

# Electric dipole moment of the neutron in the chiral quark soliton model

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Within the chiral quark soliton or Nambu–Jona-Lasinio model with a Witten-Veneziano type of  $U_A(1)$  symmetry breaking as well as with a finite  $\theta$  vacuum angle, we calculate the leading term ( $N_c^0 m_\pi^2$ ) in a systematic  $1/N_c$  expansion of the electric dipole moment of the neutron. The consistency requirement that the effects of  $\theta$  should vanish, if either the anomaly or any quark mass vanishes, is satisfied. The resulting upper limit for  $\theta$  is  $\theta < 2.8 \times 10^{-9}$ , which lies close to the upper bound obtained in the literature from other chiral models. [S0556-2821(99)03605-X]

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## I. INTRODUCTION

The neutron electric dipole moment is known to provide one of the best experimental limits on  $CP$  violating effects. One possible theoretical explanation of the neutron electric dipole moment (EDMN) is related to the  $\theta$  parameter of the QCD vacuum [1–3], which is introduced when generalizing the QCD Lagrangian by including an additional  $P$ - and  $CP$ -violating interaction

$$\mathcal{L}_{\text{QCD}}^\theta = -\theta \frac{g}{32\pi^2} \tilde{G}_{\mu\nu}^a G_{\mu\nu}^a. \quad (1)$$

More generally one could also attribute a  $CP$  violating phase to the quark mass matrix. Since via a chiral rotation they can be both combined to  $\bar{\theta} = \theta + \arg \det m$ , we will not differentiate here between them. In addition to possible  $CP$ -violating terms in the quark mass matrix this is the only extra term which may be added to the QCD Lagrangian being allowed by gauge invariance and renormalizability.

As in this kind of reasoning the neutron electric dipole moment is proportional to  $\theta$ , which is a parameter which cannot be ruled out to be different from zero in the real world. However a comparison with the experimental value of the electric dipole moment of the neutron (EDMN),  $|d_n^{\text{exp}}| < 1.1 \times 10^{-25} e \text{ cm}$ —which should vanish in the limit where  $CP$  is conserved—gives an upper bound for  $\theta$  when compared with theoretical predictions. Many calculations using different methods have been carried out in the past; for a given  $\theta$  all models provide qualitatively similar electric dipole moments of the neutron [1,2,4–8] (for effective chiral models see below in more detail). Experimentally just re-

cently [9,10] a new proposal to measure the EDMN with an improved technique appeared and a theoretical explanation of the possible finding is desirable.

The general feature which is exploited in the calculations based on the framework of effective chiral models is the possibility to transfer strong  $CP$  violation to the quark sector by making an appropriate axial rotation of the quark fields [11]. Several estimates have been performed, specifically in the  $\sigma$ -model [12], in the color dielectric model [13], the Skryme model [14,15], valence quark models (chiral bag model [16], cloudy bag model [13,17], MIT bag model [11]) and considering effective chiral Lagrangians [18,19]; they agree in the order of magnitude and give  $\theta < 10^{-9}$  (for some explicit values, cf. Table I). Nevertheless the different models and the corresponding calculations exhibit differences concerning their consistency with general theoretical constraints like chiral symmetry and the PCAC relation. The MIT bag model e.g. does not obey chiral symmetry at the bag surface, where reflected quarks change their helicity. The cloudy bag model is chirally symmetric, but with the symmetry breaking term implemented as a quark mass term and with dynamical pion fields, the PCAC relation is not given in terms of the pion field but instead in terms of the pseudoscalar isovector quark density. This in contrast to the NJL models, where the pion is a quark-antiquark state formed in the model and not put in by hand and the PCAC relation is realized explicitly.

