One loop calculation in lattice QCD with domain-wall quarks

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One loop corrections to the quark propagator are calculated in massless QCD with a domain-wall fermion. We adopt the Shamir type domain-wall fermion with an infinitely large extra dimension to describe the massless fermion. It is shown that no additative counterterm to the current quark mass is generated in this theory with infinitely many flavors, and the wave function renormalization factor of the massless quark is explicitly evaluated. We also show that an analysis with a simple mean-field approximation can explain the properties of the massless quark in numerical simulations of QCD with domain-wall quarks. [S0556-2821(98)02221-8]

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I. INTRODUCTION

The formulation of the lattice fermion in QCD with chiral symmetry is one of the most fascinating problems theoretically and practically. Although both Wilson and Kogut-Susskind (KS) fermion formulations have been popularly used for the lattice QCD simulations, some disadvantages remain in these formulations: In the Wilson fermion formulation the quark mass has an additive quantum correction and the chiral limit is reached only by fine tuning the mass parameter. As a general rule we have to take the continuum limit tuning the mass appropriately in order to simulate massless QCD. In the KS fermion formulation the number of flavors is restricted and the original flavor symmetry is broken explicitly to some residual one.

The domain-wall fermion formulation, which was originally proposed to define lattice chiral gauge theories [1], has been applied to lattice QCD [2]. This formulation is expected to have a great advantage over the previous two formulations: An advantage over the KS fermion is that the number of flavors is not fixed. This is manifest from its definition. The other advantage over the Wilson fermion is that mass renormalization is multiplicative ($m_{\rm eff}=Z_m m_{\rm tree}$). In other words, if a massless mode exists at the tree level it is stable against the quantum correction. The additive mass correction exists when the length of the extra dimension N_s of the domain-wall fermion is finite, however, it is expected to be suppressed exponentially in N_s. The stability of the massless mode is thought to be valid when we set the extra dimension to be infinitely large. This property of mass renormalization is not a trivial one, but only an intuitive discussion on it has been given so far [2]. On the other hand, recent numerical simulations suggests that the stability of the zero mode holds even nonperturbatively [3]. Therefore an analytical understanding of domain-wall QCD is now needed. The aim of this paper is to confirm the stability of the massless mode at $N_s\rightarrow\infty$ by lattice perturbation theory and to give explicitly the wave function renormalization of the quark field.

This paper is organized as follows. In Sec. II we will give the basic tools for the perturbative calculation with the domain-wall fermion. It is enough to present only the fermion propagator because other Feynman rules of the gauge interaction and gauge propagator are exactly identical to that of the ordinary Wilson fermion. In Sec. III we calculate one loop corrections to the fermion propagator. Section IV is the main part of this paper, where we discuss the renormalization of the zero mode or massless quark field. We take the diagonal basis of the mass matrix of the domain-wall fermion and see that the zero mode is stable against the one loop correction at $N_s\rightarrow\infty$. The wave function renormalization factor of the massless quark field is also given explicitly. Section V is devoted to a mean field analysis with finite N_s. We show that properties of the zero mode observed in the numerical simulation [3] are well explained in this approximation. In Sec. VI we give our conclusions and a discussion. In the appendixes some derivations of formulas used in the text are presented.

In this paper we set the lattice spacing $a=1$ and recover it through dimensionful variables as the need arises. We take the SU(N_c) gauge group with the gauge coupling constant g and the second Casimir $C_2=(N_c^2-1)/2N_c$. We set $N_c=3$ in the numerical calculations.

II. PERTURBATION THEORY WITH DOMAIN-WALL FERMION

A. Action

We adopt a domain-wall fermion of Shamir type [2] to describe massless quarks. The domain-wall fermion is a variant of the Wilson fermion with sufficiently many flavors and a special form of the mass matrix. Although it is also interpreted as a five-dimensional Wilson fermion [1], we prefer to treat it as a multiflavor system [4].

From this point of view the only difference from the Wilson fermion action is the fermion bilinear term. If we separate the QCD action for lattice perturbation theory into fermion and gauge parts,

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$$S = S_{\rm fermion} + S_{\rm gauge} + S_{\rm GF} + S_{\rm FP} + S_{\rm measure}, \tag{1}$$

the lattice gauge action $S_{\rm gauge}$, the gauge fixing and Fadeev-Popov (FP) ghost term $S_{\rm GF}+S_{\rm FP}$, the invariant measure term $S_{\rm measure}$, and the gauge-fermion interaction terms in $S_{\rm fermion}$ are exactly same as those in ordinary Wilson fermion perturbation theory [5,6] with many flavors.

The domain-wall fermion action $S_{\rm fermion}$ is written as

$$\begin{aligned} S_{\rm fermion} = & \sum_{n,m} \bar{\psi}_{m,s}(\gamma_\mu D_\mu)_{m,n}\psi_{n,s} \\ & + \bar{\psi}_{m,s} W^{+\ s,t}_{m,n} P_+ \psi_{n,t} + \bar{\psi}_{m,s} W^{-\ s,t}_{m,n} P_- \psi_{n,t} \\ & + m_q \bar{\psi}_{m,s}(\delta_{m,n}\delta_{s,N_s}\delta_{t,1}P_+ + \delta_{m,n}\delta_{s,1}\delta_{t,N_s}P_-)\psi_{n,t}, \end{aligned} \tag{2}$$

where m,n is four-dimensional space index, and $s,t = 1,\dots,N_s$ is the flavor index, for which we will take $N_s \to \infty$ limit in the one loop calculation to discuss the massless mode. Here the Dirac operator is given by

$$(\gamma_\mu D_\mu)_{n,m} = \sum_\mu \frac{1}{2}\gamma_\mu (U_{n,\mu}\delta_{n+\hat{\mu},m} - U^\dagger_{m,\mu}\delta_{n-\hat{\mu},m}), \tag{3}$$

and mass matrix $W^{\pm}_{s,t}$ is defined as

$$W^{\pm\ s,t}_{n,m} = \delta_{s\pm 1,t}\delta_{n,m} - W_{n,m}\delta_{s,t}, \tag{4}$$

where

$$\begin{aligned} W_{n,m} = & (1-M)\delta_{n,m} \\ & + \frac{r}{2}\sum_\mu (U_{n,\mu}\delta_{n+\hat{\mu},m} + U^\dagger_{m,\mu}\delta_{n-\hat{\mu},m} - 2\delta_{n,m}) \end{aligned} \tag{5}$$

is a sum of the Dirac mass term and the Wilson term, which contain gauge fields at this stage, and r is the Wilson parameter, which we set $r=-1$. The parameter m_q is the current quarks mass, but in this paper we only treat the massless QCD taking $m_q=0$. $P_\pm$ is a projection operator defined by

$$P_\pm = \frac{1\pm\gamma_5}{2}. \tag{6}$$

In our domain-wall fermion action (2) we have Dirac mass M besides the current quark mass m_q. Here we have to notice that M is not the physical quark mass but is rather an unphysical mass of the cutoff order $(1/a)$-like Wilson term. As will be mentioned later M has an important role as a parameter of the theory: choosing a suitable value for M we have a massless fermion mode for the vanishing current quark mass $(m_q=0)$ at $N_s \to \infty$.

In order to see the massless fermion mode it is more convenient to be in the momentum representation and pull out the bilinear term. The fermion action in the momentum space is written as

$$\begin{aligned} S_{\rm fermion} = & \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(-p)_s \Bigg[\sum_\mu i\gamma_\mu \sin p_\mu \\ & + W^+(p)_{s,t}P_+ + W^-(p)_{s,t}P_- \Bigg] \psi(p)_t + S_{\rm int.}, \end{aligned} \tag{7}$$

where the mass matrix has the following form,

$$W^+(p)_{s,t} = \delta_{s+1,t} - W(p)\delta_{s,t}$$

$$= \begin{pmatrix} -W(p) & 1 & & \\ & -W(p) & \ddots & \\ & & \ddots & 1 \\ & & & -W(p) \end{pmatrix}, \tag{8}$$

$$W^-(p)_{s,t} = \delta_{s-1,t} - W(p)\delta_{s,t}$$

$$= \begin{pmatrix} -W(p) & & & \\ 1 & -W(p) & & \\ & \ddots & \ddots & \\ & & 1 & -W(p) \end{pmatrix}, \tag{9}$$

$$W(p) = 1 - M - r\sum_\mu (1-\cos p_\mu). \tag{10}$$

The gauge interaction term $S_{\rm int}$ is identical to that of the Wilson fermion perturbation theory with N_s flavors.

As will be discussed in Appendix C, in spite of the presence of the Dirac mass M this fermion system has one massless fermion mode and N_s-1 excited modes with the mass of cutoff order in the $N_s \to \infty$ limit by virtue of this mass matrix form, provided that $|W(p\sim 0)|<1$ is satisfied by a suitable choice of the Dirac mass M. Here we take the momentum region $p_\mu \sim 0$ to see the zero mode with physical momenta. At the momentum $p_\mu \sim \pi$, where the doubler emerges in the naive fermion formulation, the parameter condition is not satisfied $(|W(p\sim\pi)|>1)$, so that all N_s fermion modes have masses of the cutoff order. Here please notice that the mass parameter M is only required to be in the wide range $0<M<2$ [2] and we need no fine-tuning of it. As will be seen later, the absence of the fine-tuning problem remains true at the one-loop order, though the allowed range of M for the zero mode becomes coupling-constant dependent.

