$P_{33}(1232)$ resonance contribution to the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ from an analysis of the $p(e,e'p)\pi^0$ data at $Q^2=2.8$, 3.2, and 4 $(\mathrm{GeV}/c)^2$ within the dispersion relation approach

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(Received 14 August 1998; published 1 February 1999)

Within the fixed-t dispersion relation approach we have analyzed the TJNAF and DESY data on the exclusive $p(e,e'p)\pi^0$ reaction in order to find the $P_{33}(1232)$ resonance contribution to the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. As an input for the resonance and nonresonance contributions to these amplitudes the earlier obtained solutions of the integral equations which follow from dispersion relations are used. The obtained values of the ratio $E2/M1$ for the $\gamma^*N\to P_{33}(1232)$ transition are $0.039\pm0.029, 0.121\pm0.032$, 0.04 ± 0.031 for $Q^2=2.8$, 3.2, and 4 $(\mathrm{GeV}/c)^2$, respectively. The comparison with the data at low Q^2 shows that there is no evidence for the presence of the visible perturbative QCD (PQCD) contribution into the transition $\gamma N\to P_{33}(1232)$ at $Q^2=3-4\ \mathrm{GeV}^2$. The ratio $S_{1+}^{3/2}/M_{1+}^{3/2}$ for the resonance parts of multipoles is -0.049 ± 0.029, -0.099 ± 0.041, -0.085 ± 0.021 for $Q^2=2.8$, 3.2, and 4 $(\mathrm{GeV}/c)^2$, respectively. Our results for the transverse form factor $G_T(Q^2)$ of the $\gamma^*N\to P_{33}(1232)$ transition are lower than the values obtained from the inclusive data. With increasing Q^2, $Q^4G_T(Q^2)$ decreases, so there is no evidence for the presence of the PQCD contribution here too. [S0556-2821(99)03303-2]

PACS number(s): 13.60.Le, 11.55.Fv, 11.80.Et, 25.30.Rw

I. INTRODUCTION

It is known that the question of how high Q^2 must be for perturbative QCD to dominate in exclusive processes is a subject of controversy and has caused intense debates over the past two decades (a detailed discussion of this problem can be found, for example, in Ref. [1]). Although the Q^2 behavior of the pion and nucleon form factors manifests the features which are characteristic of perturbative QCD (PQCD) beginning with small Q^2, there is no consistent quantitative description of the experimental data within PQCD. From the quantitative description of the available data it is seen that soft mechanisms play an important, and possibly dominant, role in the region of a few GeV^2. The dominance of the PQCD contribution may depend strongly on the specific reaction. For example, it is possible [1] that the asymptotic value of the leading-order helicity-conserving amplitude for the $\gamma^*N\to P_{33}(1232)$ transition is numerically small; as a result, the PQCD contribution into this transition may be suppressed over a large range of Q^2. In the present paper we will analyze experimental data on the cross sections of the exclusive reaction $p(e,e'p)\pi^0$ obtained recently at TJNAF at $Q^2=2.8$ and 4 $(\mathrm{GeV}/c)^2$ [2] and more earlier DESY data at $Q^2=3.2\,(\mathrm{GeV}/c)^2$ [3] in order to extract an information on the $\gamma^*N\to P_{33}(1232)$ transition in the region of Q2=3–4 $(\mathrm{GeV}/c)^2$. This information is useful for understanding the role of the range of $Q^2=3-4\,(\mathrm{GeV}/c)^2$ in the transition to the PQCD regime for the $\gamma^*N\to P_{33}(1232)$ transition.

The investigation of the transition $\gamma^*N\to P_{33}(1232)$, using the experimental data on the pion photoproduction and electroproduction on the nucleons, is connected with the problem of the separation of the resonance and nonresonance contributions in the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$, which carry information on this transition. These amplitudes may contain significant nonresonance contributions, a fact which was clear when the first accurate data [4,5] on the amplitude $E_{1+}^{3/2}$ at $Q^2=0$ was obtained. The energetic behavior of this amplitude, in fact, is incompatible with the resonance behavior. The first investigations of this problem [6–8] showed that it is closely related to the problem of fulfillment of the unitarity condition, which for electroproduction amplitudes in the $P_{33}(1232)$ resonance region means the fulfillment of the Watson theorem [9]:

$$M(W,Q^2)=\exp[i\delta_{1+}^{3/2}(W)]|M(W,Q^2)|. \tag{1.1}$$

Here $M(W,Q^2)$ denotes any of the multipoles under consideration, and $\delta_{1+}^{3/2}$ is the phase of the corresponding πN scattering amplitude $h_{1+}^{3/2}(W)=\sin[\delta_{1+}^{3/2}(W)]\exp[i\delta_{1+}^{3/2}(W)]$.

There are different approaches for the extraction of information on the $\gamma^*N\to P_{33}(1232)$ transition from the pion photoproduction and electroproduction data with the different forms of the unitarization of the multipole amplitudes. These approaches can be subdivided into the following groups: the phenomenological approaches [6–8,10] including the approaches based on the K-matrix formalism [11,12],

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the effective Lagrangian approaches [13–17] with different phenomenological forms of the unitarization of amplitudes, the dynamical approaches [18–24], and the approaches based on the fixed-t dispersion relations [25–27].

In this work our analysis will be based on the solutions for the multipole amplitudes $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ obtained in Ref. [27] using the fixed-t dispersion relations within the approach of Refs. [28,29]. This approach is very useful for the extraction of information on the $\gamma^*N\to P_{33}(1232)$ transition, because in a natural way it reproduces the resonance and nonresonance contributions into the multipole amplitudes, and the obtained solutions satisfy unitarity condition (1.1). Let us discuss this in more detail using the simplified version of the dispersion relations for these multipoles with the s-channel cut only, i.e., in the form which is similar to the dispersion relations in the quantum mechanics

$$M(W,Q^2)=M^B(W,Q^2)+\frac{1}{\pi}\int_{W_{\rm thr}}^{\infty}\frac{\operatorname{Im}M(W',Q^2)}{W'-W-i\varepsilon}dW'. \tag{1.2}$$

