Sachs-Wolfe effect: Gauge independence and a general expression

Jai-chan Hwang
Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Taegu, Korea and Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85740 Garching bei München, Germany

Hyerim Noh
Korea Astronomy Observatory, San 36-1, Whaam-dong, Yusung-gu, Daejon, Korea and Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Strasse 1, D-85740 Garching bei München, Germany

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We address two points concerning the Sachs-Wolfe effect: (i) the gauge independence of the observable temperature anisotropy and (ii) a gauge-invariant expression of the effect considering the most general situation of hydrodynamic perturbations. The first result follows because the gauge transformation of the temperature fluctuation at the observation event only contributes to the isotropic temperature change which, in practice, is absorbed into the definition of the background temperature. Thus, we proceed without fixing the gauge condition, and express the Sachs-Wolfe effect using the gauge-invariant variables. [S0556-2821(99)07104-0]

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The excess noise in the radio sky discovered by Penzias and Wilson in 1965 [1] was immediately recognized as the remnant of an early hot stage in our universe. We call it the cosmic microwave background radiation (CMBR). Soon after its discovery, in a fundamental paper published in 1967 [2] Sachs and Wolfe pointed out that the CMBR should show the temperature anisotropy caused by photons traveling in the perturbed metric which is associated with the large-scale structure formation processes based on gravitational instability. The dipole and higher multipole anisotropies have now been discovered [3]. There have been many studies of the Sachs-Wolfe effect in the literature [4–8]. We notice, however, the two points mentioned in the abstract may still deserve addressing. Despite its trivial nature, we found the first point has not always been well understood by workers in the field. We explain it below Eq. (10) and below Eq. (14). The general expression mentioned in the second point is in Eq. (17) which is the main result in this Brief Report. We now explain our notation and strategy.

As the metric we consider a spatially homogeneous and isotropic one with the most general spacetime-dependent perturbations (we set $c\equiv 1$):

$$ds^2=-a^2(1+2\alpha)d\eta^2-a^2(\beta_{,\alpha}+bY^{(v)}_\alpha)d\eta dx^\alpha +a^2[g^{(3)}_{\alpha\beta}(1+2\varphi)+2(\gamma_{,\alpha|\beta}+cY^{(v)}_{(\alpha|\beta)}+C^{(t)}_{\alpha\beta})]dx^\alpha dx^\beta. \tag{1}$$

The four-velocity of the fluid is $u^0\equiv a^{-1}(1-\alpha)$ and $u^\alpha \equiv a^{-1}(-k^{-1}v^{(s)|\alpha}+v^{(v)}Y^{(v)\alpha})$. $Y^{(v)}_\alpha$ and $C^{(t)}_{\alpha\beta}$ are based on $g^{(3)}_{\alpha\beta}$, and a vertical bar indicates the covariant derivative based on $g^{(3)}_{\alpha\beta}$. $Y^{(v)}_\alpha$ is a (transverse) vector harmonic function [9,10]. The (transverse-tracefree) tensor-type perturbation $C^{(t)}_{\alpha\beta}$ is invariant under the gauge transformation, and we can construct the gauge-invariant combinations for the vector-type perturbation [9]: $v^{(v)}-b\equiv v_\omega$, $v^{(v)}+c'\equiv v_\sigma$, and $v_\sigma-v_\omega\equiv\Psi$, where a prime denotes the time derivative based on η. Due to the spatial homogeneity and isotropy in the background spacetime the three perturbation types decouple and evolve independently to the linear order.

The scalar-type variables depend on the gauge transformation. Our strategy concerning the gauge is to use the several available gauge conditions as an advantage for handling problems. A certain gauge condition is suitable for handling a certain aspect of the problem. But, usually we do not know which gauge is the suitable one *a priori*. Thus, it is desirable to design the equations so that we can easily impose the fundamental gauge conditions [11,12]. We call it the *gauge-ready* approach, and the relativistic perturbation equations in the gauge-ready form needed in this work can be found in Ref. [13].

