mu=300$ MeV. For a different choice of the cut-off prescription in related models see [15,16].

III. $B\to\rho$ AND $B\to a_1$ FORM FACTORS: EVALUATION OF THE DIRECT CONTRIBUTIONS

The form factors for the semileptonic decays $B\to\rho l\nu$ and $B\to a_1 l\nu$ can be written as follows ($q=p-p'$):

$$\begin{aligned}\langle\rho^+(\epsilon(\lambda),p')|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= \frac{2V(q^2)}{m_B+m_\rho}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta - i\epsilon^*_\mu(m_B+m_\rho)A_1(q^2)\\ &+i(\epsilon^*\cdot q)\frac{(p+p')_\mu}{m_B+m_\rho}A_2(q^2)\\ &+i(\epsilon^*\cdot q)\frac{2m_\rho}{q^2}q_\mu[A_3(q^2)-A_0(q^2)],\end{aligned} \tag{22}$$

where

$$A_3(q^2)=\frac{m_B+m_\rho}{2m_\rho}A_1(q^2)-\frac{m_B-m_\rho}{2m_\rho}A_2(q^2), \tag{23}$$

and

$$\langle a_1^+(\epsilon(\lambda),p')|\overline{q'}\gamma_\mu(1-\gamma_5)b|B(p)\rangle$$
$$=\frac{2A(q^2)}{m_B+m_a}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta-i\epsilon^*_\mu(m_B+m_a)V_1(q^2)$$
$$+i(\epsilon^*\cdot q)\frac{(p+p')_\mu}{m_B+m_a}V_2(q^2)$$
$$+i(\epsilon^*\cdot q)\frac{2m_a}{q^2}q_\mu[V_3(q^2)-V_0(q^2)], \tag{24}$$

where m_a is the a_1 mass and

$$V_3(q^2)=\frac{m_B+m_a}{2m_a}V_1(q^2)-\frac{m_B-m_a}{2m_a}V_2(q^2). \tag{25}$$

We note that, having used this parametrization for the weak matrix elements [17], at $q^2=0$ the following conditions hold:

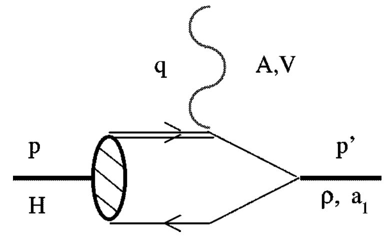

FIG. 1. Diagram for the direct contribution to the form factor $B\to\rho$ and $B\to a_1$.

$$A_3(0)=A_0(0) \tag{26}$$

$$V_3(0)=V_0(0). \tag{27}$$

The contribution we consider in this section arises from diagrams where the weak current couples directly to the quarks belonging to the light and heavy mesons (see Fig. 1).

These diagrams are computed using the rules described in the previous section. The results of a straightforward, but lengthy calculation are as follows:

$$V^D(q^2)=-\frac{m_\rho^2}{f_\rho}\sqrt{\frac{Z_H}{m_B}}(\Omega_1-mZ)(m_B+m_\rho) \tag{28}$$

$$A_1^D(q^2)=\frac{2m_\rho^2}{f_\rho}\sqrt{Z_H m_B}\frac{1}{m_B+m_\rho}[(m^2+mm_\rho\overline{\omega})Z-\overline{\omega}m_\rho\Omega_1-m_\rho\Omega_2-2\Omega_3-\Omega_4-\Omega_5-2\overline{\omega}\Omega_6] \tag{29}$$

$$A_2^D(q^2)=\frac{m_\rho^2}{f_\rho}\sqrt{\frac{Z_H}{m_B}}\left(mZ-\Omega_1-2\frac{\Omega_6}{m_\rho}\right)(m_B+m_\rho) \tag{30}$$

$$A_0^D(q^2)=-\frac{m_\rho}{f_\rho}\sqrt{Z_H m_B}\left[\Omega_1\left(m_\rho\overline{\omega}+2m\frac{q^2}{m_B^2}-\frac{r_1}{m_B}\right)+m_\rho\Omega_2\right.$$
$$\left.+2\Omega_3+\Omega_4\left(1-2\frac{q^2}{m_B^2}\right)+\Omega_5+2\Omega_6\left(\overline{\omega}-\frac{r_1}{m_B m_\rho}\right)-Z\left(m^2-m\frac{r_1}{m_B}+mm_\rho\overline{\omega}\right)\right] \tag{31}$$

where

$$\overline{\omega}=\frac{m_B^2+m_\rho^2-q^2}{2m_B m_\rho}, \tag{32}$$

and

$$r_1=\frac{m_B^2-q^2-m_\rho^2}{2} \tag{33}$$

and the functions Z, Ω_j are given by the formulas of the Appendix with $\Delta_1=\Delta_H$, $\Delta_2=\Delta_1-m_\rho\overline{\omega}$, $x=m_\rho$; m is the constituent light quark mass.

