Higgs boson mass constraints from precision data and direct searches

Michael S. Chanowitz*
Theoretical Physics Group, Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720

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Two of the nine measurements of $\sin^2\theta_{eff}^{lepton}$, the effective weak interaction mixing angle, are found to be in significant conflict with the direct search limits for the standard model (SM) Higgs boson. Using a scale factor method, analogous to one used by the Particle Data Group, we assess the possible effect of these discrepancies on the SM fit of the Higgs boson mass. The scale factor fits increase the value of $\sin^2\theta_{eff}^{lepton}$ by as much as two standard deviations. The central value of the Higgs boson mass increases as much as a factor of 2, to $\simeq 200$ GeV, and the 95% confidence level upper limit increases to as much as 750 GeV. The scale factor is based not simply on the discrepant measurements, as was the case in a previous analysis, but on an aggregate goodness-of-fit confidence level for the nine measurements and the limit. The method is generally applicable to fits in which one or more of a collection of measurements are in conflict with a physical boundary or limit. In the present context, the results suggest caution in drawing conclusions about the Higgs boson mass from the existing data. [S0556-2821(99)03707-8]

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I. INTRODUCTION

Measurements of Z boson decay asymmetries at the CERN e^+e^- collider LEP and SLAC Linear Collider SLC [1] and of the top quark mass at Fermilab [2] appear to constrain the mass of the standard model (SM) Higgs boson at the level of a factor of 2 or better. The combined fit of nine measurements of the effective leptonic weak interaction mixing angle yields $\sin^2\theta_{eff}^{lepton}=0.23148\pm0.00021$, which implies a SM Higgs boson mass $m_H=86^{+84}_{-42}$ GeV and the upper limit $m_H<260$ GeV at 95% confidence level (C.L.). In a previous Letter [3] I observed that the most precise of the nine measurements, the left-right asymmetry A_{LR}, then implied $m_H=16$ GeV and an *upper* limit $m_H<77$ GeV at 95% C.L., in contrast to the *lower* limit from direct searches, then given by $m_H>77$ GeV, also at 95% C.L. I analyzed the possible impact of this discrepancy on the SM fit of m_H using a scale factor method inspired by a method the Particle Data Group [4] (PDG) has used to combine discrepant data. The conclusion was that both the central value and the upper limit on m_H could be appreciably higher than in the conventional fit. Similar observations had been made previously, using different methods, by Gurtu [5] and Dittmaier, Schildknecht, and Weiglein [6].

The work presented here differs significantly from Ref. [3] in which the discrepancy between the A_{LR} measurement and the search limit was evaluated simply as the likelihood for a 95% C.L. upper limit at 77 GeV to be consistent with a 95% C.L. lower limit at the same mass, i.e., $2\times0.05\times0.95\simeq0.1$ or 10%. This may be a fair appraisal if we have an *a priori* reason to focus on the A_{LR} measurement, such as for instance that it provides the most precise determination of $\sin^2\theta_{eff}^{lepton}$, rather than choosing to consider it *because* we have noticed that it implies a value of m_H below the SM search limit. In the latter case we need to consider the likelihood that *any* of the nine relevant measurements of $\sin^2\theta_{eff}^{lepton}$ could fluctuate to produce a like discrepancy. It is fair to say that in this instance our attention is drawn to A_{LR} by both its precision and the fact of its conflict with the SM search limits.

It may therefore be appropriate to approach the analysis from the perspective of the consistency of the complete ensemble of nine measurements with the SM search limit. That is the perspective of the analysis presented here, in which a suitable scale factor method is proposed. The method could be applied to other physical situations in which data within a collection of measurements conflict with a limit or physical boundary. Here I will apply the method to the SM fit of m_H, using the spring 1998 data [1], which differs appreciably from the 1997 data used in the earlier analysis.

In the previous analysis the scale factor was introduced based on the goodness-of-fit C.L. between just the discrepant measurement and the limit. In the method presented here the scale factor is determined by the goodness-of-fit C.L. between the complete set of asymmetry measurements and the limit, therefore taking account of the likelihood that any measurement in the set might fluctuate into the low tail of the $\sin^2\theta_{eff}^{lepton}$ distribution. The method is then truly analogous to the PDG method, which rescales the fit uncertainty by a scale factor determined by the goodness-of-fit C.L. of the chi-squared distribution of the complete data set.

It is important to keep in mind that the analysis presented here assumes the validity of the standard model [or the minimal supersymmetric standard model (MSSM) in the decoupling limit] and that in general, without a specific theoretical framework, the electroweak radiative corrections tell us nothing about the nature of electroweak symmetry breaking. In addition to quantum corrections from the Higgs sector, the value of $\sin^2\theta_{eff}^{lepton}$ could be affected by quantum corrections from other sectors of new physics and/or from gauge boson mixing in theories with extended gauge sectors. The nature

*Email address: chanowitz@lbl.gov

of electroweak symmetry breaking can only be definitively established by direct discovery and detailed study of the Higgs sector quanta at a high energy collider. Until then anything is possible: light Higgs scalars, dynamical symmetry breaking without Higgs scalars, or even that the Higgs mechanism is not realized in nature at all. Here we assume that no new physics contributes to $\sin^2\theta_{eff}^{lepton}$ except the quantum corrections from the Higgs sector, and that any Higgs scalar decays as prescribed in the SM so that the Higgs boson search limits are applicable.

Section II is a brief review of the 1998 data and the SM fit of m_H. The uncertainties in the fit are examined for two different evaluations of $\alpha(m_Z)$ [7,8]. (The values quoted in this introductory section are based on Ref. [7].) Though the 1998 data set for $\sin^2\theta_{eff}^{lepton}$ is more internally consistent than the 1997 data, its confidence level is still not robust and it continues to exhibit discrepancies with the SM search limits. The central value of m_H implied by A_{LR} has increased to 25 GeV, but the direct search limit [9] has also increased, to $m_H>89.3$ GeV at 95% C.L., and the precision of the A_{LR} measurement has improved. Putting all these changes together there is still a significant discrepancy, with A_{LR} now implying $m_H<89.3$ GeV at 93% C.L.

A somewhat bigger discrepancy occurs in the less precise tau front-back asymmetry measurement, A_{FB}^{τ}, which implies $m_H=4$ GeV and $m_H<89.3$ GeV at 95% C.L. Although a single value of $\sin^2\theta_{eff}^{lepton}$ is typically presented for the combined leptonic front-back asymmetry, A_{FB}^{l}, the measurements of A_{FB}^{e}, A_{FB}^{μ}, and A_{FB}^{τ} are in fact quite distinct, each posing a unique set of experimental issues. As can be seen in Table II below, A_{FB}^{μ} and A_{FB}^{τ} are individually at the same level of precision as all but the two most precise measurements; so it is most natural to consider them separately.