Under the gauge transformation $\tilde{x}^a=x^a+\xi^a$, the variables transform as ($\xi^t\equiv a\xi^\eta$) [12]

$$\tilde{\varphi}=\varphi-H\xi^t,\ \ \tilde{\chi}=\chi-\xi^t,\ \ \tilde{v}=v-(k/a)\,\xi^t,\ \ \tilde{\alpha}=\alpha-\dot{\xi}^t,$$

$$\tilde{\delta}=\delta+3(1+w)H\xi^t,\ \ \delta\tilde{T}=\delta T+HT\xi^t, \tag{2}$$

where $H\equiv\dot{a}/a$ and an overdot denotes the derivative with respect to $t(dt\equiv ad\eta)$; $w\equiv p/\mu$ where μ and p are the energy density and the pressure, respectively. φ, $\chi\equiv a(\beta+\gamma')$, $v\equiv(v^{(s)}+k\beta)$, and $\delta\equiv\delta\mu/\mu$ are perturbed parts of the three-space curvature, shear, velocity, and relative density variables, respectively; all these variables are spatially gauge invariant. For the temperature $T(\mathbf{x},t)$, we decompose it into the background and perturbed parts as

$$T(\mathbf{x},t)=\bar{T}(t)+\delta T(\mathbf{x},t), \tag{3}$$

where an overbar indicates a quantity to the background order; we neglect it unless necessary. If we regard T as a scalar quantity, the perturbed part changes as $\delta\tilde{T}=\delta T-\dot{T}\xi^t$, and considering $\bar{T}\propto a^{-1}$, we have Eq. (2); some of the fundamental gauge conditions we can recognize in Eq. (2) are the uniform-curvature gauge ($\varphi\equiv 0$), the zero-shear gauge ($\chi\equiv 0$), the comoving gauge ($v\equiv 0$), the uniform-density gauge ($\delta\equiv 0$), and the uniform-temperature gauge ($\delta T\equiv 0$). Each one of these gauge conditions fixes the temporal gauge transformation property completely (i.e., $\xi^t=0$) and, thus, each variable in these gauge conditions is equivalent to a corresponding gauge-invariant combination. The synchronous gauge imposes $\alpha=0$ and fails to fix the gauge mode completely; i.e., we still have $\xi^t=\xi^t(\mathbf{x})$.

We proposed to write the gauge-invariant variables as

$$\delta_v \equiv \delta + 3(1+\mathrm{w})(aH/k)\,v,\quad \varphi_\chi \equiv \varphi - H\chi,\quad \alpha_\chi \equiv \alpha - \dot{\chi},$$
$$v_\chi \equiv v - (k/a)\,\chi,\quad \varphi_v \equiv \varphi - (aH/k)\,v,\quad \text{etc.} \tag{4}$$

δ_v becomes δ in the comoving gauge ($v\equiv 0$), etc. In this manner, using Eq. (2), we can systematically construct the corresponding gauge-invariant combination for any variable based on a gauge condition which fixes the temporal gauge transformation property completely. A given variable evaluated in different gauges can be considered as different variables, and they show different behaviors in general.

The background universe is described by

$$H^2=(8\pi G/3)\,\mu-(K/a^2)+(\Lambda/3),\quad \dot{\mu}=-3H(\mu+p), \tag{5}$$

where K and Λ are the three-space curvature and the cosmological constant, respectively. Later, it is convenient to have the following equations, derived in Ref. [13]:

$$[(k^2-3K)/a^2]\,\varphi_\chi = 4\pi G\mu\,\delta_v, \tag{6}$$

$$\dot{\varphi}_\chi + H\varphi_\chi = -4\pi G(\mu+p)\,(a/k)\,v_\chi - 8\pi G H\sigma, \tag{7}$$

$$\alpha_\chi = -\varphi_\chi - 8\pi G\sigma, \tag{8}$$

where $\sigma(\mathbf{x},t)$ indicates the anisotropic pressure.

The CMBR has a black-body distribution and the photons are redshifted during their travel from last scattering to the observer. After the last scattering, the photons are effectively collision-free and non-self-gravitating, thus follow the geodesic path in the given (perturbed) metric. The null vector tangent to the geodesic $x^a(\lambda)$ with an affine parameter λ is $k^a=dx^a/d\lambda$. We define the null energy-momentum four-vector k^a to the perturbed order as $k^0\equiv a^{-1}(\bar{\nu}+\delta\nu)$ and $k^\alpha \equiv -\bar{\nu}a^{-1}(\bar{e}^\alpha+\delta e^\alpha)$. The temperatures of the CMBR at two different points (O and E) along a single null-geodesic ray in a given observational direction is [2]

$$T_O/T_E = (k^a u_a)_O/(k^b u_b)_E, \tag{9}$$

where O is the observed event here and now and E is the emitted event at the intersection of the ray and the last scattering surface. u_a at O and E are the local four velocities of the observer and the emitter, respectively. In the large angular scale we are considering (larger than the horizon size at the last scattering era which subtends about $2\sqrt{\Omega_0}$ degree by an observer today) the detailed dynamics at last scattering is not important. The physical processes of last scattering are important in the small angular scale where we need to solve the Boltzmann equation for the photon distribution function [8].