The color dielectric model is a non-topological soliton model like the solitonic NJL model which even generates confinement dynamically via the coupling of a scalar field to the gluons. In this model Birse and McGovern [13] have calculated the pion loop contribution. The explicit chiral symmetry breaking terms were expressed there in terms of local mesonic fields instead of explicit quark mass terms in the fermion part and therefore in that case the tree level contribution to the EDMN was vanishing. The tree level contribution was however calculated in the cloudy bag model

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TABLE I. The various contributions to the EDMN in units of  $10^{-16}\theta e$  cm for the cloudy bag models (CBM), the color dielectric model (CDM), the MIT bag model (MIT), the chiral bag model ( $\chi$ BM), chiral effective Lagrangian (ECL) and the present NJL model. In the last line the power counting ( $N_c, m_\pi^2$ ) is given as far as applicable. Tree/valence level contributions in CDM do not appear due to a special choice for the explicit symmetry breaking terms. The Skyrme contribution was considered as related to the Dirac sea contribution in chiral quark models [20].

	tree/valence	pion loop	Dirac sea	total	comment
CBM [13]	-1.77	-1.14	-	-2.83	$R=1$ fm
CDM [13]	0	-1.17	0	-1.17	no quark mass
MIT [11]	-8.2			-8.2	
Skyrme [14]			-0.57-1.3	-1.87	$N_c^0, N_c^{-1}$
$\chi$ BM [16]	-2.3			-2.3	
CBM [17]	-1.8	-1.4		-3.2	
ECL [18]	-7.2			-7.2	$\phi = -11^\circ$
ECL [19]	-1.5	-3.3		-4.8	
NJL	-0.45	-	0.06	-0.40	$M=420$ MeV
	$N_c^0 m_\pi^2$	$N_c^{-1} m_\pi^2 \log m_\pi^2$	$N_c^0 m_\pi^2$		

[13,17], where the current masses are always present inside the bag radius.

The Nambu–Jona-Lasinio (NJL) model with the  $U_A(1)$ -symmetry breaking Witten-Veneziano term offers the possibility to calculate the neutron electric dipole moment in a fully self-consistent way. We are altogether able to perform a calculation of the sea and the valence contribution to the neutron electric dipole moment in a model, which actually respects PCAC (partial conservation of axial-vector current) and which has been applied rather successfully in the last years to various observables of the nucleon [20], and we check, if our result is in line with the presently generally accepted upper limit for  $\theta$ .

## II. $U_A(1)$ -SYMMETRY BREAKING IN THE $SU(3)$ -NJL MODEL

The original form of the Nambu–Jona-Lasinio (NJL) Lagrangian [21,22] with current quark mass  $m = \text{diag}(m_u, m_d, m_s)$  and supplemented with strange quark fields becomes

$$\mathcal{L}_{\text{NJL}} = \bar{q}(x)(i\partial - m)q(x) + \mathcal{L}_{\text{int}}, \quad (2)$$

where the interaction part can be written as

$$\mathcal{L}_{\text{int}} = -\frac{G}{2} [(\bar{q}(x)\lambda^a q(x))^2 + (\bar{q}(x)i\gamma_5\lambda^a q(x))^2]. \quad (3)$$

The Lagrangian  $\mathcal{L}_{\text{NJL}}$  possesses chiral  $SU_R(3) \times SU_L(3)$  in addition to  $U_A(1) \times U_V(1)$  symmetry. Whereas the chiral symmetry is spontaneously broken in the NJL model, leading to the Goldstone boson octet of pions and kaons, the  $U_A(1)$  symmetry is not broken.

