

New contributions to neutralino elastic cross sections from CP violating phases in the minimal supersymmetric standard model

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We compute the four-Fermi neutralino-quark interaction Lagrangian including contributions from the CP -violating phases in the MSSM. We find that neutralino-nucleus scattering cross sections relevant for direct detection experiments show a strong dependence on the value of the CP -violating phase associated with the μ parameter θ_μ . In some cases, for a broad range of non-zero θ_μ , there are cancellations in the cross sections which reduce the cross section by more than an order of magnitude. In other cases, there may be enhancements as one varies θ_μ . [S0556-2821(99)01705-1]

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The minimal supersymmetric standard model (MSSM) with a neutralino lightest supersymmetric particle (LSP) provides one of the better motivated candidates for the dark matter in the Universe. From observations of the dynamics of galaxies and clusters of galaxies [1], and from the constraints on the baryon density from big bang nucleosynthesis [2], it is clear that a considerable amount of non-baryonic dark matter is needed. The MSSM, with supersymmetry breaking mediated by gravitational interactions and with R -parity conservation, typically possesses a stable dark matter candidate, the LSP, which for much of the parameter space is a neutralino [a linear combination of the $SU(2)$ and $U(1)$ gauginos, and the two Higgsinos] with a mass in the range $m_\chi \sim O(1-100)$ GeV. In fact, there has been considerable progress recently in establishing strong constraints on the supersymmetric parameter space from recent runs at the CERN e^+e^- collider LEP [3,4]. These constraints provide a lower bound to the neutralino mass of ~ 40 GeV, when in addition to the bounds from experimental searches for charginos, associated neutralino production and Higgs bosons, constraints coming from cosmology and theoretical simplifications concerning the input scalar masses in the theory are invoked. (The pure experimental bound is about $m_\chi \gtrsim 30$ GeV [5].)

A major issue concerning dark matter of any kind is its detection and identification. Indeed, there are a multitude of ongoing experiments involved in the direct and indirect detection of dark matter, many with a specific emphasis on searching for supersymmetric dark matter [6]. The event rates for either direct or indirect detection depend crucially on the dark matter elastic cross section, in this case the neutralino-nucleon, or neutralino-nucleus, cross section. Because the neutralinos have Majorana mass terms, their interactions with matter are generally spin dependent, coming from an effective interaction term of the form $\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5q$. In the regions of the MSSM parameter space where the LSP is a mixture of both gaugino and

Higgsino components, or where the squarks are highly mixed [7], there is also an important contribution to the scattering cross section due to a term in the interaction Lagrangian of the form $\bar{\chi}\chi\bar{q}q$ [8] which is spin independent. These terms are particularly important for scattering off of large nuclei, where coherent nucleon scattering effects can quickly come to dominate all others.

When gaugino mass unification at the grand unified theory (GUT) scale is assumed, as is done here, the identity of the LSP in the MSSM is determined by three parameters. These are the gaugino mass, represented here as the $SU(2)$ gaugino mass M_2 at the weak scale, the Higgsino mixing mass, μ , and the ratio of Higgs vacuum expectation values (VEVs), $\tan\beta$. The interactions of the LSP with matter also depend on additional mass parameters, specifically the sfermion and Higgs boson masses, which in turn are determined from the soft supersymmetry breaking sfermion masses, trilinear and bilinear parameters, m_i , A_i , and B . It is very common to choose a common soft sfermion mass m_0 at the GUT scale, which greatly reduces the number of available parameters. In some cases, the Higgs boson soft masses are also chosen equal to the common sfermion soft masses at the GUT scale. This assumption leads to what is known as the constrained MSSM (CMSSM). In the CMSSM, two parameters, usually μ and the Higgs pseudo-scalar mass, are fixed by the condition of proper electroweak symmetry breaking. The CMSSM generally leads to a nearly pure B -ino as the LSP, and as we want to consider all neutralino compositions, we will not consider the case of universal soft Higgs boson masses, though for simplicity we will assume that the remaining (sfermion) soft masses are unified at the GUT scale.

