Type-II superstrings and new spacetime superalgebras

Makoto Sakaguchi*
Theory Division, Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki, 305-0801 Japan

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We present a geometric formulation of type-IIA and -IIB superstring theories in which the Wess-Zumino term is second order in the supersymmetric currents. The currents are constructed using supergroup manifolds corresponding to superalgebras: the type-IIA superalgebra derived from *M* algebra and the type-IIB superalgebra obtained by a *T*-duality transformation of the type-IIA superalgebra. We find that a slight modification of the type-IIB superalgebra is needed to describe *D*-string theories, in which the U(1) gauge field on the world sheet is explicitly constructed in terms of *D*-string charges. A unification of the superalgebras in a $(10+1)$-dimensional $N=2$ superalgebra is discussed too. [S0556-2821(99)07202-1]

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I. INTRODUCTION

It is now widely appreciated that super *p*-branes play an important role in nonperturbative string physics. The dynamics of the super *p*-branes is generally very difficult, but it does possess some algebraic properties. One of these is a modification of the Poincaré superalgebra in the presence of super *p*-branes.

Siegel [1] found a manifestly supersymmetric formulation of the Green-Schwarz superstring, based on a superalgebra discovered earlier by Green [2]. This superalgebra is a generalization of super Poincaré algebra, in which a new fermionic generator is contained and translations do not commute with the supercharges. He constructed a suitable set of supercurrents on the corresponding supergroup manifold and wrote down the Wess-Zumino term of the Green-Schwarz action in a manifestly supersymmetric form, without having to go one higher dimension. Bergshoeff and Sezgin showed that Siegel's formulation generalizes to higher super *p*-branes [3]. They introduced a set of new spacetime superalgebras. By introducing the new coordinates corresponding to the new generators of the underlying superalgebra and constructing the supercurrents on the supergroup manifolds, they were able to write the Wess-Zumino terms for super *p*-branes, which are $(p+1)$th order in the supercurrents and which equal the usual Wess-Zumino terms up to the total derivative terms. (These *p*-brane superalgebras $(p\geqslant 1)$ were further investigated in [4].)

It is not known whether the formulation generalizes to type-II branes: type-II superstrings, Neveu-Schwarz 5-branes (NS5-branes) and *D p*-branes ($p=$odd for the type-IIB superstring theory and $p=$even for the type-IIA superstring theory). In this paper, we show that the formulation generalizes to type-II superstrings and *D*-strings. To do this, we introduce a set of new spacetime superalgebras: the type-IIA superalgebra derived from the *M* algebra which was discovered by Sezgin [5] and the type-IIB superalgebra obtained by a *T*-duality transformation of the type-IIA superalgebra. Using the new superalgebras, we construct supercurrents on the supergroup manifolds corresponding to the superalgebras. In terms of these currents, we write down the Wess-Zumino terms, which are second order in the supercurrents. We find that one needs a slight modification of the type-IIB superalgebra in order to describe *D*-string theories, in which the U(1) gauge field on the world sheet is parametrized by coordinates associated with the *D*-string charges. The modified type-IIB superalgebra, which corresponds to the description of the type-IIB superstring and the *D*-string on an equal footing, is not related by the *T* duality to the type-IIA superalgebra obtained from the *M* algebra. As a trial to relate these superalgebras, we consider a unification of the superalgebras in a $(10+1)$-dimensional $N=2$ superalgebra.

This paper is organized as follows. We first present a review of the technology used in this paper and Siegel's formulation. In Sec. III, we derive the type-IIA superalgebra from the *M* algebra and show that the superalgebra corresponds to the type-IIA superstring theories. Next, in Sec. IV, performing a *T*-duality transformation, we obtain the type-IIB superalgebra. The algebra is shown to correspond to the type-IIB superstrings. In Sec. V, *D*-strings are found to be described by modifying the type-IIB superalgebra, in which the U(1) gauge field is parametrized by the coordinates corresponding to the *D*-string charges. In Sec. VI, considering the identities, we show that the modified type-IIB superalgebra cannot be related to the type-IIA superalgebra by *T*-duality transformations. A unification of these algebras in a $(10+1)$-dimensional $N=2$ superalgebra is discussed in Sec. VII. The last section is devoted to a summary and discussions.

II. SUPERALGEBRA AND SIEGEL'S FORMULATION

Let us denote the generators of an algebra collectively as T_A. The algebra can be written as

$$[T_A,T_B\}=f_{AB}{}^{C}T_C. \qquad (2.1)$$

In the dual basis, the Maurer-Cartan super one-forms are defined by

$$e^A=dZ^M L_M{}^A, \qquad (2.2)$$

*Email address: gu@theory.kek.jp

where dZ^M are the differentials on the group manifold. The left-invariant group vielbeins $L_M{}^A$ and the pullbacks $L_i{}^A$ are obtained by the left-invariant Maurer-Cartan form

$$U^{-1}\partial_i U=\partial_i Z^M L_M{}^A T_A = L_i{}^A T_A\,, \tag{2.3}$$

where U is a supergroup element. The Maurer-Cartan equations, which are expressed in terms of the dual forms

$$de^C=-\frac{1}{2}e^B\wedge e^A f_{AB}{}^C, \tag{2.4}$$

contain equivalent information about the algebra. Given a super q-form G, the exterior derivative acts as follows: $d(F\wedge G)=F\wedge dG+(-1)^q dF\wedge G$. The Jacobi identities are satisfied iff the integrability conditions $d^2e^A=0$ hold.

