DCC pion production from classical sources

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We propose a scenario of DCC (disoriented chiral condensate) pion production. As the simplest realization of this scenario we consider the linear sigma model $O(2)$ in two dimensions. Through a numerical evolution of the classical equation of motion we evaluate the energy exchange between the sigma and the pion fields. Using our simulations we estimate the number and spectrum of emitted pions. [S0556-2821(99)03801-1]

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I. INTRODUCTION

Recently, there has been a lot of interest in the phenomena of coherent pion production from classical sources ([1], for a review see e.g. [2,3]). It is expected that at some moment of the ion-ion or hadron-hadron collision a classical pion field is produced. This field exists in a finite volume over a finite period of time and acts as a source of the Klein-Gordon equation for the pion field. The quantization of such a system leads to states of the pion field quanta [4].

In the standard picture of heavy ion collision there are two regions of baryon number density concentration which recede from the collision center at almost the speed of light. In between we expect a place where many interesting phenomena can happen: a chiral phase transition or color deconfinement, for example. A collision of heavy ions is obviously a nonequilibrium process; therefore one can expect large fluctuations of energy density. If in some regions the local energy density is high enough, the quark-gluon plasma can be created. The opposite situation may also be of considerable importance. Let us assume that after some time, after a collision, there are regions cold enough that we can use to their description low energy degrees of freedom, namely chiral fields. For instance, such regions may be created after a rapid quark-gluon plasma expansion and cooling. Additionally let us suppose that these regions are domains of a false vacuum with the orientation of the chiral vector different from that of a usual (''true'') vacuum. In the context of chiral fields one usually uses the linear [5,6,7] or nonlinear sigma model [4,8,9]. These models describe well the low energy pion interactions. The potential of linear sigma model with explicit symmetry breaking term, given by the formula

$$V(\sigma,\vec{\pi})=\frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2-f_\pi m_\pi^2\sigma, \tag{1}$$

possesses a unique ground state. In this state the chiral vector takes the form

$$\Phi_0=(\sigma,\vec{\pi})=(f_\pi,\vec{0}). \tag{2}$$

In the absence of the pion field the potential possesses also another local minimum for which the energy is larger then that of the ground state, because of the explicit symmetry breaking term. As a domain of a false vacuum we understand the region of space where the sigma field sits in the local minimum of the potential. This region is connected with the outside true vacuum by the domain wall in which a substantial part of the energy is stored. The volume energy of the domain is proportional to its size and to the pion mass squared. Thus in the massless pion case the false vacuum becomes also the true vacuum. Nevertheless, even in this case the domain still exists since the ground state is degenerated and the inside and outside ''vacua'' can differ from each other.

Let us consider the situation that a small amount of the pion field appears inside the bubble of the false vacuum. We expect that during the evolution the inside false vacuum relaxes back to its proper outside value through the dissipation of the domain walls. The energy released in this process is transferd to the pion field and stored in the long wavelength modes. The final state of such system should consist of the sigma field oscillating with a small amplitude around its vacuum state and the amplified pion field. This field becomes a classical source of pions quanta. The strategy of described above scenario is as follows: (1) During the collision the domains of the false vacuum with a small amount of the pion field inside are created—this is our initial condition; (2) the subsequent classical evolution amplifies the pion modes through the decay of the false vacua; (3) We assume that there exists a decoupling time after which the pion and sigma fields decouple from each other; (4) After the decoupling time, the produced pion field becomes a classical pion source and one can compute the number and spectrum of produced pions.

In this paper we propose a simple, nontrivial realization of the above scenario. We consider the linear sigma model in two dimensions. We solve the classical field equation for the sigma and pion fields for a certain class of initial conditions. We follow the energy flow between the sigma and the pion fields. Using our simulation we estimate the number and the main features of the pion spectrum.

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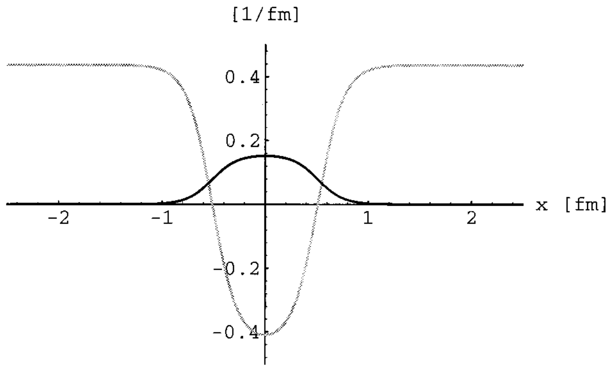

FIG. 1. The initial configuration of σ (gray curve) and π (black curve) fields for the total energy $E=400$ MeV.

In Sec. II we explain the initial configuration. In Secs. III, IV we discuss the decay of the false vacuum for massless and massive pion cases. The last section contains a simple model for disoriented chiral condensate (DCC) production.

II. INITIAL CONFIGURATION

We analyze the equations of the linear sigma-model in $1+1$ dimension that consists of two real fields σ and π:

$$\ddot{\sigma}(t,x)-\sigma''(t,x)=\lambda(v^2-\sigma^2(t,x)-\pi^2(t,x))\sigma(t,x)+H, \tag{3}$$

$$\ddot{\pi}(t,x)-\pi''(t,x)=\lambda(v^2-\sigma^2(t,x)-\pi^2(t,x))\pi(t,x),$$

where dot and prime denote the derivatives with respect to time and spatial coordinate, respectively.