It is certainly the case that our attention is drawn to A_{FB}^{τ} by the low value of m_H it implies; so in considering the conflict of A_{LR} and A_{FB}^{τ} with the search limit we must assess the goodness-of-fit of the measurements with the search limit from the perspective of the complete set of nine measurements. The scale factors computed in this way then appropriately weight the increased likelihood of outlying measurements when A_{FB}^{l} is disaggregated, with the number of $\sin^2\theta_{eff}^{lepton}$ measurements increased from 7 to 9.

Section III begins with a review of the PDG scale factor method for combining discrepant data and then presents a method to extend it to the case of measurements in conflict with a limit. The central observation of the PDG is that low C.L. data sets occur more often than expected by chance, and that historically many discrepancies are found to result from underestimated systematic errors. This should not be a surprise, since the estimation of systematic error is perhaps the most challenging task faced by experimenters in the analysis and presentation of their data. The PDG scaled error is meant to provide a more cautious interpretation of low C.L. data sets, with minimal impact on moderately discrepant data. After reviewing the motivation and formulation of the PDG scale factor, S^*, an analogous scale factor is constructed for situations in which the discrepancy is between a collection of measurements and a limit. Section III concludes with a brief discussion of the complementary relationship of the scale factor method with a recent analysis by Cousins and Feldman [10] of confidence intervals near a physical boundary. Their construction is used to determine the upper limits on m_H from the scaled fits.

Section IV presents the application of the scale factor method to the fit of m_H from the nine measurements of $\sin^2\theta_{eff}^{lepton}$. The result is a continuum of fits which differ in how the scaling is shared between the two low measurements, A_{LR} and A_{FB}^{τ}. At one extreme, it suffices to scale the uncertainty of A_{FB}^{τ} by a factor of 3 while leaving A_{LR} unmodified; in this case the effect on the fit is small. At the other extreme, when the rescaling is dominantly applied to A_{LR}, the fitted central value of m_H increases by a factor of 2 relative to the conventional fit, while the 95% C.L. upper limit (in the Cousins-Feldman construction) increases by nearly a factor of 3 relative to the conventional 95% C.L. limit. These extremes and a sample of intermediate cases are presented in Sec. IV.

The analysis in Secs. II–IV assumes a perfect search limit, $m_H>89.3$ GeV with 100% C.L. In Sec. V, I show that the results obtained in this approximation apply to the actual, less than perfect experimental limits. The conclusion relies on the sharply increased confidence level obtained by the search experiments for values of $m_H^{\rm LIMIT}$ slightly below 89 GeV.

A brief summary and discussion are given in Sec. VI.

II. THE ELECTROWEAK DATA AND THE SM HIGGS BOSON MASS

Our strategy is to focus on the most direct determination of m_H, using the measurement of $\sin^2\theta_{eff}^{lepton}$, augmented by the direct measurement of the top quark mass [by the Collider Detector at Fermilab (CDF) and D0 Collaborations] together with the value of $\alpha(m_Z)$. The effective mixing angle, $\sin^2\theta_{eff}^{lepton}$, has the greatest sensitivity to m_H with the least collateral dependence on various other quantities such as the strong coupling constant $\alpha_S(m_Z)$ or the fraction of hadronic Z decays to b quarks, R_b. From the nine measurements of $\sin^2\theta_{eff}^{lepton}$, which combine to yield $\sin^2\theta_{eff}^{lepton}=0.23148\pm0.00021$, and the conservative determination of $\alpha(m_Z)=(128.896\pm0.090)^{-1}$ by Eidelmann and Jegerlehner [7] I obtain using the state of the art radiative corrections of Degrassi *et al.* [11] $m_H=86^{+84}_{-42}$ GeV, compared with the LEP Electroweak Working Group [1] global fit value $m_H=66^{+74}_{-39}$ GeV [which also uses Ref. [7] for $\alpha(m_Z)$]. Gaussian statistics are assumed for the $\sin^2\theta_{eff}^{lepton}$ measurements, from which it follows in the SM fit that the logarithm of the Higgs boson mass, $\ln m_H$, is Gaussian distributed.

The difference between the global fit and the fit based just on the $\sin^2\theta_{eff}^{lepton}$ data is not great and is due primarily to the fact that the global fit uses the top quark mass, $m_t=171.1\pm5.1$ GeV, determined from the combination of direct and indirect measurements, while in the fit restricted to the $\sin^2\theta_{eff}^{lepton}$ data I have used the directly measured Fermilab value [4], $m_t=173.8\pm5.1$ GeV. The smaller value of m_t

TABLE I. Uncertainties in the evaluation of the natural logarithm of the SM Higgs boson mass, $\ln m_H$, from $\sin^2\theta_{eff}^{lepton}$. The two values for $\alpha(m_Z)$ and ''Total'' correspond to Refs. [7] (larger values) and [8] (smaller values).

Parameter	$\Delta(\ln m_H)$
$\sin^2\theta_{eff}^{lepton}$	0.40
$\alpha(m_Z)$	0.46 or 0.11
m_t	0.32
$\alpha_S(m_Z)$	0.02
theory	0.07
Total	0.67 or 0.52

from the indirect determination is due principally to the remnant of the R_b anomaly — since the current value of R_b is 1.6 standard deviations above the SM fit value, the global fit prefers smaller values of m_t in order to minimize the discrepancy. Because m_t and m_H are correlated in the fit, a higher value of R_b thus leads indirectly to a lower value of m_H in the global fit. Since in this paper I am assuming the validity of the standard model, the strategy followed seeks to minimize the extent of such indirect effects, which during the height of the R_b anomaly (when R_b was believed to be three standard deviations above the SM value) led to a serious distortion of the global fit of m_H [6].

The uncertainty in the SM determination of m_H is analyzed in Table I. The principal sources of uncertainty are the uncertainties in the measurements of $\sin^2\theta_{eff}^{lepton}$ and m_t, and the evaluation of the fine structure constant at m_Z. I use $\sin^2\theta_{eff}^{lepton}=0.23148\pm0.00021$ from the conventional least squares fit of the nine measurements and $m_t=173.8\pm5.1$ GeV from the current PDG fit of the Fermilab top quark mass measurements. For $\alpha(m_Z)$ I use two values $(128.896\pm0.090)^{-1}$ and $(128.933\pm0.021)^{-1}$. The former is the conservative evaluation by Eidelmann and Jegerlehner [7], while the latter, from Davier and Höcker [8], is one of several [12] recent, more optimistic evaluations, which rely on perturbative QCD down to lower energy scales. These typically have a smaller estimated error and a smaller central value, the latter implying a larger value of m_H. In this paper I will present results using both Refs. [7] and [8]. Table I also displays much smaller contributions from the QCD coupling constant, $\alpha_S(m_Z)=0.120\pm0.003$, and from uncomputed higher order corrections. For the latter I rely on the estimate of Degrassi *et al.* [11], whose compact representation of their calculations of the radiative corrections is used throughout this paper.[1] Combined in quadrature the net uncertainty in $\ln(m_H)$ is ±0.67 or ±0.52 for the two evaluations of $\alpha(m_Z)$, corresponding respectively to a factor of 2 or 1.7 uncertainty in m_H.