This  $U_A(1)$  symmetry is however broken in QCD by the anomaly and non-perturbative gauge field configurations like instantons are known to give sizeable contributions to these matrix elements. The instanton induced effective quark interaction in the instanton gas or the instanton liquid model is of

't Hooft type [23,24] and contains also flavor mixing interactions. A simpler, effective way to incorporate such an  $U_A(1)$  breaking term in the NJL model may be to add

$$\mathcal{L}_{\text{an}} = -\frac{\chi^2}{2} \left( -\theta + \frac{i}{2} (\log \det \bar{q} P_{RQ} - \log \det \bar{q} P_{LQ}) \right)^2, \quad (4)$$

which was first proposed by Witten [25] and Veneziano [26] and which is not flavor mixing. Recently it was even derived [27] from some non-local four quark interaction term which had an infrared singularity in momentum space. The mesonic spectrum is somewhat different [28] for 't Hooft and Witten-Veneziano interaction, but the different flavor mixing properties of both are not essential for the present investigation. The size of the parameter  $\theta$  gives a measure for the strength of  $CP$ -violation, which results in the bosonized form e.g. in a non-vanishing vacuum expectation value of the singlet pseudoscalar field for  $\theta \neq 0$ . The quark bosonization will be performed in the next section in more detail and results in rewriting the  $U_A(1)$  symmetry breaking expression Eq. (4) into the following form:

$$\mathcal{L}_{\text{an}} = \frac{\chi^2}{2} \left( \theta + \text{Tr} \arctan \frac{\pi}{\sigma} \right)^2. \quad (5)$$

The effect of this term for  $\theta=0$  (i.e. without  $CP$ -violation) is to change the masses of the  $\eta$  and  $\eta'$  mesons and to drive them considerably closer to their experimental values. In the general case of non-vanishing  $\theta$  the term  $\mathcal{L}_{\text{an}}$  contains linear and quadratic parts in the fields. Whereas the quadratic part contributes to the masses, as just stated, the effect of the linear part can be shifted to the quark sector. Therefore we perform an axial transformation of the quark fields to remove the linear part. The nonanomalous part of the Lagrangian is chirally invariant except of the current mass matrix.

Up to the first order in the pion field we have  $\text{Tr} \arctan(\pi/\sigma) \approx \text{Tr}(\pi/\sigma)$ , and with neglecting the flavor symmetry breaking of the scalar field, i.e. with setting  $\sigma = \sigma_a \lambda^a \approx \sigma \mathbf{1}_3$ , we get for the  $U_A(1)$  and  $CP$  non-invariant part of the effective Lagrangian

$$\mathcal{L}^{\text{an}} \approx \frac{1}{3} \frac{\chi^2 N_f^2}{\sigma^2} (\pi^0)^2 + \sqrt{\frac{2}{3}} \frac{\chi^2 N_f}{\sigma} \theta \pi^0. \quad (6)$$

Together with this contribution the  $\pi^0$ - $\pi^8$ -subsystem of the mesonic mass matrix is given by  $[m_0 \equiv (m_u + m_d)/2]$

$$\mathcal{L}_{\text{mass}}^{\text{an}} \approx (\pi^0, \pi^8) \begin{pmatrix} \frac{\mu^2}{3} \left( 2 \frac{m_0}{M_u} + \frac{m_s}{M_s} \right) + \frac{1}{3} \frac{\chi^2 N_f^2}{\sigma^2} & \frac{\mu^2}{3} \sqrt{2} \left( \frac{m_0}{M_u} - \frac{m_s}{M_s} \right) \\ \frac{\mu^2}{3} \sqrt{2} \left( \frac{m_0}{M_u} - \frac{m_s}{M_s} \right) & \frac{\mu^2}{3} \left( \frac{m_0}{M_u} + 2 \frac{m_s}{M_s} \right) \end{pmatrix} \begin{pmatrix} \pi^0 \\ \pi^8 \end{pmatrix} + \sqrt{\frac{2}{3}} \frac{\chi^2 N_f}{\sigma} \theta \pi^0. \quad (7)$$