We choose the same values of parameters as in paper [6]: $v=87.4$ MeV, $H=(119\text{ MeV})^3$ and $\lambda=20$. The parameters of the model are related to physical quantities: the pion decay constant $f_\pi=92.5$ MeV, mass of the pion $m_\pi=135$ MeV, and mass of the sigma particle $m_\sigma=600$ MeV. The parameter H is directly related to the mass of the pion: for vanishing H mass of the pion also vanishes. We take these parameters later for our model of DCC production.

An example of an initial configuration of σ and π fields is given in Fig. 1. The sigma field is described by the function [10]

$$\sigma(t,x)=\sigma_0\left(1+\tanh\frac{x-x_0-ut}{\sqrt{2}\sqrt{1-u^2}}-\tanh\frac{x+x_0+ut}{\sqrt{2}\sqrt{1-u^2}}\right), \tag{4}$$

where the parameter σ_0 is equal to vacuum expectation value of the sigma field. When $H=0$ then $\sigma_0=v$, otherwise $\sigma_0=f_\pi$. The velocity u of the domain walls is uniquely related to the initial energy of the system provided we fix the value of x_0 describing the initial size of the σ domain. The larger is the energy of the system, the closer is the velocity u to the speed of light and the steeper are the walls of the field. We take 0.5 fm as the value of x_0 which corresponds to the initial size of the well of order 1 fm. The initial pion field is given by

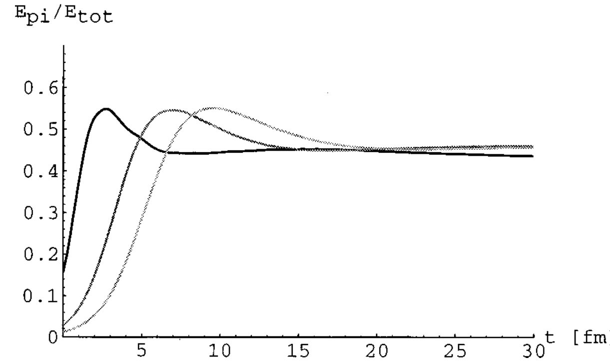

FIG. 2. Energy of the pion field E_π as a fraction of the total energy for different values of $E_{\text{tot}}=250$ MeV (black curve), 400 MeV (dark gray curve) and 500 MeV (light gray curve).

$$\pi(t,x)=N\left(\tanh\frac{x-x_0-ut}{\sqrt{2}\sqrt{1-u^2}}-\tanh\frac{x+x_0+ut}{\sqrt{2}\sqrt{1-u^2}}\right). \tag{5}$$

The normalization factor N is the most uncertain parameter. To fix the order of its magnitude we use the following condition:

$$\sigma^2(0,0)+\pi^2(0,0)=\sigma_0^2, \tag{6}$$

where σ_0 is vacuum expectation value of the sigma field. The evolution of the system is described by the equations (3). We solve them numerically using a kind of predictor-corrector scheme presented in Appendix A.

III. MASSLESS PION CASE

In the first step we consider the case with the parameter $H=0$. Now the ground state of the system is described by the relation

$$\sigma^2(t,x)+\pi^2(t,x)=v^2. \tag{7}$$

For vanishing pion field we have two independent vacua for sigma field: $\sigma_0=\pm v$. For $\pi(t,x)=0$ there exists a single kink (K) and anti-kink ($\bar{K}$) static solution:

$$\sigma_{K(\bar{K})}=\pm v\ \tanh\frac{x-x_0}{\sqrt{2}}. \tag{8}$$

Thus the function (4) approximately describes the $K\bar{K}$ configuration for large separation x_0. This configuration was investigated in detail by Campbell *et al.* [10] as the model of kink-antikink interaction in Φ^4 theory. In the absence of the pion field the $K\bar{K}$ configuration is stable—kinks can be reflected from each other or they create a long-living bound state [10]. If the pion is present it takes some energy from the sigma field. Our present consideration can be also described as a model of the $K\bar{K}$ interaction in the presence of the pion field. We choose the initial conditions in such a way that the energy of the sigma field is greater than that of the pion field.

The nonlinear interaction of the fields leads to energy-flow from one field to the other. This is shown in Fig. 2

