

Diffuse γ -ray background and primordial black hole constraints on the spectral index of density fluctuations

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We calculate the flux of γ rays emitted from primordial black holes (PBHs) which are formed by a “blue” power-law spectrum of density fluctuations in the early universe. Gamma-ray emission from such PBHs may contribute significantly to the observed extragalactic diffuse γ -ray background (DGB). Using the observed DGB flux from the imaging Compton Telescope (COMPTEL) and the Energetic Gamma Ray Experiment Telescope (EGRET) as the upper limit of γ -ray flux from PBHs, we derive the upper limit on the spectral index n of the density fluctuations. The range of initial PBH masses which can contribute to the DGB is $2 \times 10^{13} \text{ g} - 5 \times 10^{14} \text{ g}$, corresponding to a cosmic reheating temperature of $7 \times 10^7 \text{ GeV} - 4 \times 10^8 \text{ GeV}$. In this range, we find the upper limit to be $n \leq 1.23 - 1.25$. This limit is stronger than those derived from the energy density in PBHs or PBH relics and matches the value of n required to explain the cosmic microwave background anisotropy. [S0556-2821(99)05804-X]

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I. INTRODUCTION

Distinct from black holes which form by recent processes such as stellar collapses, black holes formed by mechanisms in the early universe can exist. Such black holes, named primordial black holes (PBHs), produce many interesting consequences in the early universe and can also be sources of present astrophysical events. The simplest mechanism for PBH formation is the density fluctuations in the early universe [1]. Overdense regions which strongly deviate from the background universe can evolve into black holes when the overdense regions enter the cosmological horizon. The resulting PBH mass is about the horizon mass at entry. Thus, PBH masses can be as small as about the Planck mass $M_{\text{Pl}} \approx 2 \times 10^{-5} \text{ g}$ or as large as $10^7 M_{\odot}$. In the latter case, they can bound the mass of a typical galaxy after decoupling [2]. PBHs surviving today, or their massive relics, can be sources of dark matter.

Particle emission from black holes due to Hawking evaporation [3] enlarges the role of PBHs. Interactions of the emitted particles with the matter in the universe can affect numerous early universe phenomena, such as nucleosynthesis [4], baryogenesis [5], cosmic microwave background radiation (CMBR) distortion [6], entropy production [7], diffuse γ -ray background (DGB) [5,8–10], and so on. PBHs with initial mass $\sim 5 \times 10^{14} \text{ g}$ are presently at the final stage of their evaporation [11] and may emit enormous amounts of energy. In some grand unified theory (GUT) scale theories, the relics of PBHs created with initial mass $\leq 5 \times 10^{14} \text{ g}$ and expired by today can constitute the dark matter [12].

From the effects of PBHs mentioned above, upper limits

on Ω_{BH} , the fraction of the critical energy density of the universe which can be in PBHs, are found [13]. Constraints on the spectral index of the density fluctuations have been derived from these energy density limits [14,15] and it has been shown that PBHs in general give the strongest upper bounds on the spectral index [16].

The upper limit of Ω_{BH} in PBHs with masses about $5 \times 10^{14} \text{ g}$ was also calculated from the PBH γ -ray emission [8,9]. There exists a homogeneous and isotropic γ -ray background in the universe whose origin is known to be extragalactic [17,18]. Earlier authors postulated a contribution to the DGB from the PBH γ -ray emission [5,8,9]. MacGibbon and Carr have recently updated the limit on the PBH density using the DGB measurement of the Energetic Gamma Ray Experiment Telescope (EGRET) [19] and found that $\Omega_{\text{BH}} \approx (5.1 \pm 1.3) \times 10^{-9} h^{-1.95 \pm 0.15}$ [20]. Here h is the Hubble parameter in units of $100 \text{ kms}^{-1} \text{ Mpc}^{-1}$. Similar values are deduced from the PBH antiproton and γ -ray emission if PBHs cluster along with cold dark matter in galactic halos [21,22]. The limits on Ω_{BH} in this mass range assume, though, that the density fluctuations have a scale-invariant Harrison-Zel’dovich spectrum with spectral index $n=1$. Therefore they cannot be converted into an upper limit on the spectral index. Also, in these approaches the fluctuation amplitude was not explicitly normalized. Instead, a parameter related to the PBH density was introduced and varied to match the PBH γ -ray flux to the observed DGB. However, if one normalizes the fluctuation amplitude on the scales for PBH formation to that detected by Cosmic Background Explorer (COBE) on much larger scales from the CMBR anisotropy amplitude $\delta \sim 1.9 \times 10^{-5}$ [23], it is impossible to

form a significant number of PBHs with a continuous Harrison-Zel'dovich spectrum. With the normalization to the CMBR anisotropy, significant PBH abundance is possible only if the density fluctuations have an $n > 1$ ("blue") spectrum. A blue spectrum with a constant spectral index is a valid assumption, for example, in the hybrid inflationary scenario [14].

For these reasons, we will reexamine the γ -ray flux from PBHs formed by an $n > 1$ spectrum and find the upper limit on the spectral index, using the recent DGB measurements of the imaging Compton Telescope (COMPTEL) [24] and the EGRET [19] on board the Compton Gamma Ray Observatory (CGRO) and normalizing the fluctuation amplitude to that on CMBR anisotropy scales. While previous authors [14,16] have estimated limits on n from the γ -ray emission of PBHs formed by a blue fluctuation spectrum, they have not performed the explicit calculation of the PBH γ -ray emission and matched it to the DGB, analogous to the approach of MacGibbon and Carr for $n = 1$ [9,20]. They also did not include the effect of quark and gluon production which dominates the emission above black hole temperatures of about 100 MeV [25].

In Sec. II, we review the PBH formation and PBH mass spectrum, correcting errata in Ref. [15]. The black hole evaporation and QCD fragmentation effects on the particle emission are discussed in Sec. III and the calculation of the γ -ray flux from PBHs is given in Sec. IV. The recent DGB observations are reviewed in Sec. V. Our detailed calculation of the upper limit on n is presented in Sec. VI. The paper closes with some concluding remarks in Sec. VII.