The desired transformation to remove the linear part from the action is of the form,  $q \rightarrow \exp(-i/2) \gamma_5 (\epsilon_0 \lambda^0 + \epsilon_8 \lambda^8) q$  and  $\bar{q} \rightarrow \bar{q} \exp(-i/2) \gamma_5 (\epsilon_0 \lambda^0 + \epsilon_8 \lambda^8)$ . They induce the following transformations [as  $\mathcal{O}(\epsilon) = \mathcal{O}(\theta)$  only first order is taken into account] of the  $\pi$ -fields,  $\pi^0 \rightarrow \pi^0 + \sigma \epsilon_0$  and  $\pi^8 \rightarrow \pi^8 + \sigma \epsilon_8$  and the current mass matrix (without isospin symmetry breaking, i.e.,  $m_u = m_d = m_0$ ),  $\bar{q} m q \rightarrow \bar{q} m q + \mathcal{L}_{\text{CP}}$  with  $\mathcal{L}_{\text{CP}} = \bar{q} i \gamma_5 \bar{m} \theta q$  and

$$\bar{m} = -\mathbf{1}_3 \left( \frac{1}{3} \sqrt{\frac{2}{3}} (2m_0 + m_s) \frac{\epsilon_0}{\theta} + \frac{2}{3\sqrt{3}} (m_0 - m_s) \frac{\epsilon_8}{\theta} \right) - \lambda^8 \left( \sqrt{\frac{2}{3}} (m_0 - m_s) \frac{\epsilon_0}{\theta} + \frac{1}{3} (m_0 + 2m_s) \frac{\epsilon_8}{\theta} \right). \quad (8)$$

Performing the rotation of  $\mathcal{L}_{\text{mass}}^{\text{an}}$ , Eq. (7), and demanding the linear part to vanish gives two linear equations to determine the parameters of the chiral rotation,  $\epsilon_0$  and  $\epsilon_8$ . Inserting them into Eq. (8) leads in the limit of small current masses to the flavor singlet result for  $\mathcal{L}_{\text{CP}}$ , corresponding to

$$\bar{m} = \frac{1}{\frac{1}{m_0} + \frac{1}{m_0} + \frac{1}{m_s}} (1-S) = \left[ \sum_i \frac{1}{m_i} \right]^{-1} (1-S) \quad (i = u, d, s). \quad (9)$$

The parameter  $S$  is numerically small for the physical region, of the order of magnitude of some percent. This is exactly the result which was already found in Ref. [13]. Equation (9) satisfies the constraints which are demanded in the literature (cf. e.g. [18,19] and references therein): (i)  $\bar{m}$  vanishes if one of the current masses is zero; (ii) it vanishes in absence of the anomaly, i.e. for  $\chi^2 = 0$  (then  $S = 1$ ). Since the anomaly is suppressed by  $1/N_c$  [18] the quantity  $\bar{m}$  is actually  $\mathcal{O}(N_c^{-1})$ .

The numerical value for  $\bar{m}$  follows at once from the values for  $m_0$  and  $m_s$ . Using  $m_0 = 6.1$  MeV,  $m_s = 150$  MeV and  $N_f = 3$  gives  $\bar{m} = 3.0$  MeV.

### III. THE ELECTRIC DIPOLE MOMENT OF THE NEUTRON

The expectation value of the neutron electric dipole moment is calculated in the so called chiral quark soliton model [29] also known as solitonic NJL model. This instanton motivated model was first derived by Diakonov and Petrov [30] and recently extensive reviews describing a variety of static hyperon observables calculated so far appeared [20,31]. Therefore we will go through the derivation here only very shortly. The original four quark interaction of the Nambu–Jona-Lasinio model is first bosonized by introducing auxiliary fields, carrying the quantum number of the scalar and pseudoscalar meson nonet of SU(3) [20]. In this way the four quark interaction terms are reduced to an expression only bilinear in the quarks and containing the auxiliary mesonic field. This has the advantage of making it possible to integrate out the quarks and perform a saddle point approximation for the mesonic fields which in nuclear physics corresponds to the so called Hartree approximation.

