Complete analysis of photino-mediated lepton flavor violations in generalized supersymmetric models

Yeong Gyun Kim, Pyungwon Ko, Jae Sik Lee, and Kang Young Lee
Department of Physics, Korea Advanced Institute of Science and Technology, Taejon 305-701, Korea

(Received 20 July 1998; published 10 February 1999)

We consider lepton flavor violations (LFV) mediated by a photino as a result of the nondiagonal slepton mass matrices in general supersymmetric models. Using the experimental upper bounds on $l \to l' + \gamma$ and $\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$ as constraints on the flavor changing slepton mass insertions, we predict the possible ranges of the upper limits on the branching ratios of other LFV processes such as $l \to 3l'$, $\tau \to l_{i\neq 3} + \pi^0$(or η, ρ^0, ϕ), $Z^0 \to l_i \overline{l_{j\neq i}}$, and muonium $\to$ antimuonium conversion. Most of these decays are expected to occur with small branching ratios far below the current or future experimental search limits. We also derive constraints on the flavor conserving mass insertions from the anomalous magnetic moments of the leptons. [S0556-2821(99)01203-5]

PACS number(s): 12.60.Jv, 11.30.Fs, 13.10.+q, 13.40.Em

I. INTRODUCTION

At present, the minimal supersymmetric standard model (MSSM) is widely considered as the leading candidate for physics beyond the standard model (SM) [1]. It can solve the gauge hierarchy problem by supersymmetrizing the SM (with an additional Higgs doublet). The boson loop contribution to the Higgs boson mass is cancelled by the fermion loop contribution, the latter of which comes in with opposite sign to the former. In doing so, the particle spectrum of the theory becomes doubled compared to that of the SM. One expects many new scalar particles (superpartners of the SM fermions) and new fermions (superpartners of the SM gauge bosons and Higgs bosons). These new particles should have masses around $O(100)$ GeV$-O(1)$ TeV in order to solve the gauge hierarchy problem in terms of softly broken supersymmetry. Nice features of the supersymmetric theories are their calculability using perturbation theory, and the decoupling nature of the loop effects of new (super)particles on various electroweak observables, except for the dangerous supersymmetric (SUSY) flavor changing neutral current (FCNC) and SUSY CP problems (which will be discussed shortly in more detail). Therefore the successful predictions of the SM do not change very much even if we have doubled spectrum of particles in SUSY theories modulo SUSY FCNC and CP problems.

However, in generic supersymmetric (SUSY) models, one has to pay for this extra symmetry. First of all, the lepton family numbers ($L_{i=e,\mu,\tau}$) and the baryon number (B) are no longer conserved as in SM. One can write down renormalizable superpotential which violates the L_i and B numbers and leads to too fast proton decay in conflict with the observation. Secondly, the soft mass terms for sfermions can lead to large FCNC unless certain conditions are met. In most phenomenological SUSY models, one solves the first problem by assuming R-parity conservation by hand. The second problem (SUSY FCNC) is solved by assuming that either (i) the sfermion mass matrices are proportional to the unit matrix in the flavor space [2,3], (ii) the sfermion mass matrices are proportional to the corresponding fermion mass matrices so that both can be diagonalized simultaneously [4], or (iii) assuming that the first two generation sfermions are highly degenerate and very heavy ($\gtrsim 50$ TeV$\gg M_{SUSY}$ so that they basically decouple) [5]. In the minimal supergravity (SUGRA) models with the flat Kähler metric at M_{Planck} scale, the first condition can be met, namely the squarks, sleptons, and Higgs boson are all degenerate at the Planck scale. However, when one evolves the sfermion mass parameters to the electroweak scale using renormalization group (RG), the off-diagonal elements of the squark mass matrices are induced in a calculable manner, although there is no lepton flavor violation (LFV) induced at low energy. In the lepton sector of the minimal SUGRA model, there is no LFV as in the SM since neutrinos are massless. As an example, consider the minimal SUGRA model with the flat Kähler metric. Then the scalar masses are universal (being m_0^2) at the Planck scale, whereas at the weak scale the sfermion masses change as

$$(m_{\tilde{d}}^2)_{LL}(\mu = m_Z) = m_d m_d^\dagger + m_0^2 + c_d m_u m_u^\dagger, \qquad (1)$$

as a result of renormalization [2]. Because of the last term containing $m_u m_u^\dagger$, it is not possible to diagonalize $m_d m_d^\dagger$ and $(m_{\tilde{d}}^2)_{LL}$ simultaneously. This leads to the flavor changing gluino-quark-squark vertices, which can contribute to various low energy FCNC processes. Also this (LL) mixing induces the LR and RR mixings in the minimal SUGRA models. For sleptons, on the other hand, we have

$$(m_{\tilde{l}}^2)_{LL}(\mu = m_Z) = m_l m_l^\dagger + m_0^2 + c_l m_\nu m_\nu^\dagger, \qquad (2)$$

and massless neutrinos (namely, absence of right-handed neutrinos) imply that there is no generation mixing in the slepton mass matrix. Since LR and RR transitions are proportional to the (LL) mixing, there will be no lepton family number violation in the minimal SUGRA models.[1] However, the condition of the flat Kähler metric is a strong assumption

[1] If right-handed neutrinos are included in the minimal SUGRA models [6], there can be generic LFV at electroweak scale.

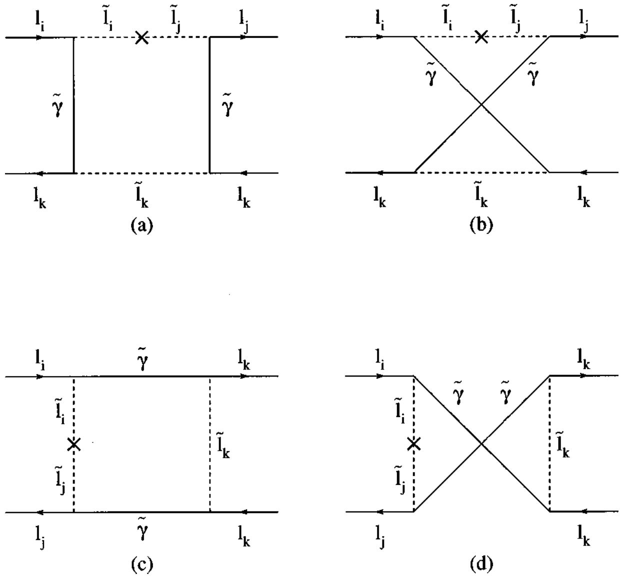

FIG. 1. Box diagrams for $\Delta L_i=1$. Here i,j,k,l are the generation indices.

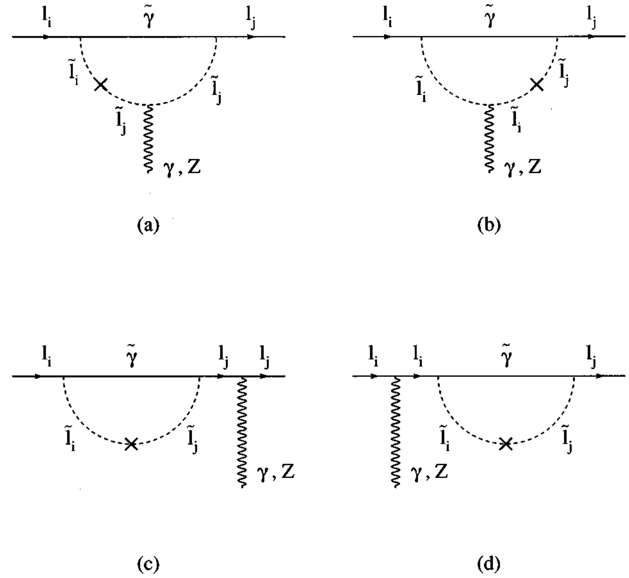

FIG. 2. Penguin diagrams for $\Delta L_i=1$. Here i,j,k,l are the generation indices.

which may not be true in general. For example, SUGRA radiative corrections to the boundary conditions at $M_{\rm Planck}$ scale induce generically $O(\sim 10\%)$ off-diagonal sfermion mass matrix elements [7]. Moreover, there is generic LFV at the high energy scale in the context of the supersymmetric grand unification theories (SUSY GUT) [8]. Therefore, one can imagine certain amount of nondiagonal sfermion mass matrix elements at the electroweak scale in general.