The calculation for the $B\to a_1$ transition is similar. The results are

$$A^D(q^2)=-\frac{m_a^2}{f_a}\sqrt{\frac{Z_H}{m_B}}\left(\Omega_1-mZ-\frac{2m}{m_a}\Omega_2\right)(m_B+m_a) \tag{34}$$

$$V_1^D(q^2)=\frac{2m_a^2}{f_a}\sqrt{Z_H m_B}\frac{1}{m_B+m_a}[(-m^2+mm_a\bar{\omega})Z+2m\Omega_1-\bar{\omega}m_a\Omega_1+(2m\bar{\omega}-m_a)\Omega_2-2\Omega_3-\Omega_4-\Omega_5-2\bar{\omega}\Omega_6] \tag{35}$$

$$V_2^D(q^2)=\frac{m_a^2}{f_a}\sqrt{\frac{Z_H}{m_B}}\left(mZ-\Omega_1-2\frac{\Omega_6}{m_a}+2\frac{m}{m_a}\Omega_2\right)(m_B+m_a) \tag{36}$$

$$V_0^D(q^2)=-\frac{m_a}{f_a}\sqrt{Z_H m_B}\left[\Omega_1\left(m_a\bar{\omega}+2m\frac{q^2}{m_B^2}-\frac{r_1'}{m_B}-2m\right)+\Omega_2\left(m_a+2m\frac{r_1'}{m_B m_a}-2m\bar{\omega}\right)+2\Omega_3+\Omega_4\left(1-2\frac{q^2}{m_B^2}\right)\right.$$
$$+\Omega_5$$
$$\left.+2\Omega_6\left(\bar{\omega}-\frac{r_1'}{m_B m_a}\right)+Z\left(m^2+m\frac{r_1'}{m_B}-mm_a\bar{\omega}\right)\right], \tag{37}$$

where now

$$\bar{\omega}=\frac{m_B^2+m_a^2-q^2}{2m_B m_a} \tag{38}$$

$$r_1'=\frac{m_B^2-q^2-m_a^2}{2}. \tag{39}$$

The previous results for the form factors can be used directly for the numerical analysis. In order to allow an easier way of using our results we have fitted these formulas by the simple parametrization:

$$F^D(q^2)=\frac{F^D(0)}{1-a_F\left(\frac{q^2}{m_B^2}\right)+b_F\left(\frac{q^2}{m_B^2}\right)^2} \tag{40}$$

for a generic form factor $F^D(q^2)$; a_F, b_F have been fitted by a numerical analysis performed up to $q^2=16$ GeV^2, both for ρ and a_1 mesons. We have collected the fitted values in Table I. We note explicitly that the results for $B\to a_1$ form factors at $q^2=0$ are proportional to the factor $(0.25\ \text{GeV}^2/f_a)$. In addition to the normalization uncertainty due to f_a, we estimate a theoretical error of 15% on these parameters.

TABLE I. Parameters of the direct contribution to the various B form factors for $B\to\rho$ and $B\to a_1$ decays. The values $F^D(0)$ for the $B\to a_1$ transition (last four columns) should be multiplied by the factor 0.25 GeV$^2/f_a$. The theoretical uncertainty is $\pm 15\%$.

	V^D	A_1^D	A_2^D	A_0^D	A^D	V_1^D	V_2^D	V_0^D
$F^D(0)$	0.83	0.69	0.81	0.33	1.62	1.13	1.13	1.13
a_F	0.93	0	0.87	2.9	1.13	0.18	1.3	1.9
b_F	0.02	0	−0.17	2.6	0.12	0.04	3.8	0.93