The generating functional of the bosonized Nambu–Jona-Lasinio model is then given by a path-integral in Euclidian space as

$$W[\omega^\dagger, \omega] = \int \mathcal{D}\bar{q} \mathcal{D}q \mathcal{D}\hat{M} \times \exp \left\{ - \int d^4x (\mathcal{L}_{\text{NJL}} + \mathcal{L}_{\text{CP}} + i\omega^\dagger q + i q^\dagger \omega) \right\}, \quad (10)$$

with  $\mathcal{L}_{\text{NJL}} = q^\dagger(x) (-i\not{\partial} + m + P_R \hat{M}(x) + P_L \hat{M}^\dagger(x)) q(x)$  and  $\hat{M} = g(\sigma + i\pi)$  where  $\hat{M}$  is the complex chiral field containing the scalar and pseudoscalar meson nonets. The  $CP$  violating part here in the quark sector is given by  $\mathcal{L}_{\text{CP}} = q^\dagger i \gamma_5 \bar{m} \theta q$ .

The soliton in this model is obtained after separation of spatial and time derivatives and defining the one particle Hamiltonian

$$H = \frac{\alpha_i \partial_i}{i} + \beta(m + g(\sigma + i\gamma_5 \pi)) + \beta i \gamma_5 \bar{m} \theta \quad (11)$$

(we set  $\gamma^i = \beta\alpha_i$ ,  $\gamma^0 = \beta$ ). Furthermore we use the usual hedgehog-ansatz for the time independent chiral field  $U(\vec{x}) \equiv \sigma(\vec{x}) + i\gamma_5\pi(\vec{x})$  and arrive at the  $SU(3)$  flavor case via a trivial embedding of the  $SU(2)$  solution, i.e. by setting [32]

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (12)$$

where  $U_0$  is the  $SU(2)$ -field. The classical equations of motion can be solved self-consistently for the chiral field  $U$ , resulting in a localized soliton with unit winding number. The energy spectrum of the Hamiltonian  $H$  for the baryon number one sector contains a discrete valence level inside a mass gap of size  $2M$  ( $M = \text{constituent quark mass}$ ) [33].

In order to obtain states with the definite quantum numbers for spin and isospin, a semiclassical quantization of the rotational zero modes is performed. This is the idea of Adkins, Nappi and Witten [34]. A detailed treatment is presented e.g. in Refs. [20,31].

The complicated time dependent ansatz for the chiral fields in this scheme is to be compensated by an appropriate rotation of the quark fields [35]. By rotating the quark fields according to  $q'(x) = A(t)q(x)$ , where  $A(t)$  is a time dependent  $SU(3)$  rotation matrix, the Lagrangian  $\mathcal{L}_E = \mathcal{L}_{NJL} + \mathcal{L}_{CP}$  becomes  $\mathcal{L}_{\text{rot}}[A(t)]$  [20]. Then the expectation value of the electric dipole density [with the quark charge matrix  $Q = e \text{diag}(2/3, -1/3, -1/3)$ ] takes on the following form:<sup>1</sup>

$$\begin{aligned} d(x) &:= \langle N | q^\dagger(x) Q x_3 q(x) | N \rangle \\ &= \int \mathcal{D}q^\dagger \mathcal{D}q \mathcal{D}A(t) [q^\dagger(x) Q x_3 q(x)] \\ &\quad \times \exp \left\{ - \int d^4x \mathcal{L}^{\text{rot}}[A(t)] \right\}. \quad (13) \end{aligned}$$

We are treating the Witten-Veneziano term here as a perturbation and Eq. (13) has to be expanded in addition in the rotational frequency  $\Omega = iA^\dagger \dot{A}$  and possible flavor symmetry breaking terms  $\sim A^\dagger m A$ , where  $m$  is the current quark mass matrix. The rotational corrections are however suppressed by  $1/N_c$  and the symmetry breaking corrections also turn out to be a next to leading order corrections. Therefore, as first attempt to determine the magnitude of the EDMN, we ignore possible  $1/N_c$  as well as current mass corrections. Solving the system without expanding in  $\theta$  would change the soliton since the term mixes states with natural and unnatural parity [20] and would make it technically rather complicated since it would involve the diagonalization of matrices twice the size. Since  $\theta$  is very small the perturbative treatment is justified here.