II. PBHS AND THEIR MASS SPECTRUM

We address PBH formation in a universe with a hard equation of state, that is $p = \gamma\rho$ with $0 < \gamma \leq 1$. Studies of the evolution of a spherical overdense region whose initial radius R is greater than the particle horizon show that for the region to collapse to a black hole, the initial density contrast of the region, δ_i , should satisfy the following condition [26]:

$$\beta^2 \left(\frac{M_i}{M_{\text{Hi}}} \right)^{-2/3} \lesssim \delta_i \lesssim \alpha^2 \left(\frac{M_i}{M_{\text{Hi}}} \right)^{-2/3}. \quad (1)$$

Here α and β are constants of the order of $\sqrt{\gamma}$, M_i is the mass contained in the region of radius R at the time t_i when the fluctuation develops, and M_{Hi} is the horizon mass at t_i . The lower bound comes from the requirement that the radius of the region at its maximum expansion should be larger than the Jean's length at that epoch. The upper bound comes from the requirement that the overdense region should not be disconnected from the universe. Since $M_i \propto R^3$ and $R \propto k^{-1}$, we will use M_i , R , and k interchangeably to represent the initial mass, size, or comoving wavelength. The PBHs form when the overdense region enters into the horizon. The resulting PBH mass, $M_{\text{BH}i}$, is approximately the horizon mass at that time t_{H} and is given by

$$M_{\text{BH}i} \approx \gamma^{3/2} M_{\text{Hi}} \frac{t_{\text{H}}}{t_i}. \quad (2)$$

$M_{\text{BH}i}$ is related to M_i via [26]

$$M_{\text{BH}i} \approx \gamma^{3\gamma/(1+3\gamma)} M_i^{(1+\gamma)/(1+3\gamma)} M_{\text{Hi}}^{2\gamma/(1+3\gamma)}. \quad (3)$$

As the universe expands, larger PBHs are formed, so that PBHs with masses less than $M_{\text{BH}i}$ coexist in the universe at time t_{H} .

We assume the density fluctuations to be Gaussian. Recently it was claimed that a non-Gaussian nature to the fluctuations may affect the PBH formation [27]. This is model dependent and does not much alter the upper limit on n [16]. If one surveys the universe with a window having size R , the smoothed density field $\delta_R(\mathbf{x})$ is defined by

$$\delta_R(\mathbf{x}) = \int d^3\mathbf{y} \delta(\mathbf{x}+\mathbf{y}) W_R(\mathbf{y}), \quad (4)$$

where $\delta(\mathbf{x}) \equiv [\rho(\mathbf{x}) - \rho_b]/\rho_b$, ρ_b is the background energy density of the universe, and $W_R(\mathbf{x})$ is the smoothing window function of size R . The dispersion σ_R , the standard deviation of the density contrast of the regions with R , is given by

$$\sigma_R^2 = \frac{1}{V_W^2} \langle \delta_R^2(\mathbf{x}) \rangle = \frac{1}{V_W^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} |\delta_{\mathbf{k}}|^2 W_{\mathbf{k}}^2(R), \quad (5)$$

where $V_W \sim R^3$ denotes the effective volume filtered by W_R , and $\delta_{\mathbf{k}}$ and $W_{\mathbf{k}}$ are the Fourier transforms of $\delta(\mathbf{x})$ and $W_R(\mathbf{x})$, respectively. For Gaussian fluctuations the probability that the region of size R has density contrast in the range $(\delta + d\delta, \delta)$ is

$$P(M_i, \delta) d\delta = \frac{1}{\sqrt{2\pi}\sigma_R} \exp\left(-\frac{\delta^2}{2\sigma_R^2}\right) d\delta. \quad (6)$$

Thus, the probability that the region with M_i collapses to a black hole is

$$P_{\text{BH}}(M_i) = \int_{\mathcal{B}}^{\mathcal{A}} P(M_i, \delta) d\delta, \quad (7)$$

with

$$\mathcal{A} = \alpha^2 \left(\frac{M_i}{M_{\text{Hi}}} \right)^{-2/3}, \quad \mathcal{B} = \beta^2 \left(\frac{M_i}{M_{\text{Hi}}} \right)^{-2/3}. \quad (8)$$

The above quantity $P_{\text{BH}}(M)$ has been interpreted as the ratio of the density in PBHs to the density of the universe. This is not strictly so because regions larger than M_i also contribute. As in our previous work [15], we proceed by omitting these contributions and find the number density of black holes produced by the collapse of regions with mass between M_i and $M_i + dM_i$ to be [15]

$$n_{\text{BH}}(M_i) dM_i = -\frac{\rho_i}{M_i} \sqrt{\frac{2}{\pi}} \frac{\mathcal{B}}{\sigma_R^2} \frac{\partial \sigma_R}{\partial M_i} \exp\left(-\frac{\mathcal{B}^2}{2\sigma_R^2}\right) dM_i. \quad (9)$$

Here $\rho_i = 3/(32\pi G t_i^2)$ is the background energy density of the universe at t_i . Unlike Ref. [15], we will not convert the

mass spectrum into a function of unsmoothed quantities but retain the use of the smoothed quantities. This is to avoid factors being dropped in the conversion.

Since fluctuations grow as $a^2(t)$ in the radiation-dominated era, where $a(t)$ denotes the cosmic expansion factor, the dispersion corresponding to σ_R becomes, at the horizon crossing,

$$\sigma_H = \left(\frac{M_i}{M_{\text{Hi}}} \right)^{2/3} \sigma_R, \quad (10)$$

whence $n_{\text{BH}}(M_i)$ can be written as

$$\begin{aligned} n_{\text{BH}}(M_i) dM_i = & - \sqrt{\frac{2}{\pi}} \frac{\rho_i}{M_i} \gamma \left[\frac{1}{\sigma_H^2} \frac{\partial \sigma_H}{\partial M_i} - \frac{2}{3} \frac{\sigma_H^{-1}}{M_i} \right] \\ & \times \exp\left(- \frac{\gamma^2}{2\sigma_H^2} \right) dM_i. \end{aligned} \quad (11)$$