IV. STRONG COUPLING CONSTANTS

In this section we compute the strong couplings $HH\rho, HS\rho, HHa_1, HSa_1$. As discussed in the Introduction they are relevant for the calculation of the polar contribution to the form factors. We parametrize these couplings by considering the following effective Lagrangians (we follow the notations introduced in [7]):

$$\mathcal{L}_{HH\rho}=i\lambda\,\text{Tr}[\bar{H}H\sigma^{\mu\nu}\mathcal{F}(\rho)_{\mu\nu}]-i\beta\ \text{Tr}(\bar{H}H\gamma^\mu\rho_\mu) \tag{41}$$

$$\mathcal{L}_{HS\rho}=-i\zeta\ \text{Tr}(\bar{S}H\gamma^\mu\rho_\mu)+i\mu\,\text{Tr}[\bar{S}H\sigma^{\mu\nu}\mathcal{F}(\rho)_{\mu\nu}] \tag{42}$$

$$\mathcal{L}_{HHa_1}=-i\zeta_A\text{Tr}(\bar{H}H\gamma^\mu a_\mu)+i\mu_A\text{Tr}[\bar{H}H\sigma^{\mu\nu}\mathcal{F}(a)_{\mu\nu}] \tag{43}$$

$$\mathcal{L}_{HSa_1}=i\lambda_A\text{Tr}[\bar{S}H\sigma^{\mu\nu}\mathcal{F}(a)_{\mu\nu}]-i\beta_A\text{Tr}(\bar{S}H\gamma^\mu a_\mu). \tag{44}$$

The strong couplings can be computed by matching the effective meson Lagrangian of Eqs. (42)–(44) with the quark-meson Lagrangian (14), i.e. considering triangular quark loops with external legs representing light and heavy mesons. The calculation is similar to the one of the previous section. The results are as follows:

$$\lambda=\frac{m_\rho^2}{\sqrt{2}g_V f_\rho}Z_H(-\Omega_1+mZ) \tag{45}$$

$$\beta=\sqrt{2}\frac{m_\rho^2}{g_V f_\rho}Z_H \times[2m\Omega_1+m_\rho\Omega_2+2\Omega_3-\Omega_4+\Omega_5-m^2Z]. \tag{46}$$

Here the functions Z,Ω_j are given by the formulas of the Appendix with $\Delta_1=\Delta_H$, $x=m_\rho$, $\omega=m_\rho/(2m_B)$ (we keep here the first $1/m_Q$ correction). Moreover,

$$\mu=\frac{m_\rho^2}{\sqrt{2}g_V f_\rho}\sqrt{Z_H Z_S}\left(-\Omega_1-2\frac{\Omega_6}{m_\rho}+mZ\right) \tag{47}$$

$$\zeta=\frac{\sqrt{2}m_\rho^2}{g_V f_\rho}\sqrt{Z_H Z_S}(m_\rho\Omega_2+2\Omega_3+\Omega_4+\Omega_5-m^2 Z). \tag{48}$$

Here the functions Z,Ω_j are given by the formulas of the Appendix with $\Delta_1=\Delta_H$, $\Delta_2=\Delta_S$, $x=m_\rho$ and $\omega=(\Delta_1-\Delta_2)/m_\rho$. For the axial-vector a_1 couplings to H and S states we find

$$\lambda_A=\frac{m_a^2}{\sqrt{2}g_A f_a}\sqrt{Z_H Z_S}\left(-\Omega_1+2\Omega_2\frac{m}{m_a}+mZ\right) \tag{49}$$

$$\beta_A=\sqrt{2}\frac{m_a^2}{g_A f_a}\sqrt{Z_H Z_S}(m_a\Omega_2+2\Omega_3-\Omega_4+\Omega_5+m^2 Z), \tag{50}$$

where Z,Ω_j are given by the formulas of the Appendix with $\Delta_1=\Delta_H$, $\Delta_2=\Delta_S$, $x=m_a$ and $\omega=(\Delta_1-\Delta_2)/m_a$. Moreover,

$$\mu_A=\frac{m_a^2}{\sqrt{2}g_A f_a}Z_H\left[m\left(Z+2\frac{\Omega_2}{m_a}\right)-\Omega_1-2\frac{\Omega_6}{m_a}\right] \tag{51}$$

$$\zeta_A=\frac{\sqrt{2}m_a^2}{g_A f_a}Z_H(-2m\Omega_1+m_a\Omega_2+2\Omega_3+\Omega_4+\Omega_5+m^2 Z), \tag{52}$$

where $\Delta_1=\Delta_H$, $x=m_a$, $\omega=m_a/(2m_B)$. Numerically we get the following results:

$$\lambda=0.60\ \text{GeV}^{-1} \qquad \lambda_A=0.85\times(0.25\ \text{GeV}^2/f_a)\ \text{GeV}^{-1}$$