B. Fermion propagator

In the next section we will calculate the one loop correction to the fermion propagator. In this subsection we set up the lattice Feynman rules for domain-wall fermion with vanishing current quark mass $(m_q=0)$.

As was discussed in the previous subsection, the domain-wall fermion action is almost the same as that of the ordinary Wilson fermions one with N_s flavors. The peculiar feature of the domain-wall fermion is the form of the fermion propagator, which is given by

$$S_F(p)_{s,t}=[i\gamma_\mu \sin p_\mu + W^+(p)P_+ + W^-(p)P_-]^{-1}_{s,t}. \tag{11}$$

The explicit form is written as

$$S_F(p)_{s,t}=[(-i\gamma_\mu \sin p_\mu + W^-)G_R(s,t)P_+ + (-i\gamma_\mu \sin p_\mu + W^+)G_L(s,t)P_-]_{st}, \tag{12}$$

where

$$G_R(s,t)\equiv\left(\frac{1}{\sin^2 p + W^+W^-}\right)_{st} = G^0(s-t)+A_{++}e^{\alpha(s+t)}+A_{+-}e^{\alpha(s-t)} + A_{-+}e^{\alpha(-s+t)}+A_{--}e^{\alpha(-s-t)}, \tag{13}$$

$$G^0(s-t)=A(e^{\alpha(N_s-|s-t|)}+e^{-\alpha(N_s-|s-t|)}), \tag{14}$$

$$\begin{pmatrix} A_{++} \\ A_{-+} \end{pmatrix} = \frac{A}{e^{\alpha N_s}(1-We^{\alpha})-e^{-\alpha N_s}(1-We^{-\alpha})} \times \begin{pmatrix} (1-We^{-\alpha})(e^{-2\alpha N_s}-1) \\ W(e^{\alpha}-e^{-\alpha}) \end{pmatrix}, \tag{15}$$

$$\begin{pmatrix} A_{+-} \\ A_{--} \end{pmatrix} = \frac{A}{e^{\alpha N_s}(1-We^{\alpha})-e^{-\alpha N_s}(1-We^{-\alpha})} \times \begin{pmatrix} W(e^{\alpha}-e^{-\alpha}) \\ (1-We^{\alpha})(1-e^{2\alpha N_s}) \end{pmatrix}, \tag{16}$$

and

$$G_L(s,t)\equiv\left(\frac{1}{\sin^2 p + W^-W^+}\right)_{st} = G^0(s-t)+B_{++}e^{\alpha(s+t)}+B_{+-}e^{\alpha(s-t)} + B_{-+}e^{\alpha(-s+t)}+B_{--}e^{\alpha(-s-t)}, \tag{17}$$

$$\begin{pmatrix} B_{++} \\ B_{-+} \end{pmatrix} = \frac{A}{e^{\alpha N_s}(1-We^{\alpha})-e^{-\alpha N_s}(1-We^{-\alpha})} \times \begin{pmatrix} e^{-\alpha}(e^{-\alpha}-W)(e^{-2\alpha N_s}-1) \\ W(e^{\alpha}-e^{-\alpha}) \end{pmatrix}, \tag{18}$$

$$\begin{pmatrix} B_{+-} \\ B_{--} \end{pmatrix} = \frac{A}{e^{\alpha N_s}(1-We^{\alpha})-e^{-\alpha N_s}(1-We^{-\alpha})} \times \begin{pmatrix} W(e^{\alpha}-e^{-\alpha}) \\ e^{\alpha}(e^{\alpha}-W)(1-e^{2\alpha N_s}) \end{pmatrix}. \tag{19}$$

Here α and A are defined as

$$\cosh\alpha \equiv \frac{1+W^2+\sin^2 p}{2W}, \tag{20}$$

$$\sinh\alpha = \frac{1}{2W}\sqrt{(1-W^2)^2+2(1+W^2)\sum \sin^2 p_\mu + \left(\sum \sin^2 p_\mu\right)^2}, \tag{21}$$

$$A\equiv\frac{1}{2W\sinh\alpha}\,\frac{1}{2\sinh(\alpha N_s)}. \tag{22}$$

Note that the argument p of W and α is suppressed throughout this paper unless necessary. Since this fermion propagator is invariant under $\alpha\to-\alpha$, we take the $\alpha>0$ without loss of generality. G_R and G_L are also symmetric in (s,t). See Appendix B for the derivation. In the one-loop calculation we use the above propagator in the $N_s\to\infty$ limit.

III. ONE LOOP CALCULATION

A. Diagrams

In this section we calculate the one loop correction to the fermion propagator, which is given by two contributions $\Sigma^{\text{tadpole}}(p)+\Sigma^{\text{half-circle}}(p)$, from diagrams in Fig. 1. The 1PI fermion two-point vertex function is given by

$$V^{(2)}_{\text{1-loop}}(p)_{s,t} = [i\gamma_\mu \sin p_\mu + W^+(p)P_+ + W^-(p)P_- - \Sigma(p)]_{s,t} \tag{23}$$

with

$$\Sigma(p)=\Sigma^{\text{tadpole}}(p)+\Sigma^{\text{half-circle}}(p). \tag{24}$$

In order to investigate the massless mode of $\Gamma^{(2)}_{\text{1-loop}}(p)_{s,t}$ in the $p_\mu\to 0$ limit, we need only the first few terms in the p_μ expansion. Since the only dimensionful quantity is the external momentum p_μ in our calculation, the higher order terms in the p_μ expansion are also higher order in a.

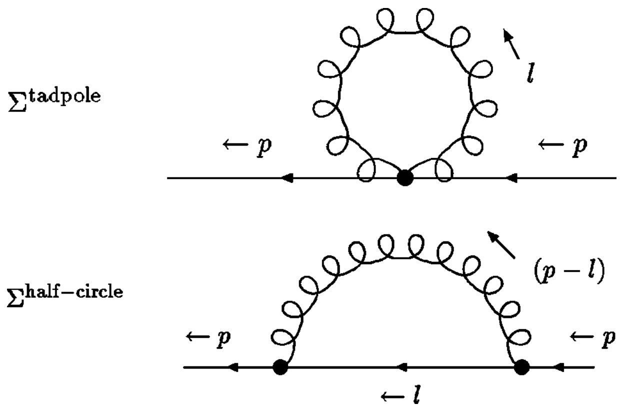

FIG. 1. Diagrams which contribute to the one-loop correction to the fermion propagator. Above: Tadpole diagram. Below: Half-circle diagram.

B. Contribution from tadpole diagram

The contribution from the tadpole diagram is written as

$$\Sigma^{\text{tadpole}} = \frac{1}{2} g^2 C_2 \sum_\mu (i\gamma_\mu \sin p_\mu - r\cos p_\mu) \times \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \frac{1}{4\sin^2(l/2)} \delta_{s,t} \tag{25}$$

$$= g^2 C_2 T \left(\frac{1}{2} i\not{p} + 2 \right) \delta_{s,t} + \mathcal{O}(p^2), \tag{26}$$

where T is the tadpole loop integral

$$T = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \frac{1}{4\sin^2(l/2)} = 0.154612. \tag{27}$$

The first term in Eq. (26) is finite, the second term linearly diverges, and the third term vanishes in the limit $a \to 0$ when we recover the lattice spacing a. We see that Σ^{tadpole} is diagonal in flavor space, and its effect is to modify the mass parameter $M \to \tilde{M} = M - 2g^2 C_2 T$ and the wave function renormalization $Z = 1 \to 1 - g^2 C_2 T/2$.

C. Contribution from half circle diagram

The contribution from the half circle diagram in the Feynman gauge

$$\Sigma^{\text{half-circle}}_{s,t} = \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \sum_\mu (-igT^a) \times \left\{ \gamma_\mu \cos\frac{1}{2}(l_\mu + p_\mu) - ir\sin\frac{1}{2}(l_\mu + p_\mu) \right\} \times S_{\mathrm{F}}(l)_{s,t} \times (-igT^a) \left\{ \gamma_\mu \cos\frac{1}{2}(l_\mu + p_\mu) - ir\sin\frac{1}{2}(l_\mu + p_\mu) \right\} \times \frac{1}{4\sin^2(p-l)/2} \tag{28}$$

cannot be calculated analytically because of its complicated dependence on the flavor indices s,t in the fermion propagator.