The MSSM is well known to contain several independent CP -violating phases. If one assumes that all of the supersymmetry breaking trilinear mass terms, A_i , are equal to A_0 at the GUT scale, then the number of independent phases reduces to 2, which one can take as θ_A and θ_μ . The phase of μ can always be adjusted so that it is equal and opposite to

that of the supersymmetry breaking bilinear mass term B , $\theta_B = -\theta_\mu$ by rotating the Higgs fields so that their vacuum expectation values are real¹ [9]. Though these phases can lead to sizable contributions to the neutron and electron dipole moments [10], it has been shown that large phases are indeed compatible with these constraints, as well as cosmological constraints on the neutralino relic density in the MSSM [12] and in the constrained MSSM [13,14]. Indeed, in the CMSSM, cancellations between different contributions to the electric dipole moments (EDMs) over a broad range in mass parameters allow for a θ_μ as large as $\sim 0.3\pi$, depending on the magnitude of A_0 and $\tan\beta$, and a θ_A which is essentially unconstrained. If we drop the assumption of universal Higgs boson masses at the GUT scale, these phases are even less constrained.

Here we will show the importance of the CP -violating phases on the elastic scattering cross-sections of neutralinos on matter. To this effect, we will calculate the four-Fermi χ -quark interaction Lagrangian with the inclusion of the CP -violating phase θ_μ for the standard spin dependent and spin

independent interactions. Here, we have chosen $\theta_{A_i} = \pi/2$ and adjusted the magnitude of the A_i (of order 1–3 TeV) in order to satisfy the bounds for the electric dipole moments of the electron and neutron. That this can be done has been demonstrated in [15] where it was shown that the neutron electric dipole moment contains separate contributions from the imaginary parts of A_u , A_d , and B . Though this can be regarded as a fine-tuning, our purpose here is to concentrate on the behavior of the elastic scattering cross section rather than the cancellation of the electric dipole moments which has been treated at length elsewhere. A complete treatment of the effective Lagrangian which includes θ_A as well as new annihilation contributions for non-zero phases will be presented elsewhere [16].

Before writing down the effective Lagrangian, it will be useful to clarify our notation. We will write the lowest mass neutralino eigenstate (the LSP) as

$$\chi = Z_{\chi_1} \tilde{B} + Z_{\chi_2} \tilde{W} + Z_{\chi_3} \tilde{H}_1 + Z_{\chi_4} \tilde{H}_2. \quad (1)$$

The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis

$$\begin{pmatrix} M_1 & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta \\ 0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta \\ -M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu \\ M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu & 0 \end{pmatrix} \quad (2)$$

depends explicitly on the Higgsino mass parameter μ , and the coefficients Z_{χ_i} all depend on the phase θ_μ . In Eq. (2), we have taken $M_1 = \frac{5}{3} \tan^2 \theta_W M_2$. The phases could in principle also enter into the calculation through the sfermion mass eigenstates. The sfermion mass² matrix can be written as

$$\begin{pmatrix} M_L^2 + m_f^2 + \cos 2\beta (T_{3f} - Q_f \sin^2 \theta_W) M_Z^2 & -m_f \bar{m}_f e^{i\gamma_f} \\ -m_f \bar{m}_f e^{-i\gamma_f} & M_R^2 + m_f^2 + \cos 2\beta Q_f \sin^2 \theta_W M_Z^2 \end{pmatrix} \quad (3)$$

where $M_{L(R)}$ are the soft supersymmetry breaking sfermion masses, which we have assumed are generation independent and generation diagonal and hence real. Because of our choice of phases, there is a non-trivial phase associated with the off-diagonal entries, which we denote by $-m_f(\bar{m}_f e^{i\gamma_f})$, of the sfermion mass² matrix, and

$$\bar{m}_f e^{i\gamma_f} = R_f \mu + A_f^* = R_f |\mu| e^{i\theta_\mu} + |A_f| e^{-i\theta_{A_f}}, \quad (4)$$

where m_f is the mass of the fermion f and $R_f = \cot \beta (\tan \beta)$ for weak isospin $+1/2$ ($-1/2$) fermions. We also define the sfermion mixing angle θ_f by the unitary matrix U which diagonalizes the sfermion mass² matrix,

$$U = \begin{pmatrix} \cos \theta_f & \sin \theta_f e^{i\gamma_f} \\ -\sin \theta_f e^{-i\gamma_f} & \cos \theta_f \end{pmatrix} \equiv \begin{pmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{pmatrix}. \quad (5)$$