Applying Eq. (3) with $\gamma=1/3$, the mass spectrum can be expressed as a function of the initial PBH mass M_{BHi} and we have

$$\begin{aligned} n_{\text{BH}}(M_{\text{BHi}}) dM_{\text{BHi}} \\ = & - \sqrt{\frac{2}{\pi}} \gamma^{7/4} \rho_i M_{\text{Hi}}^{1/2} M_{\text{BHi}}^{-3/2} \left[\frac{1}{\sigma_H^2} \frac{\partial \sigma_H}{\partial M_{\text{BHi}}} - \frac{\sigma_H^{-1}}{M_{\text{BHi}}} \right] \\ & \times \exp\left(- \frac{\gamma^2}{2\sigma_H^2} \right) dM_{\text{BHi}}. \end{aligned} \quad (12)$$

In general, tensor perturbations (gravitational waves) are also produced in inflationary scenarios and contribute to the CMBR anisotropy. However, inclusion of the tensor perturbations does not significantly affect the PBH mass spectrum if the fraction in tensor perturbations is not dominant [14]. Therefore we assume that the anisotropy is only due to the scalar fluctuation.

The four-year results of the COBE experiment resolve the horizon crossing amplitude at present to be [23]

$$\delta_0 = 1.91 \times 10^{-5} \exp[1.01(1-n)], \quad (13)$$

with $n = 1.2 \pm 0.3$. This implies a smoothed amplitude today σ_0 of 9.5×10^{-5} with a slight dependence on n [16]. We denote the mass contained at t_i in the region whose comoving scale corresponds to the present horizon scale by M_0 . Under the power-law spectrum assumption, $\sigma_H \propto k^{(n-1)/2}$ and so

$$\sigma_H = \sigma_0 \left(\frac{M_i}{M_0} \right)^{(1-n)/6}. \quad (14)$$

[Note that the spectral index n of Ref. [26] is equivalent to $(n+3)/6$ in this work.] M_0 is not the present horizon mass as it was incorrectly taken to be in Ref. [15]. That misidentification introduced further errors when converting σ_H into a

function of M_{BHi} and made the results of Ref. [15] far weaker than those of Ref. [16]. Our revised results are given in Sec. VI.

From Eq. (3), σ_H can be represented as a function of M_{BHi}

$$\sigma_H = \sigma_0 \left(\frac{M_{\text{BHi}}}{M_{\text{BHi}0}} \right)^p, \quad (15)$$

with $p=(1-n)/4$ in the radiation-dominated era ($\gamma=1/3$) and $p=(1-n)/6$ in the matter-dominated era ($\gamma=0$). Minor discrepancies, which arise in Eq. (15) at the transition into the matter-dominated era, lead to less than a 1% change in the constraint on n [16]. The PBH initial mass spectrum under the $n>1$ power-law spectrum assumption is thus described by

$$\begin{aligned} n_{\text{BH}}(M_{\text{BHi}}) dM_{\text{BHi}} = & \frac{n+3}{4} \sqrt{\frac{2}{\pi}} \gamma^{7/4} \rho_i M_{\text{Hi}}^{1/2} M_{\text{BHi}}^{-5/2} \sigma_H^{-1} \\ & \times \exp\left(- \frac{\gamma^2}{2\sigma_H^2} \right) dM_{\text{BHi}}. \end{aligned} \quad (16)$$

III. PARTICLE EMISSION FROM BLACK HOLES

Due to the Hawking effect [3], a rotating charged black hole emits particles at a rate

$$\frac{dN_s}{d\omega dt} = \frac{\Gamma_s}{2\pi} \left[\exp\left(\frac{\omega - l\Omega - q\Phi}{\kappa/2\pi} \right) + (-1)^{2s} \right]^{-1}, \quad (17)$$

per degree of particle freedom. Here, κ , Ω , and Φ are the surface gravity, angular velocity and electric potential, respectively, s is the particle spin, l is the axial quantum number or angular momentum, and q is the particle charge. The absorption probability for the emitted species Γ_s is in general a function of $\omega, \Omega, \Phi, \kappa$, together with the internal degrees of freedom and rest mass of the emitted particle. Taking the above emission rate, it has been shown that $\Omega \rightarrow 0$ before most of the black hole evaporates [28] and that a black hole with mass $\lesssim 10^6 M_\odot$ discharges faster than it evaporates [29]. Hence, it is natural to regard PBHs as Schwarzschild black holes.

At high energies, $\Gamma_s \propto \omega^2$ for massless or relativistic particles and the emission spectrum mimics the radiation from a black body of temperature $T_{\text{BH}} = \kappa/2\pi$. Noting that the surface gravity is $\kappa = 1/4GM_{\text{BH}}$ when $\Omega = \Phi = 0$, the temperature of a black hole can be defined as

$$T_{\text{BH}} = \frac{1}{8\pi GM_{\text{BH}}} \simeq 1.06 \left(\frac{M_{\text{BH}}}{10^{13} \text{ g}} \right)^{-1} \text{ GeV}. \quad (18)$$

At low energies, Γ_s does not simply scale as ω^2 but depends on other quantities mentioned above. The form of Γ_s has been explored both analytically and numerically (see references in Ref. [30]). Extensive numerical studies of the direct particle emission from black holes were done by Page [31]. Hawking emission can be thought of as a process by which a

black hole emits particles with approximately a black body radiation spectrum once the black hole temperature exceeds the rest mass of the particle. Thus, black holes with masses larger than 10^{17} g, corresponding to $T_{\text{BH}} \approx 0.1$ MeV, emit only massless particles. As the black hole mass decreases, massive particles will be emitted.

