LSND, solar and atmospheric neutrino oscillation experiments, and *R*-parity violating supersymmetry

Rathin Adhikari*
The Institute of Mathematical Sciences, C.I.T Campus, Taramani Chennai-600 113, India

Gordana Omanoviċ†
International Centre for Theoretical Physics, Strada Costiera 11, I-34013 Trieste, Italy
(Received 25 February 1998; revised manuscript received 19 October 1998; published 8 March 1999)

With only three flavors it is possible to account for various neutrino oscillation experiments. The masses and mixing angles for three neutrinos can be determined from the available experimental data on neutrino oscillation and from astrophysical arguments. We have shown here that such masses and mixing angles which can explain the atmospheric neutrino anomaly, LSND result, and the solar neutrino experimental data, can be reconciled with the *R*-parity violating supersymmetric models through lepton-number-violating interactions. We have estimated the order of magnitude for some lepton-number-violating couplings. Our analysis indicates that lepton number violation is likely to be observed in near future experiments. From the data on neutrino oscillation and the electric dipole moment of electron, under some circumstances it is possible to obtain a constraint on the complex phase of some supersymmetry breaking parameters in *R*-parity-violating supersymmetric models. [S0556-2821(99)04405-7]

PACS number(s): 14.60.St, 12.60.Jv, 14.60.Pq

I. INTRODUCTION

Although in the standard model of electroweak and strong interactions the neutrinos are massless to all orders in perturbation theory, in its extension, the neutrinos may acquire small masses with a seesaw type mechanism in the presence of sterile neutrinos. Also such masses can be present in the minimal supersymmetric model with the renormalizable lepton-number-violating terms in the Lagrangian. On the other hand, the astrophysical and cosmological considerations also strongly suggest the existence of massive neutrinos. Presently, there is some possible evidence [1] of massive neutrinos and the mixing of different flavor of neutrinos particularly coming from the anomalies observed in the solar neutrino flux [2], in the atmospheric neutrino production [3,4], and in the neutrino beams from accelerators and reactors [5]. Although some of the evidence, such as that coming from solar neutrinos and accelerator data, has been explained [6] considering one massive and two nearly massless neutrinos, it is in general difficult to fit various neutrino data considering three neutrinos as particularly the first three pieces of evidence are best fitted by three different mass gaps for neutrinos. However, the conventional approach to analyze various observed neutrino anomalies in the experiments is to parametrize those in terms of oscillation of two neutrino states only. This assumption may not hold well while fitting several observed anomalies simultaneously and the consistent three flavor mixing scheme [7] for three neutrinos to analyze various data is essential. Several authors [8–10] have tried to fit various experimental data on neutrino oscillation in the three flavor mixing scheme. It is interesting to note that, including the recent CHOOZ [11] and SuperKamiokande [3,12] results on neutrino oscillation along with other experiments in this direction, it is possible to find the mass square differences and the mixing angles for three neutrinos almost uniquely [9,10]. Furthermore, the analysis in the three flavor mixing scheme indicates sizable oscillations of electron neutrinos to tau neutrinos that should be observed by the long baseline neutrino experiments such as those utilizing a muon storage ring at Fermilab [13]. These analyses [9,10] also indicate that the solar neutrinos observed on Earth should show no Mikheyev-Smirnov-Wolfenstein (MSW) effect [1] as the large mass squared differences has been considered in those analyses. A precise measurement of the multi-GeV, ''overhead'' ($\cos\theta_z \sim 1$) events at SuperKamiokande will also be able to verify the three flavor mixing scheme [9,10] as the double ratio $R=(N_\mu/N_e)_{\text{measured}}/(N_\mu/N_e)_{\text{no oscillation}}$ for the electron and muon for those events is somewhat less than 1 in the three flavor mixing scheme but this ratio is 1 in the analysis with a single oscillation process with small mass square differences for neutrinos. However, present SuperKamiokande data are inconclusive in this low L/E region. Three flavor mixing schemes [9,10] give a very good fit to the SuperKamiokande data [3] for the double ratio for upward going events ($\cos\theta_z < -0.6$) but do not give a very good fit to the data on individual ratio for electron and muon. However, the double ratios are less sensitive to systematic errors than the individual ratios. In these analyses [9,10] the Liquid Scintillation Neutrino Detection (LSND) result has been considered as an oscillation effect rather than an unexplained background. In the near future the BooNE experiment [14] will test the same channel of neutrino oscillation as LSND with higher sensitivity and statistics. Particularly the solutions for the mass square differences and the mixing angles in the three flavor mixing scheme as obtained in Ref. [9] are not significantly contradicted by any existing experimental result and the con-

*Email address: rathin@imsc.ernet.in
†Email address: gordana@ictp.trieste.it

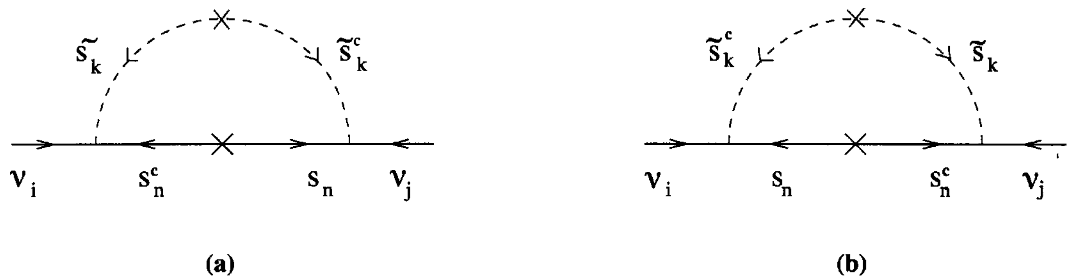

FIG. 1. One loop diagram involving L-violating couplings generating the neutrino mass.

flicting evidence is below the two sigma level. Future various experiments on neutrino oscillation and some of those experiments with higher statistics and lesser systematic errors will be able to verify the three flavor mixing scheme [9,10] and it will be certain whether we really need a fourth sterile neutrino [15]. At present, we feel that the three flavor mixing scheme for neutrinos is very interesting as it has some specific predictions as mentioned before which can be verified by experiments and it tells us about the mass squared differences and the mixing angles almost uniquely.