Here $M^B(W,Q^2)$ is the contribution of the Born term (i.e., of the nucleon and pion poles) into the multipoles. As it was discussed in more detail in Ref. [27], we can write in the integrand of Eq. (1.2) $\operatorname{Im}M(W,Q^2)=h^*(W)M(W,Q^2)$ due to the fact that the πN amplitude $h_{1+}^{3/2}(W)$ is elastic up to quite large energies. Thus, the dispersion relation (1.2) transforms into the singular integral equation which has a solution in the following analytical form (see Ref. [28], and references therein):

$$M(W,Q^2)=M^{\rm part}(W,Q^2)+c_M M^{\rm hom}(W), \tag{1.3}$$

where

$$M^{\rm part}(W,Q^2)=M^B(W,Q^2)+\frac{1}{\pi}\frac{1}{D(W)}\times\int_{W_{\rm thr}}^{\infty}\frac{D(W')h(W')M^B(W',Q^2)}{W'-W-i\varepsilon}dW' \tag{1.4}$$

is the particular solution of the singular equation, generated by the Born term, and

$$M^{\rm hom}(W)=\frac{1}{D(W)}=\exp\left[\frac{W}{\pi}\int_{W_{\rm thr}}^{\infty}\frac{\delta(W')}{W'(W'-W-i\varepsilon)}dW'\right] \tag{1.5}$$

is the solution of the homogeneous equation

$$M^{\rm hom}(W)=\frac{1}{\pi}\int_{W_{\rm thr}}^{\infty}\frac{h^*(W')M^{\rm hom}(W')}{W'-W-i\varepsilon}dW', \tag{1.6}$$

which enters the solution (1.3) with an arbitrary weight, i.e., multiplied by an arbitrary constant c_M.

The analogy with quantum mechanics shows that the solution $M^{\rm part}(W,Q^2)$ is the modification of the Born contribution produced by the πN rescattering in the final state (see Ref. [30], Chap. 9). This modification unitarizes the Born contribution which by itself is real:

$$M^{\rm part}(W,Q^2)=\exp[i\delta(W)]\times[M^B(W,Q^2)\cos\delta(W)+e^{a(W)}r(W,Q^2)], \tag{1.7}$$

where

$$r(W,Q^2)=\frac{P}{\pi}\int_{W_{\rm thr}}^{\infty}\frac{e^{-a(W')}\sin\delta(W')M^B(W',Q^2)}{W'-W}dW', \tag{1.8}$$

$$a(W)=\frac{P}{\pi}\int_{W_{\rm thr}}^{\infty}\frac{W\delta(W')}{W'(W'-W)}dW'. \tag{1.9}$$

So, $M^{\rm part}(W,Q^2)$ should be considered as the nonresonance background to the resonance contribution.

It is natural to identify with the resonance contribution the solution $M^{\rm hom}(W)$, because the dispersion relation (1.2) takes the form (1.6), when only the $P_{33}(1232)$ resonance contribution in the s channel is taken into account. This solution satisfies the unitarity condition (1.1) too,

$$M^{\rm hom}(W)=\frac{1}{D(W)}=\exp[i\delta(W)]e^{a(W)}. \tag{1.10}$$

From Eq. (1.7) it is seen that $M^{\rm part}(W,Q^2)$ has a nontrivial energy dependence. The factor at $\exp[i\delta(W)]$ in $M^{\rm part}(W,Q^2)$ is determined mainly by the first term in the brackets and changes the sign in the vicinity of the resonance. The comparison with the experiment shows that the amplitude $E_{1+}^{3/2}$ at $Q^2=0$ is described, in fact, by $M^{\rm part}(W,Q^2=0)$ [27]. Hence, this amplitude is mainly of nonresonance nature, and its nontrivial energy dependence is due to the the final state interaction in the Born term.

It is important to note that such type nonresonance contributions exist in all dynamical models [18–24]. They are produced by rescattering effects in the pole terms of these models and have the same type nontrivial energy dependence as Eq. (1.7). However, the magnitudes of these contributions are quite different, because their investigations within the models contain many model uncertainties coming from the cutoff procedures, the methods of taking into account off-shell effects, and the methods of the treatment of the gauge invariance. These uncertainties are discussed in detail in Refs. [31,32].

It is interesting that in the phenomenological approaches based on the K-matrix formalism [11,12] and in the effective Lagrangian approach of Ref. [16], with the unitarization made by the Noelle method [33] or using the K-matrix ansatz, the nonresonance contributions into the multipoles $M_{1+}^{3/2},E_{1+}^{3/2},S_{1+}^{3/2}$ have the same kind of energy dependence as Eq. (1.7). In these cases such energy behavior of the nonresonance contributions is also connected with the πN interaction in the final state.

In Refs. [25,26] at $Q^2=0$ the fixed-t dispersion relations are used in the same way as in Ref. [27]. However, the interpretation of the obtained solutions of the integral equations is different, although the results for the whole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ are the same as in Ref. [27]. In order to extract the $P_{33}(1232)$ resonance contribution in Refs. [25, 27] the method of the speed plot analysis is used. As a result, ignoring the physical nature of $M^{\text{part}}(W)$, the resonance contributions in these parts of the amplitudes are found.

In Sec. II the multipole amplitudes which are included into the fitting procedure in our analysis are listed, and the fitted parameters are specified. In Sec. III the results of our analysis of the TJNAF data at $Q^2=2.8$ and $4\ (\text{GeV}/c)^2$ [2] and of the DESY data at $Q^2=3.2\,(\text{GeV}/c)^2$ [3] are presented. The comparison with theoretical predictions and with the behavior of the amplitudes, which is characteristic of the PQCD asymptotics, is made.

II. DISPERSION RELATIONS AND PARAMETRIZATION OF MULTIPOLE AMPLITUDES

In our analysis we use the fixed-t dispersion relations for the Ball invariant amplitudes $B_1, B_2, B_3, B_5', B_6, B_8$ [34], which for the reaction $\gamma^* p \to \pi^0 p$ ($B_i^{(\pi^0 p)} = B_i^{(0)} + B_i^{(+)}$) require no subtraction:

$$\text{Re}\, B_i^{(\pi^0 p)}(s,t,Q^2) = R_i^{(p)}\left(\frac{1}{s-m^2}+\frac{\eta_i}{u-m^2}\right) + \frac{P}{\pi}\int_{s_{\text{thr}}}^{\infty} \text{Im}\, B_i^{(\pi^0 p)}(s',t,Q^2) \times \left(\frac{1}{s'-s}+\frac{\eta_i}{s'-u}\right) ds'. \tag{2.1}$$