It is easily seen that the loop integral of Eq. (28) has infrared divergence. As is in ordinary lattice perturbation theory the infrared divergence can be written in an analytic form. To do this we separate $\Sigma^{\text{half-circle}}_{s,t}$ as follows:

$$\Sigma^{\text{half-circle}}_{s,t}(p) = \Sigma^{\text{lat}}_{s,t}(p) + \Sigma^{\text{cont}}_{s,t}(p), \tag{29}$$

where

$$\Sigma^{\text{lat}}_{s,t}(p) = \Sigma^{\text{half-circle}}_{s,t}(p) - \Sigma^{\text{cont}}_{s,t}(p), \tag{30}$$

and $\Sigma^{\text{cont}}_{s,t}(p)$ is introduced to extract the infrared divergence

$$\Sigma^{\text{cont}}_{s,t}(p) = 2g^2 C_2 \int \frac{d^4 l}{(2\pi)^4} \frac{-i\not{l}(C_+ P_+ + C_- P_-)_{s,t}}{l^2 (p-l)^2} \times \theta(\pi^2 - l^2)$$

with

$$(C_+)_{s,t} = (1 - w_0^2) w_0^{s+t-2},$$

$$(C_-)_{s,t} = (1 - w_0^2) w_0^{2N_s - s - t},$$

and $w_0 = W(0)$. In order to have zero modes with the physical momentum, w_0 should be in the region $w_0^2 \leqslant 1$. This leads to the condition of M that $0 \leqslant M \leqslant 2$ [2]. Since $\Sigma^{\text{lat}}_{s,t}(p)$ is infrared finite in the $p \to 0$ limit, we can evaluate it in the p expansion:

$$\Sigma^{\text{lat}}_{s,t}(p) = \Sigma^{\text{lat}}_{s,t}(0) + p_\mu \frac{\partial \Sigma^{\text{lat}}_{s,t}}{\partial p_\mu}(0) + \mathcal{O}(p^2). \tag{31}$$

The logarithmically divergent part $\Sigma^{\text{cont}}(p)$ can be calculated analytically, while a linearly divergent and finite terms [the first and the second terms in Eq. (31)] have to be evaluated by numerical integrations of loop momenta. After a little algebra we have

$$\Sigma^{\text{half-circle}}_{s,t} = -g^2 C_2 [i\not{p}(I^+_{s,t} P_+ + I^-_{s,t} P_-) + M^+_{s,t} P_+ + M^-_{s,t} P_-], \tag{32}$$

where $I^\pm$ and $M^\pm$ are given by

$$I^{\pm}_{s,t}=I^{\pm}_{\log}(s,t)+I^{\pm}_{\rm finite}(s,t), \tag{33}$$

$$I^{\pm}_{\log}(s,t)=\frac{1}{16\pi^2}(C_{\pm})_{s,t}\left(\ln(\pi^2)+\frac{1}{2}-\ln p^2\right), \tag{34}$$

$$\begin{aligned}I^{+/-}_{\rm finite}(s,t)=&\int\frac{d^4l}{(2\pi)^4}\frac{1}{4\sin^2 l/2}\Bigg\{\frac{1}{8}\sum_{\mu}\left[\cos l_\mu(W^-G_R+W^+G_L)(s,t)+\sin^2 l_\mu(G_L+G_R)(s,t)\right]\\&+\sum_\mu\frac{\sin^2 l_\mu}{4(4\sin^2 l/2)^2}\left[(W^-G_R+W^+G_L)(s,t)+2\left(\sum_\nu\cos^2\frac{l_\nu}{2}-2\cos^2\frac{l_\mu}{2}\right)G_{L/R}(s,t)+\sum_\nu\sin^2\frac{l_\nu}{2}G_{R/L}(s,t)\right]\Bigg\}\\&-(C_{+/-})_{s,t}\int\frac{d^4l}{(2\pi)^4}\frac{1}{(l^2)^2}\theta(\pi^2-l^2),\end{aligned} \tag{35}$$

$$M^{+/-}_{s,t}=\int\frac{d^4l}{(2\pi)^4}\frac{1}{4\sin^2 l/2}\sum_\mu\left[\cos^2\frac{l_\mu}{2}(W^{+/-}G_{L/R})(s,t)-\sin^2\frac{l_\mu}{2}(W^{-/+}G_{R/L})(s,t)+\frac{1}{2}\sin^2 l_\mu(G_L+G_R)(s,t)\right]. \tag{36}$$

By the dimensional counting $I^{\pm}$ has $\ln a^2$ divergence and constant terms in a, and $M^{\pm}$ has $1/a$ linear divergence when lattice spacing a is introduced explicitly as $p_\mu\to ap_\mu$, $\Sigma\to\Sigma/a$. Although $M^{\pm}$ may have $\ln a^2$ divergence naively, it is canceled by the algebraic relation

$$[W^+(p=0)]_{s,t}(1-M)^t=0, \tag{37}$$

$$[W^-(p=0)]_{s,t}(1-M)^{-t}=0. \tag{38}$$

The logarithmic divergence $\ln a^2$ in $I^{\pm}$ is given analytically. As we can see from the form of $(C_+)_{s,t}=(1-w_0^2)w_0^{s+t-2}$, $I^+_{\log}$ is localized in the boundary $(s,t)=(1,1)$. This is because the logarithmic divergence comes from the effect of massless fermion mode which is localized in the boundary. The other one $I^-_{\log}$ is localized in the other boundary $(s,t)=(N_s,N_s)$.

The finite terms and linearly divergent terms should be calculated by repeating the numerical integration $O(N_s^2)$ times. However, as can be seen in the next section, such huge number of integrations can be avoided for the wave-function renormalization of the quark field. On the other hand, since the structures of $I^{\pm}_{\rm finite}$ and $M_{\pm}$ are useful to understand the domain-wall QCD more deeply, we will give them in a separate paper.

IV. RENORMALIZATION OF QUARKS FIELD

The result obtained in the previous section is summarized in the following form of the effective action for a two-point function with the scale $p^2=(\mu a)^2$ at one-loop level:

$$\begin{aligned}\Gamma^{(2)}=\bar{\psi}(-p)_s[i\gamma_\mu p_\mu(Z^+P_++Z^-P_-)\\+\bar{W}^+P_++\bar{W}^-P_-]_{s,t}\psi(p)_t,\end{aligned} \tag{39}$$

where

$$Z^{\pm}=1+g^2C_2(I_{\rm tad}+I^{\pm}_{\log}+I^{\pm}_{\rm finite}), \tag{40}$$

$$\bar{W}^{\pm}=W^{\pm}(0)+g^2C_2(M_{\rm tad}+M^{\pm}) \tag{41}$$

with

$$I_{\rm tad}(s,t)=-\frac{1}{2}T\delta_{s,t}=-0.077306\delta_{s,t}, \tag{42}$$

$$M_{\rm tad}(s,t)=-2T\delta_{s,t}=-0.309224\delta_{s,t}. \tag{43}$$

In Eq. (39) we expand the effective action in external momentum p_μ and pick up the relevant terms for renormalization. The expressions for $I^{\pm}_{\log}$, $I^{\pm}_{\rm finite}$, and $M^{\pm}$ were given in the previous section. In this section we consider the renormalization of zero modes, which is interpolated by the quark field: $q(p)=P_+\psi(p)_1+P_-\psi(p)_{N_s}$. Here we only present the results and give the details of derivations in Appendix C.

A. Diagonalization of mass matrix and stability of zero modes

For the renormalization of zero modes, it is better to use new basis, $\psi^d(p)$ which diagonalize the one loop level mass matrix $\bar{W}^{\pm}$. These basis are given by the relation that

$$\psi^d_s(p)=U_{s,t}P_+\psi(p)+V_{s,t}P_-\psi(p), \tag{44}$$

where unitary matrices U and V satisfy

$$[U\bar{W}^-\bar{W}^+U^\dagger]_{s,t}=M_s^2\delta_{s,t},$$

$$[V\bar{W}^+\bar{W}^-V^\dagger]_{s,t}=M_s^2\delta_{s,t}.$$

In our notation the mass eigenvalue squared M_s^2 is arranged in such a way that $M^2_{N_s}=0$, and we can take U and V real matrices without loss of generality.

We calculate U and V at one-loop level:

$$U=(1+g^2U_1)U_0, \quad V=(1+g^2V_1)V_0, \tag{45}$$

where tree level matrices U_0 and V_0 are analytically obtained in the large N_s limit as follows:

$$[U_0]_{s,t}=\begin{cases}(2/N_s)^{1/2}\sin\alpha_s(N_s+1-t) & s\neq N_s,\\ (1-w_0^2)^{1/2}w_0^{(t-1)} & s=N_s,\end{cases} \tag{46}$$

and $[V_0]_{s,t}=[U_0]_{s,N_s+1-t}$, where $w_0=1-\tilde{M}$ with $\tilde{M}=M+4(u-1)$. Here $u=1$ for naive perturbation theory, while $u=1-g^2C_2T/2$ for tadpole improved perturbation theory [7]. Hereafter we will call both cases ''the tree level'' and will not distinguish the two cases unless necessary. Note that the allowed range for the zero mode now becomes $0<\tilde{M}<2$, which is the g^2 dependent condition for M in the latter case. If we expand the mass eigenvalue squared as $(M^2)_s=(M_0^2)_s+g^2(M_1^2)_s$, the tree level one is related to the phase factor α_s such that $2w_0\cos\alpha_s=1+w_0^2-(M_0^2)_s$. This phase factor, which also satisfies $\sin\alpha_s N_s=w_0\sin\alpha_s(N_s+1)$, is explicitly given as $\alpha_s=\pi s/N_s$ in the large N_s limit. It is also shown that U_0 and V_0 diagonalize the tree level mass matrix $W_0^{\pm}=W^{\pm}(0)$ itself such that $[V_0W_0^+U_0^\dagger]_{s,t}=[U_0W_0^-V_0^\dagger]_{s,t}=(M_0)_s\delta_{s,t}$.