$$\beta=-0.86 \qquad \beta_A=-0.81\times(0.25\ \text{GeV}^2/f_a)$$

$$\mu=0.16\ \text{GeV}^{-1} \qquad \mu_A=0.23\times(0.25\ \text{GeV}^2/f_a)\ \text{GeV}^{-1}$$

$$\zeta=0.01 \qquad \zeta_A=0.15\times(0.25\ \text{GeV}^2/f_a).$$

A discussion about the theoretical uncertainties of these results is in order. We have explicitly written down the dependence on f_a of the strong coupling constants involving the light axial-vector meson, since, as noted before, this is a major source of theoretical uncertainty for these constants. Another source of spreading in the reported values is the variation of Δ_H in the range $\Delta_H=0.3$–0.5 GeV (we use $\Delta_H=0.4$ GeV in the calculation). This produces a significant uncertainty only for ζ,β_A,ζ_A since we obtain $\zeta=0.01\pm0.19$, $\beta_A=-0.81^{+0.45}_{-0.24}$ and $\zeta_A=0.15^{+0.16}_{-0.14}$ while in the other cases only a few percent variation is observed. For the other constants $\lambda,\mu,\lambda_A,\mu_A$, a theoretical uncertainty of $\pm15\%$ can be guessed. This estimate follows for example from a different evaluation of the λ parameter performed in I. For other determinations of the coupling constant λ see [18] (QCD sum rules and light cone sum rules) and [19] (this paper uses data from D^* decays together with vector meson dominance).

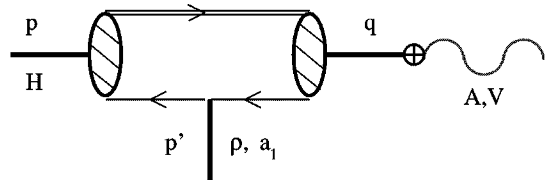

FIG. 2. Diagram for the polar contribution to the form factor $B\to\rho$ and $B\to a_1$.

V. $B\to\rho$ AND $B\to a_1$ FORM FACTORS: EVALUATION OF THE POLAR CONTRIBUTIONS

The polar contributions are given by diagrams where the weak current is coupled to B and to the light vector or axial vector meson by an intermediate heavy meson state (see Fig. 2). These diagrams, because of the heavy meson propagator, produce a typical polar behavior of the form factors, in the form

$$F^P(q^2)=\frac{F^P(0)}{1-\frac{q^2}{m_P^2}}. \tag{53}$$

This behavior is certainly valid near the pole; we assume its validity for the whole q^2 range, which can be justified on the basis of the minor numerical role of this polar contribution, as compared to the direct one, in the region of low q^2, at least for the form factors A_1^P,A_2^P (see the numerical results at the end of this section). The assumption (53) cannot be made for the form factors $A_0^P(q^2)$ and $V_0^P(q^2)$, as we discuss below, and is also less reliable for $A^P(q^2)$ and $V^P(q^2)$ (see Table II).

TABLE II. Parameters of the polar contribution to the various B form factors for $B\to\rho$ and $B\to a_1$ decays. Pole masses in GeV. The values $F^P(0)$ for the $B\to a_1$ transition (last four columns) should be multiplied by the factor $(0.25\ \text{GeV}^2/f_a)$. The theoretical uncertainty is $\pm15\%$.

	V^P	A_1^P	A_2^P	A_0^P	A^P	V_1^P	V_2^P	V_0^P
$F^P(0)$	-0.84	-0.11	-0.15	-0.019	-1.48	-0.32	-0.57	0.07
m_P	5.3	5.5	5.5	–	5.5	5.3	5.3	–

The values at $q^2=0$ in Eq. (53) can be easily computed in term of the strong coupling constants defined in the previous section and using the leptonic decay constants $\hat{F}$ and $\hat{F}^+$ that give the coupling of the intermediate states to the currents. Neglecting logarithmic corrections, $\hat{F}$ and $\hat{F}^+$ are related, in the infinite heavy quark mass limit, to the leptonic decay constant f_B and f^+ defined by

$$\langle 0|\overline{q}\gamma^\mu\gamma_5 b|B(p)\rangle = ip^\mu f_B \tag{54}$$

$$\langle 0|\overline{q}\gamma^\mu b|B_0(p)\rangle = p^\mu f^+, \tag{55}$$

by the relations $f_B=\hat{F}/\sqrt{m_B}$ and $f^+=\hat{F}^+/\sqrt{m_{B_0}}$ (B_0 is the S state with $J^P=0^+$ and $b\overline{q}$ content). These couplings have been computed in [3] with the results given in Table III for different values of the parameters.