In view of this, it is important to see how large deviation from the above conditions (i) and (ii) are allowed in the general SUSY models by the various FCNC processes at low energy. Such studies have been done previously by several authors already, mainly on the gluino-mediated FCNC in the quark sector and the photino-mediated $l_i \rightarrow l_{j\neq i}+\gamma$ [3]. Basically deviations between the first and the second families should be very small. In terms of a dimensionless parameter defined as

$$(\delta^l)_{AB} \equiv (\Delta^l)_{AB}/m_{\tilde{l}}^2, \tag{3}$$

where $m_{\tilde{l}}^2$ is a suitable average of the slepton masses, the condition that deviations between the first and the second families should be very small can be represented as following constraints [3]:

$$(\delta_{12}^l)_{LL}=O(10^{-3}), \quad \text{and} \quad (\delta_{12}^l)_{LR}=O(10^{-6}). \tag{4}$$

On the other hand, the deviations involving the third family are more loosely constrained:

$$(\delta_{13(23)}^l)_{LL}=O(1-10), \quad \text{and} \quad (\delta_{13(23)}^l)_{LR}=O(10^{-2}). \tag{5}$$

This is in part due to the less precise experimental informations on various FCNC processes involving the third family. But there are many interesting possibilities for which one can treat the third family in a different manner from the first two families. In such theories, one may expect larger deviations from the degeneracy in general, and thus expect FCNC processes with branching ratios that may be accessible in the near future.

In this work, we mainly concentrate on the photino-mediated FCNC processes in the lepton sector, which are almost parallel to the work by Gabbiani *et al.* [3]. Namely, we assume that the slepton mass matrices are not diagonal in the basis where $\tilde{l}_i - l_j - \tilde{\gamma}$ vertex is flavor diagonal. In order to simplify the analysis, we make an assumption that the lightest superparticle (LSP) is a photino $(\tilde{\gamma})$, and other neutralinos are fairly massive so that their effects are negligible compared to the LSP effects considered in this work. Finally, we assume that the off-diagonal mass matrix elements of sleptons are small enough that the mass insertion approximations are applicable. All of these assumptions are the same as Ref. [3], except for the photino mediated LFV instead of gluino mediated FCNC. In the case of gluino-mediated FCNC, the neutralino effects will be generically suppressed by α_2/α_s, so that one can safely ignore the neutralino-mediated FCNC. For the case of LFV, all the couplings of four neutralinos will be the same order of magnitude, and all the neutralino contributions to LFV should be included at the same time in principle. However, we assume that the photino is the LSP and other neutralinos are heavy and can be ignored in order to simplify our analysis. It would be straightforward, although tedious, to include 4 neutralinos altogether and make more complete our analysis.

There are a few differences between Ref. [3] and our work. First of all, we can restrict the allowed regions of the FC mass insertion by considering different processes. Different processes provide independent constraints from each other, and we need not make an assumption that there is no fortuitous cancellation between δ_{LL} and δ_{LR}, and so on. In the limit the light photino dominates the LFV, we can even predict the upper bound on some LFV decays in a completely model independent fashion. It is straightforward to relax this assumption and include all the four neutralino contributions to LFV, if necessary. We consider all the LFV processes that are studied experimentally at present. We consider LFV decays of Z^0 gauge boson, and processes involving two leptons and two quarks, such as $\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$, and $\tau \to \mu(e) +$ (a neutral meson) as well as processes involving four leptons and the LFV radiative decays.

Secondly, the authors of Ref. [3] derived constraints on the flavor conserving mass insertion δ^l_{ii} from the requirement that the SUSY one-loop contribution to the lepton mass (one loop diagram with an insertion of δ^l_{ii}) is smaller than the actual lepton mass ($\Delta m_l^{\mathrm{SUSY}} < m_l^{\mathrm{exp}}$). However, we regard this condition as an improper one, since the particle mass cannot be predicted by SM or SUSY models. On the contrary, it turns out that the anomalous magnetic moment of a lepton $[a_l \equiv (g-2)/2]$ can provide more meaningful and stronger bounds on δ^l_{ii}.

This paper is organized as follows. In Sec. II, we construct the effective Lagrangian for $\Delta L_i = 1$ and 2. The results form the basis for the calculations of transition rates for various LFV processes in the Sec. III. Constraints on the flavor conserving mass insertions from the anomalous magnetic moment are derived in Sec. IV, and the results are summarized in Sec. V.

II. EFFECTIVE LAGRANGIAN FOR $\Delta L_i = 1$ AND 2

A. $\mathcal{L}_{\mathrm{eff}}(4l)$ for $\Delta L_i = 1$

Let us first derive the effective Lagrangian for $\Delta L_i = 1$. A complete basis for $\Delta L_i = 1$ effective Lagrangian is

$$\mathcal{L}_{\mathrm{eff}}^{\Delta L_i = 1}(4l) = \sum_{i=3,5,7} [C_i O_i + C_i' O_i'], \tag{6}$$

where

$$O_3 = \overline{l_j} \gamma_\mu P_L l_i \sum_k \overline{l_k} \gamma^\mu P_L l_k,$$

$$O_5 = \overline{l_j} \gamma_\mu P_L l_i \sum_k \overline{l_k} \gamma^\mu P_R l_k,$$

$$O_7 = \frac{e}{8\pi^2} m_i \overline{l_j} \sigma^{\mu\nu} P_R l_i F_{\mu\nu}. \tag{7}$$

The operators O_i''s and the associated Wilson coefficients C_i''s are obtained from O_i's and C_i's by the exchange $L \leftrightarrow R$. Evaluating the Feynman diagrams in Figs. 1 (the box diagrams) and 2 (the penguin diagrams), and matching the full amplitudes with those in the effective theory, we get

$$C_3 = \frac{2\alpha^2}{m_{\tilde{l}}^2} (\delta^l_{ji})_{LL} [P_1(x) - 4B_1(x) - 2B_2(x)],$$

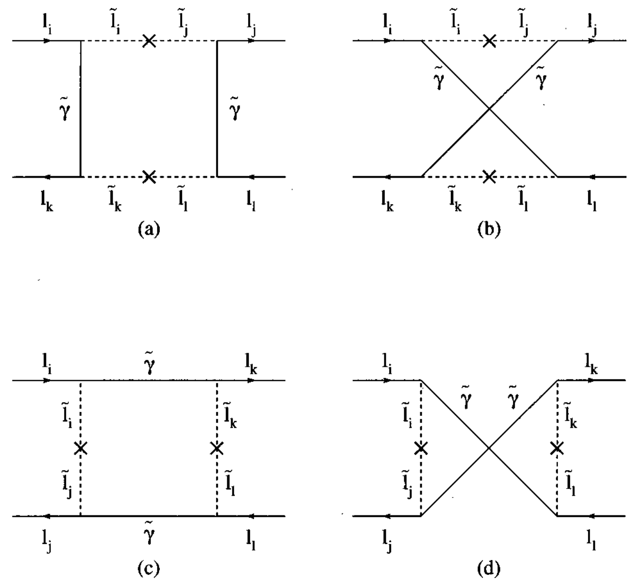

FIG. 3. Feynman diagrams for $\Delta L_i = 2$. Here i,j,k,l are the generation indices.

$$C_5 = \frac{2\alpha^2}{m_{\tilde{l}}^2} (\delta^l_{ji})_{LL} [P_1(x) + 4B_1(x) + 2B_2(x)],$$

$$C_7 = \frac{2\alpha\pi}{m_{\tilde{l}}^2} \left[(\delta^l_{ji})_{LL} M_3(x) + (\delta^l_{ji})_{LR} \frac{m_{\tilde{\gamma}}}{m_i} M_1(x) \right]. \tag{8}$$

We have neglected the final lepton mass m_j in the above expression, and $x \equiv m_{\tilde{\gamma}}^2 / m_{\tilde{l}}^2$.[2] As noted in Ref. [3], the Z-penguin contributions to $\mu \to 3e$, etc. are suppressed compared to the above by a factor of m_l^2/M_Z^2, and thus were safely ignored. Note that the δ_{LR} and δ_{RL} contribute only to O_7, and not to $O_{3,5}$, when we keep only dimension-6 operators in our effective theory.