The observed temperature along the single ray may depend on the location of the observer $\mathbf{x}_O$ (cosmic variance), and the direction of the observed ray $\mathbf{e}_O$. Similarly as in Eq. (3) we may decompose the *observed* temperature along the single ray into the background and perturbed parts as

$$T(\mathbf{x}_O,t_O;\mathbf{e}_O)=\bar{T}(\mathbf{x}_O,t_O)+\delta T(\mathbf{x}_O,t_O;\mathbf{e}_O). \tag{10}$$

Although we used similar notations in Eqs. (3), (10), it is desirable to notice the difference: Eq. (3) decomposed the temperature at spacetime points, whereas Eq. (10) decomposed the observed temperature along different directions $\mathbf{e}_O$ at observer's location $\mathbf{x}_O$. Up to this point, the decomposition in Eq. (10) still has arbitrariness as the one in Eq. (3). In the observations, however, we often take the background temperature as an averaged temperature all around the sky at the observer's location, i.e., $\bar{T}(\mathbf{x}_O,t_O)\equiv\langle T(\mathbf{x}_O,t_O;\mathbf{e}_O)\rangle_{\mathbf{e}_O}$. In this way the arbitrariness is fixed, and the remaining $\delta T|_O$ over the sky apparently coincides with the angular variation of observed temperature. Thus, $\delta T|_O$ should be independent of the gauge condition (imposed at the observer's spacetime position). Let us explain this last point below. In the temporally evolving background, $\bar{T}=\bar{T}(t)$, δT is a gauge-dependent quantity. The gauge dependence of δT should be considered in handling fluctuations at the last scattering era E. However, for δT evaluated at the observation event O, the effect of the gauge transformation $H\xi^t(\mathbf{x},t)$ evaluated at O will show no angular dependence, thus can be *absorbed* into our definition of the background temperature, and is irrelevant for the temperature anisotropy; thus, the observable temperature *anisotropy* is a concept *independent of the gauge condition* used [14]. Equivalently, since $H\xi^t|_O$ terms cancel, the difference of observed temperatures in two different directions is gauge invariant.

Perturbation analyses of the null equation ($k^a k_a=0$), the geodesic equation ($k^a{}_{;b}k^b=0$), and Eq. (9) provide the equations we need. To the background order, we have: $\bar{T}\propto\bar{\nu}\propto a^{-1}$, $\bar{e}^\alpha\bar{e}_\alpha=1$, and $\bar{e}^{\alpha\prime}=\bar{e}^\alpha{}_{|\beta}\bar{e}^\beta$. To the perturbed order, we have [for convenience, we consider the contributions from three perturbation types separately as $\delta T|_O=\delta T^{(s)}|_O+\delta T^{(v)}|_O+\delta T^{(t)}|_O$]:

$$(\delta T^{(s)}/T)|_O=(\delta T/T)|_E-(1/k)\,v_{,\alpha}e^\alpha|_E^O + \int_E^O[-\varphi'+\alpha_{,\alpha}e^\alpha-(1/a)\,\chi_{,\alpha|\beta}e^\alpha e^\beta]dy, \tag{11}$$

$$(\delta T^{(v)}/T)|_O=v_\omega Y^{(v)}_\alpha e^\alpha|_E^O-\int_E^O\Psi Y^{(v)}_{(\alpha|\beta)}e^\alpha e^\beta dy, \tag{12}$$

$$(\delta T^{(t)/T})|_O=-\int_E^O C^{(t)\prime}_{\alpha\beta}e^\alpha e^\beta dy, \tag{13}$$

where $d/dy\equiv\partial/\partial\eta-\bar{e}^\alpha\partial/\partial x^\alpha$ [thus, the integral is along the ray's null-geodesic path]. The temperature fluctuation in the last scattering era $\delta T/T|_E$, contributes to the scalar-type perturbation. Equations (12), (13) are apparently gauge-invariant. Equation (11) is written in a gauge ready form, so that we can impose any gauge condition we want. Each term on the right hand side (RHS) of Eq. (11) depends on the temporal gauge transformation and the gauge invariance of the terms altogether is not obvious; using Eq. (2) we can show that the RHS alone is not gauge invariant [apparently, the LHS is not also gauge invariant, so that the overall equation is gauge invariant]. Shortly, we will see that the observable contributions to anisotropy can be expressed in terms of gauge-invariant variables.