After some straightforward but tedious steps this formalism leads to an expression for the dipole moment of the neutron

<sup>1</sup>Here in a schematic way, see [20] for a much more pedagogical introduction.

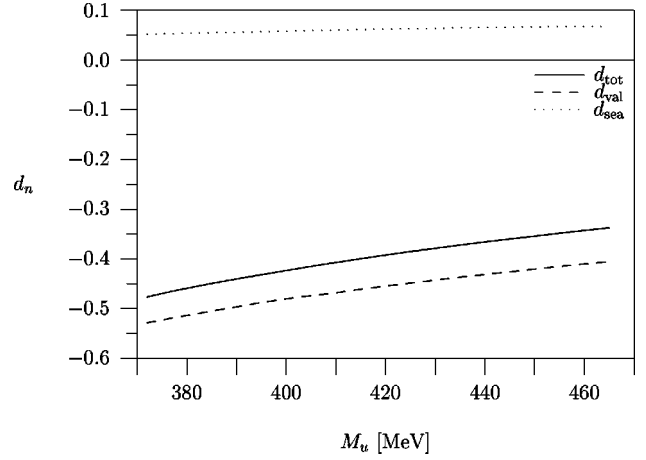


FIG. 1. The electric dipole moment of the neutron in dependence on the constituent quark mass  $M_u$ , in  $10^{-16}\theta e \text{ cm}$ .

$$\int d^3x d(x) = d_n^{\text{valence}} + d_n^{\text{sea}} \quad (14)$$

where

$$\begin{aligned} d_n^{\text{sea}} &= 2eN_c \bar{m} \theta \sum_{m < 0, n > 0} \frac{\langle n | x_3 \tau^3 | m \rangle \langle m | i\gamma_0 \gamma_5 | n \rangle}{|E_n - E_m|} \\ &\quad \times \langle N \uparrow | D_{Q_3} | N \uparrow \rangle \quad (15) \end{aligned}$$

$$\begin{aligned} d_n^{\text{valence}} &= 2eN_c \bar{m} \theta \sum_{n \neq \text{val}} \frac{\langle n | x_3 \tau^3 | \text{val} \rangle \langle \text{val} | i\gamma_0 \gamma_5 | n \rangle}{E_n - E_{\text{val}}} \\ &\quad \times \langle N \uparrow | D_{Q_3} | N \uparrow \rangle. \quad (16) \end{aligned}$$

Here the  $\langle N \uparrow | D_{Q_3}(A) | N \uparrow \rangle = 1/10$  are the matrix elements of the Wigner function  $D_{Q_3}(A)$  between two neutron states with spin up. The sums in Eqs. (15), (16) run over all positive and negative one particle energy levels; positive (negative) indices are associated with positive (negative) values of the orbital energies. A remark to the non-regularization of the sea contribution in Eq. (16): The dipole moment is given by the integral over the time component of the vector current, which in Euklidean space is related to the imaginary part of the effective action and related to the Wess Zumino term. Since the  $\theta$  term in Eq. (11) itself is Hermitian, the dipole moment calculated in the linear order in  $\theta$  and in lowest order of the rotational corrections is also related to the imaginary part and therefore finite. Therefore the expression (16) is not regularized and we checked numerically that it is indeed finite. In fact it can be shown that an expansion of the dipole moment in terms of the gradients of the chiral fields only yields finite expressions.