For the values $F^P(0)$ we obtain the following results for the $B\to\rho$ transition:

$$V^P(0) = -\sqrt{2}g_V\lambda\hat{F}\frac{m_B+m_\rho}{m_B^{3/2}} \tag{56}$$

$$A_1^P(0) = \frac{\sqrt{2m_B}g_V\hat{F}^+}{m_{B_0}(m_B+m_\rho)}(\zeta-2\mu\overline{\omega}m_\rho) \tag{57}$$

$$A_2^P(0) = -\sqrt{2}g_V\mu\hat{F}^+\frac{\sqrt{m_B}(m_B+m_\rho)}{m_{B_0}^2}, \tag{58}$$

where $\overline{\omega}=m_B/(2m_\rho)$. For $A_0^P(q^2)$, we have to implement the condition contained in Eq. (26); for instance a possible choice is

$$A_0^P(q^2) = A_3^P(0) + g_V\beta\hat{F}\frac{1}{m_\rho\sqrt{2m_B}}\frac{q^2}{m_B^2-q^2}. \tag{59}$$

For the $B\to a_1$ transition we have

$$A^P(0) = -\sqrt{2}g_A\lambda_A\hat{F}^+\frac{m_B+m_a}{m_B^{3/2}} \tag{60}$$

$$V_1^P(0) = \frac{\sqrt{2m_B}g_A\hat{F}}{m_B(m_B+m_a)}(\zeta_A-2\mu_A\overline{\omega}m_a) \tag{61}$$

TABLE III. $\hat{F}$ and $\hat{F}^+$ for various values of Δ_H. Δ_H in GeV, leptonic constants in GeV$^{3/2}$.

Δ_H	$\hat{F}$	$\hat{F}^+$
0.3	0.33	0.22
0.4	0.34	0.24
0.5	0.37	0.27

$$V_2^P(0) = -\sqrt{2}g_A\mu_A\hat{F}\frac{\sqrt{m_B}(m_B+m_a)}{m_B^2}, \tag{62}$$

where $\overline{\omega}=m_B/(2m_a)$. Similarly to the previous discussion for $A_0^P(q^2)$ we can put, for instance,

$$V_0^P(q^2) = V_3^P(0) + g_A\beta_A\hat{F}^+\frac{1}{m_a\sqrt{2m_B}}\frac{q^2}{m_{B_0}^2-q^2}. \tag{63}$$

We note that Eqs. (59) and (63) have been written down only as an example of a possible behavior of these form factors satisfying the given constraints. For massless leptons they do not contribute to the semileptonic width and can be neglected.

Numerically we obtain the results in Table II where we have also reported the values of the pole masses for all the form factors except $A_0^P(q^2)$ and $V_0^P(q^2)$ because of Eqs. (59) and (63). Similarly to the previous analyses an overall uncertainty of $\pm 15\%$ can be assumed. In Fig. 3 we plot the form factors A_1 and A_2 for the semileptonic decay $B\to\rho$. In Fig. 4 are shown the form factors A, V_1 and V_2 for the semileptonic decay $B\to a_1$. Since the behavior in Eqs. (59) and (63) is only guessed, we have not included the form factors $A_0^P(q^2)$ and $V_0^P(q^2)$ in Figs. 3 and 4; in addition $V(q^2)$ is not reported in Fig. 3 since our prediction is affected by a large error (see the discussion in the next section). Note that the theoretical error is not included in Figs. 3 and 4; one should refer to the numbers in Tables I and II.

VI. BRANCHING RATIOS AND WIDTHS

In this section we compute the branching ratios and widths for semileptonic decays using the numerical results for form factors reported in Tables I, II. Let us first compare our results for the $B\to\rho$ form factors with those obtained by other methods (see Table IV). These form factors (as well as

TABLE IV. Form factors for the transition $B\to\rho$ at $q^2=0$. Our results are compared with the outcome of other theoretical calculations: potential models, light cone sum rules (LCSR), QCD sum rules (SR), calculations involving both lattice and light cone sum rules. The large error of $V^\rho(0)$ in our approach is due to the large cancellation between the direct and polar contribution.