Similarly, the right-invariant group vielbeins $R_M{}^A$ are obtained by the right-invariant Maurer-Cartan form

$$\partial_i U U^{-1}=\partial_i Z^M R_M{}^A T_A = R_i{}^A T_A\,. \tag{2.5}$$

Using the the right-invariant group vielbeins $R_M{}^A$, supersymmetry transformations are obtained as follows. An infinitesimal transformation is written as $U'=(1+\epsilon)U$, where $\epsilon=\epsilon^A T_A$ is the transformation parameter. This implies that $\epsilon=\delta U U^{-1}=\delta Z^B R_B{}^A T_A$; one then finds that an infinitesimal transformation can be expressed as

$$\delta Z^A=\epsilon^A R_A{}^M, \tag{2.6}$$

where $R_A{}^N$ is defined by $R_M{}^A R_A{}^N=\delta_M^N$. The transformation parameter ϵ^α associated with the supercharge Q_α can be interpreted as a rigid supersymmetry transformation parameter.

Useful in calculating the left- or right-invariant Maurer-Cartan equations are the Zumino's formulas:

$$e^{-\phi}de^{\phi}=\left(\frac{1-e^{-\phi}}{\phi}\right)\wedge d\phi, \tag{2.7}$$

$$de^{\phi}e^{-\phi}=\left(\frac{e^{\phi}-1}{\phi}\right)\wedge d\phi, \tag{2.8}$$

$$e^{-\phi}\beta e^{\phi}=e^{-\phi}\wedge\beta, \tag{2.9}$$

$$e^{\phi}\beta e^{-\phi}=e^{\phi}\wedge\beta, \tag{2.10}$$

where the wedge denotes a compact expression of the commutation relation $\phi\wedge\psi=[\phi,\psi]$, $\phi^2\wedge\psi=[\phi,[\phi,\psi]]$, and $1\wedge\psi=\psi$.

Siegel [1] found a manifestly supersymmetric formulation of the Green-Schwarz superstring based on a superalgebra [2],

$$\{Q_\alpha,Q_\beta\}=(\gamma^a)_{\alpha\beta}P_a\,,\quad [Q_\alpha,P_a]=(\gamma_a)_{\alpha\beta}\Sigma^\beta, \tag{2.11}$$

in which the translation does not commute with the supercharge. Parametrizing the supergroup manifold as

$$U=e^{\phi_\alpha\Sigma^\alpha}e^{x_aP^a}e^{\theta^\alpha Q_\alpha}, \tag{2.12}$$

where coordinates $Z^A=(\phi_\alpha,x_a,\theta^\alpha)$ associate to the generators $T_A=(\Sigma^\alpha,P^a,Q_\alpha)$, the superstring action is written in terms of left-invariant pullback vielbeins,

$$I=\int d^2\xi\left[-\frac{1}{2}\sqrt{-g}g^{ij}L_i{}^aL_j{}^b\eta_{ab}-\frac{1}{2}\epsilon^{ij}L_i{}^\alpha L_{j\alpha}\right], \tag{2.13}$$

where g_{ij} and η_{ab} are the world sheet and spacetime metric, respectively, and $g=\det g_{ij}$. A nontrivial feature of this new action is that the new coordinates ϕ_α only occur as a total derivative term. Up to this total derivative term, the above action is identical to the standard Green-Schwarz superstring action. Furthermore, the Wess-Zumino term in the above action is manifestly supersymmetric, while in the usual Green-Schwarz formulation the supersymmetry is up to a total derivative term. These transformations involve $L_i{}^a$ and $L_i{}^\alpha$, which remain unchanged by the presence of the new coordinate ϕ_α. The coefficient of Wess-Zumino term is fixed so that the action is also invariant under the usual κ-symmetry transformations. Following the same line, the authors of Ref. [3] showed that p-brane actions can be constructed by using new superalgebras.

III. TYPE-IIA SUPERSTRING

We derive the type-IIA superalgebra by a dimensional reduction of the M algebra. For later use in Sec. IV, we include $D0$-brane and $D2$-brane charges in addition to supertranslation and superstring charges. We then show that the type-IIA superstring action can be constructed using the obtained type-IIA superalgebra.

A. Type-IIA superalgebra from *M* algebra

The M algebra found by Sezgin [5] is characterized by generators: supertranslation Q_M, $M2$-brane Z^{MN}, "superstring"[1] Z^M, and $M5$-brane Z^{MNOPQ}, where 11-dimensional spacetime indices $\mu,\nu,\ldots$ and Majorana spinor indices $\alpha,\beta,\ldots$ are collectively denoted as $M,N,\ldots$, so that $Q_M=(P_\mu,Q_\alpha)$, $Z^{MN}=(Z^{\mu\nu},Z^{\mu\alpha},Z^{\alpha\beta})$, etc. The Maurer-Cartan equations, containing equivalent information about the algebra, are described in terms of the dual basis: e^M, e_{MN}, e_M, and e_{MNOPQ}, respectively. It is sufficient for our present purpose to consider the former three: e^M, e_{MN}, and e_M. The corresponding part of the M algebra is (we call this algebra M algebra for simplicity throughout this paper)

$$de^\mu=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma^\mu)_{\alpha\beta}, \tag{3.1}$$

$$de_\mu=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma_\mu)_{\alpha\beta}, \tag{3.2}$$