The functions B_i's (from the box diagrams, Fig. 1), P_i's (from the penguin diagrams, Fig. 2) are defined in Ref. [3], and shown below for completeness:

$$B_1(x) = \frac{1 + 4x - 5x^2 + 4x\ln(x) + 2x^2\ln(x)}{8(1-x)^4},$$

$$B_2(x) = x\frac{5 - 4x - x^2 + 2\ln(x) + 4x\ln(x)}{2(1-x)^4},$$

$$P_1(x) = \frac{1 - 6x + 18x^2 - 10x^3 - 3x^4 + 12x^3\ln(x)}{18(x-1)^5},$$

[2]Strictly speaking, the photino LSP implies that $x < 1$. However, we consider the case $x > 1$ as well, since it would give a rough estimate of neutralino-mediated LFV's in case the photino is no longer the LSP.

$$P_2(x)=\frac{7-18x+9x^2+2x^3+3\ln(x)-9x^2\ln(x)}{9(x-1)^5},$$
$$M_1(x)=4B_1(x),$$
$$M_3(x)=\frac{-1+9x+9x^2-17x^3+18x^2\ln(x)+6x^3\ln(x)}{12(x-1)^5}. \tag{9}$$

B. $\mathcal{L}_{\rm eff}(2l-2q)$ for $\Delta L_i=1$

In order to study the $\mu^-+\mathrm{Ti}\rightarrow e^-+\mathrm{Ti}$, and $\tau\rightarrow\mu(\text{or } e)+$ (neutral meson), we need the effective Lagrangian for $l_i+q\rightarrow l_j+q$ where q denotes a specific quark flavor. From Feynman diagrams analogous to Figs. 1 and 2, we obtain

$$\mathcal{L}^{2l-2q}_{\rm penguin}=-\frac{2\alpha^2}{m^2_{\tilde{l}}}(\delta^l_{ji})_{LL}P_1(x)\overline{l_j}\gamma^\mu P_L l_i\sum_{q=u,d,s}e_q\overline{q}\gamma_\mu q+(L\rightarrow R), \tag{10}$$

$$\mathcal{L}^{2l-2q}_{\rm box}=\frac{-4\alpha^2}{m^2_{\tilde{l}}}\left[(\delta^l_{ji})_{LL}(2B_1(x)+B_2(x))\overline{l_j}\gamma^\mu P_L l_i\sum_{q=u,d,s}e_q^2\overline{q}\gamma_\mu(P_L-P_R)q-(L\leftrightarrow R)\right] \tag{11}$$

where we assume $m_{\tilde{l}}=m_{\tilde{q}}$ for simplicity.[3] Again the functions P_i's and B_i's are originated from the penguin and the box diagrams, respectively. Note that $\mathcal{L}^{2l-2q}_{\rm penguin}$ and $\mathcal{L}^{2l-2q}_{\rm box}$ can be obtained from Eqs. (6)–(8) by replacing $e_l^4\rightarrow e_l^3 e_q$ and $e_l^4\rightarrow e_l^2 e_q^2$, respectively. The penguin contribution $\mathcal{L}^{2l-2q}_{\rm penguin}$ contains the vector quark current, and thus can contribute to μ^-+ Ti $\rightarrow e^-+$ Ti, and $\tau\rightarrow l_{i\neq 3}+V(\equiv\rho^0,\phi)$. On the other hand, the box contribution $\mathcal{L}^{2l-2q}_{\rm box}$ depends only on the axial vector quark current so that it cannot contribute to the aforementioned processes, but it is relevant to the process $\tau\rightarrow l_{i\neq 3}+P(\equiv\pi^0,\eta)$. One also has to include the operator O_7 describing $l_i\rightarrow l_j+\gamma$ to the above effective Hamiltonian when calculating physical amplitude for $2l-2q$ processes.

C. $\mathcal{L}_{\rm eff}$ for $\Delta L_i=2$

In this subsection, we derive the effective Lagrangian for $\Delta L_i=2$ for completeness. This Lagrangian is relevant to the muonium $\rightarrow$ antimuonium conversion, although the resulting effect turns out to be too small. The relevant Feynman diagrams are shown in Fig. 3. The results are (we fix $i=1, j=2$ in this subsection)

$$\mathcal{L}^{\Delta L_i=2}_{\rm eff}=\sum_{i=1}^{5}C_i^{\Delta L_i=2}Q_i, \tag{12}$$

where the basis operators in the effective theory are defined as

$$Q_1=\overline{e}\gamma_\alpha P_L\mu\overline{e}\gamma^\alpha P_L\mu,$$
$$Q_2=\overline{e}\gamma_\alpha P_R\mu\overline{e}\gamma^\alpha P_R\mu,$$
$$Q_3=\overline{e}P_R\mu\overline{e}P_R\mu,$$
$$Q_4=\overline{e}P_L\mu\overline{e}P_L\mu,$$
$$Q_5=\overline{e}P_L\mu\overline{e}P_R\mu. \tag{13}$$

By matching the full theory amplitude with the effective amplitude, one can obtain the Wilson coefficients as follows:

$$C_1^{\Delta L_i=2}=\frac{\alpha^2}{m^2_{\tilde{l}}}(\delta^l_{12})^2_{LL}\left\{\frac{1}{2}\tilde{f}_6(x)+xf_6(x)\right\},$$
$$C_2^{\Delta L_i=2}=C_1\ \ (\text{with } L\leftrightarrow R),$$
$$C_3^{\Delta L_i=2}=-\frac{\alpha^2}{m^2_{\tilde{l}}}(\delta^l_{12})^2_{LR}2xf_6(x),$$
$$C_4^{\Delta L_i=2}=C_3\ \ (\text{with } L\leftrightarrow R),$$
$$C_5^{\Delta L_i=2}=\frac{\alpha^2}{m^2_{\tilde{l}}}\{(\delta^l_{12})_{LL}(\delta^l_{12})_{RR}[2\tilde{f}_6(x)+4xf_6(x)]$$
$$-(\delta^l_{12})_{LR}(\delta^l_{12})_{RL}4\tilde{f}_6(x)\}. \tag{14}$$

Here, the functions $\tilde{f}_6(x)$ and $f_6(x)$ are defined as

$$\tilde{f}_6(x)=\frac{6x(1+3x)\ln x-x^3-9x^2+9x+1}{3(x-1)^5},$$
$$f_6(x)=\frac{6(1+3x)\ln x+x^3-9x^2-9x+17}{6(x-1)^5}. \tag{15}$$

This completes the derivation of effective Lagrangians for $\Delta L_i=1$ and 2 that will be used in the following sections.

[3] In general, the function P_1, B_1 and B_2 should be generalized as functions of two variables, $x\equiv m^2_{\tilde{\gamma}}/m^2_{\tilde{l}}$ and $y\equiv m^2_{\tilde{q}}/m^2_{\tilde{l}}$ because of the difference between the slepton and squark masses.

Also, for the purpose of $Z\to l_i\overline{l_{j\neq i}}$, we present the amplitude for this decay. In this case, we need the full amplitude as given in Sec. III E.

III. ANALYTIC EXPRESSIONS AND NUMERICAL ANALYSES FOR VARIOUS LFV PROCESSES

A. $l_i\to l_j+\gamma$

The amplitude for $l_i\to l_j+\gamma^*$ can be written as

$$\mathcal{M}(l_i^-\to l_j^-+\gamma^*)=e\epsilon^{\alpha*}\overline{u_j}(p-q)[q^2\gamma_\alpha(A_1^LP_L+A_1^RP_R)+im_i\sigma_{\alpha\beta}q^\beta(A_2^LP_L+A_2^RP_R)]u_i(p) \quad (16)$$

with

$$A_1^L=-\frac{\alpha}{2\pi m_{\tilde{l}}^2}(\delta_{ji}^l)_{LL}P_1(x),$$

$$A_1^R=A_1^L\ \ (\text{with }\ L\leftrightarrow R),$$

$$A_2^R=-\frac{C_7}{4\pi^2}=-\frac{\alpha}{2\pi m_{\tilde{l}}^2}\left[M_3(x)(\delta_{ji}^l)_{LL}+\frac{m_{\tilde{\gamma}}}{m_{l_i}}M_1(x)(\delta_{ji}^l)_{LR}\right],$$

$$A_2^L=A_2^R\ \ (\text{with }\ L\leftrightarrow R). \quad (17)$$

The decay rate for $l_i\to l_j+\gamma$ is

$$\Gamma(l_i\to l_j+\gamma)=\frac{\alpha}{4}m_i^5(|A_2^L|^2+|A_2^R|^2). \quad (18)$$

Note that only the transition magnetic form factors $A_2^{L,R}$ contribute to the on-shell photon emission. The off-shell photon contribution ($A_1^{L,R}$ form factors) is relevant to the $\mu\to 3e$ and μ^-+ Ti $\to e^-+$ Ti. Normalizing it to the decay rate for $l_i\to l_j\nu_i\overline{\nu_j}$, one gets

$$B(l_i\to l_j+\gamma)=\frac{\alpha^3}{G_F^2}\frac{12\pi}{m_{\tilde{l}}^4}\times\left\{\left|M_3(x)(\delta_{ji}^l)_{LL}+\frac{m_{\tilde{\gamma}}}{m_{l_i}}M_1(x)(\delta_{ji}^l)_{LR}\right|^2+(L\leftrightarrow R)\right\}B(l_i\to l_j\nu_i\overline{\nu_j}). \quad (19)$$

One can derive the limits on δ_{ij}^l's from the experimental upper bounds listed in Table I, assuming there is no fortuitous cancellations among various terms, as in Ref. [3]; see Table II. Without such assumption, one would get a band in the $((\delta_{ij}^l)_{LL},(\delta_{ij}^l)_{LR})$ plane (see the solid lines in Fig. 4).