Note that $\eta_{21} = -\eta_{12}^*$.

The general form for the four-Fermi effective Lagrangian can be written as

$$\mathcal{L} = \bar{\chi} \gamma^\mu \gamma^5 \chi \bar{q}_i \gamma_\mu (\alpha_1 + \alpha_2 \gamma^5) q_i + \alpha_3 \bar{\chi} \chi \bar{q}_i q_i + \alpha_4 \bar{\chi} \gamma^5 \chi \bar{q}_i \gamma^5 q_i + \alpha_5 \bar{\chi} \chi \bar{q}_i \gamma^6 q_i + \alpha_6 \bar{\chi} \gamma^5 \chi \bar{q}_i q_i. \quad (6)$$

¹Note that in some cases loop effects may not allow this simple tree level rotation [11].

The Lagrangian should be summed over quark generations, and the subscript i refers to up-type $i=1$ and down-type $i=2$ quarks. Here, we shall only be concerned with the axial vector (α_2) and scalar (α_3) contributions. These coefficients are given by

$$\begin{aligned} \alpha_{2i} = & \frac{1}{4(m_{1i}^2 - m_\chi^2)} \left[\left| \eta_{11}^* \left(\frac{Y_i}{2} g' Z_{\chi_1} + g T_{3i} Z_{\chi_2} \right) + \frac{\eta_{12}^* g m_{q_i} Z_{\chi_{5-i}}}{2m_W B_i} \right|^2 + \left| -\eta_{12}^* e_i g' Z_{\chi_1}^* + \frac{\eta_{11}^* g m_{q_i} Z_{\chi_{5-i}}^*}{2m_W B_i} \right|^2 \right] \\ & + \frac{1}{4(m_{2i}^2 - m_\chi^2)} \left[\left| \eta_{21}^* \left(\frac{Y_i}{2} g' Z_{\chi_1} + g T_{3i} Z_{\chi_2} \right) + \frac{\eta_{22}^* g m_{q_i} Z_{\chi_{5-i}}}{2m_W B_i} \right|^2 + \left| -\eta_{22}^* e_i g' Z_{\chi_1}^* + \frac{\eta_{21}^* g m_{q_i} Z_{\chi_{5-i}}^*}{2m_W B_i} \right|^2 \right] \\ & - \frac{g^2}{8m_Z^2 \cos^2 \theta_W} (|Z_{\chi_3}|^2 - |Z_{\chi_4}|^2) T_{3i} \end{aligned} \quad (7)$$

$$\begin{aligned} \alpha_{3i} = & -\frac{1}{2(m_{1i}^2 - m_\chi^2)} \text{Re} \left[\left(\frac{\eta_{11}^* g m_{q_i} Z_{\chi_{5-i}}^*}{2m_W B_i} - \eta_{12}^* e_i g' Z_{\chi_1}^* \right) \left(\eta_{11}^* \left(\frac{Y_i}{2} g' Z_{\chi_1} + g T_{3i} Z_{\chi_2} \right) + \frac{\eta_{12}^* g m_{q_i} Z_{\chi_{5-i}}}{2m_W B_i} \right)^* \right] \\ & - \frac{1}{2(m_{2i}^2 - m_\chi^2)} \text{Re} \left[\left(\frac{\eta_{21}^* g m_{q_i} Z_{\chi_{5-i}}^*}{2m_W B_i} - \eta_{22}^* e_i g' Z_{\chi_1}^* \right) \left(\eta_{21}^* \left(\frac{Y_i}{2} g' Z_{\chi_1} + g T_{3i} Z_{\chi_2} \right) + \frac{\eta_{22}^* g m_{q_i} Z_{\chi_{5-i}}}{2m_W B_i} \right)^* \right] \\ & - \frac{g m_{q_i}}{4m_W B_i} \left[\text{Re}(Z_{\chi_3} [g Z_{\chi_2} - g' Z_{\chi_1}]) C_i D_i \left(-\frac{1}{m_{H_1}^2} + \frac{1}{m_{H_2}^2} \right) + \text{Re}(Z_{\chi_4} [g Z_{\chi_2} - g' Z_{\chi_1}]) \left(\frac{D_i^2}{m_{H_1}^2} + \frac{C_i^2}{m_{H_2}^2} \right) \right]. \end{aligned} \quad (8)$$