[1] This was discussed in [6] and [7].

$$de_\alpha = -e^\beta \wedge e^\mu (\gamma_\mu)_{\alpha\beta} + (1-\lambda) e^\beta \wedge e'_\mu (\gamma^\mu)_{\alpha\beta} - \frac{\lambda}{10} e^\beta \wedge e_{\mu\nu} (\gamma^{\mu\nu})_{\alpha\beta}, \tag{3.3}$$

$$de_{\mu\nu} = -\frac{1}{2} e^\alpha \wedge e^\beta (\gamma_{\mu\nu})_{\alpha\beta}, \tag{3.4}$$

$$de_{\mu\alpha} = -e^\beta \wedge e^\nu (\gamma_{\mu\nu})_{\alpha\beta} - e^\beta \wedge e_{\mu\nu} (\gamma^\nu)_{\alpha\beta}, \tag{3.5}$$

$$de_{\alpha\beta} = \frac{1}{2} e^\mu \wedge e^\nu (\gamma_{\mu\nu})_{\alpha\beta} - \frac{1}{2} e_{\mu\nu} \wedge e^\mu (\gamma^\nu)_{\alpha\beta} - \frac{1}{4} e_{\mu\gamma} \wedge e^\gamma (\gamma^\mu)_{\alpha\beta} - 2 e_{\mu\alpha} \wedge e^\gamma (\gamma^\mu)_{\beta\gamma}, \tag{3.6}$$

where the Jacobi identities are satisfied due to the identities in $(10+1)$ dimensions:

$$(\gamma_\mu)_{(\alpha\beta}(\gamma^{\mu\nu})_{\gamma\delta)} = 0, \tag{3.7}$$

$$(\gamma_\mu)_{(\alpha\beta}(\gamma^\mu)_{\gamma\delta)} + \frac{1}{10}(\gamma_{\mu\nu})_{(\alpha\beta}(\gamma^{\mu\nu})_{\gamma\delta)} = 0. \tag{3.8}$$

Throughout this paper we use a notation where a given spinor always has an upper or a lower spinor index, and never raise or lower a spinor index using the charge-conjugation matrix.

The type-IIA superalgebra is obtained by a dimensional reduction of the 11th direction, say $x^\natural$. The obtained type-IIA superalgebra is characterized by generators: supertranslation Q_A, superstring Z^A, $D0$-brane Σ, $D2$-brane Σ^{AB}, and $Z^{A'}$ and $Z^{\natural\,'}$ originated from "superstring" Z^M in 11 dimensions, where the indices $A,B,\ldots$ collectively denote ten-dimensional spacetime indices, $a,b,\ldots$ and Majorana-Weyl spinor indices, $\alpha,\beta,\ldots$ and $\dot\alpha,\dot\beta,\ldots$ with positive and negative chirality, respectively. We find that the dual forms of the generators of the type-IIA superalgebra are defined in terms of those of the generators of the M algebra as follows:

$$\begin{aligned}
\text{supertranslation } Q_A &: \quad e^A = (e^a, e^\alpha, e^{\dot\alpha}),\\
\text{superstring } Z^A &: \quad e_A = (e_{\natural a}, e_{\natural\alpha}, e_{\natural\dot\alpha}),\\
D0\text{-brane } \Sigma &: \quad e = (e^\natural),\\
D2\text{-brane } \Sigma^{AB} &: \quad e_{AB} = (e_{ab}, e_{a\alpha}, e_{a\dot\alpha}, e_{\alpha\beta}, e_{\alpha\dot\beta}, e_{\dot\alpha\dot\beta}),\\
Z^{A'} &: \quad e'_A = (e'_a, e'_\alpha, e'_{\dot\alpha}),\\
Z^{\natural\,'} &: \quad e' = (e'_\natural),
\end{aligned} \tag{3.9}$$

which is consistent with the fact that the superstring and $D2$-brane consist of a wrapped $M2$-brane and $M2$-brane, respectively. The Maurer-Cartan equations of the type-IIA superalgebra are found to be

$$de^a = -\frac{1}{2} e^\alpha \wedge e^\beta (\gamma^a)_{\alpha\beta} - \frac{1}{2} e^{\dot\alpha} \wedge e^{\dot\beta} (\gamma^a)_{\dot\alpha\dot\beta}, \tag{3.10}$$

$$de_a = -\frac{1}{2} e^\alpha \wedge e^\beta (\gamma_a)_{\alpha\beta} + \frac{1}{2} e^{\dot\alpha} \wedge e^{\dot\beta} (\gamma_a)_{\dot\alpha\dot\beta}, \tag{3.11}$$

$$de_\alpha = -e^\beta \wedge e^a (\gamma_a)_{\alpha\beta} - e^\beta \wedge e_a (\gamma^a)_{\alpha\beta}, \tag{3.12}$$

$$de_{\dot\alpha} = +e^{\dot\beta} \wedge e^a (\gamma_a)_{\dot\alpha\dot\beta} - e^\beta \wedge e_a (\gamma^a)_{\dot\alpha\dot\beta}, \tag{3.13}$$

$$de = -e^\alpha \wedge e^{\dot\beta} (1)_{\alpha\dot\beta}, \tag{3.14}$$

$$de_{ab} = -e^\alpha \wedge e^{\dot\beta} (\gamma_{ab})_{\alpha\dot\beta}, \tag{3.15}$$

