PHYSICAL REVIEW D, VOLUME 59, 061501

Possible quantum instability of primordial black holes

E. Elizalde*
Consejo Superior de Investigaciones Científicas (CSIC), Institut d'Estudis Espacials de Catalunya (IEEC), Edifici Nexus 201, Gran Capità 2-4, 08034 Barcelona, Spain
and Departament ECM and IFAE, Facultat de Física, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain

S. Nojiri†
Department of Mathematics and Physics, National Defence Academy, Hashirimizu Yokosuka 239, Japan

S. D. Odintsov‡
Consejo Superior de Investigaciones Científicas (CSIC), Institut d'Estudis Espacials de Catalunya (IEEC), Edifici Nexus 201, Gran Capità 2-4, 08034 Barcelona, Spain
and Department of Mathematics and Physics, Tomsk Pedagogical University, 634041, Tomsk, Russia§

(Received 31 August 1998; revised manuscript received 29 October 1998; published 8 February 1999)

Evidence for the possible existence of a quantum process opposite to the famous Hawking radiation (evaporation) of black holes is presented. This new phenomenon could be very relevant in the case of exotic multiple horizon Nariai black holes and in the context of common grand unified theories. This is clearly manifested in the case of the SO(10) GUT, that is here investigated in detail. The remarkable result is obtained that anti-evaporation can occur there only in the SUSY version of the theory. It is thus concluded that the existence of primordial black holes in the present Universe might be considered as evidence for supersymmetry. [S0556-2821(99)50104-5]

PACS number(s): 04.70.Dy, 04.60.−m, 04.62.+v

In his celebrated paper, Ref. [1], published over twenty years ago, Hawking made one of the most striking discoveries of the history of black hole physics: the possibility for black holes to evaporate, as a result of particle creation. This effect—which is now called the Hawking radiation process—produced a deep impact on our understanding of quantum gravity. Nowadays it is considered a classical test of the theory.

What confers this effect its great importance is the observation that Hawking radiation is a universal phenomenon. One of its most dramatic consequences is the fact that all primordial black holes should evaporate in the process of the early universe evolution.

There exists, however, an exotic class of black holes (see Ref. [2] for a review) which possess a multiple horizon and for which the *opposite* effect might occur. We have investigated this possibility in detail. Consider a nearly degenerate Schwarzschild–de Sitter black hole (the so-called Nariai black hole [3] in the specialized literature). The Schwarzschild–de Sitter black hole represents the neutral, static, spherically symmetric solution of Einstein's theory with a cosmological constant. The corresponding metric looks as follows:

$$ds^2 = -V(\tau)dt^2 + V^{-1}(\tau)d\tau^2 + \tau^2 d\Omega^2,$$
$$V(\tau) = 1 - \frac{2\mu}{\tau} - \frac{\Lambda}{3}\tau^3, \tag{1}$$

where μ is the mass and τ the radius of the black hole, $d\Omega^2$ the metric corresponding to a unit two-sphere, and Λ is the cosmological constant. It can be easily checked that the equation

$$V(\tau) = 0 \tag{2}$$

has two positive roots, τ_c and τ_b (we set $\tau_c > \tau_b$). Here, τ_c and τ_b have the meaning of a cosmological and a black-hole horizon radius, respectively. In the degenerate case (which corresponds to a black hole of maximal mass) both radii coincide and the black hole is in thermal equilibrium. It is called a Nariai black hole.

There are in fact two opposite sources contributing to this equilibrium, namely a radiation flux coming from the cosmological horizon and the Hawking evaporation originated at the black hole horizon. It is plausible that such a state might be unstable, since it could be affected by small perturbations of the geometry. This is what happens, indeed.

It was demonstrated in a recent paper by Bousso and Hawking, Ref. [4], that a nearly maximal (or nearly degenerated) Nariai black hole may not only evaporate—as shown in Ref. [1]—but also antievaporate [4]. In other words, Nariai black holes actually develop two perturbative modes: an evaporating one and an antievaporating one. The mathematical realization of this quantum process, carried out in Ref. [4], is based on the generalized Callan-Giddings-Harvey-Strominger (CGHS) model of two-dimensional dilaton quantum gravity [5], with the dilaton coupled scalars [6] coming from the so-called minimal four-dimensional scalars in the process of spherical reduction (for a basic introduction of these concepts, the reader is addressed to Ref. [7]).