Here $s=(k+p_1)^2$, $u=(k-p_2)^2$, $t=(k-q)^2$, $Q^2=-k^2$, k,q,p_1,p_2 are the four-momenta of virtual photon, pion, initial, and final protons, respectively, $\eta_1=\eta_2=\eta_6=1, \eta_3=\eta_5'=\eta_8=-1, s_{\text{thr}}=(m+\mu)^2$, m and μ are masses of the nucleon and the pion, and $R_i^{(p)}$ are the residues in the Born pole terms

$$R_1^{(\pi^0 p)} = ge(F_1^{(p)}+2mF_2^{(p)}),$$

$$R_2^{(\pi^0 p)} = -geF_1^{(p)}(Q^2),$$

$$R_3^{(\pi^0 p)} = -\frac{ge}{2}F_1^{(p)}(Q^2), \tag{2.2}$$

$$R_5'^{(\pi^0 p)} = \frac{ge}{2}(\mu - Q^2 - t)F_2^{(p)}(Q^2),$$

$$R_6^{(\pi^0 p)} = 2geF_2^{(p)}(Q^2),$$

$$R_8^{(\pi^0 p)} = geF_2^{(p)}(Q^2),$$

where in accordance with the existing experimental data we have

$$e^2/4\pi = 1/137, \quad g^2/4\pi = 14.5,$$

$$F_1^{(p)}(Q^2) = \left(1+\frac{g^{(p)}\tau}{1+\tau}\right)G_{\text{dip}}(Q^2),$$

$$F_2^{(p)}(Q^2) = \frac{g^{(p)}}{2m}\frac{G_{\text{dip}}(Q^2)}{1+\tau},$$

$$G_{\text{dip}}(Q^2) = 1/[1+Q^2/0.71\,(\text{GeV}/c)^2],$$

$$\tau = Q^2/4m^2, \quad g^{(p)} = 1.79. \tag{2.3}$$

The imaginary parts of the amplitudes $B_i^{(\pi^0 p)}(s,t,Q^2)$ we obtain using their expressions through the intermediate amplitudes f_i (the corresponding formulas are given in our earlier work [27]) which have the following decomposition over multipole amplitudes:

$$f_1 = \sum \{(lM_{l+}+E_{l+})P'_{l+1}(x) + [(l+1)M_{l-}+E_{l-}]P'_{l-1}(x)\},$$

$$f_2 = \sum [(l+1)M_{l+}+lM_{l-}]P'_l(x),$$

$$f_3 = \sum [(E_{l+}-M_{l+})P''_{l+1}(x)+(E_{l-}+M_{l-})P''_{l-1}(x)],$$

$$f_4 = \sum (M_{l+}-E_{l+}-M_{l-}-E_{l-})P''_l(x),$$

$$f_5 = \sum [(l+1)S_{l+}P'_{l+1}(x)-lS_{l-}P'_{l-1}(x)],$$

$$f_6 = \sum [lS_{l-}-(l+1)S_{l+}]P'_l(x), \tag{2.4}$$

where $x=\cos\theta$, θ is the polar angle of the pion in the c.m.s. The relations of the amplitudes f_i to the helicity amplitudes and to the cross section are also given in Ref. [27].

It is known that at $s'<s$ in the integrands of the dispersion relations (2.1) written at fixed t there is an unphysical region, where $|x'|=|\cos\theta'|>1$. In this work we analyze data in the $P_{33}(1232)$ resonance region, and, therefore, the unphysical region is close to the threshold, where the imaginary parts of the multipole amplitudes are proportional to $|\mathbf{q}|^{2l+1}$. For this reason the role of this region in our analysis is not significant. In addition, with increasing Q^2 the unphysical region becomes smaller. For example, if we analyze data at $W=1.232\,\text{GeV}$ the range of x' near threshold at $W'=1.1\,\text{GeV}$ is $[0.72-(-9.1)]$ at $Q^2=0$, $[3.3-(-2.3)]$ at $Q^2=4\ (\text{GeV}/c)^2$, and $[3.7-(-1.6)]$ when $Q^2\to\infty$.

Let us consider the parametrization of the multipole amplitudes now. For the resonance amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ we use as an input the solutions of the integral equations which follow from the dispersion relations for these amplitudes. According to these solutions obtained in Ref. [27] the resonance multipoles are sums of the particular and homogeneous solutions of the integral equations. The particular solutions which correspond to the nonresonance contributions into the multipoles have definite magnitudes fixed by the Born terms. The homogeneous solutions corresponding to the resonance contributions have definite shapes fixed by the homogeneous integral equations which correspond to the dispersion relations for $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ with the zero Born terms. The weights of these solutions are arbitrary and should be found from the requirement of the best description of the experimental data. So, the resonance multipoles bring into our analysis three fitting parameters which are the weights of the resonance contributions in the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$.

In the $P_{33}(1232)$ resonance region a significant contribution into Im f_i for the reaction $\gamma^* p \to \pi^0 p$ can give also the following combinations of the nonresonance multipole amplitudes:

$$E_{0+}^{(0)} + \frac{1}{3}E_{0+}^{1/2} + \frac{2}{3}E_{0+}^{3/2},$$

$$S_{0+}^{(0)} + \frac{1}{3}S_{0+}^{1/2} + \frac{2}{3}S_{0+}^{3/2},$$

$$M_{1-}^{3/2}, \quad \text{and} \quad S_{1-}^{3/2}. \tag{2.5}$$

This is connected with the fact that the πN phases corresponding to these multipoles are large enough, so, their imaginary parts can be significant. In order to take into account these multipoles in Im f_i, we have calculated their real parts from the Born terms, then the imaginary parts of the multipoles were found using for the corresponding πN phases the following analytical formulas:

$$\delta_{0+}^{1/2} = \frac{75|\mathbf{q}|}{1+2.5|\mathbf{q}|},$$

$$\delta_{0+}^{3/2} = -45|\mathbf{q}|[1+(2.2\mathbf{q})^2],$$

$$\delta_{1-}^{3/2} = -(6.9|\mathbf{q}|)^3, \tag{2.6}$$

where $\mathbf{q}$ is the three-momentum of the pion in the c.m.s. in the GeV units, the phases are in the degree units, and all numbers are in the GeV^{-1} units. These formulas describe well experimental data on the phases $\delta_{0+}^{1/2}, \delta_{0+}^{3/2}, \delta_{1-}^{3/2}$ [35–37] up to $E_L = (W^2 - m^2)/2m = 0.5$ GeV. At larger energies the smooth cutoff for the contributions of Eq. (2.5) was made. We have introduced in our analysis four additional fitting parameters in the form of the coefficients at the combinations (2.5) found in the above described way. These parameters were allowed to vary in the narrow region in the vicinity of 1.