	This work	Potential model [20]	LCSR [21]	SR [22]	Latt. + LCSR [23]
$V^\rho(0)$	-0.01 ± 0.25	0.45 ± 0.11	0.34 ± 0.05	0.6 ± 0.2	$0.35^{+0.06}_{-0.05}$
$A_1^\rho(0)$	0.58 ± 0.10	0.27 ± 0.06	0.26 ± 0.04	0.5 ± 0.1	$0.27^{+0.05}_{-0.04}$
$A_2^\rho(0)$	0.66 ± 0.12	0.26 ± 0.05	0.22 ± 0.03	0.4 ± 0.2	$0.26^{+0.05}_{-0.03}$
$A_0^\rho(0)$	0.33 ± 0.05	$0.29\ \pm\ 0.09$		0.24 ± 0.02	$0.30^{+0.06}_{-0.04}$

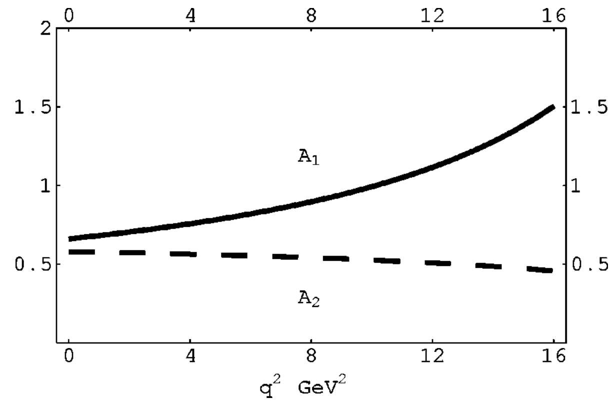

FIG. 3. Form factors A_1 (solid line) and A_2 (dashed line) for the semileptonic decay $B\to\rho l\nu$.

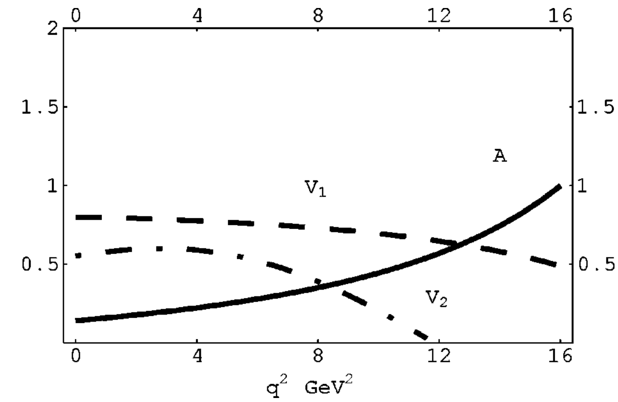

FIG. 4. Form factors A (solid line), V_1 (dashed line) and V_2 (dot-dashed line) for the semileptonic decay $B\to a_1 l\nu$.

those concerning the transition $B\to a_1$) are obtained by adding the direct and polar contributions:

$$F(q^2)=F^D(q^2)+F^P(q^2), \tag{64}$$

where $F^D(q^2)$ was introduced in Sec. III and $F^P(q^2)$ in Sec. V. Our result for the vector form factor $V^\rho(q^2)$ is affected by a large error since it arises from the sum of two terms opposite in sign and almost equal in absolute value. Apart from this large uncertainty, our results are in relative good agreement with the results of QCD sum rules, but they are in general higher than those obtained by other approaches. For the $B\to\rho l\nu$ decay width and branching ratio we obtain (using $V_{ub}=0.0032$, $\tau_B=1.56\times10^{-12}$ s)

$$\mathcal{B}(\overline{B}^0\to\rho^+ l\nu)=(2.5\pm0.8)\times10^{-4}$$

$$\Gamma_0(\overline{B}^0\to\rho^+ l\nu)=(4.4\pm1.3)\times10^{7}\ \text{s}^{-1}$$

$$\Gamma_+(\overline{B}^0\to\rho^+ l\nu)=(7.1\pm4.5)\times10^{7}\ \text{s}^{-1}$$

$$\Gamma_-(\overline{B}^0\to\rho^+ l\nu)=(5.5\pm3.7)\times10^{7}\ \text{s}^{-1}$$

$$(\Gamma_++\Gamma_-)(\overline{B}^0\to\rho^+ l\nu)=(1.26\pm0.38)\times10^{8}\ \text{s}^{-1} \tag{65}$$

where $\Gamma_0,\Gamma_+,\Gamma_-$ refer to the ρ helicities. In order to obtain the numbers in the previous formula for a different value of V_{ub}, simply multiply by the factor $|V_{ub}/0.0032|^2$, putting in any value for V_{ub}. In order to use a different value for τ_B in the branching ratio $\mathcal{B}$ multiply it by $\tau_B/(1.56\times10^{-12})$ where the τ_B value has to be expressed in seconds. The branching ratio for $B\to\rho l\nu$ is in agreement with the experimental result quoted in the Introduction, Eq. (1).