The uniqueness of the mass square differences and the mixing angles [9,10] for three neutrinos may have a strong impact on physics beyond the standard model in the way they constrain the parameters of other theories. We would like to study such an impact on the minimal R-parity-violating supersymmetric model where neutrinos can acquire mass. In supersymmetric models, R parity was introduced as a matter of convenience to prevent fast proton decay. It is now realized that the proton lifetime can be made consistent with experiment without invoking discrete R-parity symmetry. If we do not impose conservation of R parity in the model, the minimal supersymmetric standard model allows the following B- and L-violating terms in the superpotenial:[1]

$$W=\lambda_{ijk}L^iL^j(E^k)^c+\lambda'_{ijk}L^iQ^j(D^k)^c+\lambda''_{ijk}(U^i)^c(D^j)^c(D^k)^c. \tag{1.1}$$

Here L and Q are the lepton and quark doublet superfields, E^c is the lepton singlet superfield, U^c and D^c are the quark singlet superfields, and i,j,k are the generation indices. In the above, the first two terms are lepton number violating while the third term violates baryon number. For the stability of the proton, we assume that only the first two L-violating terms in the superpotential are nonzero. One may consider Z_n symmetry to remove the B-violating term in the superpotential.[2] As discussed later, L-violating couplings give rise to masses for Majorana neutrinos through one loop diagrams as shown in Fig. 1, which lead to neutrino oscillation phenomena. In this work we will show that in the R-parity-violating supersymmetric models, it is possible to obtain the required mass squared differences and the mixing angles for such massive neutrinos to explain LSND, solar, and atmospheric neutrino oscillation experiments. In our analysis, it is possible to satisfy the bound on the effective mass for the Majorana neutrinos obtained from the neutrinoless double beta decay experiment. We have estimated the magnitude of some of the lepton-number-violating couplings λ_{ijk} and λ'_{ijk} which are required to obtain the appropriate mass square differences and the mixing angles for neutrino oscillation. This kind of study was made earlier [18] in the two flavor mixing scheme with the lesser available neutrino data. Very recently some other studies [19] have also been made to analyze solar and atmospheric neutrino data in the context of supersymmetric models. However, in our work, unlike other works, we have considered solar, atmospheric neutrino oscillation experiments, as well as LSND data to reconcile with the R-parity-violating supersymmetric model. We have also discussed the case for which neutrinos may be considered as dark matter candidates.

There are stringent bounds on different λ_{ijk} and λ'_{ijk} [20] from low-energy processes [21] and very recently the product of two of such couplings has been constrained significantly from the neutrinoless double beta decay [22] and from rare leptonic decays of the long-lived neutral kaon, the muon, and the tau, as well as from the mixing of neutral K and B meson [23]. In most cases it is found that the upper bound on λ'_{ijk} and λ_{ijk} varies from 10^{-1} to 10^{-2} for a sfermion mass of order 100 GeV. For higher sfermion masses these values are even higher. In our analysis, it seems that for a sfermion mass of the order of 100 GeV various L-violating couplings are less than 10^{-2}. Considering the values of L-violating λ' couplings as obtained from our analysis and also considering the constraint obtained from the electric dipole moment of the electron it is possible to obtain constraint on the complex phase of some supersymmetry parameters. In Sec. II, we briefly discuss the constraints on masses and mixing for three neutrinos obtained from LSND, solar, and atmospheric neutrino oscillation experiments. In Sec. III, we discuss the masses of three neutrinos in the R-violating supersymmetric model and show how it is possible to reconcile the masses and the mixing angles as obtained in the three flavor mixing scheme with the R-violating supersymmetric model. We present the required values of some L-violating couplings which satisfy particularly various neutrino oscilla-

[1]One may consider another term $\mu_\alpha L^\alpha H_2$ in the superpotential [16]. However, in general this lepton number violating term can be rotated to the first two terms in the superpotential in Eq. (1.1) unless a symmetry of W does not commute with the SU(4) symmetry of L^α rotations in the field space.

[2]See Ref. [17] for other alternative approaches to forbid dimension four as well as dimension five B violating operators but keeping L violating operators in the Lagrangian.

tion experimental data and also satisfy the constraint on the effective mass for Majorana neutrinos in the neutrinoless double beta decay experiment. We compare these values with the earlier constraint on such couplings. In Sec. IV, we discuss that under some circumstances it is possible to get a constraint on the complex phase of some supersymmetry parameters such as the A parameter. In Sec. V, as concluding remarks we mention the possible implications of the obtained values of L-violating couplings in collider physics and cosmology.

II. CONSTRAINT ON NEUTRINO MASSES AND MIXING

We first mention here the necessary parameters for the three flavor neutrino oscillation. After that, following Refs. [9,10] we shall consider some specific values for the masses and mixing as solutions to satisfy various available experimental data. The neutrino flavor eigenstate is related to the mass eigenstate by

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i , \tag{2.1}$$

where $U_{\alpha i}$ are the elements of a unitary mixing matrix U, $\nu_\alpha = \nu_{e,\mu,\tau}$, and $\nu_i = \nu_{1,2,3}$. According to the standard parametrization [24] of the unitary matrix

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\delta \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\delta & c_{12}c_{23} - s_{12}s_{23}s_{13}\delta & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\delta & -c_{12}s_{23} - s_{12}c_{23}s_{13}\delta & c_{23}c_{13} \end{pmatrix}, \tag{2.2}$$

where $\delta = e^{i\delta_{13}}$ corresponds to the CP-violating phase (which will be neglected here) and c and s stand for sine and cosine of the associated angle placed as subscript. The nondiagonal neutrino mass matrix M_ν in the flavor basis is diagonalized by the unitary matrix U_ν as

$$U_\nu^T M_\nu U_\nu = D_\nu , \tag{2.3}$$

where D_ν is the diagonal mass matrix with the real eigenvalues. In the three generation neutrino mixing scheme, there are two independent mass square differences. These may be considered as Δ_{21} and Δ_{32} where

$$\Delta_{ij} = |m_i^2 - m_j^2| \tag{2.4}$$

and m_i and m_j are the neutrino mass eigenvalues. From the solar neutrino deficit one may consider [9]

$$10^{-4}\ \text{eV}^2 \leqslant \Delta_{21} \leqslant 10^{-3}\ \text{eV}^2 \tag{2.5}$$

in which the lower limit is obtained from SuperKamiokande data [3] and the upper limit is obtained from the CHOOZ experiment [11]. Keeping in mind both the atmospheric and LSND data, another mass square difference Δ_{32} can be considered as [9]

$$\Delta_{32} \approx 0.3\ \text{eV}^2 \tag{2.6}$$

in which the lower limit from the Bugey reactor constraint [25] and the upper limit from the CDHSW [26] have also been considered. In the one mass square difference dominating the other, the three flavor mixing scheme greatly simplifies [27] and one can write the probability of the observation of neutrino oscillation in LSND and SuperKamiokande in terms of Δ_{32} and the elements of U. For the solar neutrino experiment the sine squared terms containing two mass square differences in the expression for the survival probability of solar electron neutrino, can be averaged for the flight length and the energy of the neutrinos observed on Earth. In this case the probability can be written in terms of the elements of U only. As the probability of oscillation in LSND and Superkamiokande and the survival probability for solar electron neutrinos are provided by the experiments, one can solve for the three angles by which matrix U in Eq. (2.2) is defined. The results obtained by Barenboim *et al.* [9] show that four sets of solutions for three angles are possible. However, two sets of solutions can be discarded by considering the SuperKamikande zenith angle ($\cos\theta_z < -0.6$) behavior of atmospheric neutrino data for upward going events. The other two allowed set of solutions for the three angles as obtained in Ref. [9] are

$$\theta_{12} = 54.5, \quad \theta_{13} = 13.1, \quad \theta_{23} = 27.3, \tag{2.7}$$

$$\theta_{12} = 35.5, \quad \theta_{13} = 13.1, \quad \theta_{23} = 27.3. \tag{2.8}$$

The result obtained by Thun *et al.* [10] to satisfy solar, atmospheric, and LSND data almost matches the second set of solutions for three angles as mentioned in Eq. (2.8). In our analysis we shall consider either Eq. (2.7) or Eq. (2.8) for the three angles which specify the unitary matrix U in Eq. (2.2).