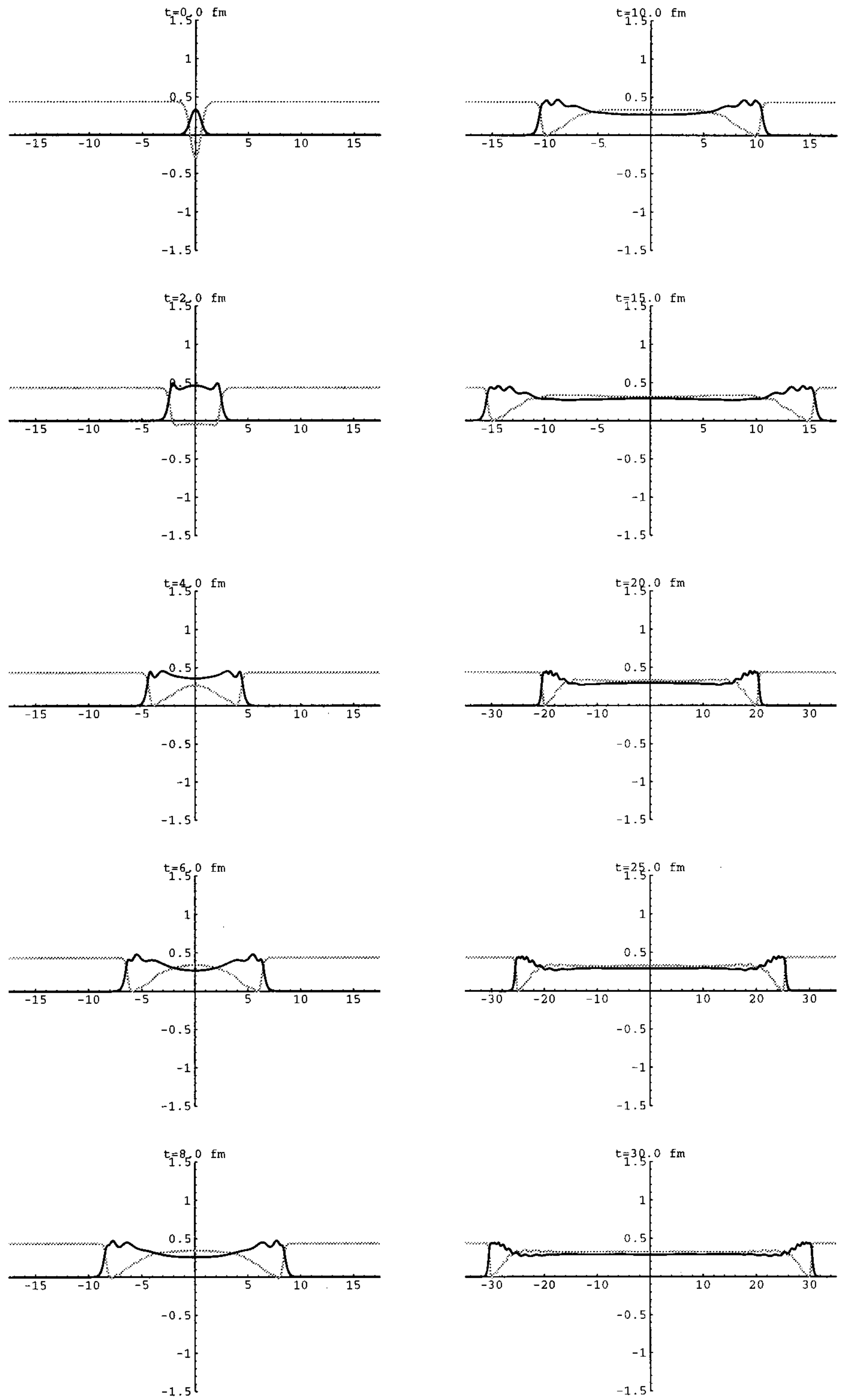

FIG. 3. Evolution of σ (gray curve) and π (black curve) fields for energy 250 MeV for massless pion case.

which presents the energy of the pion field as a function of time. In the final state about 45% of the energy is stored in the pion field and the rest of the energy remains in the sigma field (the interaction energy after a long time is relatively small).

During the evolution the system approaches its ground state. Because of the infinite degeneracy of possible vacua there is no reason to expect that the final vacuum state is the same as the initial one. Figure 3 shows a series of snapshots of the state of the system at different times. It is clearly seen

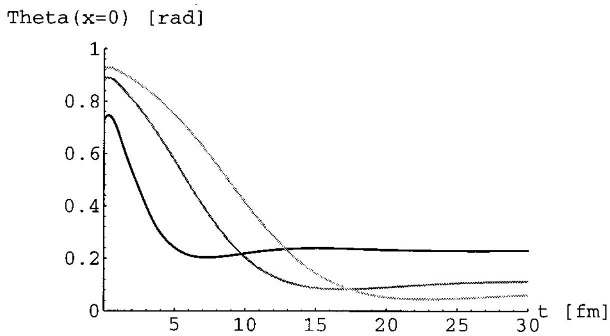

FIG. 4. Dependence of θ at $x=0$ for different values of energy: $E=250$ MeV (black curve), 400 MeV (dark gray curve) and 500 MeV (light gray curve).

that the final ground state is different from the initial one and it expands with the speed of light. One can define a variable θ:

$$\theta(t)=\arctan(\pi(t,0)/\sigma(t,0)) \tag{9}$$

to describe the orientation of the vector in (σ,π)-space at the point $x=0$. The time dependence of θ is shown in Fig. 4. It can be seen that the system goes directly to its ground state. The time of the setting of the new ground state at $x=0$ is proportional to the initial energy. The value of θ decreases to zero with an increase of the initial energy.

IV. MASSIVE PIONS CASE

In the case of H parameter different from zero the system has a unique ground state $\sigma_0=f_\pi$, $\pi_0=0$. The formula (4) does not describe the $K\bar{K}$ pair anymore. It is interesting to compare the of the system for the cases of massive and massless pion fields.

In Fig. 5 we show the energy flow from the sigma to the pion field. The pion energy density in the figure is defined as

$$\mathcal{E}=\frac{1}{2}\,(\dot{\pi}^2+\pi'^2+m_\pi^2\pi^2). \tag{10}$$

We see that at late times the energy of the pion field oscillates around the mean value of about 85%. We checked that the order of the energy flow depends very weakly on the initial pion field amplitude. However this amplitude affects the size of the oscillations. The smaller initial pion amplitude is, the larger are the oscillations. This means that the pion and sigma fields create a long-living bound state. Such a situation can violate our assumption about decoupling of pion and sigma fields. It is possible that this is only a $(1+1)$-dimensional effect although recently there have been published papers [11,12] in which authors report on similar bound states in $(2+1)$-dimensional cases. This means that the problem of bound states require further elaboration. Nevertheless our analysis shows that for not too large initial energies and not too small initial pion field amplitudes the bound state is very weak. Thus we constrained ourselves to consideration of energies below 0.6 GeV. For these configurations one can find the decoupling time in reasonable length (not more than 40–50 fm).