We now consider one-loop level mass matrix $\overline{W}^{\pm}$, which is denoted as $\overline{W}^{\pm}=W_0^{\pm}+g^2W_1^{\pm}$, where $g^2(W_1^{\pm})_{s,t}=g^2C_2(M^{\pm}+M_{\rm tad})_{s,t}+4(1-u)\delta_{s,t}$ is the one loop correction given in the previous section. To diagonalize $\overline{W}^{\mp}\cdot\overline{W}^{\pm}$ at one-loop order, U_1 and V_1 should satisfy

$$(U_1)_{s,t}(M_0^2)_t+(M_0^2)_s(U_1^\dagger)_{s,t}+(U_0W_1^-V_0^\dagger\cdot V_0W_0^+U_0^\dagger)_{s,t}+(U_0W_0^-V_0^\dagger\cdot V_0W_1^+U_0^\dagger)_{s,t}=(M_1^2)_s\delta_{s,t}, \tag{47}$$

$$(V_1)_{s,t}(M_0^2)_t+(M_0^2)_s(V_1^\dagger)_{s,t}+(V_0W_1^+U_0^\dagger\cdot U_0W_0^-V_0^\dagger)_{s,t}+(V_0W_0^+U_0^\dagger\cdot U_0W_1^-V_0^\dagger)_{s,t}=(M_1^2)_s\delta_{s,t}. \tag{48}$$

Using the fact that $(U_1,V_1)_{s,t}=-(U_1,V_1)_{t,s}$ implied by the unitarity and the reality, and $V_0W_0^+U_0^\dagger$ and $U_0W_0^-V_0^\dagger$ are diagonal, we can easily solve the above equation as

$$(U_1)_{s,t}=\frac{(M_0)_t(\tilde{W}_1)_{t,s}+(M_0)_s(\tilde{W}_1)_{s,t}}{(M_0^2)_s-(M_0^2)_t},$$

$$(V_1)_{s,t}=\frac{(\tilde{W}_1)_{s,t}(M_0)_t+(\tilde{W}_1)_{t,s}(M_0)_s}{(M_0^2)_s-(M_0^2)_t}$$

for $s\neq t$, and

$$(M_1^2)_s=2(\tilde{W}_1)_{s,s}(M_0)_s, \quad (U_1)_{s,s}=(V_1)_{s,s}=0, \tag{49}$$

where $\tilde{W}_1=V_0W_1U_0^\dagger$. The mass eigenvalue squared $M_s^2=(M_0^2)_s+g^2(M_1^2)_s$ obtained above leads to the mass eigenvalue M_s itself: $M_s=(M_0)_s+g^2(\tilde{W}_1)_{s,s}$. Note that $M_{N_s}=0$ since $(M_0)_{N_s}=0$ and $(\tilde{W}_1)_{N_s,N_s}=0$ in the large N_s limit as is shown in Appendix C. This result explicitly demonstrates the stability of the zero modes against one-loop corrections in domain-wall QCD at $N_s\to\infty$. As in the case at the tree level, it is shown that

$$(V\overline{W}^+U^\dagger)_{s,t}=(U\overline{W}^-V^\dagger)_{s,t}=M_s\delta_{s,t}+O(g^4). \tag{50}$$

B. Wave function renormalization for quark fields

After diagonalization of the mass matrix, the effective action for the zero mode field $\psi^d(p)_{N_s}=\chi_0(p)$ becomes

$$\overline{\chi}_0(-p)[i\gamma_\mu p_\mu(\tilde{Z}_+P_++\tilde{Z}_-P_-)]\chi_0(p), \tag{51}$$

where

$$\tilde{Z}_{\pm}=1-g^2C_2\frac{T}{2}+\frac{g^2C_2}{16\pi^2}\left(\log\pi^2+\frac{1}{2}-\log(\mu a)^2\right)+g^2(I_{\pm}^d)_{N_s,N_s} \tag{52}$$

with $I_+^d=C_2(U_0I_{\rm finite}^+U_0^\dagger)$ and $I_-^d=C_2(V_0I_{\rm finite}^-V_0^\dagger)$. Since the interpolating quark field $q(p)$ is expressed as $q(p)=(U_{N_s1}P_++V_{N_s,N_s}P_-)\chi_0(p)$, and $\langle\chi_0(p)\overline{\chi}_0(-p)\rangle=[(1/\tilde{Z}_+)P_++(1/\tilde{Z}_-)P_-](-i\gamma_\mu p_\mu/p^2)$, we obtain

$$\langle q(p)\overline{q}(-p)\rangle=\left[\frac{U_{N_s1}^2}{\tilde{Z}_+}P_++\frac{V_{N_s,N_s}^2}{\tilde{Z}_-}P_-\right]\frac{-i\gamma_\mu p_\mu}{p^2}. \tag{53}$$

Therefore, the renormalized quark field $Q(p)$, which satisfies $\langle Q(p)\overline{Q}(-p)\rangle=-i\gamma_\mu p_\mu/p^2$, is given by $Q(p)=[(Z_F^+)^{1/2}P_++(Z_F^-)^{1/2}P_-]q(p)$ with $Z_F^+=\tilde{Z}_+/U_{N_s1}^2$ and $Z_F^-=\tilde{Z}_-/V_{N_s,N_s}^2$. Since an explicit evaluation shows that $(I_+^d)_{N_s,N_s}=(I_-^d)_{N_s,N_s}\equiv I^d$, thus $\tilde{Z}_+=\tilde{Z}_-\equiv\tilde{Z}$, and $(U_{N_s1})^2=(V_{N_s,N_s})^2=1-w_0^2$, we finally obtain $Z_F^+=Z_F^-\equiv Z_F=\tilde{Z}/(1-w_0^2)$ where

$$\tilde{Z}=1-g^2C_2\frac{T}{2}+\frac{g^2}{16\pi^2}C_2\left(\log\pi^2+\frac{1}{2}-\log(\mu a)^2\right)+g^2I^d. \tag{54}$$

Here one unknown constant I^d is given by

$$I^d=C_2\int\frac{d^4l}{(2\pi)^4}\Bigg\{\frac{1}{32\sin^2 l/2}\sum_\mu\{\sin^2 l_\mu(\tilde G_R+\tilde G_L)+2\cos l_\mu[w_0-W(l)]\tilde G_R\}$$
$$+\sum_\mu\frac{\sin^2 l_\mu}{2(4\sin^2 l/2)^2}\Bigg[[w_0-W(l)]\tilde G_R+\Bigg(\sum_\nu\cos^2 l_\nu/2-2\cos^2 l_\mu/2\Bigg)\tilde G_L+\sum_\nu(\sin^2 l_\nu/2)\tilde G_R\Bigg]-\frac{1}{(l^2)^2}\theta(\pi^2-l^2)\Bigg\},\tag{55}$$

where

$$\tilde G_L=A\left[\tilde G-\frac{e^\alpha-W}{e^{-\alpha}-W}\frac{1}{(e^\alpha-w_0)^2}\right],$$

$$\tilde G_R=A\left[\tilde G-\frac{1}{(e^\alpha-w_0)^2}\right]$$

with

$$A=\frac{1-w_0^2}{2W\sinh\alpha},$$

$$\tilde G=\frac{\sinh\alpha_0-\sinh\alpha}{2w_0\sinh\alpha_0(\cosh\alpha_0-\cosh\alpha)},$$

and $e^{-\alpha_0}=w_0$. The numerical value of I^d is given in Table I at several values of $\tilde M$, together with the total one-loop renormalization factor Z_1 ($\tilde Z\equiv 1+g^2Z_1$) at $\mu a=1$ and the ratio of the nontadpole contribution $(Z_1)_{\rm nontad}\equiv I^d+(C_2/16\pi^2)(\log\pi^2+0.5)=I^d+0.02355$ to the total one. Note also that the tadpole contribution gives $Z_{\rm tad}\equiv -C_2T/2=-0.1031$. From this table, we see that I^d is small and depends on $\tilde M$ very weakly: The value $I^d=-0.01945$ at $\tilde M=0.05$ monotonically increases (decreases in the absolute value) to $I^d=-0.01222$ at $\tilde M=0.95$. Furthermore the nontadpole contribution $Z_{\rm nontad}$ is relatively small: 4% at $\tilde M=0.05$ and 12% at $\tilde M=0.95$, so that the tadpole contribution becomes dominant at all $\tilde M$. This justifies the use of the tree-level result with the tadpole improvement. Since $Z_1\simeq 0.1$, the one-loop correction to the Z factor is about 10% at $g^2\sim 1.0$.

V. MEAN FIELD ANALYSIS AT FINITE N_s

As seen in the previous sections, due to the presence of off-diagonal terms in the extra dimension, analysis of the one-loop correction to domain-wall quarks becomes too complicated to be easily applied to results of the numerical simulations, which should be performed on finite N_s. In this section we adopt an approximated but simpler method to analyze the effect of one-loop corrections at finite N_s. We call the method the mean field (MF) analysis since the link variable $U_{n,\mu}$ in the fermion action is simply replaced by the mean field u which is independent on n and μ. After this replacement the fermion propagator can be explicitly calculated with finite N_s and result is identical to the tree level one given in Appendix B with the replacement such that $x\rightarrow ux$ and $\cos p_\mu\rightarrow u\cos p_\mu$. In perturbation theory this is equivalent to the tree level analysis with the tadpole improvement [7], which has been shown in the previous section to give about 90% of the wave function renormalization factor at one-loop level.

Since we are interested in the zero mode at $s=1$, we set $s=t=1$ in the propagator. In this case the zero mode appears in $B_{--}e^{-2\alpha}$ of G_L, which is given at nonzero m_q by

$$B_{--}e^{-2\alpha}=\frac{(1-We^{-\alpha})(1-m_q^2)}{2W\sinh(\alpha)F},\tag{56}$$

where

$$F=We^\alpha-1+m_q^2(1-We^{-\alpha})-4m_q\cdot W\cdot\sinh(\alpha)e^{-\alpha N_s}$$
$$+e^{-2\alpha N_s}[1-We^{-\alpha}+m_q^2(We^\alpha-1)].\tag{57}$$

In the small momentum limit, this leads to

TABLE I. Value of I^d vs $\tilde M$, together with Z_1 and $(Z_1)_{\rm nontad}/Z_1$.