The neutrino oscillation experiments give us information about the mass squared differences of three neutrinos in the three flavor mixing scheme as discussed above. However, to know the mass of different neutrinos we have to consider some other experiment. The masses are generated for Majorana neutrinos in the R-violating minimal supersymmetric model, so we have to consider the constraint coming from neutrinoless double beta decay. This gives us an estimate for the masses of neutrinos. The contribution of Majorana neutrinos to the amplitude of the neutrinoless double beta decay [28] is

$$|\mathcal{M}| = \left| \sum_{i=1,2,3} U_{ei}^2 m_{\nu_i} \right| < m(0\nu\beta\beta) = 0.46 \text{ eV}.$$

To satisfy this constraint and keeping in mind that there are some uncertainties in the calculation of the nuclear matrix elements one may consider different masses of the Majorana neutrinos of the order of eV or less [29]. Considering Eqs. (2.5) and (2.6) along with this constraint it is found that there are two interesting possibilities for the masses of neutrinos. In one case, all three neutrinos have almost degenerate mass and we may consider

$$m_2 \approx 1 \text{ eV} \tag{2.9}$$

then the masses for the other two neutrinos are

$$m_1 \approx 1 \text{ eV}, \quad m_3 \approx 1.14 \text{ eV}. \tag{2.10}$$

In another case, the masses of two neutrinos are nearly degenerate whereas the third one is heavier and we may consider

$$m_2 \approx 3 \times 10^{-2} \text{ eV} \tag{2.11}$$

then the masses for the other two neutrinos are

$$m_1 \approx 2 \times 10^{-2} \text{ eV}, \quad m_3 \approx 0.55 \text{ eV}. \tag{2.12}$$

One may consider the neutrinos as candidates for the dark matter solutions also. In that case, if one assumes $\Omega = 1$ and the energy density of the neutrinos $\rho_\nu = 0.2\rho_c$ where ρ_c is the critical density in the big-bang model [30], it is desirable to have the sum of the neutrino masses around 5 eV and one may consider the nearly degenerate three masses of neutrinos given by Eqs. (2.9) and (2.10).

In our analysis we shall consider the abovementioned four interesting possible solutions for masses and mixing angles—one set of solutions from Eqs. (2.7), (2.9), and (2.10), one set of solutions from Eqs. (2.8), (2.9), and (2.10), one set from Eqs. (2.7), (2.11), and (2.12) and the other set from Eqs. (2.8), (2.11), and (2.12). We shall discuss in the next section how all these solutions can be reconciled with *R*-parity-violating supersymmetric model.

III. NEUTRINO MASS MATRIX IN *R*-VIOLATING SUPERSYMMETRIC MODEL AND CONSTRAINT ON *L*-VIOLATING COUPLINGS

The trilinear lepton-number-violating renormalizable term in the superpotential in Eq. (1.1) generates Majorana neutrino masses [16,31] through the generic one loop diagram as shown in Fig. 1 in which s and $\tilde{s}$ stand for either lepton and slepton or quark and squark, respectively. The helicity flip on the internal fermion line is necessary and that requires the mixing of $\tilde{s}$ and $\tilde{s}^c$. The contribution to the mass insertion as shown in Fig. 1, is proportional to the mass m_k of the fermionic superpartner s_k of $\tilde{s}_k$ and is also proportional to $\tilde{m}$ ($\sim A \sim \mu$, where A and μ are the SUSY-breaking mass parameter). The single diagram in Fig. 1 that contributes to the Majorana neutrino mass matrix $m_{\nu_i\nu_j}$ is

$$m_{\nu_i\nu_j} \approx \frac{\lambda_{jkn}\lambda_{ink} m_n m_k \tilde{m}}{16\pi^2 \tilde{m}_k{}^2} \tag{3.1}$$

when one considers the lepton and slepton for s and $\tilde{s}$ in the diagram in Fig. 1. Both the diagrams in (a) and (b) are to be considered together and summed to evaluate the neutrino mass matrix element. However, for $i=j$ and $k=n$, the two diagrams coincide and for that only one is to be considered. For quark and squark in the diagram the similar contribution will be obtained. However, in that case, the above contribution is to be multiplied by a color factor 3 and $\tilde{m}_k$ in the above equation is to be considered as the squark mass instead of slepton mass and the λ couplings in Eq. (3.1) are to be replaced by λ' couplings.

In constructing the neutrino mass matrix we shall consider the following things. First we shall relate squark and slepton mass as

$$\tilde{m}^2_{\text{slepton}} = \tilde{m}^2_{\text{squark}}/K, \tag{3.2}$$

where K is a number depending on the various choices of supersymmetry parameters. Different squarks have almost degenerate mass and different sleptons also have almost degenerate mass as otherwise there is a severe constraint from the flavor changing neutral current. There are 9 λ couplings and 27 λ' couplings entering the neutrino mass matrix. However, in writing each element of the neutrino mass matrix we shall consider only the leading term in terms of the magnitude of mass obtained from Eq. (3.1). We shall consider the diagram with a lepton and slepton in Fig. 1 and shall also consider the diagram with a squark and quark in Fig. 1 in each element of the neutrino mass matrix for which two different types of *L*-violating couplings appear in each element. Under this consideration only the following *L*-violating couplings appear in the neutrino mass matrix. These are $\lambda'_{133}, \lambda'_{233}, \lambda'_{333}, \lambda_{133}, \lambda_{233}, \lambda_{232}, \lambda_{132}$ couplings. The notations for the first five couplings in later discussion will be $\lambda_e^q, \lambda_\mu^q, \lambda_\tau^q, \lambda_e^l, \lambda_\mu^l$, respectively. We are ignoring the effect of other couplings in our analysis and we are assuming that the λ couplings are not much hierarchical among themselves and λ' couplings are also not much hierarchical among themselves. At the end of this section we shall make a few qualitative comments about considering other *L*-violating couplings in the mass matrix. We write the neutrino mass matrix as