Before we will go to the results let us consider the approximate explanation of the energy exchange. The equation of motion for the pion field can be written in the form

$$\ddot{\pi}(t,x)-\pi''(t,x)+m_{\rm eff}^2\pi=0 \tag{11}$$

where

$$m_{\rm eff}^2=\lambda(\sigma^2(t,x)+\pi^2(t,x)-v^2). \tag{12}$$

When the effective mass $m_{\rm eff}$ is smaller than zero one can obtain the pion field amplification for sufficiently low momenta modes. In our picture the region of negative effective mass consists of two separated regions located at the borders of the sigma well (see Fig. 6). In these places the pion field is amplified when interacting with the sigma field. During the evolution these regions recede from each other at almost speed of light and are accompanied by the pion field resulting from the pion cluster that was initially located within the sigma well. While the fields interact the energy flows to the pion modes and the sigma domain wall dissipates. When the

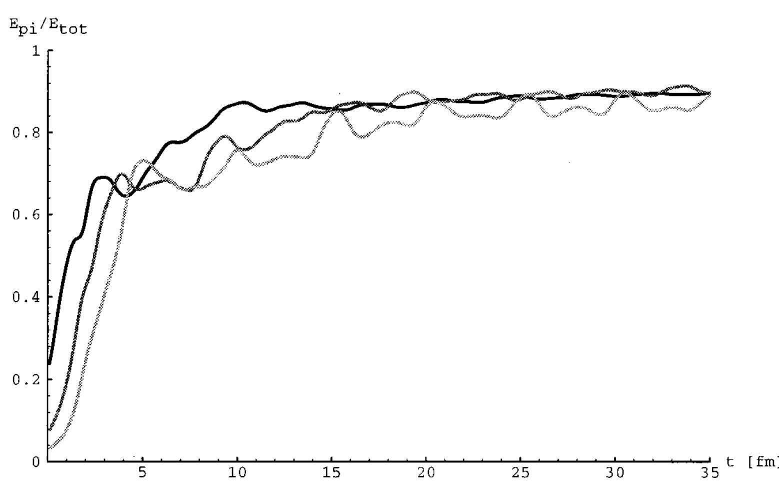

FIG. 5. Energy of the pion field E_π as a fraction of the total energy for different values of $E_{\rm tot}=250$ MeV (black curve), 400 MeV (dark gray curve) and 500 MeV (light gray curve) in the massive pions case.

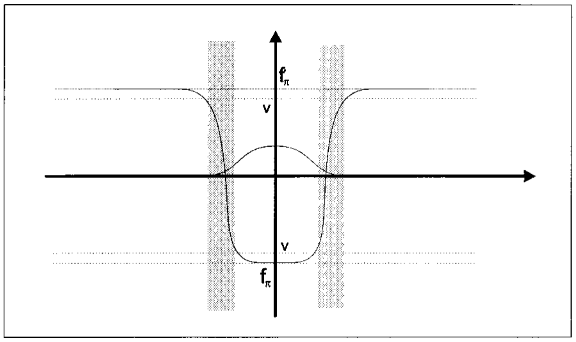

FIG. 6. The sigma well and the domain walls.

sigma domain walls vanish the fields decouple from each other but the decoupling time is rather long.

A generic example of the system evolution is given in Fig. 7. During the evolution the system tries to reach the ground state. After some time the sigma field in the interior reaches its vacuum state and oscillates around it with a small amplitude. Similarly, the pion field creates a region of field oscillating with a constant amplitude. This oscillating behavior for $x=0$ is shown in Fig. 8. It can be understood in the following way. The system possesses two fundamental oscillation frequencies m_π and m_σ corresponding to two directions of the (σ,π)-space along pion and sigma axes. After getting to the vicinity of the vacuum value the pion field starts to oscillate with a relatively large amplitude along the pion direction ($\sigma=\sigma_0$) with the frequency m_π and with a small amplitude along sigma direction with the frequency m_σ. It can be easily seen after the Fourier analysis of $\pi(t)=\pi(t,x=0)$ which is shown in Fig. 9. At late times when the amplitude of the pion field is smaller than m_π the leading frequency of the oscillation in the vicinity of $x=0$ is described by the equation

$$\ddot{\pi}(t)+\lambda(f_\pi^2-v^2)\pi(t)=0, \tag{13}$$

where $\lambda(f_\pi^2-v^2)=m_\pi^2$. It follows directly from this equation that the frequency of the oscillation is $\omega_\pi=m_\pi$. When the amplitude of the sigma field near the origin falls below f_π one can divide the field into two components $\sigma(t,x=0)=f_\pi+\delta\sigma(t)$, where $\delta\sigma(t)$ satisfies the approximate equation:

$$\ddot{\delta\sigma}(t)+\lambda f_\pi \pi(t)^2=0 \tag{14}$$

Since the $\pi(t)\sim\exp(im_\pi t)$ then one can deduce from the above equation that the leading frequency of the sigma field is $\omega_\sigma=2m_\pi$ (Fig. 10).

The amplitude of the field oscillation is getting smaller with the passing time and the two conditions for the π and σ amplitude ($|\pi(t)|\ll m_\pi$, $|\delta\sigma(t)|\ll f_\pi$) are better fulfilled and the above picture becomes more clear.

The oscillations of the pion and sigma fields near $x=0$ exponentially decrease because of the energy flow along the x direction.

V. SIMPLE MODEL FOR DCC PRODUCTION

Let us consider the following picture of nuclei collisions similar to that described in paper [9]. In the center of mass frame the nuclei approach each other with the relative velocity closed to that of light. Along the direction of motion nuclei are lorentz contracted. In the central collisions the area of colliding nuclei is $S=\pi R_A^2$, where $R_A=1.07A^{1/3}$ fm is a radius of a nucleus. Such a system possesses the cylindrical symmetry. During the collision the nuclei interact while passing through each other and then recede to infinity.