Let us now discuss the theoretical uncertainty of these results. The large error of $V^\rho(0)$ affects significantly the values of Γ_+ and Γ_-, whose errors are correlated; it has however no effect on Γ_0 and a small effect on the branching ratio, which increases at most by 8%. The theoretical uncertainties in $A_1^\rho(0)$ and $A_2^\rho(0)$ are likely to be related. To get the theoretical error in the widths we have added in quadrature the error induced by $V^\rho(0)$ and a common $\pm15\%$ error in $A_1^\rho(0)$ and $A_2^\rho(0)$.

Having used the decay $B\to\rho l\nu$ as a test of the CQM model, we can now consider the $B\to a_1 l\nu$ semileptonic decay. The results obtained are

$$\mathcal{B}(\overline{B}^0\to a_1^+ l\nu)=(8.4\pm1.6)\times10^{-4}$$

$$\Gamma_0(\overline{B}^0\to a_1^+ l\nu)=(4.0\pm0.7)\times10^{8}\ \text{s}^{-1}$$

$$\Gamma_+(\overline{B}^0\to a_1^+ l\nu)=(4.6\pm0.9)\times10^{7}\ \text{s}^{-1}$$

$$\Gamma_-(\overline{B}^0\to a_1^+ l\nu)=(0.98\pm0.18)\times10^{8}\ \text{s}^{-1} \tag{66}$$

where $\Gamma_0,\Gamma_+,\Gamma_-$ refer to the a_1 helicities. In order to obtain the results in the previous formula for different values of V_{ub} and τ_B refer to the discussion after Eq. (65). We have included in the determination of these decay widths only the normalization uncertainty arising from f_a; the lower values correspond to $f_a=0.30$ GeV^2 while the higher values to $f_a=0.25$ GeV^2. One should also take into account the theoretical errors arising from the values of the form factors at $q^2=0$; they are more difficult to estimate reliably and are not included here. In any case the overall theoretical uncertainty is larger (presumably by a factor of 2) than the one reported in the previous formula.

VII. CONCLUSIONS

The main conclusion of this paper can be read from Eqs. (66). We predict a branching ratio for the decay $\overline{B}^0\to a_1^+ l\nu$ significantly larger than the branching ratio for $\overline{B}^0\to\rho^+ l\nu$; in spite of the theoretical uncertainties inherent to the CQM model, which we have discussed in the previous sections, this is a remarkable outcome. A consequence of this result is that the $B\to a_1 l\nu$ decay channel might account for around 50% of the semileptonic $B\to X_u l\nu$ decay channel (evaluated, for example, by the parton model), whereas the $B\to\rho l\nu$ decay channel adds another 15%; given the relevance of these results for the determination of V_{ub}, it would be interesting to test these predictions in the future by other theoretical methods and, hopefully, by experimental data.

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APPENDIX

In the paper we have introduced several integrals and parameters that we list in this appendix:

$$I_0(\Delta)=\frac{iN_c}{16\pi^4}\int^{\rm reg}\frac{d^4k}{(v\cdot k+\Delta+i\epsilon)}$$

$$=\frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{ds}{s^{3/2}}e^{-s(m^2-\Delta^2)}\left(\frac{3}{2s}+m^2-\Delta^2\right)[1+\mathrm{erf}(\Delta\sqrt{s})]-\Delta\frac{N_c m^2}{16\pi^2}\Gamma\left(-1,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right) \tag{A1}$$

$$I_1=\frac{iN_c}{16\pi^4}\int^{\rm reg}\frac{d^4k}{(k^2-m^2)}=\frac{N_c m^2}{16\pi^2}\Gamma\left(-1,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right) \tag{A2}$$

$$I_1'=\frac{iN_c}{16\pi^4}\int^{\rm reg}d^4k\,\frac{k^2}{(k^2-m^2)}=\frac{N_c m^4}{8\pi^2}\Gamma\left(-2,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right) \tag{A3}$$

$$I_2=-\frac{iN_c}{16\pi^4}\int^{\rm reg}\frac{d^4k}{(k^2-m^2)^2}=\frac{N_c}{16\pi^2}\Gamma\left(0,\frac{m^2}{\Lambda^2},\frac{m^2}{\mu^2}\right) \tag{A4}$$

$$I_3(\Delta)=-\frac{iN_c}{16\pi^4}\int^{\rm reg}\frac{d^4k}{(k^2-m^2)(v\cdot k+\Delta+i\epsilon)}$$