$$N=a\begin{pmatrix} Km_\tau^2\lambda_e^{l\,2}+3m_b^2\lambda_e^{q2} & 2Km_\tau^2\lambda_e^l\lambda_\mu^l+6m_b^2\lambda_e^q\lambda_\mu^q & -2Km_\mu m_\tau\lambda_\mu^l\lambda_{132}+6m_b^2\lambda_e^q\lambda_\tau^q \\ 2Km_\tau^2\lambda_e^l\lambda_\mu^l+6m_b^2\lambda_e^q\lambda_\mu^q & Km_\tau^2\lambda_\mu^{l2}+3m_b^2\lambda_\mu^{q\,2} & -2Km_\tau m_\mu\lambda_\mu^l\lambda_{232}+6m_b^2\lambda_\mu^q\lambda_\tau^q \\ -2Km_\mu m_\tau\lambda_\mu^l\lambda_{132}+6m_b^2\lambda_e^q\lambda_\tau^q & -2Km_\tau m_\mu\lambda_\mu^l\lambda_{232}+6m_b^2\lambda_\mu^q\lambda_\tau^q & Km_\mu^2\lambda_{232}^2+3m_b^2\lambda_\tau^{q2}\end{pmatrix}, \tag{3.3}$$

where

$$a=\frac{\tilde{m}}{16\pi^2\tilde{m}_s^2} \tag{3.4}$$

and $\tilde{m}_s$ is the almost degenerate squark mass. The eigenvalues for this matrix correspond to three masses m_1, m_2, and m_3 for three Majorana neutrinos. We can write the diagonal mass matrix as

$$D_\nu=\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3\end{pmatrix}. \tag{3.5}$$

All the elements of this diagonal mass matrix can be written by considering a particular set of solutions for the masses from the earlier section. The unitary matrix U_ν in Eq. (2.2) diagonalizing the nondiagonal neutrino mass matrix in the flavor basis is also known to us if we consider a particular set of solutions for the three angles from the earlier section. As both D_ν and U_ν are known we can obtain the nondiagonal mass matrix M_ν in the flavor basis using the relation

$$U_\nu D_\nu U_\nu^T=M_\nu. \tag{3.6}$$

So for a particular set of solutions for the masses and the mixing angles discussed in the earlier section, all the elements of M_ν are known. However, this M_ν is equal to N which is also the nondiagonal mass matrix expressed in terms of different L-violating couplings. So we write

$$N=M_\nu. \tag{3.7}$$

From Eq. (3.7) we get six equations for the L-violating couplings:

$$Km_\tau^2\lambda_e^{l\,2}+3m_b^2\lambda_e^{q2}=M_\nu(1,1)/a, \tag{3.8}$$

$$2Km_\tau^2\lambda_e^l\lambda_\mu^l+6m_b^2\lambda_e^q\lambda_\mu^q=M_\nu(1,2)/a, \tag{3.9}$$

$$-2Km_\mu m_\tau\lambda_\mu^l\lambda_{132}+6m_b^2\lambda_e^q\lambda_\tau^q=M_\nu(1,3)/a, \tag{3.10}$$

$$Km_\tau^2\lambda_\mu^{l2}+3m_b^2\lambda_\mu^{q\,2}=M_\nu(2,2)/a, \tag{3.11}$$

$$-2Km_\tau m_\mu\lambda_\mu^l\lambda_{232}+6m_b^2\lambda_\mu^q\lambda_\tau^q=M_\nu(2,3)/a, \tag{3.12}$$

$$Km_\mu^2\lambda_{232}^2+3m_b^2\lambda_\tau^{q2}=M_\nu(3,3)/a, \tag{3.13}$$

after comparing the elements (1,1), (1,2), (1,3), (2,2), (2,3), and (3,3), respectively. However, there are seven L-violating couplings involved in these six equations. So we shall consider the possible value of one of the L-violating couplings from some other experiments instead of neutrino oscillation experiments for solving the above six equations to find six L-violating couplings. We shall consider particularly some value of λ_{232} lower than $0.006\tilde{m}_s/\sqrt{\tilde{m}}$ which is allowed after considering the constraint from lepton universality [20,21]. From these six equations we can determine the values of six L-violating couplings for which it is possible to reconcile LSND, solar, and atomspheric neutrino oscillation experimental data with the R-parity-violating supersymmetric model.

To determine M_ν we first consider Eqs. (2.7), (2.9), and (2.10) for the masses and the mixing angles. From Eqs. (2.9) and (2.10) we get a specific D_ν in Eq. (3.5), and from Eq. (2.7) we get a specific U_ν in Eq. (2.2). Using relation (3.6) we obtain the following form of M_ν [here and in later discussions to obtain M_ν we shall consider m_1, m_2, and m_3 in Eq. (3.5) in eV]:

$$M_\nu=\begin{pmatrix} 1.00712 & 0.0143034 & 0.0274612 \\ 0.0143034 & 1.02782 & 0.0542459 \\ 0.0274612 & 0.0542459 & 1.10499\end{pmatrix}. \tag{3.14}$$

We take $m_b=4.3\times10^9$ eV, $m_\tau=1.777\times10^9$ eV, and $m_\mu=0.105658\times10^9$ eV and solve Eqs. (3.8)–(3.13) after considering a specific M_ν in Eq. (3.14). For various allowed real values of λ_{232} lower than that mentioned earlier, the solution for other six L-violating couplings do not change by an order. We present below the the values of these couplings considering λ_{232} in the range $(10^{-6}-10^{-3})\tilde{m}_s/\sqrt{\tilde{m}}$ (here and in later discussions $\tilde{m}_s$ and $\tilde{m}$ stand for the corresponding magnitude in GeV):

$$\lambda_e^q\approx\frac{5.3\times10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}},\quad \lambda_\mu^q\approx\frac{1.3\times10^{-6}\tilde{m}_s}{\sqrt{\tilde{m}}},$$

$$\lambda_\tau^q\approx\frac{5.6\times10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \tag{3.15}$$

$$\lambda_e^l\approx\frac{7.3\times10^{-6}\tilde{m}_s}{\sqrt{K\tilde{m}}},\quad \lambda_\mu^l\approx\frac{2.3\times10^{-4}\tilde{m}_s}{\sqrt{K\tilde{m}}},$$

$$\lambda_{132}\approx\frac{4.0\times10^{-3}\tilde{m}_s}{\sqrt{K\tilde{m}}}. \tag{3.16}$$

Another set of real solutions for various L-violating couplings for the above case is given below:

$$\lambda_e^q \approx \frac{5.3\times 10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \quad \lambda_\mu^q \approx \frac{1.3\times 10^{-6}\tilde{m}_s}{\sqrt{\tilde{m}}},$$

$$\lambda_\tau^q \approx \frac{5.6\times 10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \tag{3.17}$$

$$\lambda_e^l \approx \frac{4.2\times 10^{-6}\tilde{m}_s}{\sqrt{K\tilde{m}}}, \quad \lambda_\mu^l \approx \frac{2.3\times 10^{-4}\tilde{m}_s}{\sqrt{K\tilde{m}}},$$

$$\lambda_{132} \approx \frac{3.9\times 10^{-3}\tilde{m}_s}{\sqrt{K\tilde{m}}}. \tag{3.18}$$

We have ignored the overall + or − sign for the solutions (here and in later cases also) for different L-violating couplings. Although there are different sets of solutions possible, if we ignore the small changes in the higher decimal places for different solutions then mainly the two sets of solutions are found to differ to some extent from each other particularly for the value of λ_e^l and λ_{132} and those two sets of solutions are presented above.