We assume that in the short time after the collision (~ 1 fm) part of the energy is used to create the domain of the false vacuum described by formula (4). Using the equations of motion (3) we perform the evolution of the initial fields along the direction of motion up to the time t_c when the interaction of the σ and π fields becomes negligible. At that moment (t_c) we start to treat the π field as source of free pion quanta of mass m:

$$(\Box+m^2)\hat{\pi}(t,x)=\rho(t,x), \tag{15}$$

where source ρ is derived from the classical pion field. The 4-dimensional Fourier transform of the source calculated on the mass shell

$$\tilde{\rho}(\vec{k})=\tilde{\rho}(k^0=E_k,\vec{k}), \tag{16}$$

where $E_k=\sqrt{\vec{k}^2+m^2}$ is the energy of a free particle of mass m, can be extracted from the 3-dimensional Fourier transform $\tilde{\pi}(t,\vec{k})$ of the classical pion field and its time derivative by the following relation [2]:

$$\tilde{\rho}(\vec{k})=-ie^{iE_k t}[E_k\tilde{\pi}(t,\vec{k})+i\dot{\tilde{\pi}}(t,\vec{k})]. \tag{17}$$

Using the standard methods one can solve Eq. (15) and calculate the inclusive spectrum of particles for coherent states [2,4,13]:

$$\frac{dN}{d^3k}=\frac{|\tilde{\rho}(\vec{k})|^2}{(2\pi)^3 2E_k}. \tag{18}$$

Our picture assumes that all particles from the domain are created along the direction of the collision (with zero transverse momentum). Therefore it is convenient to calculate all the quantities like the energy or the number of pions per unit of transverse area (1 fm^2). Thus the formula (18) can be reduced to

$$\frac{dN}{dk}=\frac{|\tilde{\rho}(\vec{k})|^2}{(2\pi)2E_k}, \tag{19}$$

and describes the flux of pions emitted per unit transverse area. Let us also observe that the initial energy is localized in the tube of the volume of order 1 fm^3, thus the initial energy in $1+1$ dimension corresponds to the average energy density in $(3+1)$-dimensional picture.

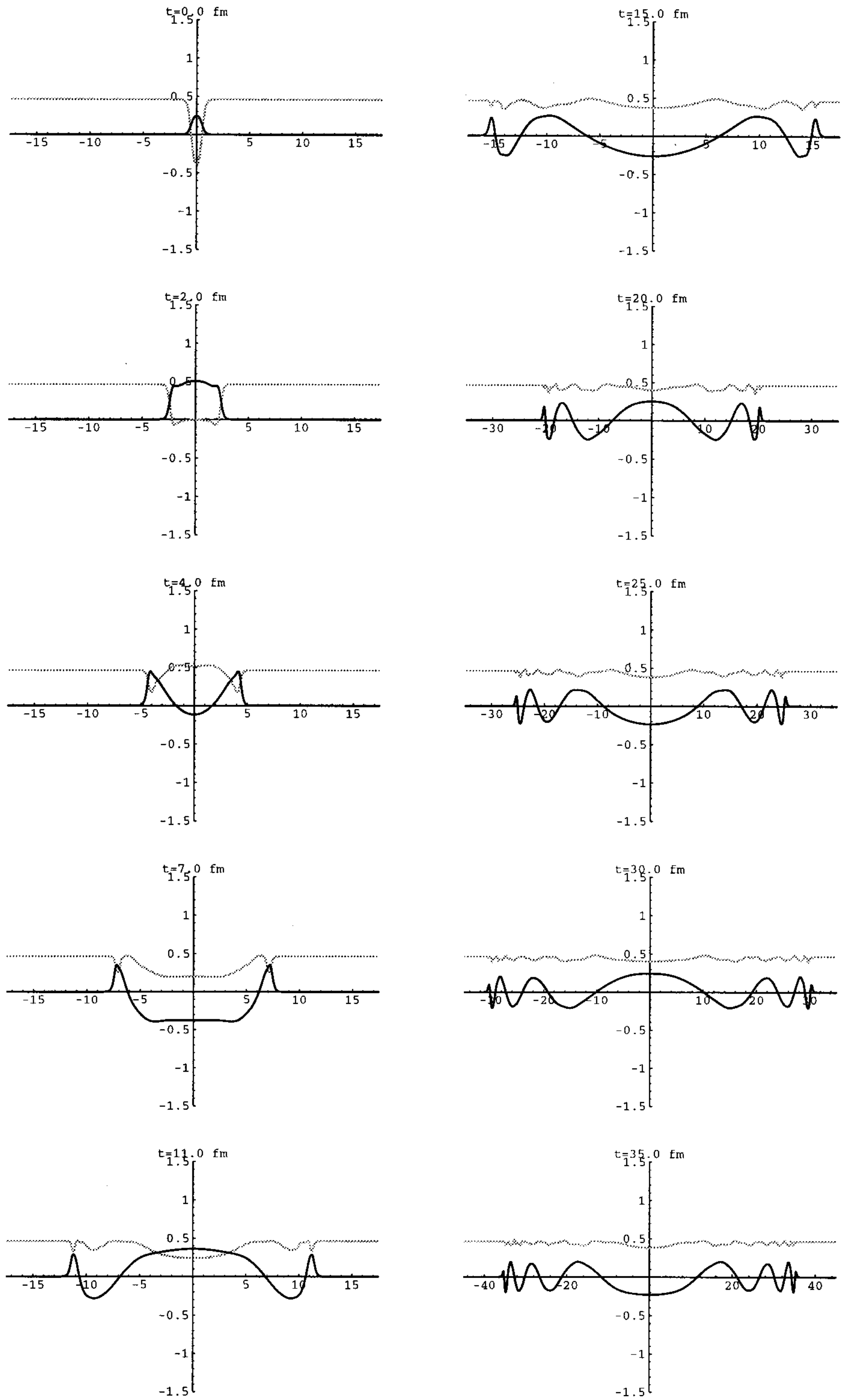

FIG. 7. Evolution of σ (gray curve) and π (black curve) fields for energy 400 MeV for massive pion case.