*Email address: elizalde@ieec.fcr.es, eli@zeta.ecm.ub.es
†Email address: nojiri@cc.nda.ac.jp
‡Email address: odintsov@tspu.edu.ru
§Present address.

The possibility of evaporation or antievaporation (i.e., increase in size) of a black hole of this kind is certainly connected with the initial conditions chosen for the perturbations. In the model of Ref. [4], use of the commonly employed Hartle-Hawking no-boundary conditions [8] shows that such black holes will most likely evaporate. We are thus left with the original situation, in this case.

The exotic Nariai black holes are not asymptotically flat. They will never appear in the process of star collapse. Nevertheless, they could actually be present in the early inflationary universe, through any of the following processes: (1) pair creation of primordial black holes during inflation [9]; (2) quantum generation due to quantum fluctuations of matter fields [10]. In any case, according to the model in Ref. [4], primordial multiple horizon black holes should quickly evaporate, and it is very unlikely that they could be detected in the present universe.

However, a different model for antievaporation of black holes has been proposed recently, Ref. [11], in which these difficulties may be overcome. In that model, the quantum effects of the conformally invariant matter have been taken into account. What is more interesting, this theory allows for the possibility of including not only scalar fields, but also fermionic and vector fields (typical of all grand unified theories, GUTs), whose classical action is

$$S=\int d^4x\sqrt{-g_{(4)}}\left[\frac{1}{2}\sum_{i=1}^{N}\left(g_{(4)}^{\alpha\beta}\partial_\alpha\chi_i\partial_\beta\chi_i+\frac{1}{6}R^{(4)}\chi_i^2\right)-\frac{1}{4}\sum_{j=1}^{N_1}F_{j\mu\nu}F_j^{\mu\nu}+\sum_{k=1}^{N_{1/2}}\bar{\psi}_k\not{D}\psi_k\right], \tag{3}$$

and its quantum correction Γ is the sum of the conformal anomaly induced action W and the action Γ' given by the Schwinger-DeWitt-type expansion:

$$\begin{aligned}
\Gamma&=W+\Gamma',\\
W&=b\int d^4x\sqrt{-g}F\sigma+b'\int d^4x\sqrt{-g}\\
&\quad\times\left\{\sigma\left[2\Box^2+4R^{\mu\nu}\nabla_\mu\nabla_\nu-\frac{4}{3}R\Box+\frac{2}{3}(\nabla^\mu R)\nabla_\mu\right]\sigma\right.\\
&\quad\left.+\left(G-\frac{2}{3}\Box R\right)\sigma\right\}-\frac{b+b'}{18}\int d^4x\sqrt{-g}\\
&\quad\times[R-6\Box\sigma-6(\nabla\sigma)(\nabla\sigma)]^2,\\
\Gamma'&=\int d^4x\sqrt{-g}\left[\left(bF+b'G+\frac{2b}{3}\Box R\right)\ln\frac{R}{\mu^2}\right]+\mathcal{O}(R^3).
\end{aligned} \tag{4}$$

Here $b=(N+6N_{1/2}+12N_1)/120(4\pi)^2$, $b'=-(N+11N_{1/2}+62N_1)/360(4\pi)^2$, F is the square of the Weyl tensor, G the Gauss-Bonnet invariant, μ is a mass-dimensional constant parameter, and we choose the metric as $ds^2=\mathrm{e}^{-2\sigma}g_{\mu\nu}dx^\mu dx^\nu$. Specially, when we assume the solution to be spherically symmetric, the metric has the form of $ds^2=\mathrm{e}^{-2\sigma}(\Sigma_{\alpha,\beta=0,1}g_{\alpha\beta}dx^\alpha dx^\beta+r_0^2d\Omega^2)$, where $d\Omega^2$ is the metric on the unit two-sphere. The parameter r_0 is introduced by hand. Note that the Schwinger-De Witt expansion is the corresponding power expansion with respect to the curvature. Having introduced the parameter r_0, the scalar curvature given by the metric $g_{\mu\nu}$ is of the order of $1/r_0^2$ if there is no singularity, as in the Nariai black hole. Therefore, if we choose r_0 to be large, the Schwinger-De Witt type expansion becomes exact.