[1]Weiglein [13] has recently estimated a somewhat larger theoretical error for the results of Ref. [11]. However, in any case the theoretical error is overwhelmed by the three dominant uncertainties in Table I.

TABLE II. Individual measurements of $\sin^2\theta_{eff}^{lepton}$ with 1σ experimental errors and their pulls with respect to the least-squares fit value $\sin^2\theta_{eff}^{lepton}=0.23148\pm0.00021$, listed in the order of the absolute value of the pulls.

Measurement	$\sin^2\theta_{eff}^{lepton}$	Pull
A_{FB}^{τ}	0.22987 (98)	-1.61
A_{LR}	0.23084 (35)	-1.57
A_{FB}^{b}	0.23211 (39)	+1.42
A_{τ}	0.23241 (80)	+1.12
$\langle Q_{FB}\rangle$	0.23210 (100)	+0.60
A_e	0.23193 (90)	+0.48
A_{FB}^{c}	0.23160 (110)	+0.12
A_{FB}^{e}	0.23164 (145)	+0.11
A_{FB}^{μ}	0.23147 (82)	+0.01

The measurements of $\sin^2\theta_{eff}^{lepton}$ have been characterized by three discrepancies, which persevere, though at a diminished level, the 1998 data. In the 1997 data the two most precise measurements, A_{LR} and A_{FB}^{b}, differed by 3.1σ (C.L. $=0.002$), and A_{LR} differed from the LEP average by 2.9σ (C.L.$=0.005$). In the 1998 data $\sin^2\theta_{eff}^{lepton}$ from A_{LR} has increased by 0.7σ while $\sin^2\theta_{eff}^{lepton}$ from A_{FB}^{b} has decreased by 0.6σ, so that the corresponding discrepancies are 2.3σ (C.L. $=0.02$) and 2.4σ (C.L.$=0.015$). The chi-squared for the nine measurements has improved from $\chi^2/N_{\rm DF}=14.5/8$ (C.L. $=0.07$) to a more acceptable $\chi^2/N_{\rm DF}=10.7/8$ (C.L.$=0.2$). The nine measurements are shown in Table II along with their ''pulls,'' defined as the number of standard deviations that each measurement differs from the least-squares fit value 0.23148 ± 0.00021. As another estimator of the consistency of the nine measurements I have used a Monte Carlo simulation to compute the confidence level to replicate the observed distribution of the *absolute values* of the pulls, obtaining a probability of 0.07.[2]

Tables III and IV [corresponding to $\alpha(m_Z)$ from Refs. [7] and [8] respectively] shows the Higgs boson mass predictions of each of the nine $\sin^2\theta_{eff}^{lepton}$ measurements listed in order of precision. For each measurement the tables display the central value for m_H, the symmetric [in $\ln(m_H)$] 90% confidence interval, and the implied probability that m_H lies *below* 89.3 GeV, which is the current 95% C.L. lower limit from the LEP direct searches [9]. To compute the confidence intervals in $\ln(m_H)$ and the implied probabilities for $m_H<89.3$ GeV we must of course include the parametric errors shown in Table I, for instance, by treating $\ln(m_H)$ as a Gaussian statistical variable for each measurement, combining in quadrature the uncertainty arising from the particular measurement of $\sin^2\theta_{eff}^{lepton}$ with the other parametric errors shown in Table I. Equivalently, as a matter of convenience, one may express the parametric errors as effective errors in

[2]That is, 0.07 is the probability that the absolute value of the largest pull is ≥1.61, the second ≥1.57, . . . , and the ninth ≥0.01.

TABLE III. SM Higgs boson mass prediction for the individual measurements, based on $\alpha(m_Z)$ from Refs. [7], listed in order of the precision of the measurements. The central value of m_H is shown along with the symmetric (in $\ln m_H$) 90% confidence interval $m_{95}^{>}, m_{95}^{<}$ and the implied probability that $m_H < 89.3$ GeV.

Measurement	m_H (GeV)	$m_{95}^{>}, m_{95}^{<}$	$P(m_H<89.3$ GeV)
A_{LR}	25	6, 100	0.93
A_{FB}^{b}	280	62, 1300	0.11
A_{τ}	500	35, 7100	0.14
A_{FB}^{μ}	83	5, 1300	0.52
A_e	200	10, 3800	0.33
A_{FB}^{τ}	4	0.2, 95	0.95
Q_{FB}	280	11, 7200	0.29
A_{FB}^{c}	110	4, 2800	0.47
A_{FB}^{e}	110	1, 12000	0.47

TABLE IV. Same as Table III but with $\alpha(m_Z)$ from Ref. [8].

Measurement	m_H (GeV)	$m_{95}^{>}, m_{95}^{<}$	$P(m_H<89.3$ GeV)
A_{LR}	33	10, 110	0.91
A_{FB}^{b}	370	100, 1400	0.04
A_{τ}	660	50, 8600	0.10
A_{FB}^{μ}	110	8, 1500	0.45
A_e	260	15, 4700	0.27
A_{FB}^{τ}	5	0.2, 120	0.93
Q_{FB}	360	15, 8800	0.24
A_{FB}^{c}	140	6, 3400	0.41
A_{FB}^{e}	150	2, 15000	0.42

$\sin^2\theta_{eff}^{lepton}$ (e.g., for fixed, known m_H) and combine them in quadrature with the experimental[3] $\delta(\sin^2\theta_{eff}^{lepton})$.

The question we wish to consider is whether and how the discrepancies of A_{LR} and A_{FB}^{τ} with the SM Higgs boson search limits should affect the SM fit of the Higgs boson mass. The first part of the question is, how big in fact is the discrepancy? The answer depends on precisely how we frame the question. If, without considering the particular central value obtained, we had an *a priori* reason to focus on a particular measurement, say on A_{LR} because it is the most precise and therefore most important single measurement in the fit, then the discrepancy could be read off from Table III or IV (though also including the effect of the less than perfect 95% confidence level of the search limit) and the analysis might then proceed as in Ref. [3]. However, it is fair to say that in the present context our attention is drawn to A_{LR} and A_{FB}^{τ} by the fact of their conflict with the search limits. In that case the appropriately framed question is, how likely is it that any two of the nine measurements could fluctuate to provide discrepancies with the search limits equal or greater than the observed discrepancies? We obtain an upper limit on that probability by assuming that the true value of m_H is precisely at the value of the direct search lower limit, $m_H = 89.3$ GeV.