In the description of the data in the $P_{33}(1232)$ resonance region the contributions of the resonances with higher masses, predominantly from the second resonance region, should be taken into account in the dispersion integrals. In the region of $Q^2 = 3-4\ (\text{GeV}/c)^2$ which we analyze in this work there is no information on the form factors of these resonances, except $S_{11}(1535)$. By this reason we begun our analysis with the DESY data which cover the second resonance region. In this analysis we had additional fitting parameters for the contributions of the amplitudes $M_{1-}^{1/2}, S_{1-}^{1/2}$ for the $P_{11}(1440)$ resonance, of the amplitudes $E_{0+}^{1/2}, S_{0+}^{1/2}$ for the $S_{11}(1535)$ resonance, and of the amplitudes $E_{2-}^{1/2}, M_{2-}^{1/2}, S_{2-}^{1/2}$ for the $D_{13}(1520)$ resonance. The contributions of these amplitudes were described in the Breit-Wigner form according to the parametrization of Ref. [38]. For the multipoles $M_{l+}, M_{l-}, E_{l+}, E_{l-}$ it has the form

$$M_{B-W}(W,Q^2) = \frac{M\Gamma(W,Q^2)}{M^2 - W^2 - iM\Gamma(W,Q^2)} \left(\frac{q_r}{|\mathbf{q}|}\right)^{l+1} \left(\frac{|\mathbf{k}|}{k_r}\right)^{l'}. \tag{2.7}$$

For the multipoles S_{l+}, S_{l-} the Breit-Wigner parametrization is

$$S_{B-W}(W,Q^2) = \frac{M\Gamma(W,Q^2)}{M^2 - W^2 - iM\Gamma(W,Q^2)} \left(\frac{q_r}{|\mathbf{q}|}\right)^{l+1} \left(\frac{|\mathbf{k}|}{k_r}\right)^{l'+1}. \tag{2.8}$$

Here $l' = l$ for $M_{l+}, M_{l-}, E_{l+}, S_{l+}$, $l' = l-2$ if $l > 1$ for E_{l-}, S_{l-}, and $l' = 1$ for S_{1-}, M and Γ are the masses and the widths of the resonances, k_r, q_r are the photon and pion three-momenta in the c.m.s. at $W = M$, and

$$\Gamma(W,Q^2) = \Gamma \left(\frac{|\mathbf{q}|}{q_r}\right)^{2l+1} \left(\frac{q_r^2 + X^2}{\mathbf{q}^2 + X^2}\right)^{l}, \tag{2.9}$$

$X = 0.35$. So, in the analysis of the DESY data there are seven additional fitting parameters which are the coefficients in Eqs. (2.7),(2.8) for the abovementioned multipole amplitudes. These parameters we consider as effective values for the description of the second resonance region, because we did not take into account backgrounds in the multipole amplitudes in this region and did not include the resonances from higher resonance regions in our analysis. Let us note, however, that the value of the amplitude E_{0+} for the resonance $S_{11}(1535)$ obtained in this analysis agrees well with the value known from the analysis of the η electroproduction data.

In the analysis of the TJNAF data, which do not cover the second resonance region, we used the results for the multipoles from this region obtained in the analysis of the DESY data with the Q^2 evolution corresponding to the results of Ref. [39]. Then the small variation of the multipoles was allowed.

III. RESULTS

The data used in our analysis are differential cross sections of π^0 production on protons at $Q^2 = 2.8$ and $4\ (\text{GeV}/c)^2$ [2] and $Q^2 = 3.2\ (\text{GeV}/c)^2$ [3]. A total 751 and 867 points which extend over an invariant mass range $W = 1.11-1.39$ GeV/c were included in the fit at $Q^2 = 2.8$ and $4\ (\text{GeV}/c)^2$, respectively. At $Q^2 = 3.2\ (\text{GeV}/c)^2$ we have included in the fit 598 points which extend from $W = 1.145$ to 1.595 GeV. The reduced χ^2 obtained in the analyses were 1.53, 1.18, and 1.35 at $Q^2 = 2.8$, 3.2, and $4\ (\text{GeV}/c)^2$, respectively. The obtained results for the multipole amplitudes

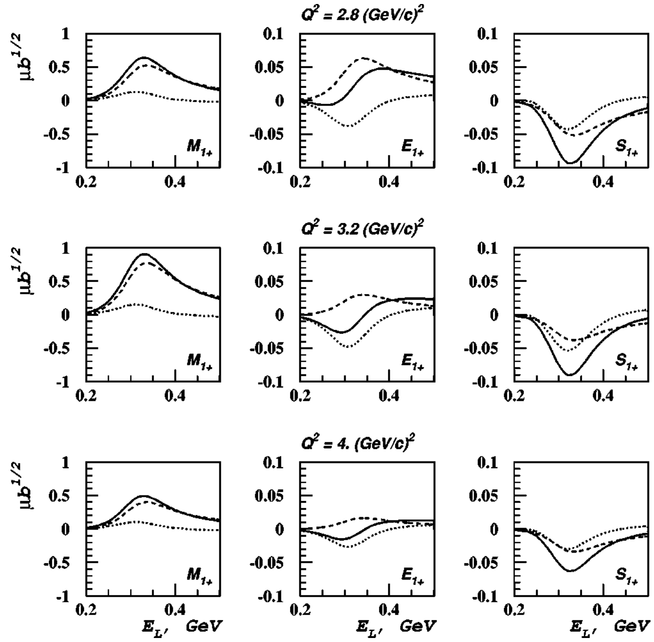

FIG. 1. Our results for the imaginary parts of the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$. Dashed curves are the resonance parts of the multipoles corresponding to the $P_{33}(1232)$ resonance contribution, dotted curves are the nonresonance background contributions, full curves are the sums of these contributions, and $E_L = (W^2 - m^2)/2m$.

$M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are presented in Fig. 1. In this figure the resonance and nonresonance contributions to these amplitudes are presented separately. It is seen that the nonresonance contributions play a significant role in the description of the amplitudes, especially for $E_{1+}^{3/2}$ and $S_{1+}^{3/2}$. In the case of $E_{1+}^{3/2}$ the sum of the resonance and nonresonance contributions gives the nontrivial energy dependence of the whole amplitude. At all investigated Q^2, $\mathrm{Im}\, E_{1+}^{3/2}$ changes sign near the resonance. So, the energy behavior of this amplitude, similar to the behavior at $Q^2 = 0$, has nonresonance character.

In the center of the $P_{33}(1232)$ resonance at $W = m_\Delta$ the resonance contributions into the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are

$$\mathrm{Im}\, M_{1+}^{3/2}(\mathrm{res}) = 0.772 \pm 0.031, \quad 0.523 \pm 0.021, \quad 0.4 \pm 0.016,$$

$$\mathrm{Im}\, E_{1+}^{3/2}(\mathrm{res}) = 0.03 \pm 0.022, \quad 0.063 \pm 0.017, \quad 0.016 \pm 0.012,$$

$$\mathrm{Im}\, S_{1+}^{3/2}(\mathrm{res}) = -0.038 \pm 0.022, \quad -0.052 \pm 0.022, \quad -0.034 \pm 0.008 \tag{3.1}$$

at $Q^2 = 2.8$, 3.2, and 4 $(\mathrm{GeV}/c)^2$, respectively.