$$=\frac{N_c}{16\,\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{ds}{s^{3/2}}e^{-s(m^2-\Delta^2)}[1+\mathrm{erf}(\Delta\sqrt{s})] \tag{A5}$$

$$I_4(\Delta)=\frac{iN_c}{16\pi^4}\int^{\rm reg}\frac{d^4k}{(k^2-m^2)^2(v\cdot k+\Delta+i\epsilon)}$$

$$=\frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{ds}{s^{1/2}}e^{-s(m^2-\Delta^2)}[1+\mathrm{erf}(\Delta\sqrt{s})], \tag{A6}$$

where Γ is the generalized incomplete gamma function and erf is the error function. Moreover, having defined

$$\sigma(x,\Delta_1,\Delta_2,\omega)=\frac{\Delta_1(1-x)+\Delta_2 x}{\sqrt{1+2\,(\omega-1)x+2\,(1-\omega)x^2}}, \tag{A7}$$

we have

$$I_5(\Delta_1,\Delta_2,\omega)=\frac{iN_c}{16\pi^4}\int^{\text{reg}}\frac{d^4k}{(k^2-m^2)(v\cdot k+\Delta_1+i\epsilon)(v'\cdot k+\Delta_2+i\epsilon)}$$
$$=\int_0^1 dx\,\frac{1}{1+2x^2(1-\omega)+2x(\omega-1)}\left[\frac{6}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}ds\ \ \sigma e^{-s(m^2-\sigma^2)}s^{-1/2}[1+\text{erf}(\sigma\sqrt{s})]\right.$$
$$\left.+\frac{6}{16\pi^2}\int_{1/\Lambda^2}^{1/\mu^2}ds e^{-s(m^2-2\sigma^2)}s^{-1}\right]. \tag{A8}$$

We also define, if $q^\mu=xv'^\mu$; $\omega=v\cdot v'$, $\Delta_2=\Delta_1-x\ \ \omega$, the formula

$$Z=\frac{iN_c}{16\pi^4}\int^{\text{reg}}\frac{d^4k}{(k^2-m^2)[(k+q)^2-m^2](v\cdot k+\Delta_1+i\epsilon)}$$
$$=\frac{I_5(\Delta_1,x/2,\omega)-I_5(\Delta_2,-x/2,\omega)}{2x}. \tag{A9}$$

We use in the text the following combinations of the previous integrals:

$$K_1=m^2Z-I_3(\Delta_2)$$

$$K_2=\Delta_1^2Z-\frac{I_3(x/2)-I_3(-x/2)}{4x}[\omega x+2\Delta_1]$$

$$K_3=\frac{x^2}{4}Z+\frac{I_3(\Delta_1)-3I_3(\Delta_2)}{4}+\frac{\omega}{4}[\Delta_1I_3(\Delta_1)-\Delta_2I_3(\Delta_2)]$$

$$K_4=\frac{x\Delta_1}{2}Z+\frac{\Delta_1[I_3(\Delta_1)-I_3(\Delta_2)]}{2x}+\frac{I_3(x/2)-I_3(-x/2)}{4}$$

$$\Omega_1=\frac{I_3(-x/2)-I_3(x/2)+\omega[I_3(\Delta_1)-I_3(\Delta_2)]}{2x(1-\omega^2)}-\frac{[\Delta_1-\omega x/2]Z}{1-\omega^2}$$

$$\Omega_2=\frac{-I_3(\Delta_1)+I_3(\Delta_2)-\omega[I_3(-x/2)-I_3(x/2)]}{2x(1-\omega^2)}-\frac{[x/2-\Delta_1\omega]Z}{1-\omega^2}$$

$$\Omega_3=\frac{K_1}{2}+\frac{2\omega K_4-K_2-K_3}{2(1-\omega^2)}$$

$$\Omega_4=\frac{-K_1}{2(1-\omega^2)}+\frac{3K_2-6\omega K_4+K_3(2\omega^2+1)}{2(1-\omega^2)^2}$$

$$\Omega_5=\frac{-K_1}{2(1-\omega^2)}+\frac{3K_3-6\omega K_4+K_2(2\omega^2+1)}{2(1-\omega^2)^2}$$

$$\Omega_6=\frac{K_1\omega}{2(1-\omega^2)}+\frac{2K_4(2\omega^2+1)-3\omega(K_2+K_3)}{2(1-\omega^2)^2}. \tag{A10}$$

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