TABLE I. Upper limits of branching ratios for LFV processes considered in the present work. The muon conversion rate on the Ti atom is normalized on the muon capture rate on the Ti atom. For the muonium conversion, see Sec. III F.

Mode	Branching ratio	Ref.
$\mu\to e\gamma$	3.8×10^{-11}	[9]
$\mu\to 3e$	1.0×10^{-12}	[9]
μ^- + Ti $\to e^-$ + Ti	6.1×10^{-13}	[9]
muonium conversion	$G_{MM}^{--}<3.0\times10^{-3}G_F$ $G_{MM}^{+-}<2.1\times10^{-3}G_F$	[10]
$\tau\to e\gamma$	2.7×10^{-6}	[15]
$\tau\to\mu\gamma$	3.0×10^{-6}	[15]
$\tau\to 3e$	2.9×10^{-6}	[17]
$\tau\to 3\mu$	1.9×10^{-6}	[17]
$\tau\to\mu e^+e^-$	1.7×10^{-6}	[17]
$\tau\to e\mu^+\mu^-$	1.8×10^{-6}	[17]
$\tau\to e\pi^0$	3.7×10^{-6}	[16]
$\tau\to\mu\pi^0$	4.0×10^{-6}	[16]
$\tau\to e\eta$	8.2×10^{-6}	[16]
$\tau\to\mu\eta$	9.6×10^{-6}	[16]
$\tau\to e\rho^0$	2.0×10^{-6}	[17]
$\tau\to\mu\rho^0$	6.3×10^{-6}	[17]
$\tau\to e\phi$	6.9×10^{-6}	[17]
$\tau\to\mu\phi$	7.0×10^{-6}	[17]

B. $\mu^-+\text{Ti}\to e^-+\text{Ti}$

The muon conversion to an electron on the titanium target is one of the most sensitive probes of LFV that may arise from physics beyond the SM. In our case, the transition amplitude for this process can be expressed as

TABLE II. Limits on $(\delta_{ij}^l)_{LL,RR,LR,RL}$ from $l_i\to l_j+\gamma$ for $m_{\tilde{l}}=100$ GeV and for different values of x assuming there is no fortuitous cancellations among various terms.

Process	Constrained δ	x	Limits
$\mu\to e\gamma$	$\lvert(\delta_{12}^l)_{LL,RR}\rvert$	0.3	4.0×10^{-3}
		0.9	7.6×10^{-3}
		3.0	1.8×10^{-2}
	$\lvert(\delta_{12}^l)_{LR,RL}\rvert$	0.3	2.0×10^{-6}
		0.9	2.3×10^{-6}
		3.0	3.8×10^{-6}
$\tau\to e\gamma$	$\lvert(\delta_{13}^l)_{LL,RR}\rvert$	0.3	2.5
		0.9	4.7
		3.0	11
	$\lvert(\delta_{13}^l)_{LR,RL}\rvert$	0.3	2.0×10^{-2}
		0.9	2.4×10^{-2}
		3.0	4.0×10^{-2}
$\tau\to\mu\gamma$	$\lvert(\delta_{23}^l)_{LL,RR}\rvert$	0.3	2.4
		0.9	4.5
		3.0	10
	$\lvert(\delta_{23}^l)_{LR,RL}\rvert$	0.3	1.9×10^{-2}
		0.9	2.3×10^{-2}
		3.0	3.8×10^{-2}

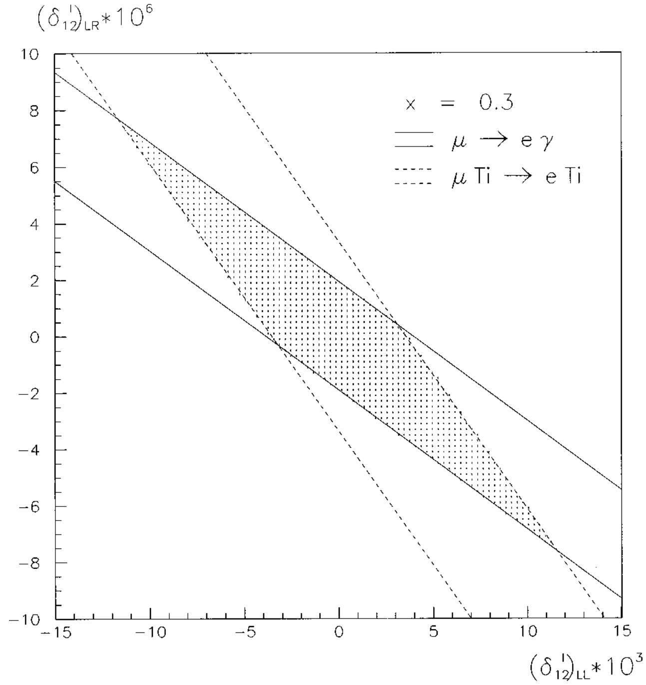

FIG. 4. Allowed regions in the $((\delta^l_{12})_{LL}, (\delta^l_{12})_{LR})$ planes for $x = 0.3$. The region between solid lines are allowed by the present experiment for $\mu \to e\gamma$ and the region between dotted lines are allowed by the present experiment for μ^- Ti$\to e^-$ Ti. Combining two experiments, only the shaded region is allowed.

$$\mathcal{M}(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) = -\frac{e^2}{q^2}\bar{e}[q^2\gamma^\alpha(A_1^L P_L + A_1^R P_R) + m_\mu i\sigma^{\alpha\beta}q_\beta(A_2^L P_L + A_2^R P_R)]\mu \times \sum_{q=u,d} e_q \bar{q}\gamma_\alpha q, \tag{20}$$

where $A_{1,2}^{L,R}$'s are defined in Eq. (17). Note that there is no box contribution to this process, since only the quark vector current is important for the coherent conversion on the Ti nucleus. Also as alluded before, there is no Z-penguin contribution to this process to the order we are working.

The transition rate is given by

$$\Gamma(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) = 4\alpha^5 \frac{Z_{\mathrm{eff}}^4}{Z} Z^2 |F(q^2 \simeq -m_\mu^2)|^2 m_\mu^5 \times [|A_1^L - A_2^R|^2 + |A_1^R - A_2^L|^2]. \tag{21}$$

For the titanium target, $Z=22$, $A=48$, $N=26$, $Z_{\mathrm{eff}}=17.6$ and $|F(q^2 \simeq -m_\mu^2)| \simeq 0.54$. The experimental limit on the transition rate is given by

$$\Gamma(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) < 6.1 \times 10^{-13} \Gamma \quad (\mu \text{ capture in Ti}), \tag{22}$$

where the muon capture rate in Ti is $\Gamma(\mu$ capture in Ti$) = (2.590 \pm 0.012) \times 10^6$/sec. Thus one gets the following upper bound:

$$|A_1^L - A_2^R|^2 + |A_1^R - A_2^L|^2 < [4.0 \times 10^{-11} \ \mathrm{GeV}^{-2}]^2. \tag{23}$$

Thus, we get

$$\frac{1}{m_{\tilde{l}}^2}\left| (\delta^l_{12})_{LL}(-P_1(x) + M_3(x)) + \frac{m_{\tilde{\gamma}}}{m_\mu}(\delta^l_{12})_{LR} M_1(x) \right| < 3.4 \times 10^{-8} \ \mathrm{GeV}^{-2}, \tag{24}$$

and similarly for the $(L \leftrightarrow R)$ case. This is another strong constraint that is independent of that from $\mu \to e\gamma$.

As we noted in the previous subsection, without assuming that there is no fortuitous cancellations among various terms, one would get a band in the $((\delta^l_{ij})_{LL}, (\delta^l_{ij})_{LR})$. One can not constrain $(\delta^l_{ij})_{LL}$ and $(\delta^l_{ij})_{LR}$ independently of each other without any assumptions. But, combining two different experiments, as one sees in Fig. 4, we obtain two different bands from $\mu \to e\gamma$ and $\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$. Only the shaded region is allowed.