Let p_τ and p_{LR} be the probabilities implied by the measurements of A_{LR} and A_{FB}^{τ} that m_H lies below 89.3 GeV. Then the upper limit on the probability that any two of nine measurements, *a* and *b*, could fluctuate into the low tail of the $\sin^2\theta_{eff}^{lepton}$ distribution such that $p_a \geqslant p_\tau$ and $p_b \geqslant p_{LR}$ is given by[4]

$$P_9(p_\tau, p_{LR}) = 1 - p_\tau^9 - 9(1-p_\tau)p_{LR}^8 . \qquad (1)$$

Equation (1) is the goodness-of-fit C.L. between the nine measurements and the direct search limit in the standard model, assuming the search limits to be perfect. Taking p_τ and p_{LR} from Tables III and IV we find $P_9(p_\tau, p_{LR}) = 0.12$ and 0.18 respectively. Though we assume here that the search limit has 100% C.L., it is shown in Sec. V that essentially the same results are obtained when the actual confidence levels of the searches are taken into account.

These confidence levels, 0.12 and 0.18, might be characterized as marginal, not big enough to be considered ‘‘robust’’ or small enough to force us to choose between the standard model and the experiments. They are in the gray area to which the Particle Data Group scaling factor S^* would apply if similar C.L.’s were obtained from the χ^2 distribution of a collection of measurements, as discussed in the next section.

III. SCALE FACTORS FOR DISCREPANT DATA

Having quantified the extent of the discrepancy between the search limit and the measurements of $\sin^2\theta_{eff}^{lepton}$ in the SM, we now consider the more difficult aspect of the question: whether and how these discrepancies should affect the SM fit of the Higgs boson mass. There is no single ‘‘right’’ answer. A maximum likelihood fit including both the precision data and the direct search data would replicate the conventional fit if the central value lies above the lower limit, m_H^{LIMIT}, from the direct searches. That is a defensible interpretation, since if the true value of m_H were near m_H^{LIMIT} we would expect values of m_H obtained from measurements of $\sin^2\theta_{eff}^{lepton}$ to lie both above and below m_H^{LIMIT}. By underweighting downward fluctuations while leaving upward fluc-

[3]The theoretical uncertainties of the very large and very small values of m_H in Tables III and IV are somewhat bigger than indicated in Table I. The largest values, $\geqslant 1$ TeV, have no precise meaning in any case. For the very small values, such as $m_H = 4$ MeV from A_{FB}^{τ}, we are really only concerned with the implied probability $P(m_H<89.3$ GeV) which only depends on the relationship between m_H and $\sin^2\theta_{eff}^{lepton}$ at $m_H = m_H^{\mathrm{LIMIT}}$ where Table I does apply.

[4]That is, $P_9(p_\tau, p_{LR})$ is the complement of the probability that all nine measurements have $p_i < p_\tau$ or that one among them has $p_i > p_\tau$ while the other eight have $p_i < p_{LR}$.

tuations at their full weight, we risk skewing the fit upward. Mindful of this risk, it is still instructive to explore the sensitivity of the fit to the weight ascribed to measurements that are in significant contradiction with the direct search limit.

Clearly the direct search limit is not irrelevant. If, for instance, the only information available were the direct search limit and the A_{LR} measurement, we would conclude that the standard model is excluded at 90% C.L. Theorists would have flooded the Los Alamos server with papers on the death of the standard model and the birth of new theories W,X,Y,Z . . . . In the SM fit the A_{LR} measurement causes m_H to shift by a factor of 2, from 170 to 85 GeV, and the 95% upper limit to fall from 570 to 260 GeV. It is fully weighted in the conventional standard model fit despite a significant contradiction with the standard model.

If the discrepancy were even greater — say, for instance, a precision measurement implying[5] $m_H = 11$ MeV with a 99.9% C.L. upper limit at 89 GeV — we would be faced with three alternatives: (1) omit the measurement from the SM fit, presuming a plausible reason exists to suspect a large systematic error, (2) disregard the search limits, presuming them to be systematically flawed in some way, or (3) abandon the standard model. On the other hand, a measurement one-half standard deviation below the lower limit, with a $\simeq 30\%$ probability to be consistent with the limit, would surely be retained at essentially full weight.

The difficult question is how to resolve the intermediate cases in which the discrepancy is significant but not so significant that we are forced to choose between the data and the SM. Assuming the validity of the search limits and of the SM we consider a method that interpolates between the extremes of cases (1) and (2) above and which allows us to explore the sensitivity of the fit to the weight assigned to the discrepant measurements.

There has been much disagreement as to how to combine inconsistent data. The mathematical theory of statistics provides no magic bullets and ultimately the discrepancies can only be resolved by future experiments. The PDG [4] has for many years scaled the uncertainty of discrepant data sets by a factor

$$S^* = \sqrt{\chi^2/(N-1)} \tag{2}$$

where N is the number of measurements being combined. They scale the uncertainty of the combined fit by the factor S^* if and only if $S^* > 1$. This is a conservative prescription, which amounts to requiring that the fit have a good confidence level, ranging from 32% for $N=2$ to $\simeq 44\%$ for $N \simeq 10$. If the confidence level is already good, the scale factor has little effect; it only has a major effect on very discrepant data. The PDG argues (see [15]) that low confidence level fits occur historically at a rate significantly greater than expected by chance, that major discrepancies are often, with time, found to result from underestimated systematic effects, and that the scaled error provides a more cautious interpretation of the data.

As an illustration we apply S^* to the determination of m_H from the nine measurements of $\sin^2\theta_{eff}^{lepton}$. The chi-squared for the nine measurements is 10.7 for 8 degrees of freedom, corresponding to C.L.=0.20. Then $S^* = \sqrt{10.7/8} = 1.16$ and the conventional fit $\sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$ is modified to 0.23148 ± 0.00024. The effect on m_H is negligible: the central value is unchanged, while the 95% C.L. upper limit increases from 255 to just 272 GeV [using [7] for $\alpha(m_Z)$]. The effect on m_H is suppressed by the fact that the experimental error from $\sin^2\theta_{eff}^{lepton}$ is dominated by the parametric error from m_t and $\alpha(m_Z)$ shown in Table I. Even for the more discrepant 1997 data, with $\chi^2 = 14.6$ for 8 degrees of freedom (DOF) and C.L.=0.07, the effect of the S^* factor is moderate, with the 95% C.L. upper limit increasing from 310 to 370 GeV.