In these expressions, $m_{1,2i}$ are the squark mass eigenvalues, $B_i = \sin \beta (\cos \beta)$ for up (down) type quarks and $C_i = \sin \alpha (\cos \alpha)$, $D_i = \cos \alpha (-\sin \alpha)$ (α is the scalar Higgs mixing angle). In the limit of vanishing CP -violating phases, these expressions agree with those in [6] and [22]. Expressions for α_{1i} , α_{4i} , α_{5i} and α_{6i} , which are suppressed by the neutralino-quark relative velocity, will be presented in [16].

Equations (7) and (8) contain contributions to the effective Lagrangian for neutralino-quark scattering from squark, Z , and both scalar Higgs boson exchange. The spin dependent contribution (from α_2) contains terms which are not suppressed by the quark mass and can be large over much of the parameter space, that is, they do not rely on the LSP being a mixed gaugino-Higgsino eigenstate, i.e. having both a large $Z_{\chi_{1,2}}$ and a large $Z_{\chi_{3,4}}$ component. In contrast, the spin independent term (from α_3) is always proportional to the quark mass and relies on either the LSP being a mixed state or significant squark mixing [7]. However, the spin independent cross section is enhanced by the effects of coherent scattering in a nucleus and can dominate over the spin dependent cross section for heavy nuclei.

The elastic scattering cross sections based on $\alpha_{2,3}$ have been conveniently expressed in [6]. The spin dependent cross section can be written as

$$\sigma_2 = \frac{32}{\pi} G_F^2 m_r^2 \Lambda^2 J(J+1) \quad (9)$$

where m_r is the reduced neutralino-nucleus mass, J is the spin of the nucleus and

$$\Lambda = \frac{1}{J} (a_p \langle S_p \rangle + a_n \langle S_n \rangle) \quad (10)$$

and

$$a_p = \sum_i \frac{\alpha_{2i}}{\sqrt{2} G_F} \Delta_i^{(p)}, \quad a_n = \sum_i \frac{\alpha_{2i}}{\sqrt{2} G_F} \Delta_i^{(n)}. \quad (11)$$

The factors $\Delta_i^{(p,n)}$ depend on the spin content of the nucleon and are taken here to be $\Delta_i^{(p)} = 0.77, -0.38, -0.09$ for u, d, s respectively [17] and $\Delta_u^{(n)} = \Delta_d^{(p)}$, $\Delta_d^{(n)} = \Delta_u^{(p)}$, $\Delta_s^{(n)} = \Delta_s^{(p)}$. The $\langle S_{p,n} \rangle$ are expectation values of the spin content in the nucleus and therefore are quite dependent on the target nucleus. We will display results for scattering off of a ^{73}Ge target for which in the shell model $\langle S_{p,n} \rangle = 0.011, 0.491$, and for ^{19}F , which has $\langle S_{p,n} \rangle = 0.415, -0.047$. For details on these quantities, we refer the reader to [6].