In Fig. 2 our results for the transverse form factor G_T of the $\gamma^* N \to P_{33}(1232)$ transition are presented in comparison with the data obtained from inclusive experiments and partly from exclusive data. These data are taken from Table 5 of Ref. [40] by a recalculation of our definition of G_T which is related to the magnetic dipole and electric quadrupole form factors of Ref. [41] by

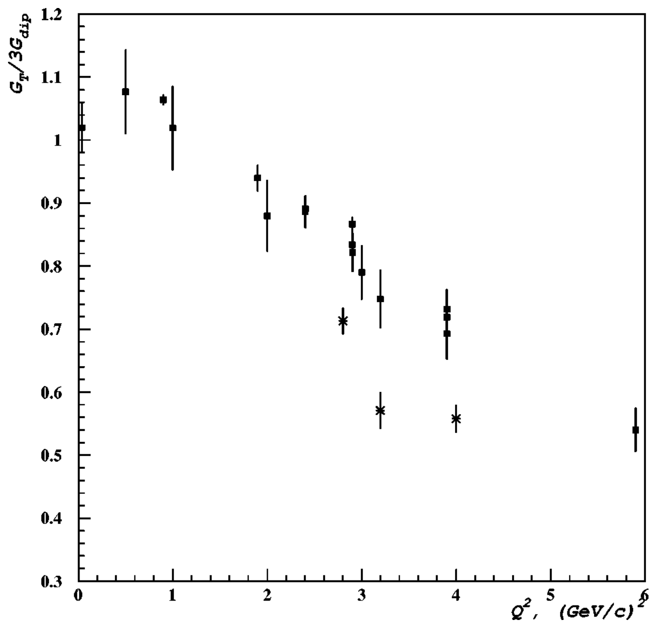

FIG. 2. Experimental data for the transverse form factor of the $\gamma N \to P_{33}(1232)$ transition defined by Eq. (3.2). The data are divided by $3G_{\mathrm{dip}}$, where $G_{\mathrm{dip}}(Q^2) = 1/[1 + Q^2/0.71\ (\mathrm{GeV}/c)^2]$. Data denoted by boxes are taken from Table 5 of Ref. [40] by recalculation for our definition of G_T; data denoted by asterisks are obtained in our analysis.

$$[G_T(Q^2)]^2 = (|G_M^*|^2 + 3|G_E^*|^2)\left(\frac{m_\Delta + m}{2m}\right)^2. \tag{3.2}$$

At large Q^2 our definition of G_T coincides with the Stoler's definition from Ref. [40]:

$$G_T^2 = G_T^2(\mathrm{Stoler})\frac{Q^2}{(m_\Delta - m)^2 + Q^2}. \tag{3.3}$$

The form factor G_T defined by Eq. (3.2) is more suitable for the description of low Q^2 data. This form factor is related to the helicity amplitudes of the $\gamma^* N \to P_{33}(1232)$ transition and to the total cross section of the reaction $\gamma^* p \to \pi N$ in the following way:

$$G_T^2 = \frac{1}{4\pi\alpha}(|A_{1/2}^p|^2 + |A_{3/2}^p|^2)\frac{2m(m_\Delta^2 - m^2)}{(m_\Delta - m)^2 + Q^2}, \tag{3.4}$$

$$\sigma(\gamma^* p \to \pi N) = 4\pi\alpha G_T^2 \frac{(m_\Delta - m)^2 + Q^2}{m_\Delta \Gamma (m_\Delta^2 - m^2)}. \tag{3.5}$$

It can be expressed through the multipoles $M_{1+} = (2A_{1+} - 3B_{1+})/4$ and $E_{1+} = (2A_{1+} + B_{1+})/4$ using Eq. (3.4) and the relations

$$A_{1+}^{3/2} = -A_{1/2}^p\left(\frac{3km}{8\Gamma\pi q m_\Delta}\right)^{1/2}, \tag{3.6}$$

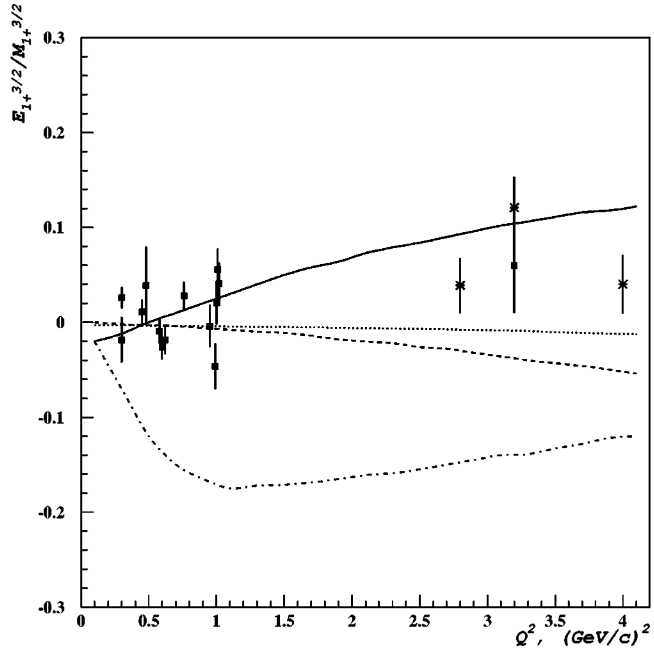

FIG. 3. Experimental data for the ratio $E^{3/2}_{1+}/M^{3/2}_{1+}$ obtained in our analysis (asterisks) and the data at low Q^2 [47] and at $Q^2=3.2\,(\mathrm{GeV}/c)^2$ from Ref. [48] in comparison with the predictions of Refs. [46] (full line), [49] (dotted line), [50] (dashed line), [51] (dash-dotted lines).

$$B^{3/2}_{1+}=A^{p}_{3/2}\left(\frac{km}{2\Gamma\pi q m_{\Delta}}\right)^{1/2}. \tag{3.7}$$

Our results for G_T in Fig. 2 are lower than other data. This is connected with the fact that they are obtained by taking into account only resonance contributions in the amplitude $M^{3/2}_{1+}$ which gives the main contribution into G_T. Our results confirm the whole tendency of the G_T data to fall more rapidly with increasing Q^2 than $1/Q^4$. Let us remind the reader that in the PQCD asymptotics G_T behaves as $1/Q^4$ [42–46]. So, there is no evidence for the presence of the PQCD contribution in G_T at $Q^2<4\,(\mathrm{GeV}/c)^2$.