It is important to note here that there is almost no change of the λ_{132} values for various real λ_{232} value in the range $(0.0-10^{-3})\tilde{m}_s/\sqrt{K\tilde{m}}$ and the two values of λ_{132} in Eqs. (3.16) and (3.18) are very close to the upper bound obtained from the experimental value of $R_\tau=\Gamma(\tau\to e\nu\bar{\nu})/\Gamma(\tau\to\mu\nu\bar{\nu})$ [20,21,32]. This indicates that there is a possibility to see the lepton universality violation in future experiments. The same comment is also true for λ_{232} coupling as almost the same real solutions for various L-violating couplings exist for the higher allowed value of λ_{232} coupling. Of course this statement is based on the present neutrino oscillation data and considering the neutrino as a Majorana particle, the main contribution in the neutrino mass matrix coming from the earlier mentioned seven L-violating couplings and the L-violating couplings considered here are real. Although in obtaining the solutions for L-violating couplings we have considered here almost degenerate mass for three neutrinos, but such higher values of λ_{132} or λ_{232} are possible for hierarchical nature of the masses of neutrinos also, as can be seen in the later part of our analysis. If we consider complex or imaginary value of λ_{232} coupling considering the experimental upper bound mentioned earlier it is possible to obtain complex solutions for other six L-violating couplings from Eqs. (3.8)–(3.13) for M_ν in Eq. (3.14). For brevity, we are not presenting those solutions of various L-violating couplings for this case of masses and mixing angles and for other cases also. However, at the end of this section we shall make a few general remarks on the complex solutions for these L-violating couplings. Existence of the possible solutions for L-violating couplings in Eqs. (3.15) and (3.16) or Eqs. (3.17) and (3.18) indicates that considering the almost degenerate mass neutrinos (which may be candidate for dark matter also) as mentioned in Eqs. (2.9) and (2.10), it is possible to reconcile LSND, solar and atmospheric neutrino oscillation data with the R-parity-violating supersymmetric model.

Next, we consider the other possible solutions for the mixing angles as stated in Eq. (2.8) and consider again the almost degenerate mass of neutrinos as mentioned in Eqs. (2.9) and (2.10). In this case, we get the following form of M_ν after using Eq. (3.6):

$$M_\nu=\begin{pmatrix} 1.00704 & 0.0143116 & 0.0274772 \\ 0.0143116 & 1.02788 & 0.054211 \\ 0.0274772 & 0.054211 & 1.105 \end{pmatrix}. \tag{3.19}$$

As in the earlier case, we again solve Eqs. (3.8)–(3.13) for specific M_ν in Eq. (3.19) and obtain the solutions for six L-violating couplings. All the solutions in this case are approximately the same as Eqs. (3.15)–(3.18). So the different set of choices for the mixing angles in Eq. (2.8) do not lead to significant change in the values of L-violating couplings.

We shall consider next the hierarchical neutrino masses as mentioned in Eqs. (2.11) and (2.12). For the three mixing angles we consider Eq. (2.7). As before using Eq. (3.6) we obtain the following form of M_ν:

$$M_\nu=\begin{pmatrix} 0.0534391 & 0.0569347 & 0.10027 \\ 0.0569347 & 0.127333 & 0.202493 \\ 0.10027 & 0.202493 & 0.417771 \end{pmatrix}. \tag{3.20}$$

Solving Eqs. (3.8)–(3.13) for specific M_ν in Eq. (3.20) we obtain the following real solutions for six L-violating couplings. We present below the values of these couplings considering λ_{232} in the range $(10^{-6}-10^{-3})\tilde{m}_s/\sqrt{\tilde{m}}$:

$$\lambda_e^q \approx \frac{8.5\times 10^{-6}\tilde{m}_s}{\sqrt{\tilde{m}}}, \quad \lambda_\mu^q \approx \frac{8.3\times 10^{-6}\tilde{m}_s}{\sqrt{\tilde{m}}},$$

$$\lambda_\tau^q \approx \frac{3.4\times 10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \tag{3.21}$$

$$\lambda_e^l \approx \frac{3.7\times 10^{-5}\tilde{m}_s}{\sqrt{K\tilde{m}}}, \quad \lambda_\mu^l \approx \frac{7.2\times 10^{-5}\tilde{m}_s}{\sqrt{K\tilde{m}}},$$

$$\lambda_{132} \approx \frac{1.8\times 10^{-3}\tilde{m}_s}{\sqrt{K\tilde{m}}}. \tag{3.22}$$

Another set of real solutions for various L-violating couplings for the above case is given below,

$$\lambda_e^q \approx \frac{1.2\times 10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \quad \lambda_\mu^q \approx \frac{8.3\times 10^{-6}\tilde{m}_s}{\sqrt{\tilde{m}}},$$

$$\lambda_\tau^q \approx \frac{3.4\times 10^{-5}\tilde{m}_s}{\sqrt{\tilde{m}}}, \tag{3.23}$$

$$\lambda_e^l \approx \frac{5.2\times 10^{-6}\tilde{m}_s}{\sqrt{K\tilde{m}}}, \quad \lambda_\mu^l \approx \frac{7.2\times 10^{-5}\tilde{m}_s}{\sqrt{K\tilde{m}}},$$

$$\lambda_{132} \approx \frac{1.2\times 10^{-3}\tilde{m}_s}{\sqrt{K\tilde{m}}}. \tag{3.24}$$

So it is seen that considering hierarchical nature of the masses of neutrinos also it is possible to reconcile LSND, solar, and atmospheric neutrino oscillation experiments with the R-violating supersymmetric model. In this case the only thing to note here is that λ_{132} is slightly lower than the earlier cases, however, not far from the present experimental bound obtained from the lepton universality violation [20,21,32].

Next we shall consider the hierarchical mass pattern of neutrinos as in the earlier case but consider the mixing angles as presented in Eq. (2.8). In that case, using Eq. (3.6) we get the following form of M_ν:

$$M_\nu = \begin{pmatrix} 0.0503506 & 0.0572644 & 0.100908 \\ 0.0572644 & 0.129869 & 0.201098 \\ 0.100908 & 0.201098 & 0.418324 \end{pmatrix}. \tag{3.25}$$

As before we solve Eqs. (3.8)–(3.13) for M_ν in Eq. (3.25) and consider the same range for λ_{232} as in earlier cases. In this case, the solutions for L-violating couplings are almost the same as before with hierarchical masses of neutrinos and we are not presenting those solutions seperately.