$\tilde M$	I^d	Z_1	$(Z_1)_{\rm nontad}/Z_1$
0.05	−0.01945(5)	−0.09897	0.041
0.10	−0.01871(5)	−0.09822	0.049
0.15	−0.01804(5)	−0.09756	0.056
0.20	−0.01744(5)	−0.09696	0.063
0.25	−0.01688(5)	−0.09640	0.069
0.30	−0.01636(5)	−0.09589	0.075
0.35	−0.01588(5)	−0.09541	0.080
0.40	−0.01544(5)	−0.09496	0.085
0.45	−0.01502(5)	−0.09454	0.090
0.50	−0.01463(5)	−0.09415	0.095
0.55	−0.01426(5)	−0.09378	0.099
0.60	−0.01392(5)	−0.09345	0.103
0.65	−0.01361(5)	−0.09313	0.107
0.70	−0.01332(5)	−0.09284	0.110
0.75	−0.01305(5)	−0.09257	0.113
0.80	−0.01281(5)	−0.09233	0.116
0.85	−0.01259(5)	−0.09211	0.119
0.90	−0.01239(5)	−0.09191	0.121
0.95	−0.01222(5)	−0.09174	0.124

$$\lim_{p^2\to 0} B_{--}e^{-2\alpha}=\frac{Z^{-1}}{p^2+m_F^2}\frac{(1-w_0^2)^2+p^2uw_0^2}{(1-w_0^2)^2+p^2u(1+w_0^2)}, \tag{58}$$

where $Z^{-1}=(1-m_q^2)/Au$, $m_F^2=B/Au$, and $w_0=1-M+4(1-u)=1-\tilde{M}$ with

$$A=\frac{1}{1-w_0^2}[1+m_q^2w_0^2-w_0(1-w_0^2)m_q^2+m_qw_0^{N_s}$$
$$\times\{2N_s(1-w_0^2)-1-w_0^2+2w_0(1-w_0^2)$$
$$-N_s(1-w_0^2)^2/w_0\}+w_0^{2N_s}\{w_0^2+m_q^2-2N_s(1-w_0^2)$$
$$-w_0(1-w_0^2)+N_s(1-w_0^2)^2/w_0\}],$$
$$B=(1-w_0^2)[m_q^2-2m_qw_0^{N_s}+w_0^{2N_s}].$$

Since the pole in the second factor of Eq. (58),

$$\frac{(1-w_0^2)^2+p^2uw_0^2}{(1-w_0^2)^2+p^2u(1+w_0^2)}, \tag{59}$$

is of the cutoff order and is larger than the physical pole in the first factor

$$\frac{Z^{-1}}{p^2+m_F^2}, \tag{60}$$

we neglect the second factor in the latter analysis.

Now we use the above formula to understand the behavior of the zero mode observed in Ref. [3]. For the value of u there are several choices. The tadpole diagram alone gives

$$u=1-g^2C_2T/2=1-0.10307g^2\simeq\exp[-0.10307g^2],$$

where we may take the bare coupling $2N_c/\beta$ or the renormalized coupling $g^2_{\overline{\rm MS}}(\pi/a)$ for g^2 in the above formula. Alternatively we may also use the ''observed'' value of u: $u=P^{1/4}$ where P is the average value of the plaquette normalized to unity. We adopt the latter one in our analysis. The configurations in Ref. [3] generated at $\beta=5.7$ and $m_qa=0.01$ by the dynamical Kogut-Susskind quark action give $P=0.5772$, which leads to $u=0.872$. In Ref. [3] two remarkable features are found for the zero mode: no zero mode is observed for $N_s=4$ and the zero mode is observed at $M=1.7$ but not at $M\leqslant 1.0$ for $N_s=10$. To explain these we calculate m_F as a function of M for both $N_s=4$ and 10 at m_q = 0, 0.01, 0.02, 0.03, and plot the results in Fig. 2, where solid lines are for $N_s=4$ and dashed lines for $N_s=10$. Four lines for each N_s correspond to $m_q=0$, 0.01, 0.02, 0.03 from below to above around $M=1.5$. The result tells us the followings. The allowed range for the light fermion is very narrow for $N_s=4$ (roughly $1.4<M<1.6$). This may be the reason why the light state could not be found in the simulation [3]. Note that the allowed range for the zero mode is $0.512<M<2.512$ in the $N_s\to\infty$ limit. Although the allowed range becomes larger for $N_s=10\,(1.1<M<1.9)$, no light state appears at $M\leqslant 1.0$, as observed in the simulation. Furthermore the order of the fermion mass m_F is reversed to the order of the current quark mass m_q at $M\leqslant 1.0$: m_F is largest at $m_q=0$. The plot also supports the fact that the zero mode is observed at $M=1.7$ in the simulation.

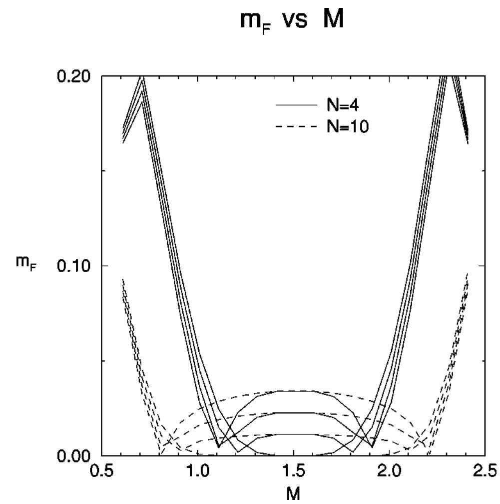

FIG. 2. The fermion mass m_F obtained in the mean-field approximation as a function of M for $N_s=4$ (solid lines) and $N_s=10$ (dashed line), at $m_q=$ 0,0.01,0.02,0.03 from below to above around $M=1.5$.

As seen in the above the behavior of the numerical simulation is understandable by the MF analysis, which can supply useful informations on the tuning of parameters in numerical simulations such as N_s, M, or m_q beforehand. For example, we may take $N_s=4$ for the simulations, which reduces the cost of both CPU time and memory a lot, if M is appropriately chosen ($M\simeq 1.5$ for $U=0.872$). Here we have to stress again that this tuning is not a fine-tuning, which is necessary in the case of the Wilson fermion, for massless quarks.

VI. CONCLUSION AND DISCUSSION

In this paper we calculated one-loop correction to the fermion propagator in the massless lattice QCD formulated via domain-wall fermions. We showed that the zero mode is stable against the one-loop correction: no additative counterterm to the quark mass is generated in the large N_s limit, and no fine-tuning of M is necessary to obtain massless quarks. This property is very different from and superior to the ordinary Wilson fermion formulation. We explicitly calculated the wave-function renormalization factor for the massless quarks and show that the tadpole contribution becomes dominant at all $\tilde{M}$. We also adopted the mean-field analysis to this model, demonstrating that it can qualitatively explain data obtained in the numerical simulation [3].

Although our results strongly indicate that the domain-wall QCD can avoid the fine-tuning problem of the quark mass, the mechanism which gives the zero mode in this formulation has not been fully understood yet. Since our proof for the stability of the zero mode contains an explicit calculation at one loop $[(\widetilde{W_1})_{Ns,N_s}=0]$, it cannot be easily carried over to higher orders. The result of numerical simulation [3] suggests that the zero mode is also stable against the nonperturbative dynamics. There may be a yet unknown symmetry which ensures the existence of zero mode in the large N_s limit. Finding such a symmetry is important for our understanding of the formulation.[1]

In this paper only the wave-function renormalization factor is explicitly evaluated. Based on the method developed in this paper, it is possible to calculate more complicated quantities such as renormalization factors for the quark mass, currents, and four-fermi operators, which are necessary to get the continuum physics from numerical simulations. The results of this paper also suggest that the smeared quark operator $q^{\rm smear}=\Sigma_s(w_0^s P_+\psi_s+w_0^{N_s-s}P_-\psi_s)$ may give better signals than $q=P_+\psi_1+P_-\psi_{N_s}$ does, since it has a larger overlap to zero modes.

Note added: After this work was completed, there appeared a new paper [10], in which the stability of the zero mode is generally considered.

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APPENDIX A: ACTION AND FEYNMAN RULES

The gauge part of the action is exactly same as that of the ordinary lattice QCD action [6]:

$$S_{\rm gauge}=\sum_n \sum_{(\mu\nu)} -\frac{\beta}{N_s}{\rm Re}\ \ {\rm tr}(U_{n,\mu}U_{n+\hat\mu,\nu}U^\dagger_{n+\hat\nu,\mu}U^\dagger_{n,\nu}), \tag{A1}$$

$$S_{\rm GF}=\sum_n \frac{1}{2\alpha}\left[\nabla_\mu A^a_\mu\left(n+\frac{1}{2}\hat\mu\right)\right]^2, \tag{A2}$$

$$S_{\rm FP}=\sum_{n,\mu}(\overline{c}^a_{n+\hat\mu}-\overline{c}^a_n)\left\{c^b_{n+\hat\mu}E^{-1}_{ba}\left[gA_\mu\left(n+\frac{1}{2}\hat\mu\right)\right]\right.$$
$$\left.-E^{-1}_{ab}\left[gA_\mu\left(n+\frac{1}{2}\hat\mu\right)\right]c^b_n\right\}, \tag{A3}$$

$$S_{\rm measure}=-\frac{1}{2}\sum_n\sum_\mu {\rm tr}\ln$$
$$\times\left(\frac{1-\cos\{gA^c_\mu[n+(1/2)\hat\mu 1]{\rm ad}(T^c)\}}{\{gA^c_\mu[n+(1/2)\hat\mu]{\rm ad}(T^c)\}^2}\right)_{ab}, \tag{A4}$$

where g is the coupling of the SU(N_c) gauge, $\beta=2N_c/g^2$. α is the gauge parameter. The actions $S_{\rm FP}$ and $S_{\rm measure}$ is not needed in our calculation at one-loop level.