In the conventional viewpoint, it is natural to assume that elementary particles such as quarks and gluons, rather than composite hadrons, are directly emitted from black holes once the emission energy exceeds the QCD confinement scale Λ_{QCD} . In this picture, pions are only directly emitted from black holes in the energy range between 100 MeV and Λ_{QCD} and are produced by quark and gluon decay above Λ_{QCD} . Taking into account the number of the emitted species, the mass loss rate of a black hole can be written as [11]

$$\frac{dM_{\text{BH}}}{dt} = -5.34 \times 10^{25} \phi(M_{\text{BH}}) M_{\text{BH}}^{-2} \text{g sec}^{-1}, \quad (19)$$

where $\phi(M_{\text{BH}})$, a function of the number of directly emitted species, is normalized to unity for $M_{\text{BH}} \gg 10^{17}$ g. Black holes with masses $5 \times 10^{14} \text{ g} \leq M_{\text{BH}} \leq 10^{17} \text{ g}$ emit e^\pm , neutrinos and photons and have initially $\phi(M_{\text{BH}}) = 1.569$. If black holes can emit three lepton families, six quark flavors, the photon and direct pions, then $\phi(M_{\text{BH}}) \leq 13.9$. Including the emission of weak gauge and Higgs bosons, $\phi(M_{\text{BH}}) \leq 15.4$ for $T_{\text{BH}} \leq 100$ GeV and $M_{\text{BH}} \geq 10^{11}$ g. At higher energies or in nonstandard models such as supersymmetry or superstrings, $\phi(M_{\text{BH}})$ may be greater but in general remains less than 100.

Integrating Eq. (19), the black hole lifetime τ_{evap} is found to be [11]

$$\begin{aligned} \tau_{\text{evap}} &\approx 1.2 \times 10^3 \frac{G^2 M_{\text{BH}}^3}{\phi(M_{\text{BH}})} \\ &= 6.24 \times 10^{-27} M_{\text{BH}}^3 \phi(M_{\text{BH}})^{-1} \text{ sec}. \end{aligned} \quad (20)$$

The jetlike fragmentation and hadronization of the quarks and gluons evaporated above Λ_{QCD} drastically change the observable spectrum of emitted particles. The evaporated quarks and gluons fragment into further quarks and gluons which then compose themselves into hadrons on distances greater than $\Lambda_{\text{QCD}}^{-1}$ in the jet frame. These particles further decay into the astrophysically stable particles—photons, neutrinos, electrons, and protons and their antiparticles. MacGibbon and Webber have shown that this picture is analogous to the decay of quark and gluon jets in e^+e^- accelerator events and calculate the instantaneous flux of particles for $0.2 \text{ GeV} \leq T_{\text{BH}} \leq 100 \text{ GeV}$ by convolving Eq. (17) with the HERWIG QCD jet code [25]. Their results differ strongly from those of previous works which omitted QCD emissions and particle decays. They find that the black hole emission at these temperatures is dominated by the jet fragmentation products. In the case of photons, the primary peak in the black hole emission is due to the decay of jet-produced π^0 and occurs around 67 MeV. The position of the photon peak does not shift significantly with the black hole temperature. In contrast, the photons directly emitted by the black

hole, not resulting from jet decay, appear at $\approx 5T_{\text{BH}}$ with fluxes 4 to 5 orders less than the flux at the jet-dominated peak. The jet-produced photons were omitted in previous estimates of the γ -ray emission from PBHs formed by $n > 1$ density fluctuations [14,16]. In this paper, we fit the instantaneous γ -ray fluxes from $T_{\text{BH}} = 0.2 - 100$ GeV black holes which are shown in Fig. 4 of Ref. [25] and derived including quark and gluon emission. We then use these results to calculate the γ -ray flux from PBHs.

IV. GAMMA RAYS FROM PBHS

To calculate the flux of γ rays from PBHs, it is important to know how many PBHs have existed in the universe. This can be determined from the PBH initial mass spectrum, which in turn depends on the fluctuation amplitude and spectral index, and t_i , the time when the fluctuations develop. Previous works considered only the case in which the fluctuations are described by the Harrison-Zel'dovich spectrum and hid the effect of fluctuation amplitude and t_i [5,8–10]. However, it is impossible to form a significant number of PBHs with the Harrison-Zel'dovich spectrum if the fluctuation amplitude on PBH formation scales is normalized to the amplitude found by the COBE experiment on much larger scales. With normalization to the CMBR anisotropy, substantial PBH formation is possible only if the fluctuations increase on small scales. This occurs if the fluctuations satisfy an $n > 1$ power-law spectrum. We will assume that the fluctuations follow a power-law spectrum and calculate the γ -ray flux from PBHs with the initial mass spectrum given by Eq. (16).

Since the number density of PBHs decreases as R^{-3} , the number density of PBHs at the time $t_1 \geq t_H$ is

$$n'_{\text{BH}}(t_1) \approx \left(\frac{R_1}{R_i}\right)^{-3} \int_{M_*(t_1)}^{M_{\text{BH}i}} n_{\text{BH}}(M_{\text{BH}i}) dM_{\text{BH}i}, \quad (21)$$

where $M_*(t_1)$, the initial mass of a PBH whose lifetime is t_1 , is given by

$$M_*(t_1) \approx \left[\frac{\phi[M_*(t_1)]}{6.24 \times 10^{-27}} \left(\frac{t_1}{1 \text{ sec}} \right) \right]^{1/3} \text{ g}. \quad (22)$$

We denote the instantaneous γ -ray flux from a black hole with mass M_{BH} as $f_\gamma(M_{\text{BH}}, \omega)$. At t_1 , PBHs with initial mass $M_{\text{BH}i}$ have evaporated down to a mass

$$M_{\text{evap}} \approx [M_{\text{BH}i}^3 - 1.6 \times 10^{26} \phi(M_{\text{BH}i}) t_1]^{1/3} \quad (23)$$

and are emitting photons with flux $f_\gamma(M_{\text{evap}}, \omega)$. The angular frequency at emission ω is redshifted by the expansion of the universe to a present angular frequency ω_0 of

$$\omega = \frac{R_0}{R_1} \omega_0. \quad (24)$$

Thus the total flux per unit solid angle at today, t_0 , of γ -rays emitted from PBHs is

$$\frac{dJ}{d\omega_0} = \frac{1}{4\pi} \int_{t_{\min}}^{t_0} \left(\frac{R_0}{R_1}\right) \left(\frac{R_1}{R_i}\right)^{-3} dt_1 \times \int_{M_*(t_1)}^{M_{\text{BH1}}} f_\gamma(M_{\text{evap}}, \omega) n_{\text{BH}}(M_{\text{BH}i}) dM_{\text{BH}i}, \quad (25)$$

where t_{\min} is the earliest time after inflation at which PBHs form.