In Figs. 3,4 our results for the ratios $E^{3/2}_{1+}/M^{3/2}_{1+}$ and $S^{3/2}_{1+}/M^{3/2}_{1+}$ corresponding to the resonance contributions to $M^{3/2}_{1+},E^{3/2}_{1+},S^{3/2}_{1+}$ are presented together with the data at

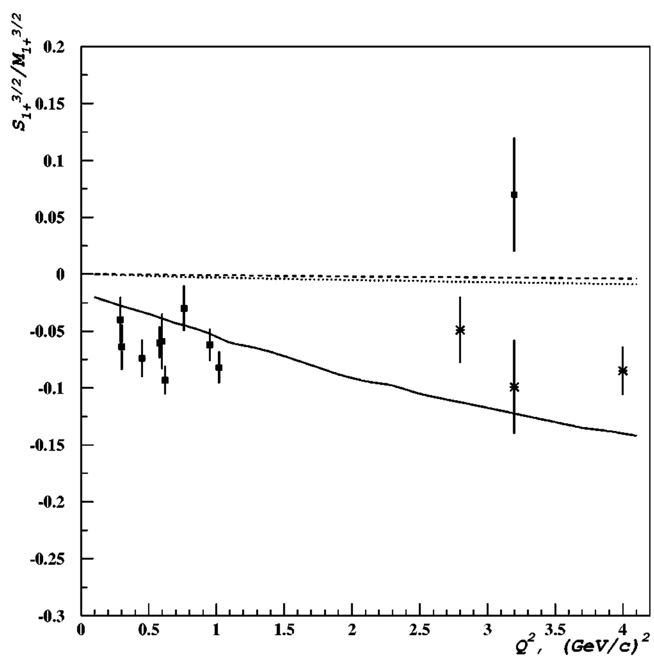

FIG. 4. Experimental data for the ratio $S^{3/2}_{1+}/M^{3/2}_{1+}$ obtained in our analysis (asterisks) and the data at low Q^2 [47] and at $Q^2=3.2\,(\mathrm{GeV}/c)^2$ from Ref. [48] in comparison with the predictions of Refs. [46] (full line), [49] (dotted line), [50] (dashed line).

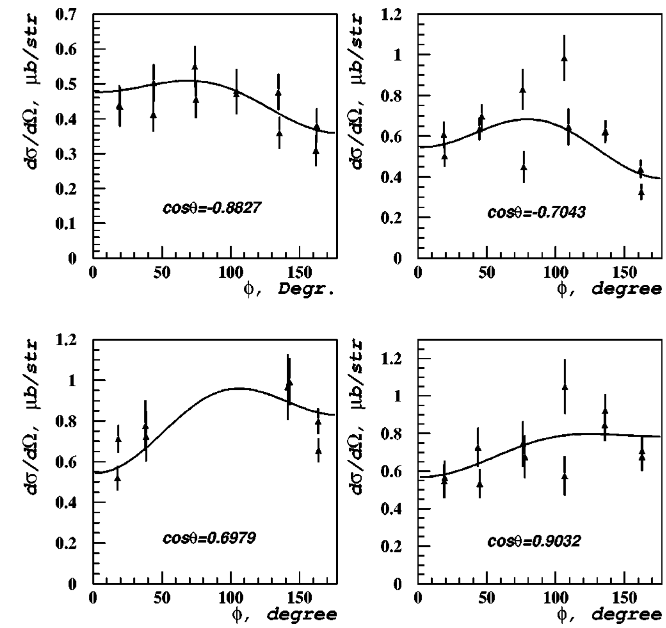

FIG. 5. Comparison of our results for ϕ distributions with the TJNAF data [2] at $W=1.235$ GeV, $Q^2=2.8\,(\mathrm{GeV}/c)^2$, $\epsilon=0.56$.

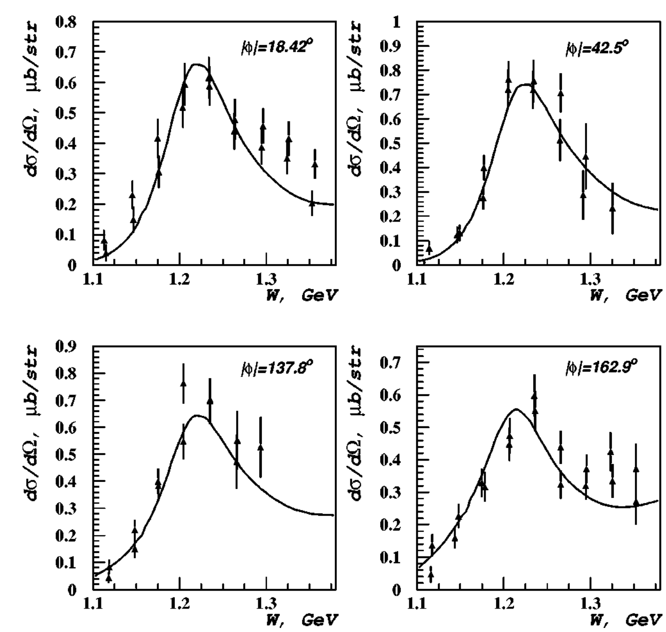

FIG. 6. Comparison of our results for energy dependence of the cross sections with the TJNAF data [2] at $Q^2=2.8\,(\mathrm{GeV}/c)^2$, $\cos\theta=0.7$, $\epsilon=0.56$.

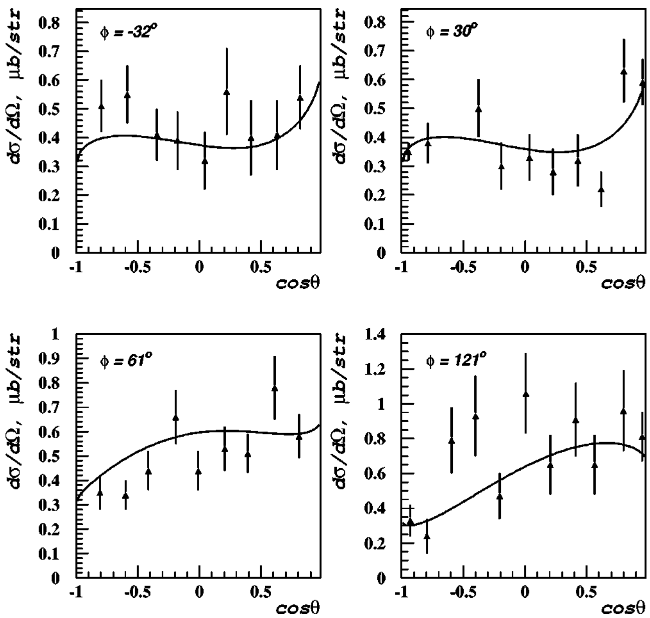

FIG. 7. Comparison of our results for angular distributions with the DESY data [3] at $W=1.235$ GeV, $Q^2=3.2\,(\mathrm{GeV}/c)^2$, $\epsilon=0.89$.