The momentum spectrum depends on time because of the interaction between the pion and sigma fields. After the time t_c the spectrum stabilizes itself and the pion field starts to behave like a free field.

In the sequence of snapshots (Fig. 11) we show the time evolution of the spectrum for initial energy 0.255 GeV. We found that for this energy the time t_c is rather long, of the order of ~20–25 fm. Using our lattice we are able to obtain a stable final configuration for the energies up to 0.6 GeV.

In the Figs. 12,13 we show the generic examples of the spectrum after time $t=35$ fm for initial energies 0.412 GeV and 0.506 GeV. The numbers of emitted pions are $N=0.9,1.4,1.8$ for the initial energies 0.255, 0.412, 0.506 GeV, respectively, which gives a linear growth in the first

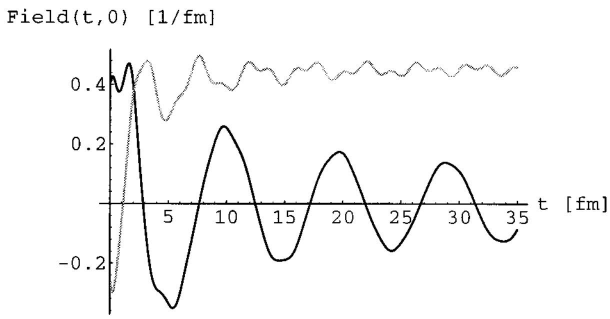

FIG. 8. Time dependence of the pion (black curve) and sigma (gray curve) field at point $x=0$.

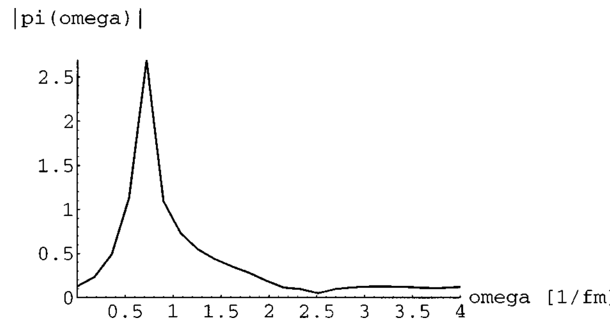

FIG. 9. Fourier analysis of the pion field time dependence at $x=0$. The sharp peak corresponds to the value of the pion mass (135 MeV).

approximation. From the given spectra we can also conclude that for higher initial energy, pions have a tendency to fill the lower momentum states. For the energy 0.255 GeV about 35% of all produced pions is located in the first bin (momentum below 0.1 GeV) while for energy 0.506 GeV it is 65%.

VI. CONCLUSIONS

In this paper we considered the interaction of the pion and sigma fields in the $O(2)$ model in two dimensions. We solved the classical equations of motion for the case of the massless $(H=0)$ and massive $(H\neq 0)$ pion field. At the initial moment we prepare our system as the domain of the false vacuum described by the sigma ''well,'' of length ~ 1 fm and the nonzero pion field inside it. The energy of the pion field we choose much smaller than the energy contained in the sigma field.

The main numerical results are the following.

For $H=0$ during the evolution the pion field is amplified to about 40% of the total energy of the system. In this case, when the ground state is degenerated, the system during the evolution switches from one ground state to another one.

For the massive case the pions take about 85% of the total energy. We checked that the order of the amplification does not depend on the initial energy (at least up to $E=0.6$ GeV) and the size of the pion field normalization.

On the other hand the initial amount of pions influences the decoupling time of the pion and sigma fields. This comes from the fact that for sufficiently small amount of the pion field the system has a tendency to create a long-living bound state. This effect may have $(1+1)$-dimensional origin. The answer if this is the case we will left to our future work.

In this paper we rather concentrate on possible application of the scenario described in the Introduction to the production of the DCC. Despite of the fact that we consider only the simplest $(1+1)$-dimensional model we can still draw some model independent physical conclusions.

The pion field is amplified during the classical evolution through the decay of the false vacuum.

Created pions have a tendency to fill the lower momentum states. The higher is initial energy the more pions condense in the first bin (below 0.1 GeV).

Basing on our simulations we expect the production of ~ 2 pion/fm^2 of the cross area for initial energy density $\epsilon=0.5$ GeV/fm^3. 65 percent of these pions are in the first bin.

If one can take into account in more realistic picture a transverse flow of pions than the signal from the produced cluster could be detected in central rapidity region. This happens when the laboratory frame coincides with the rest frame of the initial domain of the false vacuum.

The time of stabilization of the spectrum depends on the amount of the initial pion field and initial energy, and is rather long (e.g., of the order of 20–25 fm for relatively small energy value 0.255 GeV). This result depends, as we think, on the number of dimensions. In higher dimensional case the decoupling time would be shorter. Nevertheless this fact should not spoiled the physical predictions listed above, because the most important energy flow takes place at initial time of the evolution. The small number of emitted pions suggests that a large signal of final pions requires a large energy of deformed vacuum or the domains of large sizes or both.

In the future work we would like to extend our simulation to $3+1$ dimension with additional assumption of cylindrical symmetry of the system. We are planning to consider more realistic full isotopic space content of the pion fields. Also more realistic initial conditions are of importance. The last but not least is a possible application of described scenario to ''Baked Alaska'' picture. In the most simple form it is essentially $(1+1)$-dimensional model. In this situation the problem of the long-living bound states are of great importance.