Photons emitted by the black holes may interact with ambient matter in the universe via many processes and lose energy or be cut off during propagation. If a photon effectively interacts τ times during flight, the photon flux is attenuated by a factor of $e^{-\tau}$. Therefore, the actual photon flux reaching Earth at the present time is

$$\frac{dJ}{d\omega_0} = \frac{1}{4\pi} \int_{t_{\min}}^{t_0} \left(\frac{R_0}{R_1}\right) \left(\frac{R_1}{R_i}\right)^{-3} dt_1 \times \int_{M_*(t_1)}^{M_{\text{BH1}}} e^{-\tau} f_\gamma(M_{\text{evap}}, \omega) n_{\text{BH}}(M_{\text{BH}i}) dM_{\text{BH}i}. \quad (26)$$

The number of interactions τ , known as the optical depth, depends on the energy and the time or redshift z at emission. A detailed treatment on the optical depth is given in Sec. VI.

V. EXTRAGALACTIC DIFFUSE GAMMA-RAY BACKGROUND

Since its first discovery in the 0.1–2 MeV range by detectors on the lunar probes [32], the homogeneous and isotropic diffuse γ -ray flux, whose origin is extragalactic, has been observed in numerous satellite and balloon-borne experiments. The SAS-2 satellite provided the first clear evidence for the existence of an extragalactic γ -ray background between 30–150 MeV [18]. Several Apollo and balloon-borne experiments also saw evidence of a bump in the few MeV range in excess of the extrapolated x-ray continuum [33].

A number of models were proposed to explain the early measurements of the extragalactic spectrum. Active galaxies, which can be observable sources of discrete extragalactic γ -ray emission when they are located close to our Galaxy, are believed to contribute at least in part [34]. Another model which has been considered is matter-antimatter annihilation at the boundaries of superclusters [35]. In this model, the MeV bump is attributed to the redshifted peak of π^0 decays at 67 MeV. PBH γ -ray emission has also been proposed as a contributor to the DGB flux. In some exotic models, PBH emission may additionally explain the MeV bump [9]. However, none of the scenarios for the DGB production, by themselves, is sufficient to explain the measured flux and spectrum.

With higher sensitivity and wide-field of view, the detectors on the CGRO enlarge the detection range and gather

important data on the DGB. The DGB flux measured by the COMPTEL [24] at 1–30 MeV is now compatible with power-law extrapolations of the measured flux at lower and higher energies. The results below about 9 MeV are preliminary but the 2–9 MeV flux is far less than previously measured and no MeV bump is seen in this region at the levels reported previously. This weakens the need to explain an MeV feature.

We parametrize the preliminary COMPTEL results [24] in the range 0.8–30 MeV by the best-fit power law function given in Ref. [36]:

$$\left. \frac{dJ}{d\omega_0} \right|_{\text{obs}} = 6.40 \times 10^{-3} \left(\frac{\omega_0}{1 \text{ MeV}} \right)^{-2.38} \times [\text{cm}^2 \text{ sr MeV}]^{-1} \text{ (COMPTEL)}. \quad (27)$$

In the range 30 MeV–100 GeV, the EGRET experiment finds the DGB flux to be well described by the single power-law function [19]

$$\left. \frac{dJ}{d\omega_0} \right|_{\text{obs}} = k \left(\frac{\omega_0}{451 \text{ MeV}} \right)^{-\alpha} [\text{cm}^2 \text{ s sr MeV}]^{-1} \text{ (EGRET)} \quad (28)$$

with $k = (7.32 \pm 0.34) \times 10^{-9}$ and $\alpha = 2.10 \pm 0.03$. No large scale spatial anisotropy or deviations in the energy spectrum is discernible in the extragalactic component above 30 MeV. The observed flux above 10 MeV, and possibly up to 100 GeV, may be explained by unresolved blazars [19]. Below 10 MeV, the measurements still have large uncertainties and the exact nature of the emission is not well understood.

VI. CONSTRAINTS ON THE SPECTRAL INDEX

We now derive the constraints on the spectral index from the condition that the PBH γ -ray flux should not be larger than the observed DGB flux. With the normalization of the fluctuation amplitude to the CMBR anisotropy, PBH formation is limited to the epoch when the fluctuation arises. This time is related to the reheating temperature in inflationary models by [37]

$$t_{i\text{RH}} = 0.301 g_*^{-1/2} \frac{M_{\text{Pl}}}{T_{\text{RH}}} \sim \left(\frac{T_{\text{RH}}}{\text{MeV}} \right)^{-2} \text{ sec}. \quad (29)$$

Here $g_* \sim 100$ counts the degrees of freedom of the constituents in the early universe. The minimum initial PBH mass corresponding to T_{RH} is

$$M_{\text{RH}} \approx \frac{1}{8} \gamma^{3/2} M_{\text{Pl}} \left(\frac{T_{\text{RH}}}{T_{\text{Pl}}} \right)^{-2}. \quad (30)$$

PBHs created before the onset of reheating will be diluted to an insignificant density during inflation. Since we are considering the case $n > 1$, the resulting PBH initial mass spectrum has a very narrow mass range. Thus we will make the approximation that the photons are solely emitted by PBHs whose initial mass is M_{RH} .