By solving the quantum effective equations of motion derived from Eqs. (3) and (4), we can find the quantum analogue of the Nariai black hole, which has constant scalar curvature $R=R_0$ and radius $\mathrm{e}^\sigma=\mathrm{e}^{\sigma_0}$:

$$\begin{aligned}
R_0&=\left[2+[\ln(\mu r_0)]^{-1}\left(\frac{2b+3b'}{b}+\frac{9}{512\pi^2bG\Lambda}\right)\right]\\
&\quad+\mathcal{O}([\ln(\mu r_0)]^{-1}),\\
\sigma_0&=-\ln(\mu r_0)+\frac{1}{2}\ln\left(\frac{3\mu^2}{2\Lambda}\right)\\
&\quad+[\ln(\mu r_0)]^{-1}\frac{\mu^2}{8\Lambda}\left(\frac{2b+3b'}{b}+\frac{9}{512\pi^2bG\Lambda}\right)\\
&\quad+\mathcal{O}([\ln(\mu r_0)]^{-1}).
\end{aligned} \tag{5}$$

Here we assume r_0 to be large. Furthermore, we can find the perturbation around the solution. It is given by an eigenfunction of the Laplacian in the two-dimensional hyperboloid, which corresponds to the subspace in the Nariai black hole, given by radial and time coordinates. The fate of the perturbed black hole is governed by the eigenvalue of the Laplacian and we find that antievaporation can occur only if

$$2N+7N_{1/2}>26N_1. \tag{6}$$

Owing to the fact that the equations of motion in the model by Bousso and Hawking contain only second-order derivatives, antievaporation is excluded there by the no-boundary condition of Hartle and Hawking. The quantum effective equations of motion given by Eqs. (3) and (4), however, contain fourth-order derivatives and there the antievaporation phenomenon can be consistent with the no-boundary condition.

As an example take, for instance, the usual SO(10) ground unified theory (GUT), which would be a typical model in the early universe. First, we consider the nonsupersymmetric model of [12], with 16×3 (generation) fermions and $16+120+(10\text{ or }126)=136$ or 262 Higgs scalars (the numbers are the dimensions of the representations). Therefore we obtain $2N+7N_{1/2}=608$ or 860, and the adjoint vector fields ($N_1=45$) give a contribution of $26N_1=1170$. Thus, we find that Eq. (6) cannot be satisfied. The situation is, however, drastically changed when we consider the supersymmetric model. In the naive supersymmetric extension of the above model, we have contributions from squarks, Higgsinos, and gauginos. Including them, we find that $2N+7N_{1/2}=1971$ or 3105, and Eq. (6) is then certainly satisfied, since the contribution from the vector fields does not change from the one of the nonsupersymmetric model. The above structure is not modified either in the various extensions of the SO(10) model that have been considered more recently. That is, in all these cases for the nonsupersymmetric model Eq. (6) is *not* satisfied, since the contribution from the vector fields dominates, but for the supersymmetric models, the contribution from the Higgsino in the large dimensional representations dominates and Eq. (6) *is* satisfied. Then, under the hypothesis that the no-boundary initial condition applies, at least some primordial multiple horizon black holes are going to antievaporate.