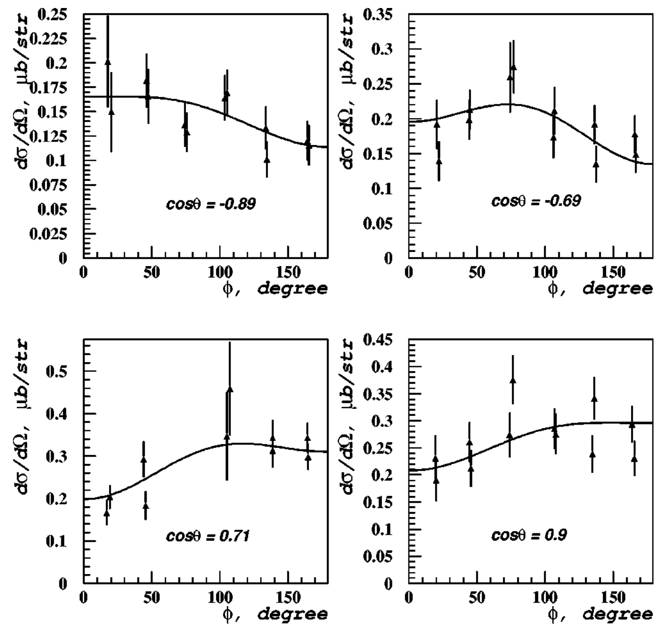

FIG. 9. Comparison of our results for ϕ distributions with the TJNAF data [2] at $W=1.235$ GeV, $Q^2=4\,(\mathrm{GeV}/c)^2$, $\epsilon=0.51$.

smaller Q^2 [47]. We have also presented the data points at $Q^2=3.2\,(\mathrm{GeV}/c)^2$ obtained from the DESY data in Ref. [48], assuming that the multipoles $M_{1+}^{3/2}, E_{1+}^{3/2}, S_{1+}^{3/2}$ are described by the sums of the resonance contributions taken in the Breit-Wigner form and the smooth nonresonance backgrounds.

It is known that the information on the Q^2 evolution of the ratio $E_{1+}^{3/2}/M_{1+}^{3/2}$ may play an important role in the investigation of mechanisms of the transition to the QCD asymptotics. The conservation of quark helicities in the asymptotic region of QCD leads to the asymptotic relation $E_{1+}^{3/2}/M_{1+}^{3/2} \to 1, Q^2 \to \infty$ [42–46]. In contrast with this, at $Q^2=0$ quark model predicts the strong suppression of $E_{1+}^{3/2}/M_{1+}^{3/2}$ which is confirmed by experiment. Thus, the transition from the nonperturbative region of QCD to the PQCD asymptotics should be characterized by a striking change of the behavior of this ratio. Summarizing our results one can say that the ratio $E_{1+}^{3/2}/M_{1+}^{3/2}$ is positive at $Q^2=2.8-4\,(\mathrm{GeV}/c)^2$. However, the

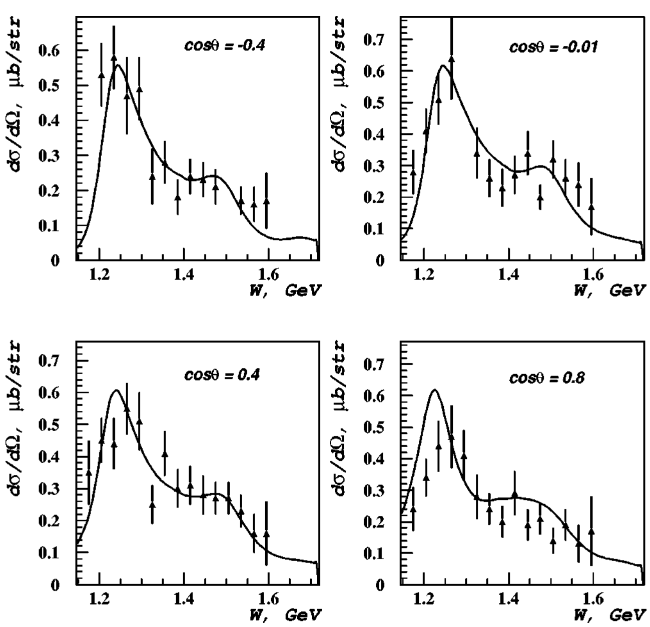

FIG. 8. Comparison of our results for energy dependence of the cross sections with the DESY data [3] at $Q^2=3.2\,(\mathrm{GeV}/c)^2$, $\phi=61.5°$, $\epsilon=0.89$.

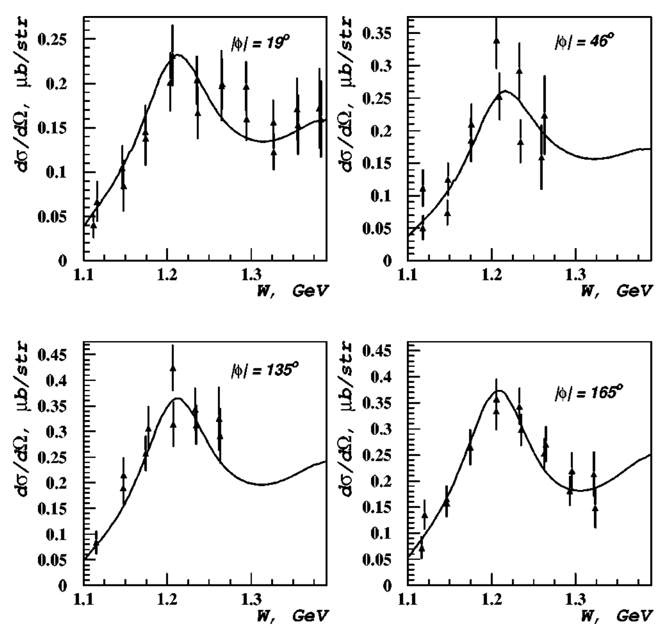

FIG. 10. Comparison of our results for energy dependence of the cross sections with the TJNAF data [2] at $Q^2=4\,(\mathrm{GeV}/c)^2$, $\cos\theta=0.7$, $\epsilon=0.51$.

magnitude is small, and the comparison with the data at low Q^2 does not show a visible change in the behavior of this ratio with increasing Q^2. Therefore, there is no evidence for the presence of the visible PQCD contribution into the transition $\gamma^* N \to P_{33}(1232)$ at $Q^2 = 2.8-4$ (GeV/c)^2.