$$de_{a\alpha} = -e^{\dot\beta} \wedge e^b (\gamma_{ab})_{\alpha\dot\beta} + e^\beta \wedge e (\gamma_a)_{\alpha\beta} - e^\beta \wedge e_{ab} (\gamma^b)_{\alpha\beta} + e^{\dot\beta} \wedge e_a (1)_{\alpha\dot\beta}, \tag{3.16}$$

$$de_{a\dot\alpha} = -e^\beta \wedge e^b (\gamma_{ab})_{\dot\alpha\beta} - e^{\dot\beta} \wedge e (\gamma_a)_{\dot\alpha\dot\beta} - e^{\dot\beta} \wedge e_{ab} (\gamma^b)_{\dot\alpha\dot\beta} - e^\beta \wedge e_a (1)_{\dot\alpha\beta}, \tag{3.17}$$

$$de_{\alpha\beta} = +e \wedge e^a (\gamma_a)_{\alpha\beta} - \frac{1}{2} e_{ab} \wedge e^a (\gamma^b)_{\alpha\beta} - \frac{1}{2} e_b \wedge e (\gamma^b)_{\alpha\beta} - \frac{1}{4} e_{a\gamma} \wedge e^\gamma (\gamma^a)_{\alpha\beta} - \frac{1}{4} e_{a\dot\gamma} \wedge e^{\dot\gamma} (\gamma^a)_{\alpha\beta} - 2 e_{a\alpha} \wedge e^\gamma (\gamma^a)_{\beta\gamma} - 2 e_\alpha \wedge e^{\dot\gamma} (1)_{\beta\dot\gamma}, \tag{3.18}$$

$$de_{\alpha\dot{\beta}}=+\frac{1}{2}e^a\wedge e^b(\gamma_{ab})_{\alpha\dot{\beta}}+\frac{1}{2}e_a\wedge e^a(1)_{\alpha\dot{\beta}}-\frac{1}{4}e_{\gamma}\wedge e^{\gamma}(1)_{\alpha\dot{\beta}}-\frac{1}{4}e_{\dot{\gamma}}\wedge e^{\dot{\gamma}}(1)_{\alpha\dot{\beta}}-e_{a\alpha}\wedge e^{\dot{\gamma}}(\gamma^a)_{\dot{\beta}\dot{\gamma}}+e_{\alpha}\wedge e^{\gamma}(1)_{\dot{\beta}\gamma}$$
$$-e_{a\dot{\beta}}\wedge e^{\gamma}(\gamma^a)_{\alpha\gamma}-e_{\dot{\beta}}\wedge e^{\dot{\gamma}}(1)_{\alpha\dot{\gamma}}, \tag{3.19}$$

$$de_{\dot{\alpha}\dot{\beta}}=-e\wedge e^a(\gamma_a)_{\dot{\alpha}\dot{\beta}}-\frac{1}{2}e_{ab}\wedge e^a(\gamma^b)_{\dot{\alpha}\dot{\beta}}-\frac{1}{2}e_b\wedge e(\gamma^b)_{\dot{\alpha}\dot{\beta}}-\frac{1}{4}e_{a\gamma}\wedge e^{\gamma}(\gamma^a)_{\dot{\alpha}\dot{\beta}}-\frac{1}{4}e_{a\dot{\gamma}}\wedge e^{\dot{\gamma}}(\gamma^a)_{\dot{\alpha}\dot{\beta}}-2e_{a\dot{\alpha}}\wedge e^{\dot{\gamma}}(\gamma^a)_{\dot{\beta}\dot{\gamma}}$$
$$+2e_{\dot{\alpha}}\wedge e^{\gamma}(1)_{\dot{\beta}\gamma}, \tag{3.20}$$

$$de'_a=-\frac{1}{2}e^{\alpha}\wedge e^{\beta}(\gamma_a)_{\alpha\beta}-\frac{1}{2}e^{\dot{\alpha}}\wedge e^{\dot{\beta}}(\gamma_a)_{\dot{\alpha}\dot{\beta}}, \tag{3.21}$$

$$de'_{\alpha}=-e^{\beta}\wedge e^a(\gamma_a)_{\alpha\beta}-e^{\dot{\beta}}\wedge e(1)_{\alpha\dot{\beta}}+(1-\lambda)e^{\beta}\wedge e'_a(\gamma^a)_{\alpha\beta}+(1-\lambda)e^{\dot{\beta}}\wedge e'(1)_{\alpha\dot{\beta}}-\frac{\lambda}{10}e^{\dot{\beta}}\wedge e_{ab}(\gamma^{ab})_{\alpha\dot{\beta}}$$
$$-\frac{\lambda}{5}e^{\beta}\wedge e_a(\gamma^a)_{\alpha\beta}, \tag{3.22}$$

$$de'_{\dot{\alpha}}=-e^{\dot{\beta}}\wedge e^a(\gamma_a)_{\dot{\alpha}\dot{\beta}}+e^{\beta}\wedge e(1)_{\dot{\alpha}\beta}+(1-\lambda)e^{\dot{\beta}}\wedge e'_a(\gamma^a)_{\dot{\alpha}\dot{\beta}}-(1-\lambda)e^{\beta}\wedge e'(1)_{\dot{\alpha}\beta}-\frac{\lambda}{10}e^{\beta}\wedge e_{ab}(\gamma^{ab})_{\dot{\alpha}\beta}$$
$$+\frac{\lambda}{5}e^{\dot{\beta}}\wedge e_a(\gamma^a)_{\dot{\alpha}\dot{\beta}}, \tag{3.23}$$

$$de'=-e^{\alpha}\wedge e^{\dot{\beta}}(1)_{\alpha\dot{\beta}}, \tag{3.24}$$

where we used the following relations [8]: $(1)_{\alpha\dot{\beta}}=-(1)_{\dot{\beta}\alpha}$ and $(\gamma_{ab})_{\alpha\dot{\beta}}=+(\gamma_{ab})_{\dot{\beta}\alpha}$. The type-IIA superalgebra is closed due to the following identities:

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)}=0, \tag{3.25}$$

$$(1)_{\dot{\alpha}(\beta}(\gamma^a)_{\gamma\delta)}+(\gamma^a{}_b)_{\dot{\alpha}(\beta}(\gamma^b)_{\gamma\delta)}=0, \tag{3.26}$$

$$-(1)_{\alpha(\dot{\beta}}(1)_{\dot{\gamma})\delta}+\frac{2}{5}(\gamma_a)_{\alpha\delta}(\gamma^a)_{(\dot{\beta}\dot{\gamma})}+\frac{1}{10}(\gamma_{ab})_{\alpha(\dot{\beta}}(\gamma^{ab})_{\dot{\gamma})\delta}$$
$$=0, \tag{3.27}$$

and those in which the undotted spinor indices are exchanged for the dotted ones. The last identity (3.27) is needed to satisfy the Jacobi identity for primed dual forms originating from ‘‘superstring’’ in the M algebra.

B. Type-IIA superstring

We now consider a subalgebra of the type-IIA superalgebra generated by the following generators: $T_A=\{P_a,Q_{\alpha},Q_{\dot{\alpha}},Z^a,Z^{\alpha},Z^{\dot{\alpha}}\}$. The algebra turns out to be

$$\{Q_{\alpha},Q_{\beta}\}=(\gamma^a)_{\alpha\beta}P_a+(\gamma_a)_{\alpha\beta}Z^a,$$

$$\{Q_{\dot{\alpha}},Q_{\dot{\beta}}\}=(\gamma^a)_{\dot{\alpha}\dot{\beta}}P_a-(\gamma_a)_{\dot{\alpha}\dot{\beta}}Z^a,$$

$$[P_b,Q_{\beta}]=2(\gamma_b)_{\alpha\beta}Z^{\alpha},\quad [Z^b,Q_{\beta}]=2(\gamma^b)_{\alpha\beta}Z^{\alpha},$$

$$[P_b,Q_{\dot{\beta}}]=-2(\gamma_b)_{\dot{\alpha}\dot{\beta}}Z^{\dot{\alpha}},\quad [Z^b,Q_{\dot{\beta}}]=2(\gamma^b)_{\dot{\alpha}\dot{\beta}}Z^{\dot{\alpha}}, \tag{3.28}$$

which is closed due to the identity (3.25) and the undotted indices replaced by dotted spinor ones.

We show that the type-IIA superstring action can be constructed from the above algebra. In this sense, we refer to the algebra as type-IIA superstring algebra. The supergroup manifold is parametrized as

$$U=e^{z_aZ^a}e^{\zeta_{\dot{\alpha}}Z^{\dot{\alpha}}}e^{\zeta_{\alpha}Z^{\alpha}}e^{x^aP_a}e^{\theta^{\dot{\alpha}}Q_{\dot{\alpha}}}e^{\theta^{\alpha}Q_{\alpha}}, \tag{3.29}$$

and the left-invariant pullback supergroup vielbeins are obtained as

$$L_i{}^{\alpha}=\partial_i\theta^{\alpha},\qquad L_i{}^{\dot{\alpha}}=\partial_i\theta^{\dot{\alpha}}, \tag{3.30}$$

$$L_i{}^a=\partial_i x^a+\frac{1}{2}(\bar{\dot{\theta}}\gamma^a\partial_i\dot{\theta})+\frac{1}{2}(\bar{\theta}\gamma^a\partial_i\theta), \tag{3.31}$$

$$L_{ia}=\partial_i z_a-\frac{1}{2}(\bar{\dot{\theta}}\gamma_a\partial_i\dot{\theta})+\frac{1}{2}(\bar{\theta}\gamma_a\partial_i\theta), \tag{3.32}$$

$$L_{i\alpha}=\partial_i\zeta_{\alpha}+2\partial_i z_a(\bar{\theta}\gamma^a)_{\alpha}+2\partial_i x^a(\bar{\theta}\gamma_a)_{\alpha}$$
$$+\frac{2}{3}(\bar{\theta}\gamma^a\partial_i\theta)(\bar{\theta}\gamma_a)_{\alpha}, \tag{3.33}$$

$$L_{i\dot{\alpha}}=\partial_i\zeta_{\dot{\alpha}}+2\partial_i z_a(\bar{\dot{\theta}}\gamma^a)_{\dot{\alpha}}-2\partial_i x^a(\bar{\dot{\theta}}\gamma_a)_{\dot{\alpha}} -\frac{2}{3}(\bar{\dot{\theta}}\gamma^a\partial_i\dot{\theta})(\bar{\dot{\theta}}\gamma_a)_{\dot{\alpha}}, \tag{3.34}$$

which are invariant under the following supersymmetry transformations:

$$\delta\theta^\alpha=\epsilon^\alpha,\quad \delta\theta^{\dot{\alpha}}=\epsilon^{\dot{\alpha}},$$

$$\delta x^a=-\frac{1}{2}(\bar{\epsilon}\gamma^a\theta)-\frac{1}{2}(\bar{\dot{\epsilon}}\gamma^a\dot{\theta}),$$

$$\delta z_a=-\frac{1}{2}(\bar{\epsilon}\gamma^a\theta)+\frac{1}{2}(\bar{\dot{\epsilon}}\gamma^a\dot{\theta}),$$

$$\delta\zeta_\alpha=-2x^a(\bar{\epsilon}\gamma_a)_\alpha-2z_a(\bar{\epsilon}\gamma^a)_\alpha+\frac{2}{3}(\bar{\epsilon}\gamma_a\theta)(\bar{\theta}\gamma^a)_\alpha,$$

$$\delta\zeta_{\dot{\alpha}}=+2x^a(\bar{\dot{\epsilon}}\gamma_a)_{\dot{\alpha}}-2z_a(\bar{\dot{\epsilon}}\gamma^a)_{\dot{\alpha}}-\frac{2}{3}(\bar{\dot{\epsilon}}\gamma_a\dot{\theta})(\bar{\hat{\dot{\theta}}}\gamma^a)_{\dot{\alpha}}. \tag{3.35}$$

In the case where we do not denote the spinor indices explicitly, it is always understood that they have their standard position, e.g., $(\gamma_a\theta)_\alpha=(\gamma_a)_{\alpha\beta}\theta^\beta$, $(\bar{\theta}\gamma_a\partial_i\theta)=\theta^\alpha(\gamma_a)_{\alpha\beta}\partial_i\theta^\beta$, etc:

We find that the type-IIA superstring action can be constructed as

$$I=\int d^2\xi\Bigg[-\frac{1}{2}\sqrt{-g}g^{ij}L_i{}^aL_j{}^b\eta_{ab} -\frac{1}{2}\epsilon^{ij}\left(L_i{}^aL_{ja}+\frac{1}{4}L_i{}^\alpha L_{j\alpha}+\frac{1}{4}L_i{}^{\dot{\alpha}}L_{j\dot{\alpha}}\right)\Bigg], \tag{3.36}$$

where g_{ij} and η_{ab} are the world sheet and spacetime metric, respectively, and $g=\det g_{ij}$. The last three terms in the action constitute the manifestly supersymmetric form of the Wess-Zumino term. The coefficient of the Wess-Zumino term is determined so that the action enjoys fermionic κ symmetry:

$$\delta_\kappa\theta^\alpha=(1+\Gamma)^\alpha{}_\beta\kappa^\beta,\quad \delta_\kappa\theta^{\dot{\alpha}}=(1-\Gamma)^{\dot{\alpha}}{}_{\dot{\beta}}\kappa^{\dot{\beta}},$$

$$\delta_\kappa x^a=\frac{1}{2}(\delta_\kappa\bar{\theta}\gamma^a\theta)+\frac{1}{2}(\delta_\kappa\bar{\dot{\theta}}\gamma^a\dot{\theta}), \tag{3.37}$$

where $\Gamma=[1/(2\sqrt{-g})]\epsilon^{ij}L_i{}^aL_j{}^b\gamma_{ab}$.

The two-form b is defined from the Wess-Zumino term in the action as

$$b=-\frac{1}{2}\left(L^a\wedge L_a+\frac{1}{4}L^\alpha\wedge L_\alpha+\frac{1}{4}L^{\dot{\alpha}}\wedge L_{\dot{\alpha}}\right). \tag{3.38}$$

The three-form $h=db$ is calculated as

$$h=-\frac{1}{2}L^a\wedge L^\alpha\wedge L^\beta(\gamma_a)_{\alpha\beta}+\frac{1}{2}L^a\wedge L^{\dot{\alpha}}\wedge L^{\dot{\beta}}(\gamma_a)_{\dot{\alpha}\dot{\beta}}. \tag{3.39}$$

Note that all of the dependence on the new fermionic coordinates has dropped out from the expression for h. In fact, the antisymmetric part of the action is the well-known Wess-Zumino term of the type-IIA superstrings up to total derivative terms:

$$\int\epsilon^{ij}b_{ij}=\int\frac{1}{2}\epsilon^{ij}\Bigg[\partial_i x^a((\bar{\theta}\gamma_a\partial_j\theta)-(\bar{\dot{\theta}}\gamma_a\partial_j\dot{\theta})) +\frac{1}{2}(\bar{\dot{\theta}}\gamma^a\partial_i\dot{\theta})(\bar{\theta}\gamma_a\partial_j\theta)\Bigg]. \tag{3.40}$$

As a result, we conclude that the type-IIA superstring action is constructed from the type-IIA superstring algebra (3.28).

IV. TYPE-IIB SUPERSTRING

In this section, the type-IIB superalgebra is constructed as the T dual of the type-IIA superalgebra. The type-IIB superalgebra includes D-string charges as well as superstring charges. We show that the type-IIB superstring action is constructed by means of the type-IIB superalgebra.