C. $l_i \to 3l_j$ and $l_i \to l_{j\neq k} l_k \bar{l}_k$

The amplitude for $l_i^- \to l_j^- l_j^+ l_j^- (\equiv 3l_j)$ can be calculated from the effective Lagrangians, Eqs. (6)–(8). It can be written as the sum of the electromagnetic penguin and the box contributions:

$$\mathcal{M}_{\mathrm{penguin}} = \overline{u_j(p_1)}[q^2\gamma^\alpha(A_1^L P_L + A_1^R P_R) + m_{l_j} i\sigma^{\alpha\beta}q_\beta(A_2^L P_L + A_2^R P_R)]u_i(p)\frac{e^2}{q^2}\overline{u_j(p_2)}\gamma_\alpha v_j(p_3) - (p_1 \leftrightarrow p_2) \tag{25}$$

$$\begin{aligned}\mathcal{M}_{\mathrm{box}} = {} & B_1^L e^2 \overline{u_j(p_1)}\gamma^\alpha P_L u_i(p)\overline{u_j(p_2)}\gamma_\alpha P_L v_j(p_3) + B_1^R e^2 \overline{u_j(p_1)}\gamma^\alpha P_R u_i(p)\overline{u_j(p_2)}\gamma_\alpha P_R v_j(p_3) \\ & + B_2^L e^2[\overline{u_j(p_1)}\gamma^\alpha P_L u_i(p)\overline{u_j(p_2)}\gamma_\alpha P_R v_j(p_3) - (p_1 \leftrightarrow p_2)] \\ & + B_2^R e^2[\overline{u_j(p_1)}\gamma^\alpha P_R u_i(p)\overline{u_j(p_2)}\gamma_\alpha P_L v_j(p_3) - (p_1 \leftrightarrow p_2)].\end{aligned} \tag{26}$$

Here the box form factors B's are given by

$$B_1^L = -\frac{2\alpha}{\pi m_{\tilde{l}}^2}(\delta^l_{ji})_{LL}(2B_1(x) + B_2(x))$$

$$
\begin{aligned}
B_2^L &= -\frac{1}{2}B_1^L,\\
B_1^R &= B_1^L \quad (\text{with } L\leftrightarrow R),\\
B_2^R &= B_2^L \quad (\text{with } L\leftrightarrow R).
\end{aligned}
\tag{27}
$$

The decay rate for $l_i^- \to l_j^- l_j^- l_j^+$ is

$$
\begin{aligned}
\Gamma(l_i^-\to l_j^- l_j^- l_j^+) = \frac{\alpha^2}{32\pi}m_{l_i}^5\Bigg[&|A_1^L|^2+|A_1^R|^2-2(A_1^L A_2^{R*}+A_2^L A_1^{R*}+\text{H.c.})+(|A_2^L|^2+|A_2^R|^2)\left(\frac{16}{3}\ln\frac{m_{l_i}}{2m_{l_j}}-\frac{14}{9}\right)+\frac{1}{6}(|B_1^L|^2+|B_1^R|^2)\\
&+\frac{1}{3}(|B_2^L|^2+|B_2^R|^2)+\frac{1}{3}(A_1^L B_1^{L*}+A_1^L B_2^{L*}+A_1^R B_1^{R*}+A_1^R B_2^{R*}+\text{H.c.})\\
&-\frac{2}{3}(A_2^R B_1^{L*}+A_2^R B_2^{L*}+A_2^L B_1^{R*}+A_2^L B_2^{R*}+\text{H.c.})\Bigg].
\end{aligned}
\tag{28}
$$

In case $j\neq k$, one has to remove the term with $p_1\leftrightarrow p_2$ from the above amplitude and divide the B_1^L and B_1^R terms by a factor of 2. Then, the decay rate becomes

$$
\begin{aligned}
\Gamma(l_i^-\to l_j^- l_k^- l_k^+) = \frac{\alpha^2}{48\pi}m_{l_i}^5\Bigg[&|A_1^L|^2+|A_1^R|^2-2(A_1^L A_2^{R*}+A_2^L A_1^{R*}+\text{H.c.})+(|A_2^L|^2+|A_2^R|^2)\left(8\ln\frac{m_{l_i}}{m_{l_k}}-12\right)+\frac{1}{8}(|B_1^L|^2+|B_1^R|^2)\\
&+\frac{1}{2}(|B_2^L|^2+|B_2^R|^2)+\frac{1}{4}(A_1^L B_1^{L*}+A_1^R B_1^{R*}+\text{H.c.})+\frac{1}{2}(A_1^L B_2^{L*}+A_1^R B_2^{R*}+\text{H.c.})\\
&-\frac{1}{2}(A_2^R B_1^{L*}+A_2^L B_1^{R*}+\text{H.c.})-(A_2^R B_2^{L*}+A_2^L B_2^{R*}+\text{H.c.})\Bigg].
\end{aligned}
\tag{29}
$$

We calculate the branching ratios for $\mu\to e\gamma$ and $\mu\to 3e$ in the allowed region shown in Fig. 4 for $x=0.3$, 0.9, and 3.0 assuming δ's are real. For $x=0.3$, the decay rate for $\mu\to 3e$ is dominated by the term which is proportional to $(|A_2^L|^2+|A_2^R|^2)$, namely $\mu\to e\gamma\to 3e$. So, there is a strong correlation between the decay rates for $\mu\to e\gamma$ and $\mu\to 3e$, see Eq. (18) and Fig. 5(a). The solid line in Fig. 5(a) denotes this correlation. For larger x, this correlation becomes weaker and disappears. From Fig. 5, we observe that the branching ratio for $\mu\to e\gamma\to 3e$ is smaller than the present upper bounds for $x<1$ and the region of high $B(\mu\to e\gamma)$ and low $B(\mu\to e\gamma\to 3e)$ is not allowed in the models under consideration.

Similar analyses could be done for $\tau\to 3e$ and $\tau\to 3\mu$. In the case of τ decay, there are no independent experiments like $\mu^-+\text{Ti}\to e^-+\text{Ti}$ for $\mu\to e\gamma$ decay at present. So, one could not make good predictions for τ decays at present. [See the discussions below Eq. (34).]

D. $\tau\to l_{i=1,2}$+(neutral meson)

In this subsection, we consider the LFV in tau decays into a lighter lepton (e or μ) plus a light meson such as π^0, η and ρ^0. Different decays depend on different form factors so that each decay mode deserve its own study. Because of the limited numbers of tau leptons that have been accumulated up to now, the typical upper limits on the branching ratios of LFV tau decays are of order $\sim 10^{-6}$. The limits on LFV tau decays may be improved in the future at tau-charm factories or B factories. Therefore, it is important to study every possible LFV tau decay in various LFV models beyond the SM. In this subsection, we consider tau decays into a lighter lepton (e or μ) plus one light meson such as π^0, η, ρ^0 or ϕ.

The amplitude for $\tau\to l_{1=1,2}$+(neutral pseudoscalar meson $\equiv$P such as π^0,η, etc.) can be derived from the effective Lagrangian (11) induced by the box diagrams:

$$
\begin{aligned}
&\mathcal{M}(\tau(k,s)\to l_i(k',s')+P(p))\\
&\quad=\frac{4\alpha^2}{m_{\tilde l}^2}(\delta_{i3}^l)_{LL}(2B_1(x)+B_2(x))\overline{l_i}\gamma^\alpha P_L\tau\\
&\qquad\times\sum_q e_q^2\langle P(p)|\overline{q}\gamma_\alpha\gamma_5 q|0\rangle+(L\to R).
\end{aligned}
\tag{30}
$$

Using the PCAC relations, and assuming that $|\eta\rangle=|(u\overline{u}+d\overline{d}-2s\overline{s})/\sqrt{6}\rangle$, one gets

$$
\langle P(p)|\sum_q e_q^2\overline{q}\gamma_\alpha\gamma_5 q|0\rangle = i\frac{1}{3}C_P f_\pi p_\alpha,
\tag{31}
$$

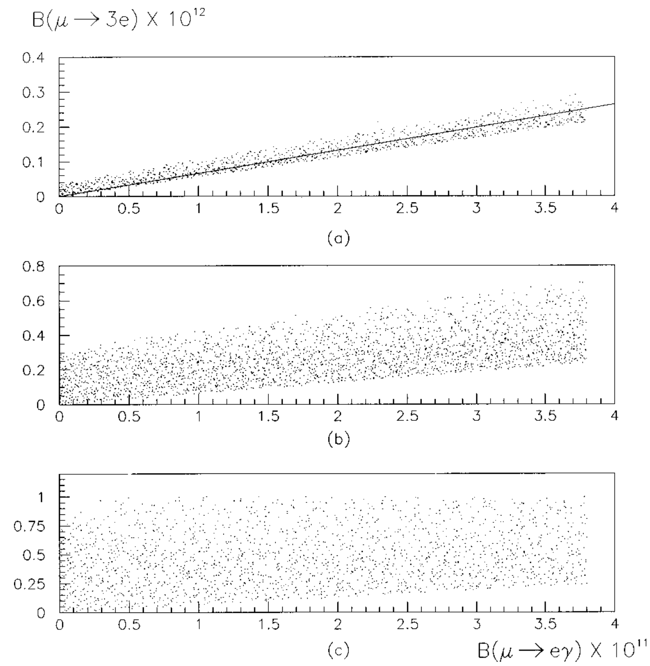

FIG. 5. The branching ratios for $\mu\to e\gamma$ and $\mu\to 3e$ in the allowed region by the present experiment for $\mu\to e\gamma$ and μ^- Ti $\to e^-$ Ti for $x=0.3$ (a), $x=0.9$ (b), and $x=3$ (c) assuming δ's are real. The solid line in (a) denotes the correlation between the decay rates for $\mu\to e\gamma$ and $\mu\to 3e$.