In Figs. 3,4 the predictions obtained in the light cone relativistic quark model in Refs. [45,46] and in the relativized versions of the quark model in Refs. [49,50] are presented. It is seen that the predictions of Refs. [45,46] are not in bad agreement with the data. We have also presented the predictions from Ref. [51], where an attempt is made to find some approximate formula for the ratio $E^{3/2}_{1+}/M^{3/2}_{1+}$, which connects the quark model prediction at $Q^2 = 0$ with the PQCD asymptotics. One of the curves, which corresponds to a larger asymptotic value of $A_{1/2}$, describe the data quite well.

Figures 5–10 are presented to show the typical agreement of our results with experimental data for the differential cross sections $d\sigma/d\Omega = d\sigma/d\phi d\cos\theta$ at definite values of the polarization factor ϵ of the virtual photon. θ and ϕ are the polar and azimuthal angles of the pion according to the virtual photon in the c.m.s.

IV. DISCUSSION

In this work we have analyzed the TJNAF [2] and DESY [3] data on the cross sections of the exclusive reaction $p(e,e'p)\pi^0$ at $Q^2 = 2.8$, 3.2, and 4 (GeV/c)^2 and found the $P_{33}(1232)$ resonance contribution into the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$. As an input for the resonance and nonresonance contributions into these amplitudes the solutions of the integral equations for the multipoles obtained in Ref. [27] were used. These integral equations follow from the dispersion relations for $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$, if we take into account the unitarity condition for the multipoles. As was discussed in the Introduction on the example of the simplified version of the dispersion relations for the multipoles with the s-channel cut only, the solutions of the integral equations for $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ contain two parts which have an interpretation in terms of the resonance and nonresonance contributions into the multipoles. One part is the particular solution of the integral equations generated by the Born term. This part is the modification of the Born contribution, produced by the πN rescattering in the final state; we consider it as the nonresonance background contribution. It has a definite magnitude fixed by the Born term. The other part of the solution corresponds to the homogeneous parts of the integral equations. We identify it with the resonance contributions. These solutions have the definite shapes fixed by the dispersion relations and arbitrary weights which determine the resonance contributions to $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$. These weights were fitting parameters in our analyses and were found from experiment.

The dispersion relations for the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$, which were investigated in Ref. [27], in addition to the integrals over the s-channel cut in Eq. (1.2) also contain integrals over the u-channel cut. The u-channel cut brings the contributions of other multipoles into the dispersion relations, namely, the contributions of nonresonance multipoles and the couplings of the resonance miltipoles with each other. At $Q^2 = 0$ these contributions were evaluated in Ref. [29], where it was found that the contributions of nonresonance multipoles are negligibly small and the couplings of the resonance multipoles with each other are reasonably small. Using the results for the resonance multipoles obtained in this paper and the values of the nonresonance multipoles $E_{0+}, S_{0+}, M_{1-}, S_{1-}$, evaluated via the procedure described in Sec. II, one can estimate additional contributions coming from other multipoles in the dispersion relations for $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ at $Q^2 = 3-4$ (GeV/c)^2. Our estimations show that these additional contributions can be neglected and, therefore, the assumption on the smallness of these contributions made in Ref. [27] is correct. There are also high-energy contributions into dispersion integrals. The calculations made in Ref. [27] had shown that at $Q^2 = 0$ these contributions can be neglected in comparison with the contributions of the Born terms. The information at $Q^2 \neq 0$ is not enough to estimate such contributions; the solutions of integral equations in Ref. [27] were obtained under assumption that high-energy contributions into the dispersion relations for $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ can be neglected. In the future, when experimental data in the whole resonance region will be available, this assumption can be checked. If it will be found that the high-energy contributions are important, a new analysis in the $P_{33}(1232)$ resonance region, taking into account these contributions, will be necessary.

Let us draw attention to the following point too. The contributions of the diagram, corresponding to the process $\gamma^* N \to P_{33}(1232) \to \pi N$, into the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ we identify with the solutions of the homogeneous parts of the integral equations which follow from the dispersion relations for these amplitudes. The rescattering effects connected with the πN interaction in the final state modify the $\pi N P_{33}(1232)$ vertex in this diagram. A conclusion on the form of this modification can be made using the results of the dynamical model of Ref. [20], if the amplitude $h^{3/2}_{1+}$ of πN scattering is the pure resonance amplitude. According to these results in this case the factor at $1/(W - m_\Delta - i\Gamma/2)$ for $\gamma^* N \to P_{33}(1232) \to \pi N$ is equal to the product of the vertex $\gamma^* N P_{33}(1232)$ and the dressed vertex $\pi N P_{33}(1232)$. The dressed vertex $\pi N P_{33}(1232)$ can be found from experimental data on the width of the $P_{33}(1232) \to \pi N$ decay. This fact was used in the derivation of the relations (3.6),(3.7), which connect the helicity amplitudes $A^p_{1/2}, A^p_{3/2}$ and the resonance parts of the amplitudes $A^{3/2}_{1+}, B^{3/2}_{1+}$ [i.e., of the amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}$ (3.1)]. Our results for the transverse form factor G_T of the $\gamma^* N \to P_{33}(1232)$ transition presented in Fig. 2 are found from Eq. (3.4) using these relations between $A^p_{1/2}, A^p_{3/2}$ and $M^{3/2}_{1+}$, $E^{3/2}_{1+}$ (3.1).

The situation is more complicated if the amplitude $h^{3/2}_{1+}$ contains a nonresonance background. In this case it is reasonable to assume that the ratios of the resonance parts of the multipole amplitudes $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ are equal to the ratios of the vertices $\gamma^* N P_{33}(1232)$ for these amplitudes, i.e., the final state interaction modifies the P_{33}(1232) resonance contributions into $M^{3/2}_{1+}, E^{3/2}_{1+}, S^{3/2}_{1+}$ in the same way. This assumption is confirmed by the results obtained in Ref. [22] within dynamical model. Therefore, our results for the ratios $E^{3/2}_{1+}/M^{3/2}_{1+}$ and $S^{3/2}_{1+}/M^{3/2}_{1+}$, presented in Figs. 3,4 can be reliably identified with the corresponding ratios for the $\gamma^* N \rightarrow P_{33}(1232)$ transition. The same statement is right for the ratios of the multipole amplitudes at different values of Q^2, i.e., for example, for the ratios of G_T at different values of Q^2.

ACKNOWLEDGMENTS

We are grateful to V. Burkert, B. Mecking, and P. Stoler for interest and stimulating discussions. One of us (I.G.A.) has been helped enormously by the interest and insightful questions of N. Isgur. I.G.A. is also thankful to A. V. Radyushkin and N. L. Ter-Isaakyan for useful discussions. The hospitality at Jefferson Lab, where the main part of this work was accomplished, is gratefully acknowledged by I.G.A. S.G.S. thanks the U.S. Department of Energy for support under Contract No. DE-AC05-84ER40150. This work was partly supported by INTAS Grant No. 93-283ext.

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