A. Type-IIB superalgebra and *T* duality

In order to obtain type-IIB superalgebra, we consider a T-duality transformation of the type-IIA superalgebra. We denote the ninth spacelike direction as $x^\sharp$ with respect to which T duality is performed. The type-IIB superalgebra is generated by supertranslations Q_A, superstring Z^A, and D-string Σ^A, which can be expressed in terms of generators of the type-IIA superalgebra. Since the type-IIB superalgebra is generated by generators with undotted spinor indices, the chirality of the fermionic generators with dotted spinor indices $\dot{\alpha}$ in the type-IIA superalgebra is flipped by multiplying $\gamma^\sharp$, as was done in Ref. [9]. We denote chirality flipped spinor indices as $\tilde{\alpha}$ and $(a,\alpha,\tilde{\alpha})$ as A collectively.

It turns out to be easy to perform the T-duality transformation by using the Maurer-Cartan equations. We found that the dual forms $\hat{e}^A$, $\hat{e}_A$, and $\hat{e}'_A$ of generators Q_A, Z^A, and Σ^A, respectively, can be written in terms of those of the generators of the type-IIA superalgebra as follows:

$$\text{supertranslation } Q_A:\quad \hat{e}^A=(\hat{e}^a=(e^i,e_\sharp),\ \hat{e}^\alpha=e^\alpha,\ \hat{e}^{\tilde{\alpha}}=\gamma^\sharp e^{\dot{\alpha}}),$$

$$\text{superstring } Z^A:\quad \hat{e}_A=(\hat{e}_a=(e_i,e^\sharp),\ \hat{e}_\alpha=e_\alpha,\ \hat{e}_{\tilde{\alpha}}=\gamma^\sharp e^{\dot{\alpha}}),$$

$$D\text{-string}\ \ \Sigma^A: \quad \hat{e}'_A = (\hat{e}'_a = (e_{\sharp i}, -e),\ \ \hat{e}'_\alpha = e_{\sharp\alpha},\ \ \hat{e}'_{\tilde{\alpha}} = \gamma^\sharp e_{\sharp\dot{\alpha}}), \tag{4.1}$$

where i runs, except for $\sharp$. We use a notation in which the primed dual form corresponds to a dual form for D-string charges. Since the dual forms $e_{\alpha\beta}$, $e_{\alpha\dot{\beta}}$, and $e_{\dot{\alpha}\dot{\beta}}$ of the type-IIA superalgebra are parts of the type-IIB $D3$-brane charges, we neglect them here. The resulting Maurer-Cartan equations for the type-IIB superalgebra are found to be (dropping carets)

$$de^a = -\frac{1}{2}e^\alpha \wedge e^\beta (\gamma^a)_{\alpha\beta} - \frac{1}{2}e^{\tilde{\alpha}} \wedge e^{\tilde{\beta}} (\gamma^a)_{\tilde{\alpha}\tilde{\beta}}, \tag{4.2}$$

$$de_a = -\frac{1}{2}e^\alpha \wedge e^\beta (\gamma_a)_{\alpha\beta} + \frac{1}{2}e^{\tilde{\alpha}} \wedge e^{\tilde{\beta}} (\gamma_a)_{\tilde{\alpha}\tilde{\beta}}, \tag{4.3}$$

$$de_\alpha = -e^\beta \wedge e^a (\gamma_a)_{\alpha\beta} - e^\beta \wedge e_a (\gamma^a)_{\alpha\beta}, \tag{4.4}$$

$$de_{\tilde{\alpha}} = +e^{\tilde{\beta}} \wedge e^a (\gamma_a)_{\tilde{\alpha}\tilde{\beta}} - e^{\tilde{\beta}} \wedge e_a (\gamma^a)_{\tilde{\alpha}\tilde{\beta}}, \tag{4.5}$$

$$de'_a = -e^\alpha \wedge e^{\tilde{\beta}} (\gamma_a)_{\alpha\tilde{\beta}}, \tag{4.6}$$

$$de'_\alpha = -e^{\tilde{\beta}} \wedge e^a (\gamma_a)_{\alpha\tilde{\beta}} - e^\beta \wedge e'_a (\gamma^a)_{\alpha\beta}, \tag{4.7}$$

$$de'_{\tilde{\alpha}} = -e^\beta \wedge e^a (\gamma_a)_{\tilde{\alpha}\beta} - e^{\tilde{\beta}} \wedge e'_a (\gamma^a)_{\tilde{\alpha}\tilde{\beta}}, \tag{4.8}$$

where the Jacobi identities are satisfied due to the well-known identity $(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)} = 0$. Here the tildes on the spinor indices of γ matrices are not written because the identity is not affected by the tildedness of the spinors. But if one wants to see how the identity is used, one finds that the following identities are used:

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)} = 0, \quad (\gamma_a)_{\tilde{\alpha}(\beta}(\gamma^a)_{\gamma\delta)} = 0, \tag{4.9}$$

and those in which the tilded spinor indices are exchanged for the untilded spinor indices. Note that the numbers of tildes in the identities are 0, 4, 1, 3 and the identity with two tilded spinor indices is absent. Since the T dual of the Maurer-Cartan equations (3.21)–(3.24) cannot be rearranged in a covariant form, we drop them here. We return to this point later in Sec. VI.