We wish to construct an analogous method for situations in which the discrepancy is between some of a collection of measurements and a limit or physical boundary. In analogy to the χ^2 confidence level for S^* our point of departure is the goodness-of-fit (GOF) C.L. between the measurements and the limit, for instance, Eq. (1) for the case at hand. The method is to rescale the errors of the measurements that conflict with the limit by factors that increase the GOF C.L. of the rescaled data to a robust minimum value. Following the PDG the minimum C.L. is chosen to equal the C.L. corresponding to $\chi^2 = N-1$ for $N-1$ degrees of freedom. Regarding the limit as an additional degree of freedom we have $N=10$ for the nine measurements and the limit. The minimum C.L. is then 0.44, corresponding to $\chi^2 = 9$ with 9 DOF.

Since there are two discrepant measurements, there are in general two different scale factors, S_τ and S_{LR}. In the notation of Eq. (1) the GOF C.L. requirement is

$$P_9(p'_\tau, p'_{LR}) = 0.44, \tag{3}$$

where p'_τ and p'_{LR} are the values of p_τ and p_{LR} after rescaling,

$$\delta(\sin^2\theta_{eff}^{lepton})_\tau \rightarrow S_\tau\, \delta(\sin^2\theta_{eff}^{lepton})_\tau \tag{4}$$

and

$$\delta(\sin^2\theta_{eff}^{lepton})_{LR} \rightarrow S_{LR}\, \delta(\sin^2\theta_{eff}^{lepton})_{LR}. \tag{5}$$

Equation (3) imposes one constraint, leaving a one dimensional parameter space within the (S_τ, S_{LR}) plane to consider.

Before turning to the electroweak data, we conclude this section with a general formulation of the method. Consider a collection of N measurements of a physical quantity x,

$$\{x_i, \delta_i\} \quad i = 1, \ldots, N \tag{6}$$

where the x_i are the individual measured values and δ_i are the one standard deviation uncertainties. Suppose there is an exact lower limit or physical boundary (this assumption is relaxed in Sec. V for the Higgs boson search limits),

[5] In fact, parity violation in atomic cesium currently implies $m_H \sim 11$ MeV (MeV is not a typographical error) though only 1.2σ from 89 GeV [14]. Its weight in the combined fit would be negligible.

$$x_{\rm TRUE} > x_{\rm LIMIT}, \tag{7}$$

and that $n \leqslant N$ of the measurements fall below the limit,

$$x_i < x_{\rm LIMIT} \quad i=1,\ldots,n$$
$$x_i > x_{\rm LIMIT} \quad i=n+1,\ldots,N. \tag{8}$$

Furthermore assume, in analogy to p_τ and p_{LR} defined above, that the probability density function (PDF) associated with each of the n low measurements, $PDF_i(x-x_i,\delta_i)$, implies a probability p_i that the measurement conflicts with the limit (7),

$$p_i = \int_{-\infty}^{x_{\rm LIMIT}} PDF_i(x-x_i,\delta_i)\,dx. \tag{9}$$

By analogy with Eq. (1) we compute an upper bound on the GOF C.L. between the N measurements and the limit. We order the n low measurements such that $p_1 > p_2 > \cdots > p_n$. The upper bound is then obtained by assuming

$$x_{\rm TRUE} = x_{\rm LIMIT} \tag{10}$$

and computing the probability that *any* n of the N measurements, designated by ordered integer n-tuples $\{a_1,\ldots,a_n\}$ chosen from the integers $\{1,\ldots,N\}$, ordered such that $p_{a_1} > p_{a_2} > \ldots > p_{a_n}$, satisfy the condition

$$p_{a_i} \geqslant p_i \tag{11}$$

for all $i=1,\ldots,n$.

The combined PDF for the N independent measurements is

$$PDF_N(\{x-x_i,\delta_i\}) = \prod_{i=1}^{N} PDF_i(x-x_i,\delta_i). \tag{12}$$

Finally we can write the upper bound on the GOF C.L. between the N measurements and the limit in the general form

$$P_N(p_1,\ldots,p_n) = \sum_{\{a_1,\ldots,a_n\}} \int_D PDF_N(\{x_{a_i} - x_{\rm LIMIT},\delta_{a_i}\})\,dx_{a_1}\cdots dx_{a_n} \tag{13}$$

where the sum is over all ordered integer n-tuples $\{a_1,\ldots,a_n\}$ chosen from the integers $\{1,\ldots,N\}$ and the domain of integration D is defined by the condition

$$\frac{x_{\rm LIMIT}-x_{a_i}}{\delta_{a_i}} \geqslant \frac{x_{\rm LIMIT}-x_i}{\delta_i} \tag{14}$$

for all $i=1,\ldots,n$.

Equations (12)–(14), in all their obtuse generality, are just a straightforward generalization of the GOF C.L. $P_9(p_1,p_2)$ given explicitly in Eq. (1). The general statement of the method now closely follows that example. We require a minimum GOF C.L.

$$P_N(p_1,\ldots,p_n) \geqslant P_{\rm MIN} \tag{15}$$

where $P_{\rm MIN}$ is the confidence level corresponding to the chi-squared distribution with $\chi^2 = N$ for N degrees of freedom. If Eq. (15) is satisfied by the data we combine the data without further ado. If Eq. (15) is not obeyed, we rescale the errors of the n low measurements,

$$\delta_i \to \delta_i' = S_i \delta_i, \tag{16}$$

so that the p_i defined in Eq. (9) are replaced by p_i',

$$p_i' = \int_{-\infty}^{x_{\rm LIMIT}} PDF_i(x-x_i,\delta_i')\,dx, \tag{17}$$

such that the GOF C.L. for the scaled data satisfies the requirement,

$$P_N(p_1',\ldots,p_n') = P_{\rm MIN}. \tag{18}$$

The condition equation (18) is satisfied by an $n-1$ dimensional subspace of the space of n-tuples $(S_1,\ldots,S_n)$.

This section concludes with a brief discussion of the relationship of the scale factor method to the Cousins-Feldman definition of confidence intervals near a physical boundary [10]. They observe that the standard construction of confidence intervals near a physical boundary is flawed, in that it leads to intervals that in some instances ''under-cover'' (i.e., correspond to less than the nominal probability) and which have discontinuities as a function of the central value that are artifacts of the construction. Particularly germane to the method presented here is their observation that near a boundary the conventional construction confuses two aspects of the fit that are or should be conceptually distinct: that is, the goodness-of-fit C.L. between the measurement and the limit is typically assessed based on the extent that the conventional confidence intervals obtained from the fit overlap the region allowed by the boundary or limit. In contrast, the usual procedure for combining data (away from a boundary) uses the minimum of the chi-squared distribution to asses goodness-of-fit, while the confidence intervals are obtained quite independently from the shape of the chi-squared distribution.