The momentum representation of gauge part is

$$S_{\rm gauge}+S_{\rm GF}=\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}A^a_\mu(-p)$$
$$\times\left[4\sin^2\frac{p}{2}\delta_{\mu\nu}-\left(1-\frac{1}{\alpha}\right)4\sin\frac{p_\mu}{2}\sin\frac{p_\nu}{2}\right]$$
$$\times A^a_\nu(p)+\cdots, \tag{A5}$$

here the ellipsis denotes the gluon self-interactions which do not come into play in our calculation.

The fermion-gauge interaction terms in the momentum representation is

$$S_{\rm int}=\sum_{n=1}^\infty\int\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4l_1}{(2\pi)^4}\cdots$$
$$\times\frac{d^4l_n}{(2\pi)^4}(2\pi)^4\delta^4(k+p+l_1+\cdots+l_n)$$
$$\times\frac{i^n}{n!}g^nA^{a_1}_\mu(l_1)\cdots A^{a_n}_\mu(l_n)\overline{\psi}(k)_sT^{a_1}\cdots T^{a_n}$$
$$\times\left[\frac{\gamma_\mu}{2}[e^{i/2(p_\mu-k_\mu)}-(-)^ne^{-i/2(p_\mu-k_\mu)}]\right.$$
$$\left.-\frac{r}{2}[e^{i/2(p_\mu-k_\mu)}+(-)^ne^{-i/2(p_\mu-k_\mu)}]\right]\psi(p)_s. \tag{A6}$$

The domain-wall fermion propagator was already given by Eq. (12).

The fermion gluon interaction vertices are given by Eq. (A6). Although there are an infinite number of interactions in lattice perturbation theory, only two of them are needed for the present purpose. One of them is the fermion interaction vertex with one gluon field, which is given by

$$V_1(k,p;l,a;\mu)=-igT^a\left\{\gamma_\mu\cos\frac{1}{2}(-k_\mu+p_\mu)\right.$$
$$\left.-ir\sin\frac{1}{2}(-k_\mu+p_\mu)\right\}. \tag{A7}$$

The other is the vertex with two gluon fields, given by

[1] After this work was completed, it was pointed out that the Ginsparg-Wilson relation [8] implies an exact symmetry to forbid the current quark mass term [9]. The unknown symmetry may be related to it.

$$V_2(k,p;l_1,a,l_2,b;\mu)$$
$$=\frac{1}{2}g^2\frac{1}{2}\{T^a,T^b\}\left\{i\gamma_\mu \sin\frac{1}{2}(-k_\mu+p_\mu)\right.$$
$$\left.-r\cos\frac{1}{2}(-k_\mu+p_\mu)\right\}\delta_{\mu\nu}. \tag{A8}$$

The gluon propagator is given by

$$G^{ab}_{\mu\nu}(p)=\frac{1}{4\sin^2 p/2}\left[\delta_{\mu\nu}-(1-\alpha)\frac{4\sin p_\mu/2\sin p_\nu/2}{4\sin^2 p/2}\right]\delta_{ab}. \tag{A9}$$

We set $\alpha=1$ in this paper.

APPENDIX B: DERIVATION OF FREE FERMION PROPAGATOR

In this appendix we derive the free fermion propagator, used in the text. For later use in perturbative analyses of this model, nonzero current quark mass m_q for finite N_s is considered. See also Refs. [4,11,12]. We also derive the propagator with Majorana mass terms, which becomes important for the lattice definition of the $N=1$ supersymmetric model via domain-wall fermions [13,14].

1. Propagator with nonzero m_q

The free fermion propagator has the following form:

$$S_{\rm F}(p)_{s,t}=[(-i\gamma_\mu\sin p_\mu+W_m^-)G_R(s,t)P_+$$
$$+(-i\gamma_\mu\sin p_\mu+W_m^+)G_L(s,t)P_-]_{st},$$

where

$$G_R(s,t)\equiv\left(\frac{1}{\sin^2 p+W_m^+W_m^-}\right)_{st}$$

and

$$G_L(s,t)\equiv\left(\frac{1}{\sin^2 p+W_m^-W_m^+}\right)_{st}$$

with

$$(W_m^+)_{s,t}=(W^+)_{s,t}+m_q\delta_{s,N_s}\delta_{t,1}$$

and

$$(W_m^-)_{s,t}=(W^-)_{s,t}+m_q\delta_{s,1}\delta_{t,N_s}. \tag{B1}$$

We first consider G_R. The following equation is satisfied for G_R:

$$\sum_t [(x+W^+W^-)_{s,t}+m_q(W_{s1}^+\delta_{tN_s}+\delta_{s,N_s}W_{1t}^-)$$
$$+m_q^2\delta_{s,N_s}\delta_{tN_s}]G_R(t,u)=\delta_{su} \tag{B2}$$

with $x=\sin^2 p$. Therefore, except $s=N_s$ or 1, this equation is satisfied by

$$G_R(s,t)=G(s,t)+A_{++}e^{\alpha(s+t)}+A_{+-}e^{\alpha(s-t)}$$
$$+A_{-+}e^{\alpha(-s+t)}+A_{--}e^{\alpha(-s-t)}, \tag{B3}$$

where

$$G(s,t)=A(e^{\alpha(N_s-|s-t|)}+e^{-\alpha(N_s-|s-t|)}) \tag{B4}$$

becomes a special solution to the equation $(x+W^+W^-)G_R=1$, with

$$\cosh\alpha\equiv\frac{1+W^2+x}{2W},$$

$$A\equiv\frac{1}{2W\sinh\alpha}\frac{1}{2\sinh(\alpha N_s)}, \tag{B5}$$

and other terms are general solutions to the equation $(x+W^+W^-)G_R=0$. We can fix their coefficients $A_{\pm\pm}$ by a boundary condition at $s=1$:

$$(x+W^2+1)G_R(1,t)-W\cdot G_R(2,t)$$
$$-W\cdot m_q\cdot G_R(N_s,t)=\delta_{1t}, \tag{B6}$$

which is simplified to

$$G_R(0,t)-m_qG_R(N_s,t)=0, \tag{B7}$$

and another boundary condition at $s=N_s$:

$$(x+W^2)G_R(N_s,t)-W\cdot G_R(N_s-1,t)$$
$$-W\cdot m_q\cdot G_R(1,t)+m_q^2G_R(N_s,t)=\delta_{N_s,t}, \tag{B8}$$

which is reduced to

$$G_R(N_s,t)-W\cdot G_R(N_s+1,t)$$
$$+W\cdot m_qG_R(1,t)-m_q^2G_R(N_s,t)=0. \tag{B9}$$

Plugging Eq. (B3) into Eqs. (B7) and (B9) leads to

$$\begin{pmatrix} 1-m_q e^{\alpha N_s} & 1-m_q e^{-\alpha N_s} \\ e^{\alpha N_s}(1-We^{\alpha}-m_q^2+Wm_q e^{\alpha(1-N_s)}) & e^{-\alpha N_s}(1-We^{-\alpha}-m_q^2+Wm_q e^{\alpha(N_s-1)}) \end{pmatrix}\begin{pmatrix} A_{++} & A_{+-} \\ A_{-+} & A_{-}- \end{pmatrix}$$
$$=-A\begin{pmatrix} e^{-\alpha N_s}-m_q & e^{\alpha N_s}-m_q \\ 1-We^{-\alpha}-m_q^2+Wm_q e^{-\alpha(N_s+1)} & (1-We^{\alpha}-m_q^2+Wm_q e^{\alpha(N_s+1)}) \end{pmatrix}. \tag{B10}$$

Solving this we obtain

$$\begin{pmatrix} A_{++} \\ A_{-+} \end{pmatrix}=\frac{A}{F}\begin{pmatrix} (e^{-2\alpha N_s}-1)(1-We^{-\alpha})(1-m_q^2) \\ 2W\ \sinh(\alpha)[1-2m_q\cosh(\alpha N_s)+m_q^2] \end{pmatrix},$$

$$\begin{pmatrix} A_{+-} \\ A_{--} \end{pmatrix}=\frac{A}{F}\begin{pmatrix} 2W\ \sinh(\alpha)[1-2m_q\cosh(\alpha N_s)+m_q^2] \\ (1-e^{2\alpha N_s})(1-We^{\alpha})(1-m_q^2) \end{pmatrix},$$

where

$$F=e^{\alpha N_s}[1-We^{\alpha}+m_q^2(We^{-\alpha}-1)] +4Wm_q\ \sinh(\alpha)+e^{-\alpha N_s}[We^{-\alpha}-1+m_q^2(1-We^{\alpha})]. \tag{B11}$$