The photon interactions in the matter-dominated era, relevant to the DGB observations, are Compton scattering, pair production, photoionization, and photon-photon interactions. Via these processes and cosmological redshift, the energy of the emitted photons is degraded. Zdziarski and Svensson have studied the attenuation of γ -ray flux at cosmological distance [38]. They found that the maximum redshift from which photons can be detected today peaks at $z_{\text{max}} \approx 700$ and for present energies $1 \text{ MeV} \leq \omega_0 \leq 1 \text{ GeV}$. All photons emitted at higher redshifts are cut off by interactions [i.e., $\tau(\omega_0, z) > 1$] and do not reach Earth. This means that PBHs which completely evaporated before $z_{\text{max}} \approx 700$ cannot contribute to the present DGB. From Eq. (20), this corresponds to a minimum detectable initial PBH mass of about $2 \times 10^{13} \text{ g}$ and a reheating temperature of $T_{\text{RH}} \approx 4 \times 10^8 \text{ GeV}$. Noting that $M_{*}(t_0) \approx 5 \times 10^{14} \text{ g}$, the range of PBHs which can contribute to the observed DGB is then $2 \times 10^{13} \text{ g} - 5 \times 10^{14} \text{ g}$ and the corresponding reheating temperature range is $7 \times 10^7 \text{ GeV} - 4 \times 10^8 \text{ GeV}$. Outside the range $1 \text{ MeV} \leq \omega_0 \leq 1 \text{ GeV}$, the maximum redshift for a given energy $z_{\text{max}}(\omega_0)$ is less than 700 and depends on the details of the interactions. We take $z_{\text{max}}(\omega_0)$ from Ref. [38]. Only photons for which $\tau < 1$ are here included and we regard these photons as being free from attenuation. The integrated γ -ray flux from PBHs does not depend significantly, though, on the details of the optical depth.

We now proceed to calculate the integrated PBH γ -ray flux, Eq. (26), using the instantaneous emission from individual black holes f_γ obtained by fitting the simulations of Ref. [25]. Our results are shown in Fig. 1. It can readily be seen that the γ -ray flux from PBHs cannot fully explain the observed DGB flux although the PBH emission may contribute significantly to the observed DGB flux around 10–100 MeV. In the $n > 1$ case, the PBH flux arises from the lifetime emission of PBHs with initial mass M_{RH} , whereas the Harrison-Zel'dovich spectrum produces a broad range of initial PBH masses. In both cases, the PBH flux falls off as roughly ω_0^{-3} above 100 MeV [5,8,9]. This high-energy tail mainly comes from the lifetime direct photon emission in the most recent evaporation epoch [11]. At low energies, where the flux is strongly determined by QCD jet fragmentation, the flux spectrum for $n > 1$ does not scale as ω_0^{-1} , as for $n = 1$, but instead flattens out due to the narrow PBH initial mass range. In addition, the turnover in the spectrum occurs at lower energies as the reheating temperature increases and M_{RH} decreases. This is because the turnover corresponds to the redshifted peak emission of an M_{RH} black hole emitting at its initial temperature.

From the constraint that the PBH γ -ray flux cannot be larger than the observed DGB flux, we derive the upper limits of the spectral index in the range $7 \times 10^7 \text{ GeV} \leq T_{\text{RH}} \leq 4 \times 10^8 \text{ GeV}$ (Fig. 2). The upper limit on n is

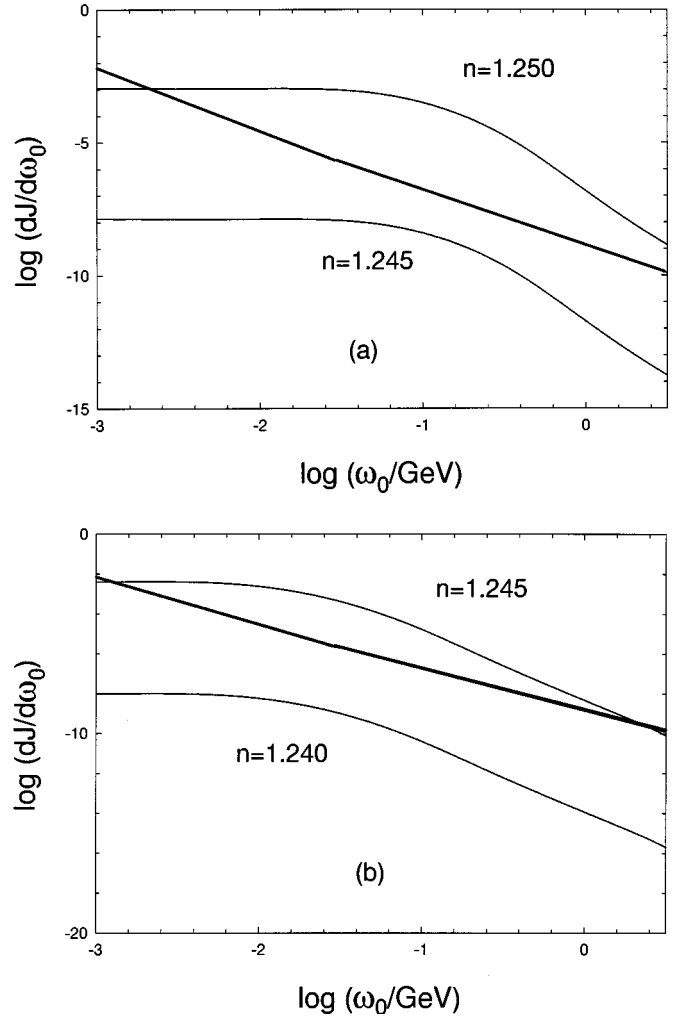


FIG. 1. The integrated γ -ray flux from PBHs, $dJ/d\omega_0$ in units of $(\text{cm}^2 \text{ s sr MeV})^{-1}$, for (a) $T_{\text{RH}} = 7 \times 10^7 \text{ GeV}$ corresponding to $M_{\text{BH}} = 5 \times 10^{14} \text{ g}$, and (b) $T_{\text{RH}} = 10^8 \text{ GeV}$ corresponding to $M_{\text{BH}} = 2 \times 10^{14} \text{ g}$. The bold lines are the observed flux $dJ/d\omega_0|_{\text{obs}}$ from the COMPTEL ($1 \text{ MeV} < \omega_0 < 30 \text{ MeV}$) and the EGRET ($30 \text{ MeV} < \omega_0 < 100 \text{ GeV}$).

$$n \lesssim 1.23 - 1.25. \quad (31)$$

Even though photons emitted in models with higher T_{RH} suffer more interactions and larger redshifts, the exponential dependence of the PBH mass spectrum, Eq. (16), implies that the number density of PBHs strongly increases with T_{RH} for a given spectral index. Thus, the upper limit on n decreases as the reheating temperature grows. That is, the constraint on n becomes stronger as the reheating temperature increases. These values are similar to the upper limits obtained from the deuterium destruction constraint in the lower mass range $10^9 \text{ g} \leq M_{\text{BH}} \leq 10^{13} \text{ g}$ [16].