Now, we concentrate on the scalar-type contribution in Eq. (11). In hydrodynamic perturbation based on Einstein gravity, it is known that only certain variables in certain gauge conditions correctly reproduce the Newtonian behaviors in the pressureless limit: the density perturbation variable in the comoving gauge (δ_v), and the perturbed potential and the perturbed velocity variables in the zero shear gauge (φ_χ and v_χ) show the correct behavior of the corresponding Newtonian ones [15,9]. These correspondences apply in *general scales* (including the superhorizon scale) considering the general K and Λ [13]: $v_\chi \leftrightarrow \delta v$, $-\varphi_\chi \leftrightarrow \delta\Phi$, where δv and $\delta\Phi$ are the Newtonian velocity and potential fluctuations, respectively.

Using these variables Eq. (11) can be written in a more suggestive form:

$$(\delta T^{(s)}/T)|_O = (\delta T/T)|_E - (1/k)\, v_{\chi,\alpha} e^\alpha|_E^O - (\alpha_\chi + H\chi)|_E^O + \int_E^O (\alpha_\chi - \varphi_\chi)' dy. \quad (14)$$

The gauge-dependent terms on the RHS are identified: the first and $H\chi$ terms are gauge dependent. Since the $-(\alpha_\chi + H\chi)$ term *evaluated at O* (here and now) does not show the angular dependence, it can be *absorbed* into the definition of the background temperature; this point was noted in Ref. [6]. The combination of remaining two gauge-dependent variables, $(\delta T/T + H\chi)|_E$, is a gauge-invariant combination $\delta T_\chi/T|_E$. As a matter of fact, by moving $-(\alpha_\chi + H\chi)|_O$ to the left-hand side (LHS) we can make a gauge-invariant form $(\delta T_\chi^{(s)}/T + \alpha_\chi)|_O$. However, since the added terms only contribute to the isotropic temperature changes those do not contribute to the observed angular variation of temperature in Eq. (10) (*with* $\bar{T}$ defined as the all-sky average); equivalently, the variation of the observed temperature with directions is gauge invariant. Similarly, one can evaluate Eq. (14) in any gauge condition with the same "observable" anisotropy. In this sense the *observable temperature anisotropy* on the LHS of Eq. (14) is *gauge independent*.

Absorbing the isotropic contributions to $\bar{T}(\mathbf{x}_O, t_O)$, we have

$$(\delta T^{(s)}/T)|_O = -k^{-1} v_{\chi,\alpha} e^\alpha|_O + (1/k) v_{\chi,\alpha} e^\alpha|_E + (\alpha_\chi + \delta T_\chi^{(s)}/T)|_E + \int_E^O (\alpha_\chi - \varphi_\chi)' dy. \quad (15)$$

The RHS is apparently gauge invariant. In the literature, the four terms on the RHS are often called: the Doppler effect due to the observer's movement, the Doppler effect due to the movement of the photon-emitting plasma along the line-of-sight, the Sachs-Wolfe (SW) effect, and the integrated Sachs-Wolfe (ISW) effect, respectively, [16].

Now, we reexpress the SW and the ISW terms using φ_χ which has the close analogy with the Newtonian gravitational potential. In order to relate the temperature fluctuation with the coexisting matter at E, we take an ansatz

$$(\delta T/T)|_E \equiv (\{\delta/[3(1+\mathrm{w})]\} + e_T)|_E, \quad (16)$$

where $e_T(\mathbf{x},t)$ is apparently gauge invariant and can be regarded as the deviation of the temperature fluctuation from the adiabaticity with the coexisting matter fluctuation; we may call it the entropic temperature fluctuation [17]. By considering e_T we can handle the effects from the multicomponent hydrodynamic situation [18].