Similarly, we can write the spin independent cross section as

$$\sigma_3 = \frac{4m_r^2}{\pi} [Z f_p + (A-Z) f_n]^2 \quad (12)$$

where

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} f_{Tq}^{(p)} \alpha_{3q} / m_q + \frac{2}{27} f_{TG}^{(p)} \sum_{q=c,b,t} \alpha_{3q} / m_q \quad (13)$$

and a similar expression for f_n . The parameters $f_{Tq}^{(p)}$ are defined by $\langle p | m_q \bar{q} q | p \rangle = m_p f_{Tq}^{(p)}$, while $f_{TG} = 1 - (f_{Tu} + f_{Td} + f_{Ts})$ [18]. We have adopted $f_{Tq}^{(p)} = 0.019, 0.041, 0.14$ for

u, d, s and for $f_{Tq}^{(n)} = 0.023, 0.034, 0.14$ [19]. The cross sections derived from Eqs. (8) and (13) approximate the squark exchange contributions for heavy quarks [20] and neglect the effect of twist-2 operators; however the change from a more careful treatment of loop effects for heavy quarks and the inclusion of twist-2 operators is numerically small [21].

We are now ready to show the importance of the phases. As we noted earlier, we will restrict our parameter choices to universal gaugino masses and universal sfermion masses at the GUT scale. We will also choose $\tan \beta = 3$ throughout to make it easier to remain consistent with recent constraints on the Higgs boson mass of about 78 GeV (for this value of $\tan \beta$) [23]. We will also choose $m_0 = 100$ GeV throughout. Because of the running of the RGE, this leads to typical squark masses of ~ 450 GeV for $M_2 = 150$ GeV. Finally, we have chosen the value of the pseudo-scalar Higgs boson mass to be 300 GeV.

We begin our discussion by focusing on the spin dependent contribution from α_2 . For this case, we consider the scattering of neutralinos on fluorine, for which the spin dependent contribution typically dominates by a factor of about 20 [6]. In Fig. 1(a), we show the contributions from different quarks to a_p given in Eq. (11), as a function of the CP -violating phase θ_μ , for $M_2 = 150$ GeV and $|\mu| = 500$ GeV. In this case, both the contributions from squark exchange and Z exchange are significant. The signs of the individual α_{2i} 's are all positive; however, the sign of the contribution to a_p is different for the u quark than for the d and s quarks due to the different sign in Δ_u relative to Δ_d and Δ_s . As one can see, there are important cancellations which can dramatically reduce the spin dependent cross section. We note that since this sign difference in the Δ 's is generic, this effect does not depend heavily on the spin structure of the nucleon.

The total value of the spin dependent cross section $\sigma_2(\theta_\mu)$ for χ - ^{19}F scattering is shown by the solid curve in Fig. 1(b), normalized to the value of the spin dependent cross section at $\theta_\mu = 0$. For these values of the MSSM parameters, the neutralino is predominantly a \tilde{B} with a mass $m_\chi \approx 75$ GeV. The relic density is about $\Omega h^2 \approx 0.15$. As one can see, there is an important dependence on θ_μ and a cancellation in σ_2 leading to a decrease in the cross section by at least an order of magnitude for $\theta_\mu/\pi = 0.2$ – 0.3 . The presence of such a large cancellation in the spin dependent cross section over a range in θ_μ and its position in θ_μ depend on the MSSM parameters. For comparison, we also show by the dashed curve, the spin independent cross section σ_3 for the same MSSM parameters and for scattering on fluorine, which also exhibits a similar reduction near $\theta_\mu/\pi = 0.3$ – 0.4 . Note that the neutralino relic density is not strongly dependent on θ_μ since the \tilde{B} mass is insensitive to μ . Furthermore, the \tilde{B} relic density depends primarily on the annihilation through slepton exchange (since squarks are heavier when universal sfermion masses are assumed at the GUT scale and $m_0 \leq M_2$). Because slepton mixing is small [the off-diagonal elements in Eq. (3) are proportional to the lepton masses] the dependence on the CP -violating phases is also small [13].