Also shown in Fig. 2 are the weaker limits on n derived from the maximum allowable energy density in PBHs. We plot the upper limits found from the requirement that the PBH energy density does not overclose the universe at any

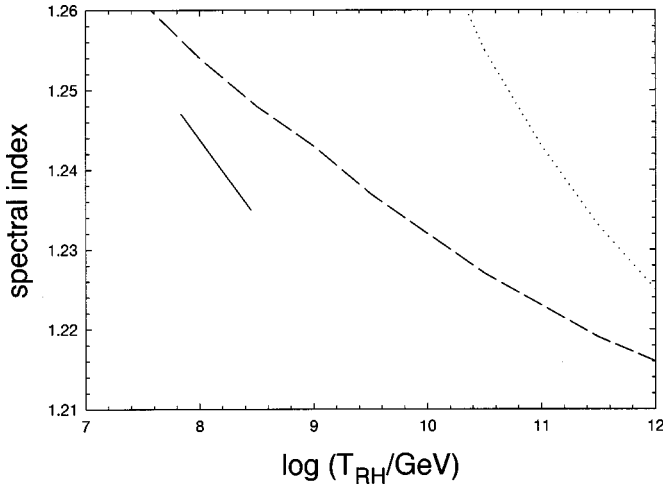


FIG. 2. The upper limits on the spectral index. The solid line between 7×10^7 GeV– 4×10^8 GeV is obtained from the condition that the PBH γ -ray flux should not exceed the observed DGB flux. The dashed line is obtained from the condition that $\Omega_{\text{BH}} < 1$ throughout the history of the universe (case I). The dotted line is obtained from the condition that $\Omega_{\text{relic}} < 1$ (case II).

epoch $\Omega_{\text{BH}} < 1$ (case I, the dashed line), and the similar restriction on any present relic density in PBHs which did not evaporate completely but left residual masses of about the Planck mass $\Omega_{\text{relic}} < 1$ (case II, the dotted line). The latter constraint strengthens somewhat if the relic mass is greater than M_{pl} [14,15]. Constraint case I applies regardless of whether PBHs evolve into massive relics. The new upper limits on n from the energy densities are much tighter than those of Refs. [14,15] and decrease as T_{RH} grows. In Ref. [16], upper limits on n were found from the condition that the present PBH density and any relic density satisfy $\Omega_{\text{BH0}} < 1$ and $\Omega_{\text{relic}} < 1$, respectively. There it was shown that the constraint $\Omega_{\text{BH0}} < 1$ is weaker than the relic constraint case II. However, when we now extend that condition to require that the PBH density fraction at any epoch does not overclose the universe, we find that case I is stronger than case II if $T_{\text{RH}} \lesssim 10^{13}$ GeV. The energy density in PBHs or PBH relics give weaker constraints than the DGB because the upper limit on Ω_{BH} from the PBH γ -ray flux is far less than 1. Because of the vast difference between the scales on which the CMBR anisotropy occurs and the scales on which PBHs form, the limit on the spectral index obtained from PBH emission is highly insensitive to the true value of the CMBR anisotropy or Ω_{BH} .

VII. CONCLUSIONS

In this paper, we calculate the γ -ray flux from PBHs formed by density fluctuations in the early universe and compare it with the observed extragalactic DGB flux. Previous works considered the case in which the density fluctuations have an $n = 1$ Harrison-Zel'dovich spectrum and did not explicitly normalize the fluctuation amplitude. If the fluctuation

amplitude on the scales of PBH formation is normalized to that on large scales deduced from the COBE observations of the CMBR anisotropy, PBHs cannot form in cosmologically significant numbers from a Harrison-Zel'dovich spectrum. Thus, we describe the fluctuations by an $n > 1$ power-law spectrum and find the upper limit on n from the condition that the γ -ray flux from PBHs should not be larger than the DGB flux. The smallness of the fluctuation amplitude from the CMBR anisotropy limits PBH formation to the time when the fluctuations develop. In inflationary models, this time is related to the reheating time. To model the γ -ray emission, we fit previous simulations of the emission from individual black holes which included QCD fragmentation and decays. Due to the interactions of γ -rays with the background matter in the universe, only PBHs surviving later than $z_{\text{max}} \lesssim 700$ can contribute to the DGB flux observed today. The initial mass range of PBHs relevant to the observed DGB is then 2×10^{13} g $\lesssim M_{\text{BH}} \lesssim 5 \times 10^{14}$ with a corresponding cosmic reheating temperature between 7×10^7 GeV $\lesssim T_{\text{RH}} \lesssim 4 \times 10^8$ GeV. We find the resulting upper limit on n in this range to be $n \lesssim 1.23$ – 1.25 . Our constraint on n is stronger than those obtained by requiring that the energy density in PBHs does not overclose the universe at any epoch ($\Omega_{\text{BH}} < 1$) and that found by requiring that any present PBH relic density similarly does not overclose the universe ($\Omega_{\text{relic}} < 1$). The upper limit on Ω_{BH} implied by the PBH γ -ray flux is far less than 1. If the fluctuation amplitude is constrained by the CMBR anisotropy and PBH emission, the upper limit on n is fine-tuned and highly insensitive to the precise upper limit on Ω_{BH} or the precision in the CMBR measurement.

Recently, Niemeyer and Jedamzik [39] have argued, supported by preliminary numerical simulations, that subhorizon mass PBHs may form in considerable numbers at any formation epoch. For Gaussian fluctuations, they deduce that the PBH initial mass distribution at a given formation time peaks at about 0.6 times the horizon mass and extends from much smaller masses up to the horizon mass. Such a distribution would have an effect on our limits similar to raising T_{RH} .

We also note that it has been proposed that the emitted quarks and gluons from a black hole may interact and form a photosphere around the black hole above black hole temperatures of a few GeV [40]. This scenario, however, remains controversial. In the photosphere model, the flux would be more concentrated around 100 MeV than if the quarks and gluons directly fragment into hadrons [41]. The high-energy tail from the PBH distribution would also scale as ω_0^{-4} , not ω_0^{-3} . Photosphere formation may somewhat weaken the constraint on n but such changes would be small due to the fine-tuned nature of the constraint.