Using Eqs. (5)–(8) we can express the SW and ISW terms in Eq. (15) using φ_χ

$$\frac{\delta T^{(s,\mathrm{SW,ISW})}}{T}\bigg|_O = \left\{ \left[-1 + \frac{H^2}{4\pi G(\mu+p)} \right] (\varphi_\chi + 8\pi G\sigma) + \frac{H^2}{4\pi G(\mu+p)} \left(\frac{\dot{\varphi}_\chi}{H} + \frac{k^2 - 3K}{3a^2H^2} \varphi_\chi \right) + e_T \right\}\bigg|_E - 2\int_E^O (\varphi_\chi + 4\pi G\sigma)' dy. \quad (17)$$

In this form, we considered the general K, Λ, and $p(\mu)$ in the background, and the general $e(\mathbf{x},t)$ (the entropic pressure), σ, and e_T in the perturbation. In an ideal fluid (thus, $e = 0 = \sigma$), the general super-sound-horizon scale solution for φ_χ is presented in Ref. [13]

$$\varphi_\chi(\mathbf{x},t) = 4\pi G C(\mathbf{x}) \frac{H}{a} \int_0^t \frac{a(\mu+p)}{H^2} dt + \frac{H}{a} d(\mathbf{x}), \quad (18)$$

where $C(\mathbf{x})$ and $d(\mathbf{x})$ are integration constants indicating the relatively growing and decaying modes, respectively. Remarkably, this solution is *valid* on scales larger than Jeans scale for the general K, Λ, and generally time-varying $p(\mu)$. In the near flat case (thus, ignoring K terms), we have a powerful conserved quantity in the super-sound-horizon scale $\varphi_v(\mathbf{x},t) = C(\mathbf{x})$, with the vanishing leading decaying mode. The structural seed originated from the quantum fluctuation during the inflation era provides the initial condition for $C(\mathbf{x})$ and it is conserved during the super-sound-horizon scale evolution independently of changing equation of state, changing gravity theories, and the horizon crossing [13,19].

For $K = 0 = \Lambda$ and $w = \mathrm{const}$, the growing mode of φ_χ in Eq. (18) remains constant (we ignored e and σ). Thus, ignoring the decaying mode, we have

$$\frac{\delta T^{(s,\mathrm{SW})}}{T}\bigg|_O = \left\{ \left[-\frac{1+3\mathrm{w}}{3(1+\mathrm{w})} + \frac{2}{9(1+\mathrm{w})} \left(\frac{k}{aH} \right)^2 \right] \varphi_\chi + e_T \right\}\bigg|_E, \quad (19)$$

and the ISW term vanishes. The large observed angular scale corresponds to the superhorizon scale at the time of last scattering, and the effect from $(k/aH)^2$ term becomes subdominant. Thus, in the large angular scale, assuming the pressureless era at E, and ignoring $e_T|_E$, we finally have

$$(\delta T^{(s,\mathrm{SW})}/T)|_O = -\tfrac{1}{3}\varphi_\chi|_E = \tfrac{1}{3}\delta\Phi|_E, \quad (20)$$

which is the commonly quoted result derived in Ref. [2]. Notice, however, the various levels of assumptions used to have Eq. (20): we assumed, a single component, pressureless $(p=0)$, adiabatic $(e_T=0)$, ideal fluid $(e=0=\sigma)$, with $K=0=\Lambda$, and vanishing transient mode at E for the SW term, and along the ray's path from E to O for the ISW term.

Equation (17) expresses the SW and the ISW effects in the very general situation [20]. In addition to this, we also have two Doppler terms in Eq. (15) and the vector and tensor contributions in Eqs. (12), (13). These altogether contribute to the observed temperature anisotropy [2]. Attempts to explain the result in Eq. (20) in pedagogic ways, e.g., Ref. [22], usually involve gauge-dependent interpretations [23] with limited implications, and should be read with due caution. Many works in the literature start by fixing a certain gauge condition [2,4,5] or by using combinations of the gauge-invariant variables [6,7,21]. The final results for the observed temperature anisotropy are bound to be the same as ours, because, as we have shown, the concept is *observationally gauge independent*.

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[23] In the literature we find many analyses based on either the synchronous gauge $(\alpha\equiv 0)$ [2,4] or the zero-shear gauge $(\chi\equiv 0)$ [5]. The often mentioned ''gauge-dependent'' wisdoms are the following [we assume the same conditions used to get Eq. (20)]: (i) In the zero-shear gauge, using Eqs. (6)–(8), we can show $\delta T_\chi/T=\frac{2}{3}\varphi_\chi$, thus after compensation with $\alpha_\chi(=-\varphi_\chi)$ in Eq. (15) we have Eq. (20). (ii) In a pressureless medium the synchronous gauge is effectively the same as the comoving gauge $(v\equiv 0)$. Evaluating Eq. (16) in the comoving gauge, and using Eq. (6) we can show $\delta T_v/T=\frac{1}{3}\delta_v=\frac{2}{9}(k/aH)^2\varphi_\chi$ at E, and thus neglible compared with φ_χ at E.