They propose confidence intervals which rectify these shortcomings, at the cost of relaxing the upper limits near the boundary. In particular, their confidence intervals only have support in the allowed region, leaving the assessment of goodness-of-fit as a separate issue. In this paper I use a goodness-of-fit estimator, $P_N(p_1,\ldots,p_n)$, which is quite distinct from the confidence intervals that are the output of the fit. Rather the goodness-of-fit estimator is computed at the outset and is then used to constrain the scale factors that determine the final fit and confidence intervals. The upper limits on m_H obtained from the scaled fits are given with the Cousins-Feldman construction, though for comparison the conventionally defined limits are also provided.

TABLE V. Fits based on $\alpha(m_Z)$ from Ref. [7]. The first line is the conventional fit while the other lines display scaled fits that meet the 44% minimum goodness-of-fit confidence level for the measurements and search limit. For each fit, specified by the pair of scale factors S_τ, S_{LR}, the table displays the fitted value of $\sin^2\theta_{eff}^{lepton}$ with 1σ uncertainty, the central value of m_H, the conventional 95% C.L. upper limit, m_{95}, and the Cousins-Feldman [10] 95% C.L. upper limit, $m_{95}^{\rm CF}$.

S_{LR}	S_τ	$\sin^2\theta_{eff}^{lepton}$	m_H	m_{95}	$m_{95}^{\rm C.F.}$
1	1	0.23148 (21)	85	260	320
1	3.51	0.23155 (22)	97	300	370
1.11	2.27	0.23160 (22)	105	320	400
1.26	1.87	0.23165 (23)	117	370	460
1.42	1.74	0.23170 (24)	127	410	510
1.59	1.71	0.23173 (24)	137	440	550
1.78	1.68	0.23177 (25)	146	480	600
2.01	1.28	0.23177 (25)	147	480	600
2.50	1.16	0.23180 (26)	154	510	640
∞	1.06	0.23186 (27)	175	590	750

IV. SCALED STANDARD MODEL FITS

In this section the scale factor method is applied to the SM Higgs boson mass fit. We indicate how the scaled fit is obtained and present the results. The results in this section are obtained under the assumption that the search limit is perfect, i.e., $m_H > 89.3$ GeV at 100% C.L. In Sec. V, I show that essentially the same results follow from the actual data of the search experiments, as a result of the rapidly rising confidence level for exclusion limits below 89.3 GeV.

The results are shown in Tables V and VI and in Fig. 1. Consider for instance the results using the more conservative evaluation [7] of $\alpha(m_Z)$, shown in Table V and in the solid curves in Fig. 1. Recall from Sec. II that the goodness-of-fit C.L. between the nine measurements and a perfect lower limit at 89.3 GeV is 12%. Table V displays a selection of scaled fits with GOF C.L. of 44%. At one extreme the A_{LR} measurement is unscaled, $S_{LR}=1$, while $S_\tau=3.5$. The effect on the SM fit is negligible: the central value and 95% C.L. upper limits for m_H increase by just $\simeq 15\%$. At the other extreme, if we attempt to leave A_{FB}^τ unscaled, $S_\tau=1$, we find that even if A_{LR} is removed from the fit, $S_{LR}\to\infty$, the GOF C.L. is 39%. At this extreme in order to reach 44% it is necessary to set $S_\tau=1.06$ and $S_{LR}\to\infty$. The effect on the fit is maximal: the central value increases to $m_H=175$ GeV and the 95% C.L. upper limit increases to 750 GeV.

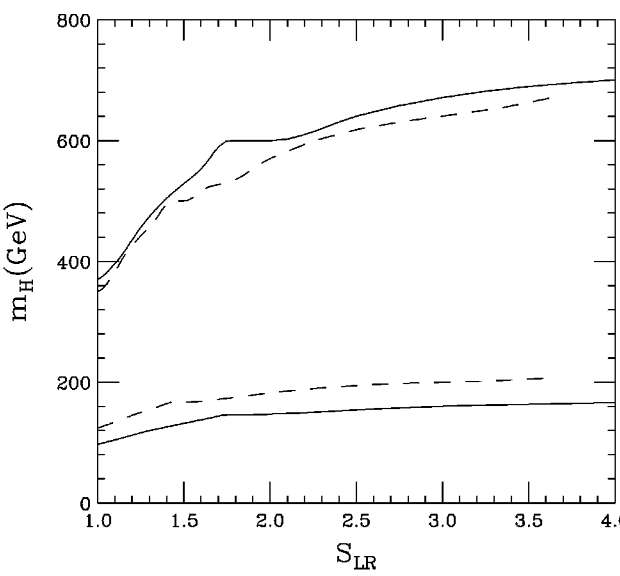

FIG. 1. Scaled fits that meet the minimum goodness-of-fit criterion. The central value and 95% C.L. upper limit for the Higgs boson mass are plotted as a function of the scale factor for $\sin^2\theta_{eff}^{lepton}$ from A_{LR}. Solid and dashed lines correspond to the evaluations of $\alpha(m_Z)$ from Refs. [7] and [8] respectively.

The scaled fits are obtained numerically, as described below. Consider for instance the entry in Table V with $S_{LR}=1$. From Table III we see that $p'_{LR}=p_{LR}=0.932$. Equation (1), $P_9(p'_\tau, 0.932)=0.44$, is solved numerically to obtain $p'_\tau=0.684$. Assuming Gaussian statistics we then deduce from the Gaussian distribution that $\sin^2\theta_{eff}^{lepton}$ from A_{FB}^τ lies $0.475\delta_\tau'^{\rm TOTAL}$ below $\sin^2\theta_{eff}^{lepton}=0.23151$, the latter being the value of $\sin^2\theta_{eff}^{lepton}$ corresponding to $m_H^{\rm LIMIT}=89.3$ GeV. Here ''TOTAL'' in $\delta_\tau'^{\rm TOTAL}$ denotes the sum in quadrature of the rescaled experimental error δ'_τ and the parametric error from the sources shown in Table I,

$$\delta_\tau'^{\rm TOTAL}=\sqrt{\delta_\tau'^2+\delta_P^2}. \tag{19}$$

Taking $\sin^2\theta_{eff}^{lepton}=0.22987$ from A_{FB}^τ we then obtain $\delta_\tau'^{\rm TOTAL}=(0.23151-0.22987)/0.475=0.00345$. Using Ref. [7] the effective parametric error, expressed as an equivalent uncertainty in $\sin^2\theta_{eff}^{lepton}$ is 0.00028, so that[6] $\delta'_\tau=0.00344$, from which we finally obtain $S_\tau=\delta'_\tau/\delta_\tau=0.00344/0.00098=3.51$.