If the future neutrino oscillation experiments with higher sensitivity and more data support the three flavor mixing scheme as mentioned in Sec. II and the L-violating couplings are real, it is expected that experiments on lepton universality violation in the future will find a signal for the values of λ_{132} or λ_{232} couplings at the level required by our analysis. However, if no signals are found for those values of L-violating couplings, particularly λ_{132} coupling, then the explanation for that may be the following. In that case normally it will be expected that λ_{132} coupling is very small. However, then if one considers again those six equations (3.8)–(3.13) considering λ_{132} as effectively zero it is found that for the various cases for masses and mixing angles, real values of λ_{232} have to be always of the order of $10^{-3}\tilde{m}_s/\sqrt{K\tilde{m}}$. However, if the signal for λ_{232} is also not seen through τ-universality violation, it will be necessary to check the role of other couplings for the analysis of neutrino masses and mixing angles. As under this circumstance, λ_{132} and λ_{232} will be smaller, we may consider the terms next to the leading order in mass in the various elements of the neutrino mass matrix in Eq. (3.3). As the mass factor associated with those other nonleading contributions will be less in magnitude, it is expected that the magnitude of some other coupling should be somewhat higher, as λ_{132} is, to reproduce the similar forms of M_ν mentioned earlier and the lepton number violation, in that case, should be observed through that coupling. Depending on the results of future experiments on neutrino oscillation, tau universality violation, etc., the analysis with other such couplings may be important.

We would like to make a few remarks on the complex solutions for various L-violating couplings. Earlier in obtaining all the abovementioned solutions for different L-violating couplings we have considered value of λ_{232} coupling in the range $(10^{-6}-10^{-3})\tilde{m}_s/\sqrt{K\tilde{m}}$. However, if one considers the value of λ_{232} in the range $(4.0-6.0)10^{-3}\tilde{m}_s/\sqrt{K\tilde{m}}$ which is very near to the experimental upper bound [20], then from Eqs. (3.8)–(3.13) considering a different form of M_ν as mentioned earlier, one will obtain the complex solutions for various L-violating couplings. Furthermore, if one considers imaginary or complex values of λ_{232}, one will obtain the complex solutions for various L-violating couplings. Depending on the various choices of the values of λ_{232} one may obtain from Eqs. (3.8)–(3.16) the various solutions for different L-violating couplings with various possible complex phases. For brevity, we have not presented various possible complex solutions. For some complex phases the solutions may not be allowed depending on the constraint from the value of the electric dipole moment of the electron. We will discuss this in the next section.

The solutions for all λ and λ' couplings are obtained in terms of the parameters $\tilde{m}$, $\tilde{m}_s$, and K. Here $\tilde{m}$ is the supersymmetry breaking mass parameter which is expected to lie in the range of $O(100\text{ GeV})$ to $O(\text{TeV})$. To compare our constraint on λ and λ' couplings with the earlier constraints [20] we shall consider $K\approx 1$ which means the slepton mass does not differ much from the squark mass and we shall also consider both the squark mass and $\tilde{m}$ of $O(100\text{ GeV})$. We shall consider those solutions of L-violating couplings for which complex phases are negligible. The earlier constraints from the tau universality violation [21] are $\lambda_{132}\leq 0.06$, $\lambda_{232}\leq 0.06$, and $\lambda_{233}=\lambda_\mu^l\leq 0.06$ which in our analysis from neutino oscillation, are found to be $O(0.01\text{–}0.04)$, $O(0.01)$,[3] and $O(0.0007\text{–}0.0023)$, respectively. The earlier constraint from $R_l=\Gamma_{\text{hadron}}(Z^0)/\Gamma_l(Z^0)$ [33], $\lambda'_{333}=l_\tau^q\leq 0.26$, and $\lambda'_{233}=l_\mu^q\leq 0.39$ which in our analysis are $O(0.0003-0.0005)$ and $O[(1-8)\times 10^{-5}]$, respectively. The earlier constraints on $\lambda'_{133}=\lambda_e^q$ obtained from the constraint from neutrino mass [34] are $\lambda_e^q\approx 0.002$ and $\lambda_{133}=\lambda_e^l\approx 0.004$ which in our analysis are $O(0.00008\text{–}0.00053)$ and $O(0.00004\text{–}0.00037)$, respectively. So our analysis indicates somewhat lower values of various L-violating couplings than the upper bound on these couplings obtained from other experiments. Furthermore, if one considers $\tilde{m}$ to be nearer to the TeV region then these values of L-violating couplings will be further lowered. The upper bounds on these couplings obtained from the experimental data on neutral currents, β decay [21], muon de-

[3]When we consider a very small value of λ_{132} which can be neglected in the neutrino mass matrix in Eq. (3.3), we get this solution for λ_{232} from Eqs. (3.8)–(3.13). Otherwise various solutions for λ_{232} are possible as mentioned earlier.

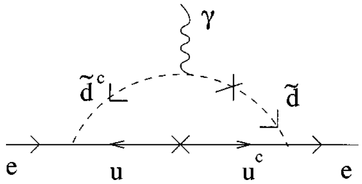

FIG. 2. One loop diagram contributing to the electric dipole moment of the electron.

cay ($\mu\to e\gamma, \mu\to e\overline{e}e$) [35], or tau decay ($\tau\to\mu\gamma, \tau\to e\gamma$) [36], etc., are somewhat higher than the values required in our analysis.

IV. CONSTRAINT ON COMPLEX PHASE OF SUPERSYMMETRY BREAKING PARAMETER

In the standard model the electric dipole moment of the electron is much smaller than the present experimental bound $d_e < 10^{-26}e$ cm [37]. So the new sources of CP violation which occurs in the supersymmetric model can be studied on the basis of the electric dipole moment of the electron [38–40]. In the minimal supersymmetric standard model apart from the Yukawa couplings there are several complex parameters such as three gaugino masses corresponding to SU(3), SU(2), and U(1) groups, the mass parameter m_H in the bilinear term in the Higgs superfields in the superpotential, and dimensionless parameters A and B in the trilinear and bilinear terms of the scalar fields. With a suitable redefinition of the fields, some of these parameters can be made real but some others cannot, such as the A parameter [39]. The complex A will contribute to the electric dipole moment (EDM) of the electron. Furthermore, if we consider the complex λ' couplings, the complex phase associated with those will also contribute to the EDM of the electron. There will be various diagrams in the R-parity-violating supersymmetry for the EDM of the electron [40]. But the significant contribution to the EDM comes from the one loop diagram containing top quark in the loop as shown in Fig. 2. There will be a diagram containing massive neutrinos in the loop. However, the masses of neutrinos are quite small in our discussion and we are ignoring those types of diagrams as there will be a lesser contribution to the EDM of the electron. In terms of complex phases we can write A_f and λ'_{ijk} as

$$A_{u,d} = |A|\exp(i\alpha_{A_{u,d}}), \quad \lambda'_{ijk} = |\lambda'_{ijk}|\exp(i\beta) \qquad (4.1)$$