In other words, primordial black holes might exist for a much longer period [if we consider some reasonable supersymmetry (SUSY) GUT as a realistic theory] than it had been expected [1]. Primordial black holes would have grown, for some time, until other effects could have stopped the process. Some could even have survived and be present in our universe. Then, if these primordial black holes were observed, they would constitute an indirect evidence of supersymmetry. The rate P of pair creation of black holes has appeared in [9]:

$$P=\exp\left(-\frac{\pi}{G\Lambda_{eff}}\right). \tag{7}$$

In the Euclidean path integral, the weight of the probability is given—in the semiclassical approximation—by substituting the classical black hole solution $g_{\mu\nu}^{\text{classical}}$, after Wick rotating $t\to i\tau$ [τ has a period of $\sqrt{(1/\Lambda)}(\pi/2)$ for Nariai black hole and $\sqrt{(3/\Lambda)}(\pi/2)$ for anti–de Sitter space], into the action S in Eq. (3) and exponentiating the action $e^{-S(g_{\mu\nu}^{\text{classical}})}$. Equation (7) is evaluated from the ratio of the weights for the classical Nariai space and anti–de Sitter space: $P=e^{-S(g_{\mu\nu}^{\text{classical Nariai}})+S(g_{\mu\nu}^{\text{classical anti–de Sitter}})}=\exp(-\pi/G\Lambda)$. In the inflational universe, the effective cosmological constant Λ_{eff} in Eq. (7) is typically given by the square of the GUT scale, 10^{16} GeV; then the exponent $\pi/G\Lambda_{eff}$ in Eq. (7) is of the order of 10^6 and pair production would be suppressed. But the magnitude of the exponent depends on the model of inflation used. If our universe were pair created as in the original work by Hartle and Hawking [8], in the very early universe, the energy density could be of the order of the Planck scale and then Λ_{eff} would be of the order of unity. In such a situation, the pair-creation of the Nariai black hole would not be suppressed. The important consequence being then, that the pair created black hole would *not* evaporate and could survive in the present universe.

In summary, we conclude that adopting the SO(10) model (in any of its several versions) and under the hypothesis that the no-boundary initial condition applies, we could obtain a proof of supersymmetry by finding evidence of the existence of antievaporating black holes. One may argue that, even if they existed, the probability of finding just one black hole of this kind would be very low. However, the extreme importance of supersymmetry for all string, brane, and conformal theories, and the like, and also the extreme difficulty in proving, by *any* other means, that such a strongly broken symmetry as SUSY is in fact a symmetry of nature (and not merely a beautiful idea), confer a relevant status to the results we have here obtained.

Again, we concede that the process of antievaporation might in fact be rather exotic and only limited to the very early times of the inflationary universe. In the own words of Stephen Hawking: *I regard antievaporation as a pathology*. Notwithstanding that, the model above has appeared as a brand new mathematical solution of plausible quantum gravity equations under the conditions that are usually assumed to be the natural ones during the evolution of the early universe. It would not be wise to discard such a solution—and the physical scenario it gives rise to—before an observational quest is consistently pursued. Moreover, antievaporation might also lie on the basis of other cosmological effects (like inflation) in the very early universe. This is presently being investigated by the authors and the results will be reported elsewhere.

One of us (S.D.O.) would like to thank Professor S. W. Hawking for the kind hospitality at DAMTP and for very interesting discussions and MEC (Spain) for financial support. This investigation has been supported by DGICYT (Spain), project PB96-0925.

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1974).
[2] S. W. Hawking, *Hawking on the Big Bang and Black Holes* (World Scientific, Singapore, 1993).
[3] H. Nariai, Sci. Rep. Tohoku Univ., Ser. 1 **35**, 62 (1951).
[4] R. Bousso and S. W. Hawking, Phys. Rev. D **57**, 2436 (1998).
[5] C. Callan, S. Giddings, J. Harvey, and A. Strominger, Phys. Rev. D **45**, 1005 (1992).
[6] E. Elizalde, S. Naftulin, and S. D. Odintsov, Phys. Rev. D **49**, 2852 (1994).
[7] I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (IOP Publishing, Bristol, 1992).
[8] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).
[9] R. Bousso and S. W. Hawking, Phys. Rev. D **54**, 6312 (1996).
[10] A. A. Bytsenko, S. Nojiri, and S. D. Odintsov, Phys. Lett. B (to be published).
[11] S. Nojiri and S. D. Odintsov, Phys. Rev. D (to be published), hep-th/9804033.
[12] M. S. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. **B128**, 506 (1977).