The fits for the intermediate cases are obtained similarly, by fixing either S_τ or S_{LR} and computing the other. Equivalently, one may choose a grid in p'_τ or p'_{LR} and compute the other, from which all other quantities in the fit can be obtained. (The latter was the procedure actually followed to construct Tables V and VI).

Except for a small ''central plateau'' it is clear from the tables and figure that the value of m_H is dominated by S_{LR}, as expected from the importance of A_{LR} in the fit. In Table V the ''central plateau'' occurs between $S_{LR}=1.75$ and $S_{LR}=2.01$, for which the inverse effects of increasing S_{LR} and decreasing S_τ cancel one another. At the extreme of Table V, with $S_{LR}\to\infty$, the value of $\sin^2\theta_{eff}^{lepton}$ is greater than the conventional fit value by two standard deviations, while the central value of m_H is increased by one standard deviation. The shift in m_H is smaller than the shift in $\sin^2\theta_{eff}^{lepton}$ because of the diluting effect of the parametric error in Table I.

Table VI and the dashed lines in Fig. 1 are based on $\alpha(m_Z)$ from Ref. [8]. They display the same general features

[6]The parametric error is negligible compared to δ'_τ but is important relative to more precise measurements such as δ_{LR}.

TABLE VI. As in Table V but with $\alpha(m_Z)$ from Ref. [8].

S_{LR}	S_τ	$\sin^2\theta_{eff}^{lepton}$	m_H	m_{95}	m_{95}^{CF}
1	1	0.23148 (21)	112	260	310
1	1.84	0.23154 (21)	124	295	350
1.07	1.71	0.23157 (22)	131	310	370
1.18	1.60	0.23161 (23)	143	350	420
1.31	1.57	0.23165 (23)	155	385	460
1.45	1.54	0.23169 (24)	167	420	500
1.62	1.18	0.23170 (24)	169	430	520
1.75	1.12	0.23171(24)	173	440	530
2.00	1.08	0.23174 (25)	182	470	570
3.62	1.00	0.23181 (26)	207	550	670

as the fits based on [7]. The central values for m_H are larger while the 95% C.L. upper limits are smaller, because Ref. [8] finds larger $\alpha(m_Z)$ but with smaller claimed uncertainty, and the latter effect dominates the former in the determination of the upper limit. Because the central values are larger, the discrepancies with the search limits are somewhat reduced (cf. Tables III and IV) and consequently the scale factors are smaller. In the extreme case it is possible to satisfy the GOF C.L. requirement of 44% for $S_\tau = 1$ and finite S_{LR}. The fit in that case, with $S_{LR} = 3.6$, yields $m_H = 207$ GeV and $m_H < 670$ GeV at 95% C.L.

V. INCLUDING THE SEARCH LIMIT CONFIDENCE LEVELS

In the previous sections we regarded the search limit, $m_H > 89.3$ GeV, as an absolute boundary, neglecting the fact that it carries a less than perfect 95% confidence level. In this section we will see that the finite confidence level has a negligible effect on the scaled fits and that the results presented in Sec. IV apply to the actual experimental situation.

The conclusion follows from the rather steep dependence of the Higgs boson search limit confidence level as a function of $m_H^{\rm LIMIT}$. For instance, preliminary data [16] from the ALEPH experiment show that the confidence level for $m_H > m_H^{\rm LIMIT}$ is 95% at $m_H^{\rm LIMIT} = 88$ GeV, rising to 99% at 83 GeV and to 99.9% at 78 GeV. These values are conservative since they follow from just one of the four LEP experiments. Furthermore, the conclusion reached below, that the results of Sec. IV apply to the real experimental limits, does not depend at all sensitively on the values quoted above for $m_H^{\rm LIMIT}$ at 99% and 99.9%, since the dependence on $m_H^{\rm LIMIT}$ is logarithmic.

To get an upper limit on the correction to the ''perfect search limit'' results of Sec. IV we consider fits using the evaluation of $\alpha(M_Z)$ claiming greater precision [8], since those fits are most sensitive to the value of $m_H^{\rm LIMIT}$. Consider the goodness-of-fit C.L. for the unscaled data. We refine the notation, making explicit the dependence of the probabilities p_i defined in Eq. (9) on $m_H^{\rm LIMIT}$, by writing $p_\tau(m_H^{\rm LIMIT})$ and $p_{LR}(m_H^{\rm LIMIT})$. Notice from Eq. (9) that these probabilities are defined for perfect search limits. The actual goodness-of-fit C.L. can be obtained by weighting the value for a perfect search limit at 89.3 GeV by its actual 95% C.L., i.e., 0.95×0.181, and then integrating over the corresponding larger goodness-of-fit C.L.'s for smaller values of $m_H^{\rm LIMIT}$, $P_9(p_\tau(m_H^{\rm LIMIT}), p_{LR}(m_H^{\rm LIMIT}))$, weighted by the probability measure given by the derivative of the experimental search limit confidence level with respect to $m_H^{\rm LIMIT}$.

TABLE VII. The goodness-of-fit confidence level between the nine $\sin^2\theta_{eff}^{lepton}$ measurements and the direct search limit for $m_H^{\rm LIMIT}$ corresponding to experimental confidence levels of 95%, 99%, and 99.9%. The GOF C.L.'s, $P_9(p_\tau, p_{LR})$, are computed assuming perfect search limits at each $m_H^{\rm LIMIT}$, as discussed in the text. Reference [8] is used for $\alpha(m_Z)$.

Search limit C.L.	$m_H^{\rm LIMIT}$(GeV)	p_τ	p_{LR}	$P_9(p_\tau, p_{LR})$
95%	89.3	0.933	0.910	0.181
99%	83	0.928	0.894	0.225
99.9%	78	0.924	0.878	0.264

In practice it suffices to obtain an upper limit by approximating the integral by a discrete sum over a few regions, representing the goodness-of-fit C.L. for each region by the maximum for the region, which occurs at the lower boundary of the region in $m_H^{\rm LIMIT}$. In the present instance just two regions will suffice, corresponding to the 99% and 99.9% limits quoted above. To an accuracy of ± 0.001 the upper limit on the true goodness-of-fit C.L. is given by

$$\begin{aligned} P_9^{\rm COMBINED} = {} & 0.95 P_9(p_\tau(89.3\ {\rm GeV}), p_{LR}(89.3\ {\rm GeV})) \\ & + 0.049 P_9(p_\tau(83\ {\rm GeV}), p_{LR}(83\ {\rm GeV})) \\ & + 0.001 P_9(p_\tau(78\ {\rm GeV}), p_{LR}(78\ {\rm GeV})). \end{aligned} \tag{20}$$

The relevant values of p_τ, p_{LR} and P_9 are given in Table VII. Substituting those values into Eq. (20) we find that the actual GOF C.L. is bounded above by 0.183 with an uncertainty ± 0.001. This value differs hardly at all from the 0.181 C.L. that corresponds to a perfect search limit at 89.3 GeV.