with $C_{\pi^0}=1$, $C_\eta=1/\sqrt{3}$ and $f_\pi=93$ MeV. Then the decay rate for this decay is given by

$$\Gamma(\tau\to l_i+P)=\frac{m_\tau^3 f_\pi^2}{18\pi}\frac{\alpha^4}{m_{\tilde{l}}^4}(2B_1+B_2)^2C_P^2 \times\{|(\delta_{i3}^l)_{LL}|^2+|(\delta_{i3}^l)_{RR}|^2\}. \tag{32}$$

The decay rate depends on $(\delta_{i3})_{LL}^l$ and $(\delta_{i3})_{RR}^l$, it does not depend on $\delta_{LR,RL}$.

The amplitudes for $\tau\to l_{i=1,2}+$ (neutral vector meson $\equiv V$ such as ρ^0,ϕ) can be calculated using the effective Lagrangian (6), (10) induced by the penguin diagrams:

$$\mathcal{M}(\tau(k,s)\to l_i(k',s')+V(p,\epsilon^*)) \equiv\frac{2\alpha^2}{m_{\tilde{l}}^2}C_Vf_Vm_V[A_L^\tau\overline{l_i}\gamma^\alpha P_L\tau+(L\leftrightarrow R)]\epsilon_\alpha^*, \tag{33}$$

where

$$A_L^\tau=(\delta_{i3}^l)_{LL}\left\{-P_1(x)+\frac{m_\tau^2}{m_V^2}M_3(x)\right\}+\frac{m_\tau^2}{m_V^2} \times\frac{m_{\tilde{\gamma}}}{m_\tau}(\delta_{i3}^l)_{LR}M_1(x), \tag{34}$$

and similarly for A_R^τ. This decay is a complete analogue of μ^-+ Ti $\to e^-+$ Ti at the parton level. So we expect that we can constrain δ_{LL} and δ_{LR} without any assumptions combining $\tau\to l+\gamma$ and $\tau\to l+V$ in the future. When writing the amplitude in the above form, we have used the definition of the vector meson decay constant f_V:

$$\sum_q \langle V|e_q\overline{q}\gamma^\alpha q|0\rangle=C_Vf_Vm_V\epsilon^{\alpha *}, \tag{35}$$

with $C_{\rho^0}=1$, $C_\phi=-1/3$ and $f_{\rho^0}=153$ MeV, $f_\phi=237$ MeV. The decay rate for $\tau\to l_i+V$ is

$$\Gamma(\tau\to l_i+V)=\frac{f_V^2(m_\tau^2-m_V^2)(m_\tau^4+m_\tau^2m_V^2-2m_V^4)}{8\pi m_\tau^3} \times\frac{\alpha^4}{m_{\tilde{l}}^4}C_V^2\{|A_L^\tau|^2+|A_R^\tau|^2\}. \tag{36}$$

We calculate the branching ratios for $\tau\to l+P$ and $\tau\to l+V$ from the constraints shown in Table II. The results are shown in Tables III and IV. Most of these decays are ex-

TABLE III. Upper limits for the branching ratios of $\tau\to l+P$ from the constraints shown in Table II.

Process	x	Branching ratio
$\tau\to e\pi$	0.3	0.61×10^{-12}
	0.9	0.38×10^{-9}
	3.0	0.24×10^{-8}
$\tau\to\mu\pi$	0.3	0.56×10^{-12}
	0.9	0.34×10^{-9}
	3.0	0.20×10^{-8}
$\tau\to e\eta$	0.3	0.20×10^{-12}
	0.9	0.13×10^{-9}
	3.0	0.81×10^{-9}
$\tau\to\mu\eta$	0.3	0.19×10^{-12}
	0.9	0.12×10^{-9}
	3.0	0.67×10^{-9}

TABLE IV. Upper limits for the branching ratios of $\tau\to l+V$ from the constraints shown in Table II.

Process	x	Branching ratio
$\tau\to e\rho^0$	0.3	0.32×10^{-7}
	0.9	0.42×10^{-7}
	3.0	0.42×10^{-7}
$\tau\to\mu\rho^0$	0.3	0.30×10^{-7}
	0.9	0.38×10^{-7}
	3.0	0.34×10^{-7}
$\tau\to e\phi$	0.3	0.29×10^{-8}
	0.9	0.39×10^{-8}
	3.0	0.43×10^{-8}
$\tau\to\mu\phi$	0.3	0.26×10^{-8}
	0.9	0.35×10^{-8}
	3.0	0.36×10^{-8}

pected to occur with small branching ratios far below the current or future experimental search limits. As such, it establishes the necessary amount of tau leptons in order to probe the LFV from nondiagonal slepton mass matrix.

E. $Z^0 \to l_i \overline{l}_{j\neq i}$

The photino-mediated LFV can generate the LFV decays of Z bosons. The amplitude for $Z \to l_i l_j$ decays are given by

$$\mathcal{M} = -\left(\frac{\alpha}{2\pi}\right)\left(\frac{8m_Z^2 G_F}{\sqrt{2}}\right)^{1/2}\frac{(v_l+a_l)}{2}[(\overline{u}(p')P_L v(p))((\delta_{ij}^l)_{LL}X_{LL}^{\mu ij}(z)-(\delta_{ij}^l)_{RL}X_{RL}^{\mu ij}(z))+(\overline{u}(p')P_R\gamma^\mu v(p))((\delta_{ij}^l)_{LL}Y_{LL}^{ij}(z) + (\delta_{ij}^l)_{LR}Y_{RL}^{ij}(z))]\cdot\boldsymbol{\epsilon}_\mu(Z)+((v_l+a_l)\leftrightarrow(v_l-a_l),P_L\leftrightarrow P_R), \tag{37}$$

where $z = m_{\tilde{\gamma}}^2/m_{\tilde{l}}^2$ and

$$X_{LL}^{\mu ij}(z) = \frac{m_l}{m_{\tilde{l}}}\left(\frac{q^\mu}{m_{\tilde{l}}}F_5(z) - \frac{p^\mu}{m_{\tilde{l}}}F_6(z)\right),$$

$$X_{RL}^{\mu ij}(z) = \frac{m_{\tilde{\gamma}}}{m_{\tilde{l}}}\left(\frac{q^\mu}{m_{\tilde{l}}}G_1(z) + \frac{p^\mu}{m_{\tilde{l}}}G_2(z)\right),$$

$$Y_{LL}^{ij}(z) = F_1(z) + \frac{m_l^2}{m_{\tilde{l}}^2}F_2(z) - F_3(z) - \frac{m_l^2}{m_{\tilde{l}}^2}F_4(z),$$

$$Y_{RL}^{ij}(z) = \frac{m_{\tilde{\gamma}}m_l}{m_{\tilde{l}}^2}G_4(z), \tag{38}$$

with

$$v_l = -\frac{1}{2} + 2\sin^2\theta_W, \quad a_l = -\frac{1}{2}, \tag{39}$$

and m_l is the mass of the heavier lepton in the final state. The functions $F_i(z)$ and $G_i(z)$ are defined as follows:

$$\frac{1}{m_{\tilde{l}}^2}F_1(z) = \int_0^1 dx\int_0^{1-x}dy\frac{1-x}{xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2},$$

$$\frac{1}{m_{\tilde{l}}^4}F_2(z) = \int_0^1 dx\int_0^{1-x}dy\frac{x(1-x)(1-x-y)}{(xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2)^2},$$

$$\frac{1}{m_{\tilde{l}}^2}F_3(z) = \int_0^1 dx\frac{x^2}{(1-x)m_{\tilde{\gamma}}^2+xm_{\tilde{l}}^2},$$

$$\frac{1}{m_{\tilde{l}}^4}F_4(z) = \int_0^1 dx\frac{x^3(1-x)}{((1-x)m_{\tilde{\gamma}}^2+xm_{\tilde{l}}^2)^2},$$

$$\frac{1}{m_{\tilde{l}}^4}F_5(z) = \int_0^1 dx\int_0^{1-x}dy\frac{(1-x)(1-x-y)(1-2y)}{(xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2)^2},$$