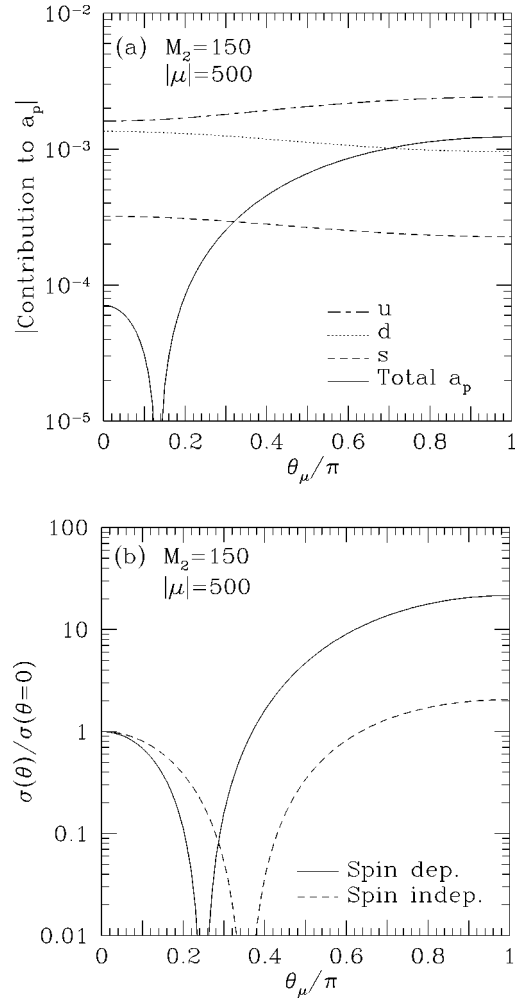


FIG. 1. For elastic scattering off of ^{19}F , (a) the absolute value of the contributions to a_p from individual quarks as a function of θ_μ , and (b) spin dependent (solid) and spin independent (dashed) cross sections as a function of θ_μ , and normalized to the cross sections at $\theta_\mu = 0$.

The spin independent cross section is dominant in much of the parameter space for scattering off of heavy nuclei. In Fig. 2, we consider the scattering of neutralinos on ^{73}Ge , for $M_2 = 150$ GeV and $|\mu| = 250$ GeV. In Fig. 2(a), we show the relative contributions to f_n . The dominant contributions to f_n come from Higgs exchange, and from Eqs. (8) and (13) one sees that contributions from up-type quarks and down-type quarks are simply scaled by the appropriate f_T 's. The solid line shows the total f_n (which is close to the total f_p), and again one sees significant cancellations, near $\theta_\mu/\pi = 0.15$. The cancellation in the total f_n occurs at a different place from that of the individual contributions because of the relative signs of the latter which are not shown. The signs of the up-type contributions differ from those of the down-type (at $\theta_\mu \sim 0, \pi$) and change sign at $\theta_\mu/\pi \sim 0.6$ and 0.4 respectively. In Fig. 2(b), we show by the dashed curve the total value of the spin independent cross section $\sigma_3(\theta_\mu)$, again normalized to $\sigma_3(0)$. Here again, we see a strong dependence on the CP -violating phase θ_μ , and for $\theta_\mu \sim 0.15$, there is again a strong cancellation in the total scattering cross