and the mixing angle for the left and right squark in the familiar way

$$\tan 2\theta = 2|A_u|m_u/(\mu_L^2 - \mu_R^2). \qquad (4.2)$$

Following Ref. [40] and assuming a different λ'_{ijk} containing another complex phase as mentioned in Eq. (4.1) we can write the EDM of the electron from Fig. 2 as

$$d_e \approx -\sin\theta\cos\theta[(\cos^2\beta - \sin^2\beta)\sin\alpha_{A_d} + \cos\beta\sin\beta\cos\alpha_{A_d}]|\lambda'_{1jk}|^2 \times \frac{2|A_d|}{3\tilde{m}_s^3} m_{u_j}[F_1(x_k) + 2F_2(x_k)]10^{-17}e \text{ cm}, \qquad (4.3)$$

where $x_k = (m_{d_k}/\tilde{m}_s)^2$ and the loop integrals F_1 and F_2 are expressed in terms of x_k as

$$F_1(x_k) = \frac{1}{2(1-x_k)^2}\left(1 + x_k + \frac{2x_k \ln x_k}{1-x_k}\right),$$

$$F_2(x_k) = \frac{1}{2(1-x_k)^2}\left(3 - x_k + \frac{2\ln x_k}{1-x_k}\right). \qquad (4.4)$$

A_d and $\tilde{m}_s$ in Eq. (4.3) correspond to the magnitude of those quantities expressed in GeV. We are particularly interested in $j=3$ and $k=3$ in Eq. (4.3). We obtained a solution for $\lambda_e^q = \lambda'_{133}$ in Sec. III to explain the neutrino physics data, so we would like to constrain the complex phases associated with A in Eq. (4.3) here. Considering λ_e^q as real or complex and writing it as $C\tilde{m}_s/\sqrt{\tilde{m}}$ in the form obtained in Sec. III (where C is some value depending on the type of solutions), A, and $\tilde{m}_s$, both from 100 GeV to 1 TeV, it is found that the constraint on the complex phase α_{A_d} and β is

$$[(\cos^2\beta - \sin^2\beta)\sin\alpha_{A_d} + \cos\beta\sin\beta\cos\alpha_{A_d}]\sin\theta\cos\theta \lesssim \frac{(2-16)\times 10^{-14}}{|C|^2}. \qquad (4.5)$$

In the case for which there is no contribution to the EDM in Eq. (4.3) is $\beta = -\alpha_A/2$. For the complex solutions of λ_e^q for which $|C|$ is not less than about 10^{-6} we can make the following statements. For $\beta\approx\pi/4$ and $\alpha_{A_d}\approx\pi/2$ the above inequality can be satisfied for any value of θ. However, such a large phase for A is not possible as the EDM of the electron will get a contribution from other diagrams involving charginos and neutralinos at the one loop level [38] cancelling this possibility. So $\beta\approx\pi/4$ is not possible for any value of θ. So those sets of complex solutions for different L-violating couplings should not be considered when the complex phase associated with λ_q^e is found to be approximately $\pi/4$ and $|C|$ satisfies the above condition. Let us consider that λ'_{1jk} is real and $\theta = \pi/4$. In this case it is seen from Sec. III that $C \approx 10^{-5}$ and the complex phase for the A parameter $\alpha_{A_d} < 3.2\times 10^{-3}$. Without any specific choice of the mixing angle θ one can constrain only the combination of β, α_A, and θ as shown in Eq. (4.5).

V. CONCLUSION

We have shown here that in the minimal supersymmetric model with R-parity-violating trilinear terms in the superpotential in Eq. (1.1) it is possible to obtain the appropriate mass square differences and the mixing angles as required to explain the LSND, atmospheric, and solar neutrino oscillation experimental data in the three flavor mixing scheme for neutrinos. The validity of the three flavor mixing scheme will be verified in the near future experiments on neutrino oscillation as mentioned in the Introduction. The masses for three Majorana neutrinos are generated at one loop level as shown in Fig. 1 and it is possible to satisfy the constraint on the masses and the mixing angles coming from the neutrinoless double beta decay. In each element of the neutrino mass matrix in Eq. (3.3) we have considered two leading terms in terms of the magnitude of masses in Eq. (3.1) coming from the diagram with a slepton and lepton and also coming from the diagram with a quark and squark in Fig. 1. Under this consideration it is interesting to note that for real values of various L-violating couplings at least one of the couplings λ_{132} or λ_{232} is expected to be quite high and very near the experimental upper bound coming from the τ-universality violation [20,21,32]. Apart from these two particular couplings for some of the L-violating couplings the magnitudes are such that it might be possible to observe such a L-violating interaction at the Fermilab Tevatron or at DESY ep collider HERA. At the Tevatron after squark pair production some squarks will decay to a LSP (say a neutralino) and others will decay via the $L^iL^jE^{k^c}$ operator giving a multilepton signal [41]. At HERA one can see a R-violating supersymmetry signal for the $L^iQ^jD^{k^c}$ operator [42] through resonant squark production and its subsequent decay to the electron or positron and neutrino giving the signal of high p_T electron or high p_T positron or missing p_T for neutrino. The basic requirement for the observation of such signal is that lightest supersymmetric particle (LSP) has to decay inside the detector [41,42]:

$$\lambda,\lambda' \gtrsim 10^{-5}\left(\frac{m_{\tilde{l},\tilde{q}}}{100\ \text{GeV}}\right)^2,$$

where $m_{\tilde{l},\tilde{q}}$ stand for the squark and the slepton mass. From the above condition it is seen that if we consider $\tilde{m}$ of the order of the squark mass $\tilde{m}_s$ for a squark mass or slepton mass of the order of 300 GeV, it may be possible to observe an L-violating signal for those couplings discussed in Sec. III for which $\lambda,\lambda' > 5\times10^{-6}\tilde{m}_s/\sqrt{\tilde{m}}$; whereas for squark or slepton masses of the order of TeV the condition is $\lambda,\ \lambda' > 3\times10^{-5}\tilde{m}_s/\sqrt{\tilde{m}}$. So for various couplings considered in our analysis in Sec. III, it might be possible to observe an L-violating signal.

If one considers the baryogenesis in the early universe at the grand unified theory (GUT) scale, after the generation of asymmetry to satisfy the out of equilibrium condition one requires L-violating couplings $\sim 10^{-7}$ for a squark mass in the 100 GeV to 1 TeV range [43] which is significantly smaller than the values of some of the couplings obtained in Sec. III. So if one would like to satisfy the neutrino physics experimental data in the three flavor mixing scheme, it seems that in the R-violating supersymmetric scenario the generation of the baryonic asymmetry near the electroweak scale is more favored where the constraint on the L-violating couplings are not so severe [44]. We have shown that in the R-violating supersymmetric models the neutrino can be considered as dark matter candidates also. Our analysis also indicates that to satisfy various experimental data on neutrino oscillation, the lepton-number-violating couplings are constrained in such a way that some combinations of left and right squark mixing angles and the complex phases of some supersymmetry parameters—particularly that of the A parameter is constrained.