Using standard techniques the numerical calculation of Eqs. (15), (16) can be performed. The NJL model in the standard formulation [20] fixes its parameters to  $m_\pi$ ,  $m_K$  and  $f_\pi$  in the meson sector and then has the constituent quark mass  $M_u$  as the only free parameter. Figure 1 shows our result for the electric dipole moment of the neutron for a physical range of constituent quark masses  $M_u$ . We see that the valence contribution is by far the dominant one and the

sea contribution does not play a crucial role. Furthermore one can see that in our model the dipole moment varies only smoothly with the constituent quark mass  $M_u$ , so that small variations of  $M_u$  do not alter the result considerably. This is in contrast to Ref. [17], where in the cloudy bag model with small variations of the bag radius a wide range of values for the dipole moment can be produced.

Finally for a typical value of  $M_u = 420$  MeV, which reproduces basically all static nucleon observables [20,31], we find

$$d_n = -0.4 \times 10^{-16} \theta e \text{ cm}. \quad (17)$$

Together with the current experimental limit for the neutron electric dipole moment [36],

$$|d_n^{\text{exp}}| < 1.1 \times 10^{-25} e \text{ cm}, \quad (18)$$

we get the following upper bound for the parameter  $\theta$ , i.e. for the strength of  $CP$ -violation in the QCD Lagrangian

$$\theta < 3 \times 10^{-9}. \quad (19)$$

#### IV. CONCLUSION

Generally, in the literature in the framework of effective chiral theories agreement is found for the order of magnitude of  $d_n$  (cf. Table I).

It has been pointed out in Ref. [13] that the contributions from the loop and the valence terms in the cloudy bag model with the correct implementation of the anomaly reinforce each other so that in none of the available models there is a cancellation mechanism for this EDMN up to now.

The present chirally symmetric NJL model, which we have investigated, contains the explicit chiral symmetry breaking current masses in the fermion determinant. Therefore we have been able to calculate the direct or valence level contribution to the dipole moment. Furthermore, since in our model also the polarization of the Dirac sea is consistently taken into account, we get in addition a sea contribution to the dipole moment which is as far as  $N_c$  counting is concerned of the same order  $\mathcal{O}(N_c^0 m_\pi^2)$  as the valence contribution. So we have been able to perform an estimate from valence and sea parts in a model which actually respects PCAC and therefore outperforms some of the older ap-

proaches. Our sea contribution however has a different sign compared to the valence term, which offers the possibility of cancellation effects and therefore relax the constraint on  $\theta$ ; but it actually turns out to be small in the full range of parameter space which is given by reasonable values for the constituent quark mass. If we would go to very large constituent quark masses  $M \sim 700$  MeV it is well known that the valence level enters the Dirac sea and the sea contribution becomes large. This is also the case for the dipole moment. Since a fixed chiral profile is determined by the dimensionless quantity  $MR$ , where  $R$  is a typical size of the chiral field, the scenario above can also be achieved by having chiral fields with rather large extensions in which case the NJL model is very closely related to the Skyrme model [20]. However most of the observables of the model become rather unphysical in this regime.

Numerically our result of  $d_n = -0.4 \times 10^{-16} \theta e \text{ cm}$  leads to  $\theta < 3 \times 10^{-9}$  and therefore gives the same order of magnitude as other comparable estimates, but it seems to be somewhat less restrictive for the size of  $\theta$ . The valence contribution in the cloudy bag model (CBM) e.g. is significantly larger, but it scales like the bag radius  $R^2$  and is much more sensitive to the model input parameter than the present NJL result.

Another point is that in the present model  $1/N_c$  corrections from SU(3) collective excitations could be large (as has been shown in [14] for the Skyrme model) and dominate the lowest order result. It is known that for the vector charges in SU(3) to satisfy the Gell-Mann–Nishijima formula [37] the linear  $1/N_c$  corrections are essential. Technically this contribution would however be well beyond the scope of the present calculation. Finally pion loop contributions, which are suppressed by  $1/N_c$  but contain the non-analytic  $m_\pi^2 \log m_\pi^2$  contributions, could give another sizeable effect in the present model similar than they do in the color dielectric model (CDM) [13].

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