Similarly, plugging the general solution for G_L

$$G_L(s,t)=G(s,t)+B_{++}e^{\alpha(s+t)}+B_{+-}e^{\alpha(s-t)} +B_{-+}e^{\alpha(-s+t)}+B_{--}e^{\alpha(-s-t)} \tag{B12}$$

into the boundary conditions

$$G_L(N_s+1,t)-m_q G_L(1,t)=0, \tag{B13}$$

$$G_L(1,t)-W\cdot G_L(0,t)+W\cdot m_q G_L(N_s,t)-m_q^2 G_L(1,t)=0, \tag{B14}$$

we finally obtain

$$\begin{pmatrix} B_{++} \\ B_{-+} \end{pmatrix}=\frac{A}{F}\begin{pmatrix} (e^{-2\alpha N_s}-1)e^{-\alpha}(e^{-\alpha}-W)(1-m_q^2) \\ 2W\ \sinh(\alpha)[1-2m_q\cosh(\alpha N_s)+m_q^2] \end{pmatrix},$$

$$\begin{pmatrix} B_{+-} \\ B_{--} \end{pmatrix}=\frac{A}{F}\begin{pmatrix} 2W\ \sinh(\alpha)[1-2m_q\cosh(\alpha N_s)+m_q^2] \\ (1-e^{2\alpha N_s})e^{\alpha}(e^{\alpha}-W)(1-m_q^2) \end{pmatrix}.$$

2. Propagator with the Majorana mass term at N_s

For an application of the free fermion propagator obtained in the domain-wall model, we consider a model with the Majorana mass term on the antiboundary at $s=N_s$, which has been proposed for a lattice definition of the $N=1$ super-Yang-Mills theory [13,14]. In this subsection we derive the fermion propagator with the Majorana mass term, though some of the results have already been used in Ref. [14]. We set $m_q=0$ hereafter.

A free fermion action of the model with the Majorana mass m_0 can be written in momentum space as

$$S=\frac{1}{2}\overline{\Psi}(-p)_s D_{s,t}(p)\Psi(p), \tag{B15}$$

where

$$\Psi_s(p)=[\psi_s(p),\overline{\psi}_s(p)],\quad \overline{\Psi}_s(p)=\begin{pmatrix} \overline{\psi}_s(p) \\ \psi_s(p) \end{pmatrix}, \tag{B16}$$

and

$$D(p)=T_0(p)+m_0X =\begin{pmatrix} D_0(p) & 0 \\ 0 & -D_0(-p)^T \end{pmatrix}+m_0\delta^2 P_+ IP_- \tag{B17}$$

with $(\delta^2)_{s,t}\equiv\delta_{s,N_s}\delta_{N_s,t}$, and

$$P_+=\begin{pmatrix} P_+ & 0 \\ 0 & P_- \end{pmatrix},\quad P_-=\begin{pmatrix} P_- & 0 \\ 0 & P_+ \end{pmatrix},$$

$$I=\begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 & \sigma_2 & 0 \\ 0 & 0 & 0 & \sigma_2 \\ \sigma_2 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \end{pmatrix}$$

in terms of 8×8 matrices. Here

$$D_0(p)=i\gamma_\mu\ \sin p_\mu+W^+P_++W^-P_-, \tag{B18}$$

is an inverse of the massless free fermion propagator in the domain-wall QCD.

By expanding D^{-1} in m_0 and rearranging it we obtain

$$D^{-1}=\sum_{n=0}^{\infty}(-T_0^{-1}m_0P_+IP_-\delta^2)^n T_0^{-1} =\sum_{n=0}^{\infty}(-m_0)^n T_0^{-1}\delta P_+ Z^{n-1}IP_-\delta T_0^{-1}, \tag{B19}$$

where $Z=IP_-\delta T_0^{-1}\delta P_+$.

Using $Z^2=-x[G_R(p)_{N_s,N_s}]^2P_+$ and summing over n, we finally get

$$D^{-1}=T_0^{-1}+[-m_0T_0^{-1}\delta P_+IP_-\delta T_0^{-1}+m_0^2T_0^{-1}\delta P_+ZIP_-\delta T_0^{-1}]\frac{1}{1+m_0^2x[G_R(p)_{N_s,N_s}]^2}. \tag{B20}$$

Explicitly this formula gives, in terms of 2×2 block notations,

$$\begin{aligned}D(p)_{11}^{-1}&=-D(-p)_{22}^{-1}=\langle\psi(p)\overline{\psi}(-p)\rangle\\&=(-i\gamma_\mu\sin p_\mu)[Z_+(p)P_++Z_-(p)P_-]\\&\quad+M_+(p)P_++M_-P_-,\end{aligned} \tag{B21}$$

where

$$Z_+(p)_{s,t}=G_R(p)_{s,t}-\frac{m_0^2xG}{1+m_0^2xG^2}G_R(p)_{s,N_s}G_R(p)_{N_s,t},$$

$$Z_-(p)_{s,t}=G_L(p)_{s,t}+\frac{m_0^2G}{1+m_0^2xG^2}\times[W^-G_R(p)]_{s,N_s}[G_R(p)W^+]_{N_s,t},$$

$$M_+(p)_{s,t}=[W^-G_R(p)]_{s,t}-\frac{m_0^2xG}{1+m_0^2xG^2}\times[W^-G_R(p)]_{s,N_s}G_R(p)_{N_s,t},$$

$$M_-(p)_{s,t}=[W^+G_L(p)]_{s,t}-\frac{m_0^2xG}{1+m_0^2xG^2}\times G_R(p)_{s,N_s}[G_R(p)W^+]_{N_s,t}$$

with $G\equiv G_R(p)_{N_s,N_s}$. Similarly

$$\begin{aligned}[D(p)_{12}^{-1}]_{s,t}&=\langle\psi(p)_s\psi(-p)_t\rangle\\&=\frac{m_0}{1+m_0^2xG^2}[xG_R(p)_{s,N_s}G_R(p)_{N_s,t}I_2P_-\\&\quad-i\sin p_\mu\gamma_\mu G_R(p)_{s,N_s}[G_R(p)W^+]_{N_s,t}I_2P_+\\&\quad+i\sin p_\mu\gamma_\mu[W^-G_R(p)]_{s,N_s}G_R(p)_{N_s,t}I_2P_-\\&\quad+[W^-G_R(p)]_{s,N_s}[G_R(p)W^+]_{N_s,t}I_2P_+],\end{aligned} \tag{B22}$$

and

$$\begin{aligned}[D(p)_{21}^{-1}]_{s,t}&=\langle\overline{\psi}(p)_s\overline{\psi}(-p)_t\rangle\\&=\frac{m_0}{1+m_0^2xG^2}[xG_R(p)_{s,N_s}G_R(p)_{N_s,t}I_2P_+\\&\quad+i\sin p_\mu\gamma_\mu^TG_R(p)_{s,N_s}[G_R(p)W^+]_{N_s,t}I_2P_-\\&\quad-i\sin p_\mu\gamma_\mu^T[W^-G_R(p)]_{s,N_s}G_R(p)_{N_s,t}I_2P_+\\&\quad+[W^-G_R(p)]_{s,N_s}[G_R(p)W^+]_{N_s,t}I_2P_-].\end{aligned} \tag{B23}$$

See Ref. [14] for an application of this result.

APPENDIX C: PROPERTIES OF DIAGONALIZATION MATRICES

In this appendix we derive several properties of diagonalization matrices U and V which are used for the renormalization of quarks fields. Let us consider the tree level diagonalization of matrices $(W_0^{\mp}\cdot W_0^{\pm})$. To diagonalize $(W_0^{\mp}\cdot W_0^{\pm})$, we have to solve the eigenvalue problems $(W_0^{\mp}\cdot W_0^{\pm})_{s,t}\phi_\pm^i(t)=(M_0^2)_i\phi_\pm^i(s)$, then U_0 and V_0 are given by normalized eigenvectors $\phi_\pm:(U_0)_{s,t}=\phi_+^s(t)$ and $(V_0)_{s,t}=\phi_-^s(t)$. The two eigenstate equations lead to the same equation

$$-w_0[\phi_\pm^i(s+1)+\phi_\pm^i(s-1)]+[1+w_0^2-(M_0^2)_i]\phi_\pm^i(s)=0, \tag{C1}$$

but with different boundary conditions

$$-w_0\phi_+^i(0)+\phi_+(1)=0,\quad \phi_+^i(N_s+1)=0 \tag{C2}$$

or

$$-w_0\phi_-^i(N_s+1)+\phi_-(N_s)=0,\quad \phi_-^i(0)=0. \tag{C3}$$

Therefore, once $\phi_+^i(s)$ is known, the other is easily obtained through $\phi_-^i(s)=\phi_+^i(N_s+1-s)$. Hereafter we consider $\phi_+^i(s)$ only and drop the suffices $+$ and i.

There are two types of solutions to the eigenstate equation. For $(M_0^2)_i\leqslant(1-w_0)^2$ we have a damping solution $\phi(s)=Ae^{-\alpha s}$ with $\cosh\alpha=[1+w_0^2-(M_0^2)_i]/2w_0$. The first boundary condition leads to $e^{-\alpha}=w_0$. This implies $(M_0^2)=0$, and therefore $\phi(s)$ is nothing but the zero mode solution of the domain-wall QCD. For this solution w_0 should satisfy $w_0^2\leqslant 1$ $(0\leqslant M\leqslant 2)$. The other boundary condition can be satisfied in the large N_s limit. The normalization constant becomes $A=(1-w_0^2)^{1/2}$. Note that there are no other damping solutions which satisfy the first boundary condition.