Since the scaled data are less precise, the correction due to the actual confidence limits of the searches will be even smaller and is therefore also perfectly negligible for the scaled fits. (I have verified this by applying the above analysis to some of the scaled fits, including the most sensitive case, from Table VI with $S_{LR} = 1$.) In fact, the numerical error in calculating Tables V and VI is of order 0.01, much bigger than the 0.002 correction from the finite confidence level of the search limits. We conclude that the fits shown in Tables V and VI do in fact reflect the actual experimental confidence levels of the direct search limits.

VI. CONCLUSIONS

Motivated by the observation that within the SM framework two of the nine measurements of $\sin^2\theta_{eff}^{lepton}$ are individually in significant conflict with the SM Higgs boson direct search limit, we constructed a scale factor method based on an aggregate goodness-of-fit confidence level between the complete set of nine measurements and the limit. Like an analogous scale factor used for many years by the Particle Data Group, the scale factor proposed here is intended to account for the possibility of underestimated systematic effects. It is applicable to other physical situations in which some of a set of measurements are in conflict with a physical boundary or experimental limit. Applied to the SM Higgs boson mass, the scaled fits exhibit the dependence of the fit on the weight accorded to the two measurements that are in conflict with the search limits. The fits in which the weight of A_{LR} is reduced allow a central value of m_H as large as $\simeq 200$ GeV and a 95% C.L. upper limit as large as 750 GeV. Relative to the conventional least-squares fit, the central value of $\sin^2\theta_{eff}^{lepton}$ increases by as much as two standard deviations while m_H increases by as much as one standard deviation.

It should be clear that the scale factor method proposed here, like that of the PDG, cannot be regarded as providing ''definitive'' fits (if there even is such a thing). Furthermore, the method has problems beyond those it may share with the PDG's S^*. As noted in Sec. III the method biases the fit appreciably to the extent that the very low measurments are low as a consequemce of statistical fluctuations. And when more than one measurement is very low, as in the case considered here, the method does not produce a unique result but only a range of possibilities. In addition, both the PDG fits and those discussed here depend on the arbitrary choice of the minimum confidence level, C.L.$_{MIN}$, chosen in this paper by the same criterion used by the PDG to fix S^*. (Though not considered here or by the PDG, one could explore the dependence of the fits on the choice of C.L.$_{MIN}$.) The most that can be said — all that is claimed — is that the scale factor provides an admittedly imperfect instrument with which to assess the possible impact on the fit of discrepancies that fall in the gray area between robust confidence and obvious inconsistency. In the end only more experimental results can tell us what is really going on.

There is a tendency to think that the value of $\sin^2\theta_{eff}^{lepton}$ is only of interest as a prognosticator of the Higgs boson mass, so that it will be of only secondary interest after or if a Higgs boson is discovered. This view underestimates the importance of $\sin^2\theta_{eff}^{lepton}$ as a fundamental probe of a variety of new physics, not simply restricted to the Higgs sector. By comparing the measured value of $\sin^2\theta_{eff}^{lepton}$ with the value predicted by the directly measured mass of the Higgs boson, we would have a probe of other possible new physics, such as for instance extended gauge sectors or nonsinglet heavy quanta. It would therefore be regrettable if the brilliant program of precision studies of Z particle properties were to conclude with some measure of uncertainty as to how definitively the value of $\sin^2\theta_{eff}^{lepton}$ has been determined.

There are a variety of possible explanations for the anomalies that have affected the measurements of $\sin^2\theta_{eff}^{lepton}$, both the internal inconsistencies, which have diminished but continue to exist as of this writing, and the inconsistencies with the search limits that are the subject of this paper. They may in fact simply be the result of bad luck, chance fluctuations. They may result from underestimated systematic errors among some of the measurements. Or they may represent real effects and be harbingers of new physics. Hopefully the situation will be clarified by further experimental work, beginning with new data and/or analyses to be presented at the 1998 conferences.

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[1] LEP Collaboration, ''A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model,'' Report No. LEPEWWG/98-01, 1998.
[2] CDF Collaboration and D0 Collaboration, Brendan Bevensee, in the proceedings of 33rd Rencontres de Moriond: QCD and High Energy Hadronic Interactions, Les Arcs, France, 1998, Report No. FERMILAB-CONF-98-155-E, 1998.
[3] M. Chanowitz, Phys. Rev. Lett. **80**, 2521 (1998).
[4] Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996); 1998 web update (http://pdg.lbl.gov/).
[5] A. Gurtu, Phys. Lett. B **385**, 415 (1996).
[6] S. Dittmaier, D. Schildknecht, and G. Weiglein, Phys. Lett. B **386**, 247 (1996).
[7] S. Eidelmann and F. Jegerlehner, Z. Phys. C **67**, 585 (1995).
[8] M. Davier and A. Höcker, Phys. Lett. B **435**, 427 (1998).
[9] C. Rembser, presented at The Pheno-CTEQ Symposium 98—Frontiers of Phenomenology: From Non-Perturbative QCD to New Physics, 1998 (unpublished).
[10] G. Feldman and R. Cousins, Phys. Rev. D **57**, 3873 (1998).
[11] G. Degrassi, P. Gambino, M. Passera, and A. Sirlin, Phys. Lett. B **418**, 209 (1998).
[12] For instance see J. Kühn and M. Steinhauser, Phys. Lett. B **437**, 425 (1998); J. Erler, Phys. Rev. D **59**, 054008 (1999).
[13] G. Weiglein, *Zeuthen Workshop on Elementary Particle Theory: Loops and Legs in Gauge Theories*, Rheinsberg, Germany, 1998 [Acta Phys. Pol. B **29**, 2735 (1998)].
[14] A. Deandrea, Phys. Lett. B **409**, 277 (1997).
[15] Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996), p. 3.
[16] ALEPH Collaboration, P. Janot *et al.*, Report No. ALEPH 98-029 (Preliminary), 1998. See also P. Dornan, ALEPH Status Report to LEPC, 1997 (unpublished).