$$\frac{1}{m_{\tilde{l}}^4}F_6(z) = \int_0^1 dx\int_0^{1-x}dy\frac{2x(1-x)(1-x-y)}{(xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2)^2}, \tag{40}$$

and

$$\frac{1}{m_{\tilde{l}}^4}G_1(z)=\int_0^1 dx\int_0^{1-x}dy\frac{(1-x)(1-2y)}{(xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2)^2},$$

$$\frac{1}{m_{\tilde{l}}^4}G_2(z)=\int_0^1 dx\int_0^{1-x}dy\frac{-2x(1-x)}{(xm_{\tilde{\gamma}}^2+(1-x)m_{\tilde{l}}^2+y(x+y-1)m_Z^2)^2},$$

$$G_3(z)=\frac{-1+z-\log z}{(1-z)^2},$$

$$G_4(z)=\frac{1+4z-5z^2+4z\log z+2z^2\log z}{2(1-z)^4}=M_1(z). \tag{41}$$

The branching ratio of $Z\to l_i l_{j\neq i}$ processes is given by

$$\begin{aligned}\mathrm{Br}(Z\to l_i l_{j\neq i})&=\mathrm{Br}(Z\to e^+e^-)\frac{\Gamma(Z\to l_i l_j)}{\Gamma(Z\to e^+e^-)}\\&=\mathrm{Br}(Z\to e^+e^-)\left(\frac{\alpha}{2\pi}\right)^2\frac{1}{2(v_l^2+a_l^2)}\left[(v_l+a_l)^2\left(|(\delta_{ij}^l)_{LL}|^2(F_1(z)-F_3(z))^2+|(\delta_{ij}^l)_{LR}|^2\frac{m_{\tilde{\gamma}}^2m_Z^2}{8m_{\tilde{l}}^4}G_2^2(z)\right)(v_l-a_l)^2\right.\\&\left.\times\left(\left|(\delta_{ij}^l)_{RR}\right|^2(F_1(z)-F_3(z))^2+|(\delta_{ij}^l)_{RL}|^2\frac{m_{\tilde{\gamma}}^2m_Z^2}{8m_{\tilde{l}}^4}G_2^2(z)\right)\right],\end{aligned} \tag{42}$$

where $B(Z^0\to e^+e^-)=3.366\%$ and

$$\Gamma(Z\to e^+e^-)=\frac{G_F m_Z^3}{12\pi\sqrt{2}}\cdot 2(v_l^2+a_l^2).$$

The upper limits on the LFV Z decays are [9]

$$B_{\mathrm{exp}}(Z\to e\mu)<2.5\times 10^{-6},$$

$$B_{\mathrm{exp}}(Z\to e\tau)<7.3\times 10^{-6},$$

$$B_{\mathrm{exp}}(Z\to \mu\tau)<1.0\times 10^{-5}. \tag{43}$$

The associated constraints on δ^l's are so loose that they are useless. Using the constraints obtained in the previous subsections, we find that the upper limits on the branching ratios for the LFV Z decays are less than $10^{-7(8)}$ for $Z\to\mu(e)+\tau$, and 10^{-10} for $Z\to e\mu$, which are far below the present experimental results. Any observations of LFV Z decays with $B>10^{-7}$ will indicate that the source of LFV should be different from the nondiagonal slepton mass matrix elements.

F. Muonium → antimuonium conversion

Now let us consider the muonium → antimuonium conversion. The current experimental upper limit on the transition probability in the external magnetic field $B_{\mathrm{ext}}=0.1$ T is [10]

$$P_{\mathrm{exp}}(M\to\overline{M})<8.2\times 10^{-11}\ \ (90\%\ \ \mathrm{C.L.}). \tag{44}$$

When this process is described the following effective Lagrangian,

$$\mathcal{L}_{\mathrm{eff}}^{M\to\overline{M}}=\frac{G_{--}}{\sqrt{2}}(\overline{e}\mu)_{V-A}(\overline{e}\mu)_{V-A}+\frac{G_{+-}}{\sqrt{2}}(\overline{e}\mu)_{V+A}(\overline{e}\mu)_{V-A}, \tag{45}$$

it is known that the effective couplings $G_{\mp -}$ are constrained as

$$G_{--}<3.0\times 10^{-3}G_F,\quad \mathrm{or}\quad G_{+-}<2.1\times 10^{-3}G_F, \tag{46}$$

assuming only one of them is nonvanishing.

Now we can derive the muonium $(M\equiv\mu^+e^-)\to$ antimuonium $(\overline{M}\equiv\mu^-e^+)$ conversion induced by the $\Delta L_i=2$ effective Lagrangian (12). Note that our model predicts that

$$G_{\mp -} \sim \frac{\alpha^2}{m_{\tilde{l}}^2} (\delta_{12}^l)^2 \times \tilde{f}_6(x) \ \ [\text{or} \ \ x f_6(x)]$$
$$\lesssim 4.6 \times 10^{-4} (\delta_{12}^l)^2 G_F, \tag{47}$$

because $x f_6$ and $\tilde{f}_6(x)$ are far less than one. So the effect of the nondiagonal slepton mass matrix on the muonium conversion is totally negligible. In other words, if one observes the muonium $\rightarrow$ antimuonium conversion, it would imply that the origin of the associated lepton flavor violation should be something different from what we consider in this work, for example R-parity violation [11] or dilepton gauge boson, etc. [12].

IV. CONSTRAINTS ON FLAVOR CONSERVING MASS INSERTION FROM THE LEPTON ANOMALOUS MAGNETIC MOMENTS

In this section, we consider the limits on the flavor conserving mass insertion $(\delta_{ii}^l)_{LR}$ that are derivable from the anomalous magnetic moments of leptons. In Ref. [3], this quantity was constrained from the condition $\Delta m_l^{\rm SUSY} < m_{\rm phys}$, where the $\Delta m_l^{\rm SUSY}$ is calculated by a flavor conserving mass insertion and is finite:

$$\Delta m_l^{\rm SUSY} = -\frac{2\alpha}{4\pi} m_{\tilde{\gamma}} \ \text{Re} \ (\delta_{ii}^l)_{LR} I(x), \tag{48}$$

where $I(x) = (-1 + x - x \ln x)/(1-x)^2$. The physical mass $m_{\rm phys}$ will be given by $m_{\rm bare} + \delta m_{\rm c.t.} + \Delta m_l^{\rm SUSY}$, where $\delta m_{\rm c.t.}$ is the mass renormalization counterterm in the MSSM. However its finite part is arbitrary, and one has to assume that there is no large cancellation between it and $\Delta m_l^{\rm SUSY}$ in order to make use of it. In other words, renormalizable couplings cannot be calculated from the first principle without any ambiguity. Therefore, the condition that $\Delta m_l^{\rm SUSY} < m_{\rm phys}$ may be a plausible assumption, but it is by no means on the firm ground like the constraints considered in the previous sections. On the other hand, the anomalous magnetic moment of a lepton is calculable in the SM and any other renormalizable field theories without any ambiguity. So it is meaningful to require $a_l^{\rm SM} + a_l^{\rm SUSY} = a_l^{\rm exp}$, which we adopt in the following.[4]

The anomalous magnetic moment of a lepton $l (\equiv a_l)$ is defined as

$$a_l \equiv \left(\frac{g-2}{2} \right) = F_2(0), \tag{49}$$

where $F_2(q^2)$ is the magnetic form factor of a lepton:

$$\mathcal{M}(l_i(p,s) \rightarrow l_i(p',s') + \gamma(q,\epsilon))$$
$$= \overline{u(p',s')} \left[F_1(q^2) \gamma^\mu + i F_2(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2 m_i} \right] u_i(p,s) \epsilon_\mu^*(q). \tag{50}$$

In order to derive the SUSY contribution to the anomalous magnetic moment, one cannot use the effective Lagrangian presented in Sec. II A, since we assumed that the final lepton mass is negligible when we derived Eq. (6). One has to go back to the original expression for the $l_i \rightarrow l_j + \gamma$ with $m_i = m_j$. It is straightforward to show that the flavor-conserving mass insertion (δ_{ii}^l) induces

$$a_l^{\rm SUSY} = -\frac{\alpha}{\pi} \frac{m_i^2}{m_{\tilde{l}}^2} \left[P_3(x) + M_3(x) (\delta_{ii}^l)_{LL} + \frac{m_{\tilde{\gamma}}}{m_i} (\delta_{ii}^l)_{LR} M_1(x) \right], \tag{51}$$

where the function $P_3(x)$ is given by

$$P_3(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4}, \tag{52}$$

and $x \equiv m_{\tilde{\gamma}}^2 / m_{\tilde{l}}^2$ as before, and $M_{1,3}(x)$'s are defined in Sec. II. Here we have assumed that $(\delta_{ii}^l)_{LL} = (\delta_{ii}^l)_{RR}$, and $(\delta_{ii}^l)_{LR} = (\delta_{ii}^l)_{RL}$. The first term in Eq. (51) arises from the slepton-photino loop without any mass insertion. There is also another term proportional to m_i^2 coming from one insertion of $(\delta_{ii}^l)_{LL}$. However, this is suppressed compared to the above by an additional factor of $m_i / m_{\tilde{l}}$, and thus can be safely neglected. From this expression, one can get the constraint on x and $(\delta_{ii}^l)_{LR}$ for a given value of $m_{\tilde{l}}$.