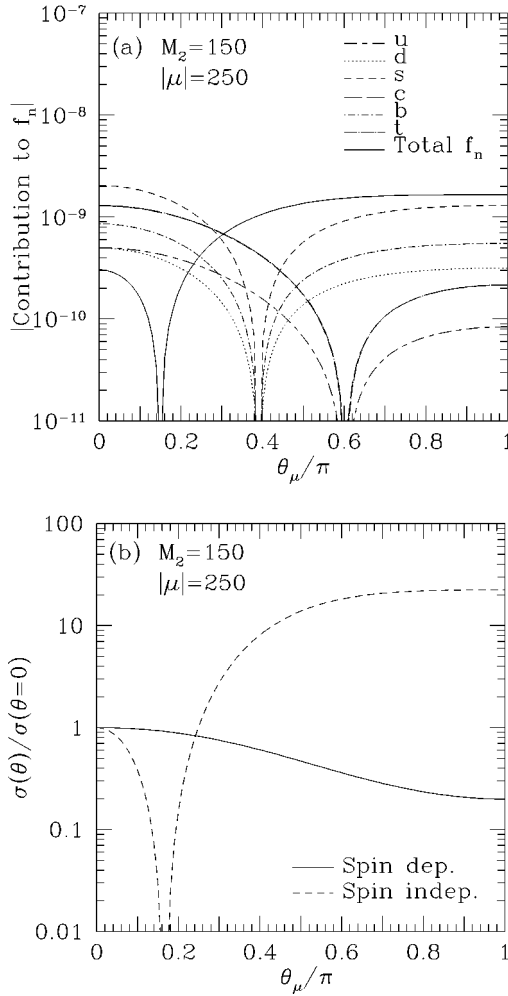


FIG. 2. For elastic scattering off of ^{73}Ge , (a) the absolute value of the contributions to f_n from individual quarks as a function of θ_μ , and (b) spin dependent (solid) and spin independent (dashed) cross sections as a function of θ_μ , and normalized to the cross sections at $\theta_\mu=0$.

section. Note that in this case, the spin dependent cross section (shown by the solid curve) simply connects the $\theta_\mu=0$ and $\theta_\mu=\pi$ limits monotonically.

Finally, in Fig. 3, we show that the θ_μ dependence of the cross sections does not always lead to cancellations and a diminishing of the cross section. Indeed, while we generally do find a strong dependence on θ_μ , in some cases this dependence leads to an enhancement of the cross section. In Fig. 3, MSSM parameters were chosen as $M_2=130$ GeV,

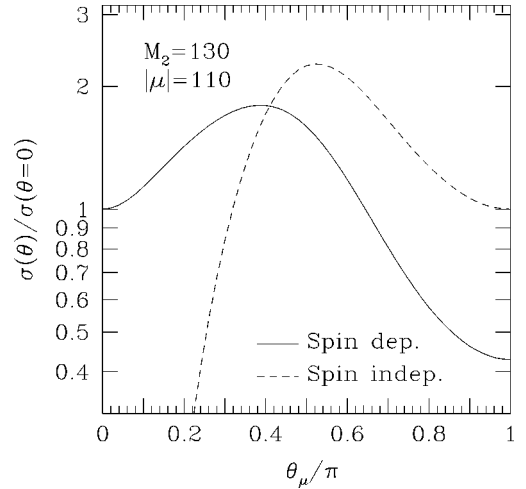


FIG. 3. An example of a θ_μ dependence which shows an enhancement in the χ - ^{19}F scattering cross section rather than a cancellation. Note that the cross sections are normalized differently in this case.

$|\mu|=110$ GeV, $\tan\beta=2$ and $m_0=1500$ GeV (to satisfy the Higgs boson mass constraint), and we now normalize the spin independent cross section to the cross section at $\theta_\mu=\pi$. As one can see, the dependence of the cross section on θ_μ is not monotonic. The large variance in the cross section from $\theta_\mu=0$ to π is largely due to the fact that the neutralino mass varies rapidly for these parameters, from 26 to 70 GeV. Note that for $\theta_\mu/\pi \lesssim 0.5$, the chargino mass (which is also strongly dependent on θ_μ) is below the current experimental constraint of about 91 GeV.

We have shown that the cross sections for elastic neutralino-nucleus scattering relevant for the detection of supersymmetric dark matter are strongly dependent on the CP -violating phase θ_μ associated with the Higgs boson mixing mass μ in the MSSM. For particular MSSM parameters, the value of the phase θ_μ can lead to either strong cancellations or in some cases enhancements to the cross section and ultimately the detection rate. The full dependence on the MSSM parameters M_2 , μ , and $\tan\beta$ as well as A_0 and its associated phase θ_A will be presented elsewhere [16].

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