ACKNOWLEDGMENTS

One of us (R.A) would like to thank E. Lisi for a nice discussion on neutrino oscillation experiments. Both of us would like to thank G. Senjanovic and F. Vissani for discussions at the early stage of this work. One of us (R.A) wishes to thank A. Smirnov also for discussions during the Extended Workshop on the Highlights in Astroparticle Physics held at ICTP.

[1] A. Yu. Smirnov, in *The proceedings of the 28th International Conference on High Energy Physics* Warsaw, Poland, 1996, hep-ph/9611465.

[2] B. T. Cleveland *et al.*, Nucl. Phys. B (Proc. Suppl.) **38**, 47 (1995); Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); J. N. Bahcall, S. Basu, and M. H. Pinsonneault, Phys. Lett. B **433**, 1 (1998).

[3] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Lett. B **433**, 9 (1988); **436**, 33 (1998); Phys. Rev. Lett. **81**, 1158; (1998) **81**, 4279(E) (1998); **81**, 1562 (1998).

[4] K. S. Hirata *et al.*, Phys. Lett. B **280**, 146 (1992).

[5] C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995); **77**, 3082 (1996).

[6] K. S. Babu, J. C. Pati, and F. Wilczek, Phys. Lett. B **359**, 351 (1995); **364**, 251 (1995).

[7] H. Minakata, Phys. Rev. D **52**, 6630 (1995); S. Goswami, K. Kar, and A. Raychaudhuri, Int. J. Mod. Phys. A **12**, 781 (1997); A. Acker and S. Pakvasa, Phys. Lett. B **397**, 209 (1997); G. L. Fogli, E. Lisi, D. Montanino, and G. Scioscia, Phys. Rev. D **56**, 4365 (1997); V. Barger, T. J. Weiler, and K. Whisnant, hep-ph/9807319.

[8] C. Y. Cardall and G. M. Fuller, Phys. Rev. D **53**, 4421 (1996); T. Teshima and T. Sakai, hep-ph/9805386.

[9] G. Barenboim and F. Scheck, Phys. Lett. B **440**, 332 (1998).

[10] R. P. Thun and S. McKee, Phys. Lett. B **439**, 123 (1998).

[11] CHOOZ Collaboration, M. Appolonio *et al.*, Phys. Lett. B **420**, 397 (1998); Mohan Narayan, G. Rajasekaran, and S. Uma Sankar, Phys. Rev. D **58**, 031 301 (1998).

[12] T. Kitamura and T. Nakatsuka, Nuovo Cimento A **103**, 1443 (1990).

[13] S. Geer, Phys. Rev. D **57**, 6989 (1998); V. Barger, K. Whisnant, and T. J. Weiler, Phys. Lett. B **427**, 97 (1998); V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys. Rev. D **58**, 093 016 (1998).
[14] E. Church *et al.*, nucl-ex/9706011.
[15] S. M. Bilenky, C. Giunti, and W. Grimus, Nucl. Phys. B (Proc. Suppl.) **666**, 404 (1998).
[16] H. P. Nilles and N. Polonsky, Nucl. Phys. **B484**, 33 (1997), and references therein for the neutrino mass generation in supersymmetric models.
[17] C. S. Aulakh and R. N. Mohapatra, Phys. Lett. **119B**, 136 (1982); R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986).
[18] K. Enqvist, A. Masiero, and A. Riotto, Nucl. Phys. **B373**, 95 (1992).
[19] M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, Phys. Rev. D **57**, R5335 (1998); E. J. Chun *et al.*, hep-ph/9807327; V. Bednyakov, A. Faessler, and S. Kovalenko, hep-ph/9808224; O. C. W. Kong, hep-ph/9808304.
[20] H. Dreiner, in *Perspectives on Supersymmetry*, edited by G. L. Kane (World Scientific, Singapore, in press), hep-ph/9707435.
[21] V. Barger, G. F. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989)
[22] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **75**, 2276 (1995).
[23] D. Choudhury and P. Roy, Phys. Lett. B **378**, 153 (1996).
[24] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[25] B. Achkar *et al.*, Nucl. Phys. **B434**, 503 (1995).
[26] F. Dydak *et al.*, Phys. Lett. **134B**, 281 (1984).
[27] G. L. Fogli, E. Lisi, and G. Scioscia, Phys. Rev. D **52**, 5334 (1995).
[28] L. Baudis *et al.*, Phys. Lett. B **407**, 219 (1997).
[29] T. Tomada, A. Faessler, K. W. Schmid, and F. Grümmer, Nucl. Phys. **A452**, 591 (1986); J. Engel, P. Vogel, X. Ji, and S. Pittel, Phys. Lett. B **225**, 5 (1989).
[30] J. R. Primack, *Dark Matter and Structure Formation*, Presented at the Jerusalem Winter School 1996, astro-ph/9707285.
[31] R. Barbieri, M. M. Guzzo, A. Masiero, and D. Tommasini, Phys. Lett. B **252**, 251 (1990).
[32] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **70**, 561 (1996).
[33] G. Bhattacharya, J. Ellis, and K. Sridhar, Mod. Phys. Lett. A **10**, 1583 (1995); G. Bhattacharya, D. Choudhury, and K. Sridhar, Phys. Lett. B **355**, 193 (1995); J. Ellis, S. Lola, and K. Sridhar, *ibid.* **408**, 252 (1997).
[34] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); R. M. Godbole, P. Roy, and X. Tata, *ibid.* **B401**, 67 (1993); A. I. Belesev *et al.*, Phys. Lett. B **350**, 263 (1995).
[35] Particle Data Group [24].
[36] A. Masiero, Workshop on Tau Lepton Physics, Orsay, France, 1990 (unpublished).
[37] K. Abdullah *et al.*, Phys. Rev. Lett. **65**, 2347 (1990).
[38] Y. Kizukuri and N. Oshimo, Phys. Rev. D **46**, 3025 (1992).
[39] M. Aoki, T. Kodoyoshi, A. Sugamoto, and N. Oshimo, Proceedings of the KEK meetings on ''*CP* violation and its origin,'' hep-ph/9706500.
[40] M. Frank and H. Hamidian, J. Phys. G **24**, 2203 (1998).
[41] D. P. Roy, Phys. Lett. B **283**, 270 (1992).
[42] J. Butterworth and H. Dreiner, Nucl. Phys. **B397**, 3 (1993); T. Konand and T. Kobayashi, Phys. Lett. B **270**, 81 (1991).
[43] B. A. Campbell, S. Davidson, J. Ellis, and K. Olive, Phys. Lett. B **256**, 457 (1991); H. Dreiner and G. G. Ross, Nucl. Phys. **B410**, 188 (1993).
[44] A. Masiero and A. Riotto, Phys. Lett. B **289**, 73 (1992); U. Sarkar and R. Adhikari, Phys. Rev. D **55**, 3836 (1997).