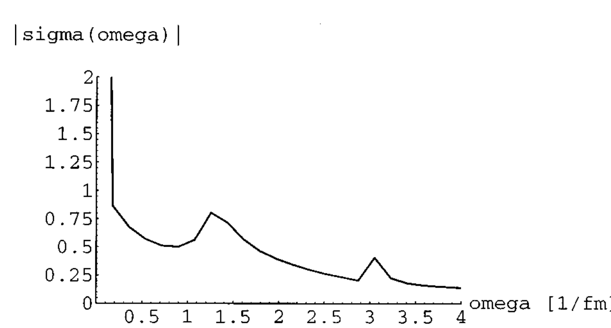

FIG. 10. Fourier analysis of the sigma field time dependence at $x=0$. The peak at $\omega=0$ corresponds to nonzero value of the sigma field in the ground state. The second peak corresponds to the value of the sigma mass (600 MeV).

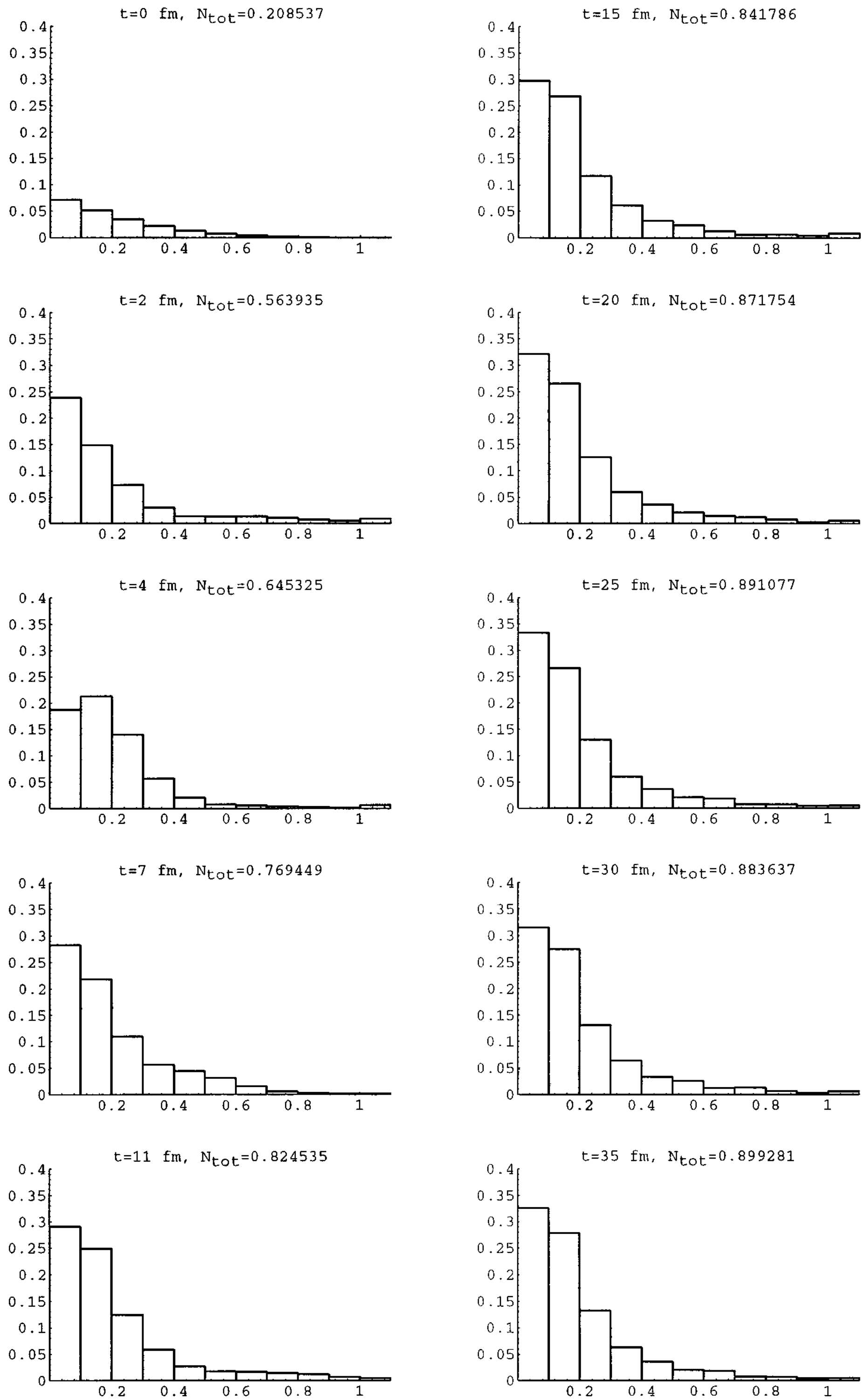

FIG. 11. Momentum spectra of produced pions for energy 250 MeV calculated at different times *t*. The horizontal axes are scaled in GeV. N_{tot} denotes the total number of produced pions.