If the eigenvalue is in the region $(1-w_0)^2\leqslant(M_0^2)_i\leqslant(1+w_0)^2$, we have an oscillating solution $\phi(s)=Ae^{i\alpha s}+Be^{-i\alpha s}$ with $\cos\alpha=[1+w_0^2-(M_0^2)_i]/2w_0$. The two boundary conditions imply

$$\begin{pmatrix} e^{i\alpha}-w_0 & e^{-i\alpha}-w_0 \\ e^{i\alpha(N_s+1)} & e^{-i\alpha(N_s+1)} \end{pmatrix} \times \begin{pmatrix} A \\ B \end{pmatrix} = 0. \tag{C4}$$

The existence of the nontrivial solution requires $w_0 \sin \alpha(N_s+1) = \sin \alpha N_s$, which leads to $\phi(s) = -Ae^{i\alpha(N_s+1)} \sin \alpha(N_s+1-s) \equiv A_0 \sin \alpha(N_s+1-s)$. Without loss of generality we can take real A_0, and the normalization condition gives $A_0=(2/N_s)^{1/2}[1+O(1/N_s)]$. Setting $\alpha=a/N_s$ we reduce the equation for α to $w_0 \sin a = \sin a$ in the large N_s limit. The solutions $a=\pi n$ with integer n to this equation is translated to N_s-1 independent solutions: $\alpha = \pi n/N_s$ with $n=1,2,\ldots,N_s-1$. (Note that $0 \leqslant \alpha \leqslant \pi$ since $\sin \alpha \geqslant 0$.) Therefore, all eigenvalues and eigenvectors are now obtained, giving

$$[U_0]_{s,t} = \begin{cases} (2/N_s)^{1/2} \sin \alpha_s(N_s+1-t) & s \neq N_s, \\ (1-w_0^2)^{1/2} w_0^{(t-1)} & s = N_s, \end{cases} \tag{C5}$$

where $\alpha_s = \pi s/N_s$, and $[V_0]_{s,t} = [U_0]_{s,N_s+1-t}$.

Next we prove some properties of U_0 and V_0. It is noted that U_0 and V_0 can also diagonalize $W_0^{\pm}$:

$$(V_0 W_0^+ U_0^\dagger)_{s,t} = \delta_{s,t} f_s, \tag{C6}$$

where

$$f_s = \begin{cases} w_0 \cos \alpha_s(N_s+1) - \cos \alpha_s N_s & s \neq N_s, \\ 0 & s = N_s. \end{cases} \tag{C7}$$

Using the equation for α_s $(s \neq N_s)$ we can show

$$\begin{aligned} f_s^2 &= w_0^2 + 1 - 2w_0[\sin \alpha_s N_s \sin \alpha(N_s+1) \\ &\quad + \cos \alpha_s N_s \cos \alpha_s(N_s+1)] \\ &= w_0^2 + 1 - 2w_0 \cos \alpha_s = (M_0^2)_s. \end{aligned} \tag{C8}$$

This proves $f_s = (M_0)_s$ for all s.

It is also important to note that $U_0\,(V_0)$ diagonalizes $I_{\log}^+\,(I_{\log}^-)$ terms, since

$$\begin{aligned} (U_0 w_0^{s+t-2} U_0^\dagger)_{s,t} &= \delta_{s,t}\delta_{s,N_s}(1-w_0^2) \\ &\quad \times \sum_{s,t} w_0^{2(s+t-2)} + O(1/N_s) \\ &= \delta_{s,t}\delta_{s,N_s}(1-w_0^2)^{-1} + O(1/N_s). \end{aligned} \tag{C9}$$

Now let us consider quantities including g^2 contributions. We first show that U and V diagonalize $\overline{W}^{\pm}$ at this order:

$$\begin{aligned} V \cdot \overline{W}^+ \cdot U^\dagger &= (1+g^2V_1)V_0(W_0^+ + g^2 W_1^+)U_0^\dagger(1+g^2U_1^\dagger) \\ &= V_0 \cdot W_0^+ U_0^\dagger + g^2\{V_1 V_0 W_0^+ U_0^\dagger \\ &\quad + V_0 W_0 U_0^\dagger U_1^\dagger + \tilde{W}_1{}^+\}, \end{aligned} \tag{C10}$$

where the coefficient of the g^2 term is simplified to

$$\begin{aligned} &(V_1)_{s,t}(M_0)_t + (M_0)_s(U_1^\dagger)_{st} + (\tilde{W}_1{}^+)_{s,t} \\ &= (\tilde{W}_1{}^+)_{s,t}\left(1 + \frac{(M_0^2)_t - (M_0^2)_s}{(M_0^2)_s - (M_0^2)_t}\right) = 0 \end{aligned} \tag{C11}$$

for $s \neq t$, and becomes $(\tilde{W}_1{}^+)_{s,s}$ for $s=t$. Equation (C10) then becomes

$$V \cdot \overline{W}^+ \cdot U^\dagger = (M_0 + g^2 \tilde{W}_1{}^+)\mathbf{1}.$$

It is necessary for the stability of the zero mode to show that $(\tilde{W}_1{}^+)_{N_s,N_s}=0$. This can be proven as follows:

$$(\tilde{W}_1{}^+)_{N_s,N_s} = (1-w_0^2)\sum_{s,t} w_0^{N_s-s}(W_1^+)_{s,t} w_0^{t-1}, \tag{C12}$$

where W_1^+ is composed of the sum G_L, G_R, $W_0^+ G_L$ and $W_0^- G_R$ but

$$\sum_{s,t} w_0^{N_s-s} G(s,t) w_0^{t-1} = O(N_s w_0^{N_s}) \to 0$$

for all $G=G_L$, G_R, $W_0^+ G_L$ and $W_0^- G_R$ in the large N_s limit. Therefore Eq. (C12) vanishes.

For the wave-function renormalization factor we have to know

$$U_{N_s,1} = V_{N_s,N_s} = (U_0)_{N_s,1} + g^2 \sum_{t \neq N_s} (U_1)_{N_s,t}(U_0)_{t,1}.$$

Fortunately, since $(U_0)_{t,1} = (2/N_s)^{1/2} \sin \alpha_t(1-1) = 0$, there is no order g^2 contribution and it becomes $U_{N_s,1} = (1-w_0^2)^{1/2}$.

Finally we would like to evaluate $I_{\pm}^d$ from $I_{\text{finite}}^{\pm}$. If we define

$$\langle F(s,t)\rangle_U \equiv \sum_{s,t} (U_0)_{N_s,s} F(s,t)(U_0)_{N_s,t},$$

and

$$\langle F(s,t)\rangle_V \equiv \sum_{s,t} (V_0)_{N_s,s} F(s,t)(V_0)_{N_s,t},$$

we can show

$$\langle e^{-\alpha|s-t|}\rangle_{U,V} = (1-w_0^2)\frac{\sinh \alpha_0 - \sinh \alpha}{2w_0 \sinh \alpha_0(\cosh\alpha_0 - \cosh\alpha)}$$

with $e^{-\alpha_0} = w_0$,

$$\langle e^{-\alpha(s+t-2)}\rangle_U = \langle e^{-\alpha(2N_s-s-t)}\rangle_V = (1-w_0^2)\frac{e^{2\alpha}}{(e^{\alpha}-w_0)^2},$$

and

$$\langle e^{-\alpha(s+t-2)}\rangle_V = \langle e^{-\alpha(2N_s-s-t)}\rangle_U = 0.$$

Using these formulas we obtain

$$\langle G_L(s,t)\rangle_U=\frac{1-w_0^2}{2W\sinh\alpha}\left[\frac{\sinh\alpha_0-\sinh\alpha}{2w_0\sinh\alpha_0(\cosh\alpha_0-\cosh\alpha)}-\frac{e^{\alpha}-W}{e^{-\alpha}-W}\frac{1}{(e^{\alpha}-w_0)^2}\right]=\langle G_R(s,t)\rangle_V\equiv\tilde{G}_L, \tag{C13}$$

$$\langle G_R(s,t)\rangle_U=\frac{1-w_0^2}{2W\sinh\alpha}\left[\frac{\sinh\alpha_0-\sinh\alpha}{2w_0\sinh\alpha_0(\cosh\alpha_0-\cosh\alpha)}-\frac{1}{(e^{\alpha}-w_0)^2}\right]=\langle G_L(s,t)\rangle_V\equiv\tilde{G}_R, \tag{C14}$$

and

$$\langle W_0^+G_L(s,t)\rangle_{U,V}=\langle W_0^-G_R(s,t)\rangle_{U,V}=(w_0-W)\tilde{G}_R.$$

The explicit expression for $I^{\pm}_{\rm finite}$ is reduced to the final result in terms of $\tilde{G}_{L/R}: I^d_+=I^d_-\equiv I^d$ where

$$I^d=C_2\int\frac{d^4l}{(2\pi)^4}\left\{\frac{1}{32\sin^2 l/2}\sum_\mu\{\sin^2 l_\mu(\tilde{G}_R+\tilde{G}_L)+2\cos l_\mu[w_0-W(l)]\tilde{G}_R\}\right.$$
$$\left.+\sum_\mu\frac{\sin^2 l_\mu}{2(4\sin^2 l/2)^2}\left[[w_0-W(l)]\tilde{G}_R+\left(\sum_\nu\cos^2 l_\nu/2-2\cos^2 l_\mu/2\right)\tilde{G}_L+\sum_\nu(\sin^2 l_\nu/2)\tilde{G}_R\right]-\frac{1}{(l^2)^2}\theta(\pi^2-l^2)\right\}. \tag{C15}$$

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