The current status of the anomalous magnetic moments for e and μ are as follows [14]:

$$\left. \begin{aligned} a_e^{\rm SM} &= 1159652156.4(1.2) \times 10^{-12} \\ a_e^{\rm exp} &= 1159652188.25(4.24) \times 10^{-12} \end{aligned} \right\} \rightarrow a_e^{\rm new} = 31.85(4.4) \times 10^{-12}$$

$$\left. \begin{aligned} a_\mu^{SM} &= 116591711(94) \times 10^{-11} \\ a_\mu^{\rm exp} &= 1165923(8.5) \times 10^{-9} \end{aligned} \right\} \rightarrow a_\mu^{\rm new} = 5.9(8.6) \times 10^{-9}. \tag{53}$$

[4]There can be also potentially important contributions from chargino-sneutrino loop (in addition to the heavier neutralinosslepton loops), especially when charginos and sneutrinos are light [13].

TABLE V. Allowed ranges for the flavor conserving mass insertion $(\delta^l_{ii})_{LR}$ from the anomalous magnetic moment of a lepton for $m_{\tilde{l}}=100$ GeV. We show two cases $i=1(e)$ and $i=2(\mu)$ only, since the anomalous magnetic moment of a tau lepton is poorly measured.

x	$(\delta^l_{11})_{LR}$	$(\delta^l_{22})_{LR}$
0.3	-3.0×10^{-2}–-2.3×10^{-2}	-6.0×10^{-2}–1.0×10^{-2}
0.9	-3.6×10^{-2}–-2.7×10^{-2}	-7.0×10^{-2}–1.2×10^{-2}
3.0	-5.9×10^{-2}–-4.5×10^{-2}	-0.11–2.0×10^{-2}

Here, the SM predictions for electron and muon anomalous magnetic moments include the one loop electroweak corrections and the two loop leading log terms, as well as QED corrections including the hadronic vacuum polarization and the hadronic light-by-light scattering [14]. We ignored the tau anomalous magnetic moment here, since the experimental value begins to probe the lowest order QED correction at the present. The resulting constraints on $(\delta^l_{ii})_{LR}$'s for $i=1,2$ are shown in Table V. Comparing with the constraints obtained in Ref. [3], our constraints are more reliable and even stronger for the case of muon.

The imaginary part of the flavor conserving mass insertion is constrained by the electric dipole moment (EDM) of a lepton, as discussed in Ref. [3]. The bound from electron EDM is very strong,

$$|\mathrm{Im}(\delta^l_{11})_{LR}| \sim \ (\text{a few } \times 10^{-7}). \tag{54}$$

V. CONCLUSION

In conclusion, we considered the LFV in general SUSY models, where the slepton mass matrices are not diagonal in the basis where $l-\tilde{l}-\tilde{\gamma}$ is diagonal. We worked in the mass insertion approximations in which $(\delta^l_{ij})_{LR}$'s constitute the suitable parameters that characterize the strengths of LFV. There are strong constraints on some of these parameters from $l_i \to l_j + \gamma$ and $\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$. Using these constraints, we predict the upper limits on other LFV processes such as $l_i \to 3 l_j$, $\tau \to \mu$ (or e)+(neutral meson), $Z \to l_i l_{j\neq i}$ and the muonium → antimuonium conversion.

LFV processes considered in this work are sensitive probes of the slepton mass matrices which are related with the SUSY breaking mechanism. Any positive LFV signal would herald the existence of some new physics beyond the SM, and if the predictions in this work are violated, then one has to think of another source of LFV other than that through the nondiagonal slepton mass matrices. In particular, if one imposes the constraints from $l_i \to l_j + \gamma$ and $\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}$, then the expected ranges for other LFV processes are well below the current limits and the level to be achieved in the near future. If some LFV processes are observed at rates higher than those predicted in this work, the source of LFV would not be likely to be photino-mediated. For example, presence of some R-parity violating couplings can lead to quantitatively different predictions [11] from those made in this work.

The constraints on the flavor conserving mass insertions were derived from the anomalous magnetic moments of leptons. These bounds are to be considered more sensible than those obtained from $\Delta m_{\mathrm{SUSY}} < m_{\mathrm{exp}}$, since the renormalizable couplings in the renormalizable field theories cannot be calculated from the given Lagrangian. Our constraints still imply that the diagonal slepton masses should be almost degenerate, especially for the first two generations. One has to speculate why this should be the case in general supersymmetric models.

Finally, let us comment on our assumption that the LSP is a pure photino, and other neutralinos are heavy enough so that their effects might be ignored. In order to do more complete analyses for given neutralino spectra (i.e., for given M_1, M_2, μ and $\tan\beta$), one can easily include the effects of all the neutralinos in principle, and do the similar analyses as presented in this work. This is possible, since the neutralino spectrum is independent of the slepton spectra. Our approach adopted in this work can be regarded as a first step to such complete analyses. The qualitative features of our predictions would not change very much. In other words, our predictions are expected to be correct within an order of magnitude.

ACKNOWLEDGMENTS

The authors thank Dr. S.Y. Choi for illuminating discussions on the Feynman rules in the presence of Majorana neutrinos. This work is supported in part by KOSEF through Center for Theoretical Physics at Seoul National University, by KOSEF Contract No. 971-0201-002-2, by the Ministry of Education through the Basic Science Research Institute, Contract No. BSRI-97-2418 (P.K.), and KAIST Center for Theoretical Physics and Chemistry (K.Y.L., P.K.).

[1] H. P. Nilles, Phys. Rep. **110**, 1 (1984); H. E. Haber and G. L. Kane, *ibid.* **117**, 75 (1985); J. A. Bagger, TASI-95 lectures, hep-ph/9604232; M. Dine, hep-ph/9612389; S. Martin, hep-ph/9709356.

[2] J. Donoghue, H. P. Nilles, and D. Wyler, Phys. Lett. **128B**, 55 (1983); L. Hall, V. Kostelecky, and S. Raby, Nucl. Phys. **B267**, 415 (1986); H. Georgi, Phys. Lett. **169B**, 231 (1986); F. Gabbiani and A. Masiero, Nucl. Phys. **B322**, 235 (1989); J. S. Hagelin, S. Kelly, and T. Tanaka, *ibid.* **B415**, 293 (1994).

[3] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996), and references therein.

[4] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993).

[5] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B **388**, 588 (1996); G. Dvali and A. Pomarol, Phys. Rev. Lett. **77**, 3728 (1996).
[6] J. Hisano *et al.*, Phys. Lett. B **357**, 579 (1995).
[7] K. Choi, J. S. Lee, and C. Muñoz, Phys. Rev. Lett. **80**, 3686 (1998).
[8] R. Barbieri and L. J. Hall, Phys. Lett. B **338**, 212 (1994).
[9] K. P. Jungman, hep-ex/9806003 and references therein.
[10] L. Willmann *et al.*, hep-ex/9807011.
[11] A. Halprin and A. Masiero, Phys. Rev. D **48**, 2987 (1993); Jihn E. Kim, P. Ko, and D.-G. Lee, *ibid.* **56**, 100 (1997).
[12] H. Fujii, S. Nakamura, and K. Sasaki, Phys. Lett. B **299**, 342 (1993); P. Ko, Nuovo Cimento **A107**, 809 (1994); W. S. Hou, Nucl. Phys. B (Proc. Suppl.) **51A**, 40 (1996).
[13] M. Carena, G. F. Giudice, and C. E. M. Wagner, Phys. Lett. B **390**, 234 (1997).
[14] T. Kinoshita, Talk at the VIIth Blois Workshop, Seoul, Korea (1997).
[15] CLEO Collaboration, K. W. Edwards *et al.*, Phys. Rev. D **55**, R3919 (1997).
[16] CLEO Collaboration, G. Bonvicini *et al.*, Phys. Rev. Lett. **79**, 1221 (1997).
[17] CLEO Collaboration, D. W. Bliss *et al.*, Phys. Rev. D **57**, 5903 (1998).