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APPENDIX A: THE NUMERICAL SCHEME

We solve the equations of motion (1) step by step on a 1-dimensional lattice of 10000–50000 points. We use the time step $dt = 0.01$ fm and the spatial step $dx = 1.2dt$. We have developed a numerical algorithm for solving the equa-

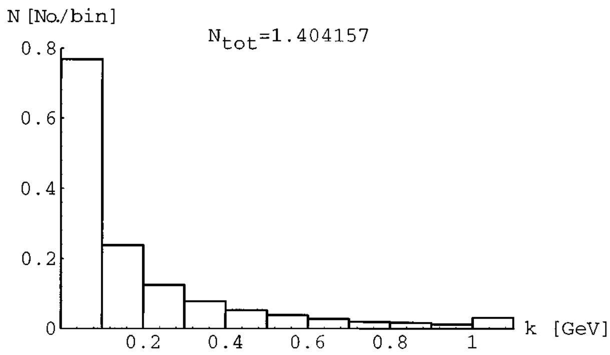

FIG. 12. Momentum spectrum of pion produced for energy 412 MeV.

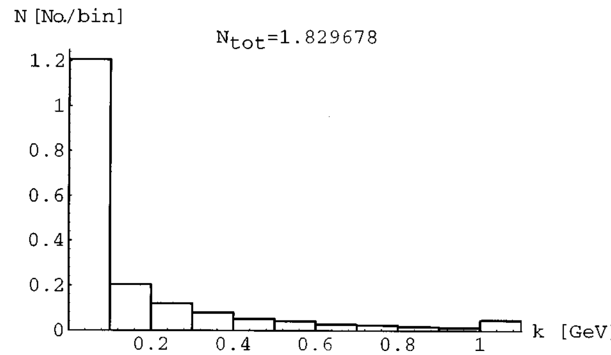

FIG. 13. Momentum spectrum of pion produced for energy 506 MeV.

tions that is based on a *predictor-corrector* scheme.

We start our evolution at $t=0$ fm. At each time step we evaluate predictory values of fields $\tilde{\sigma}_i^{n+1}$ and $\tilde{\pi}_i^{n+1}$ using an explicit method:

$$\frac{\tilde{\sigma}_i^{n+1}-2\sigma_i^n+\sigma_i^{n-1}}{dt^2}-\frac{\sigma_{i+1}^n-2\sigma_i^n+\sigma_{i-1}^n}{dx^2}=\lambda\sigma_i^n W_i^n+H, \tag{A1}$$

$$\frac{\tilde{\pi}_i^{n+1}-2\pi_i^n+\pi_i^{n-1}}{dt^2}-\frac{\pi_{i+1}^n-2\pi_i^n+\pi_{i-1}^n}{dx^2}=\lambda\pi_i^n W_i^n, \tag{A2}$$

where the upper index is a temporal one, the lower index is a spatial one, and $W_i^n=v^2-(\sigma_i^n)^2-(\pi_i^n)^2$. Then we use the predicted values in an implicit method for evaluating the corrected values of fields σ_i^{n+1} and π_i^{n+1}:

$$\frac{\sigma_i^{n+1}-2\sigma_i^n+\sigma_i^{n-1}}{dt^2}-\frac{\sigma_{i+1}^n-2\sigma_i^n+\sigma_{i-1}^n}{dx^2}$$

$$=\lambda[\alpha\sigma_i^{n+1}(v^2-(\sigma_i^{n+1})^2-(\tilde{\pi}_i^{n+1})^2)+\alpha\sigma_i^{n-1}W_i^{n-1}+(1-2\alpha)\sigma_i^n W_i^n]+H, \tag{A3}$$

$$\frac{\pi_i^{n+1}-2\pi_i^n+\pi_i^{n-1}}{dt^2}-\frac{\pi_{i+1}^n-2\pi_i^n+\pi_{i-1}^n}{dx^2}$$

$$=\lambda[\alpha\pi_i^{n+1}(v^2-(\sigma_i^{n+1})^2-(\tilde{\pi}_i^{n+1})^2)+\alpha\pi_i^{n-1}W_i^{n-1}+(1-2\alpha)\pi_i^n W_i^n], \tag{A4}$$

where α is a numerical constant less than $\frac{1}{2}$. In our calculations we have taken $\alpha=0.3$.

The presented numerical algorithm conserves the total energy of the system at the level of accuracy of the order of 10^{-3}.

[1] G. Amelino-Camelia, J. D. Bjorken, and S. E. Larsson, Phys. Rev. D **56**, 6942 (1997); Z. Huang and X.-N. Wang, *ibid.* **49**, 4335 (1994); S. Gavin and B. Mueller, Phys. Lett. B **329**, 486 (1994); M. Asakawa, Z. Huang, and X.-N. Wang, Phys. Rev. Lett. **74**, 3126 (1995); A. Bialas, W. Czyz, and M. Gmyrek, Phys. Rev. D **51**, 3739 (1995); J. Randrup, Phys. Rev. Lett. **77**, 1226 (1996); Nucl. Phys. **A590**, 581c (1995).
[2] J. D. Bjorken, Acta Phys. Pol. B **28**, 2773 (1997).
[3] J.-P. Blaizot and A. Krzywicki, Acta Phys. Pol. B **27**, 1687 (1996).
[4] A. A. Anselm, Phys. Lett. B **217**, 169 (1989).
[5] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).
[6] K. Rajagopal and F. Wilczek, Nucl. Phys. **B404**, 577 (1993).
[7] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D **50**, 442 (1994).
[8] K. L. Kowalski and C. C. Taylor, CWRUTH-92-6 (1992) (hep-ph/9211282).
[9] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D **46**, 246 (1992).
[10] D. K. Campbell, J. F. Schonfeld, and C. A. Wingate, Physica D **9**, 1 (1983).
[11] V. A. Gani *et al.*, Report No. ITEP-43-97 (hep-ph/9712526).
[12] J. N. Hormuzdiar and S. D. H. Hsu, Phys. Rev. C **58**, 1165 (1998); Report No. OITS-652 (1998) (hep-ph/9805382).
[13] A. A. Anselm and M. G. Ryskin, Phys. Lett. B **266**, 482 (1991).