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- [1] Ya. B. Zel'dovich and I. D. Novikov, *Astron. Zh.* **43**, 758 (1966) [*Sov. Astron.* **10**, 602 (1966)]; S. W. Hawking, *Mon. Not. R. Astron. Soc.* **152**, 75 (1971).
- [2] M. P. Ryan, *Astrophys. J. Lett.* **177**, L79 (1972).
- [3] S. W. Hawking, *Nature (London)* **248**, 30 (1974); *Commun. Math. Phys.* **43**, 199 (1975).
- [4] S. Miyama and K. Sato, *Prog. Theor. Phys.* **59**, 1012 (1978); D. Lindley, *Mon. Not. R. Astron. Soc.* **193**, 593 (1980); T. Rothman and R. Zatzner, *Astrophys. Space Sci.* **75**, 2291 (1981).
- [5] B. J. Carr, *Astrophys. J.* **206**, 8 (1976).
- [6] P. D. Naselsky, *Pisma Astron. Zh.* **4**, 387 (1978) [*Sov. Astron. Lett.* **4**, 209 (1978)].
- [7] Ya. B. Zel'dovich and A. A. Starobinsky, *Pis'ma Zh. Éksp. Teor. Fiz.* **24**, 616 (1976) [*JETP Lett.* **24**, 571 (1976)].
- [8] D. N. Page and S. W. Hawking, *Astrophys. J.* **206**, 1 (1976); G. F. Chapline, *Nature (London)* **253**, 251 (1975).
- [9] J. H. MacGibbon and B. J. Carr, *Astrophys. J.* **371**, 447 (1991).
- [10] F. Halzen *et al.*, *Nature (London)* **353**, 807 (1991).
- [11] J. H. MacGibbon, *Phys. Rev. D* **44**, 376 (1991).
- [12] J. H. MacGibbon, *Nature (London)* **329**, 308 (1987).
- [13] I. D. Novikov, A. G. Polnarev, A. A. Starobinsky, and Ya. B. Zel'dovich, *Astron. Astrophys.* **80**, 104 (1979).
- [14] B. J. Carr, J. H. Gilbert, and J. E. Lidsey, *Phys. Rev. D* **50**, 4853 (1994).
- [15] H. I. Kim and C. H. Lee, *Phys. Rev. D* **54**, 6001 (1996).
- [16] A. M. Green and A. R. Liddle, *Phys. Rev. D* **56**, 6166 (1997).
- [17] R. Ramaty and R. E. Lingenfelter, *Annu. Rev. Nucl. Part. Sci.* **32**, 235 (1982); see also, P. Sreekumar, F. W. Stecker, and S. C. Kappadath, [astro-ph/9709258](https://arxiv.org/abs/astro-ph/9709258).
- [18] C. E. Fichtel *et al.*, *Astrophys. J.* **198**, 163 (1975).
- [19] P. Sreekumar *et al.*, *Astrophys. J.* **494**, 523 (1998).
- [20] J. H. MacGibbon and B. J. Carr (in preparation).
- [21] K. Maki, T. Mitsui, and S. Orito, *Phys. Rev. Lett.* **76**, 3474 (1996).
- [22] E. L. Wright, *Astrophys. J.* **459**, 487 (1996).
- [23] K. Gorski *et al.*, *Astrophys. J. Lett.* **464**, L11 (1996).
- [24] S. C. Kappadath *et al.*, *Astron. Astrophys., Suppl. Ser.* **120**, 619 (1996); S. C. Kappadath *et al.*, *Proceedings of the 4th Compton Symposium, Williamsburg, VA, 1997* (unpublished).
- [25] J. H. MacGibbon and B. R. Webber, *Phys. Rev. D* **41**, 3052 (1990).
- [26] B. J. Carr, *Astrophys. J.* **201**, 1 (1975).
- [27] J. S. Bullock and J. R. Primack, *Phys. Rev. D* **55**, 7423 (1997).
- [28] D. N. Page, *Phys. Rev. D* **14**, 3260 (1976).
- [29] G. W. Gibbons, *Commun. Math. Phys.* **44**, 245 (1975).
- [30] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [31] D. N. Page, *Phys. Rev. D* **13**, 198 (1976); **16**, 2402 (1977).
- [32] J. R. Arnold *et al.*, *J. Geophys. Res.* **67**, 4876 (1962); A. E. Metzger *et al.*, *Nature (London)* **204**, 766 (1964).
- [33] J. I. Trombka *et al.*, *Astrophys. J.* **212**, 925 (1977); R. S. White *et al.*, *ibid.* **218**, 920 (1977); V. Schönfelder *et al.*, *ibid.* **240**, 350 (1980).
- [34] A. W. Strong, A. W. Wolfendale, and D. M. Worrall, *J. Phys. A* **9**, 1553 (1976); C. E. Fichtel, G. A. Simpson, and R. J. Thompson, *Astrophys. J.* **222**, 833 (1978); G. F. Bignami *et al.*, *ibid.* **232**, 649 (1979).
- [35] F. W. Stecker, D. L. Morgan, and J. Bredkamp, *Phys. Rev. Lett.* **27**, 1469 (1971); F. W. Stecker, *Nature (London)* **273**, 493 (1978).
- [36] G. D. Kribs and I. Z. Rothstein, *Phys. Rev. D* **55**, 4435 (1997).
- [37] E. Kolb and M. Turner, *The Early Universe* (Addison-Wesley, Redwood, CA, 1990).
- [38] A. A. Zdziarski and R. Svensson, *Astrophys. J.* **344**, 551 (1989).
- [39] J. C. Niemeyer and K. Jedamzik, *Phys. Rev. Lett.* **80**, 5481 (1998).
- [40] A. F. Heckler, *Phys. Rev. D* **55**, 480 (1997).
- [41] A. F. Heckler, *Phys. Rev. Lett.* **78**, 3430 (1997).