B. Type-IIB superstring

We show that the type-IIB superstring action can be constructed from the type-IIB superalgebra obtained in Sec. IV A. We start with writing down the type-IIB superalgebra:

$$\{Q_\alpha, Q_\beta\} = (\gamma^a)_{\alpha\beta} P_a + (\gamma_a)_{\alpha\beta} Z^a,$$

$$\{Q_{\tilde{\alpha}}, Q_{\tilde{\beta}}\} = (\gamma^a)_{\tilde{\alpha}\tilde{\beta}} P_a - (\gamma_a)_{\tilde{\alpha}\tilde{\beta}} Z^a,$$

$$\{Q_\alpha Q_{\tilde{\beta}}\} = (\gamma_a)_{\alpha\tilde{\beta}} \Sigma^a,$$

$$[P_a, Q_\beta] = 2(\gamma_a)_{\alpha\beta} Z^\alpha + 2(\gamma_a)_{\tilde{\alpha}\beta} \Sigma^{\tilde{\alpha}},$$

$$[P_a, Q_{\tilde{\beta}}] = -2(\gamma_a)_{\tilde{\alpha}\tilde{\beta}} Z^{\tilde{\alpha}} + 2(\gamma_a)_{\alpha\tilde{\beta}} \Sigma^\alpha,$$

$$[Z^a, Q_\beta] = 2(\gamma^a)_{\alpha\beta} Z^\alpha, \quad [Z^a, Q_{\tilde{\beta}}] = 2(\gamma^a)_{\tilde{\alpha}\tilde{\beta}} Z^{\tilde{\alpha}},$$

$$[\Sigma^a, Q_\beta] = 2(\gamma^a)_{\alpha\beta} \Sigma^\alpha, \quad [\Sigma^a, Q_{\tilde{\beta}}] = 2(\gamma^a)_{\tilde{\alpha}\tilde{\beta}} \Sigma^{\tilde{\alpha}}. \tag{4.10}$$

Parametrizing the supergroup manifold by

$$U = e^{z_a Z^a} e^{\zeta_{\tilde{\alpha}} Z^{\tilde{\alpha}}} e^{\zeta_\alpha Z^\alpha} e^{y_a \Sigma^a} e^{\phi_{\tilde{\alpha}} \Sigma^{\tilde{\alpha}}} e^{\phi_\alpha \Sigma^\alpha} e^{x^a P_a} e^{\theta^{\tilde{\alpha}} Q_{\tilde{\alpha}}} e^{\theta^\alpha Q_\alpha}, \tag{4.11}$$

we obtain the pullback vielbeins of the left-invariant supergroup as follows:

$$L_i{}^\alpha = \partial_i \theta^\alpha, \quad L_i{}^{\tilde{\alpha}} = \partial_i \theta^{\tilde{\alpha}}, \tag{4.12}$$

$$L_i{}^a = \partial_i x^a + \frac{1}{2}(\bar{\theta}\gamma^a \partial_i \theta) + \frac{1}{2}(\bar{\tilde{\theta}}\gamma^a \partial_i \tilde{\theta}), \tag{4.13}$$

$$L_{ia} = \partial_i z_a + \frac{1}{2}(\bar{\theta}\gamma^a \partial_i \theta) - \frac{1}{2}(\bar{\tilde{\theta}}\gamma^a \partial_i \tilde{\theta}), \tag{4.14}$$

$$L_{i\alpha} = \partial_i \zeta_\alpha + 2\partial_i z_a (\bar{\theta}\gamma^a)_\alpha + 2\partial_i x^a (\bar{\theta}\gamma_a)_\alpha + \frac{2}{3}(\bar{\theta}\gamma^a \partial_i \theta)(\bar{\theta}\gamma_a)_\alpha, \tag{4.15}$$

$$L_{i\tilde{\alpha}} = \partial_i \zeta_{\tilde{\alpha}} + 2\partial_i z_a (\bar{\tilde{\theta}}\gamma^a)_{\tilde{\alpha}} - 2\partial_i x^a (\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}} - \frac{2}{3}(\bar{\tilde{\theta}}\gamma^a \partial_i \tilde{\theta})(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}}, \tag{4.16}$$

$$L'_{ia} = \partial_i y_a + (\bar{\tilde{\theta}}\gamma_a \partial_i \theta), \tag{4.17}$$

$$L'_{i\alpha} = \partial_i \phi_\alpha + 2\partial_i x^a (\bar{\tilde{\theta}}\gamma_a)_\alpha + 2\partial_i y_a (\bar{\theta}\gamma^a)_\alpha + (\bar{\theta}\gamma^a \partial_i \theta)(\bar{\tilde{\theta}}\gamma_a)_\alpha + \frac{1}{3}(\bar{\tilde{\theta}}\gamma^a \partial_i \tilde{\theta})(\bar{\tilde{\theta}}\gamma_a)_\alpha, \tag{4.18}$$

$$L'_{i\tilde{\alpha}} = \partial_i \phi_{\tilde{\alpha}} + 2\partial_i x^a (\bar{\theta}\gamma_a)_{\tilde{\alpha}} + 2\partial_i y_a (\bar{\tilde{\theta}}\gamma^a)_{\tilde{\alpha}} + (\bar{\tilde{\theta}}\gamma^a \partial_i \theta)(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}} + \frac{1}{3}(\bar{\theta}\gamma^a \partial_i \theta)(\bar{\theta}\gamma_a)_{\tilde{\alpha}}. \tag{4.19}$$

The supersymmetry transformations are found to be

$$\delta\theta^\alpha = \epsilon^\alpha, \quad \delta\theta^{\tilde{\alpha}} = \epsilon^{\tilde{\alpha}}, \tag{4.20}$$

$$\delta x^a = -\frac{1}{2}(\bar{\epsilon}\gamma^a \theta) - \frac{1}{2}(\bar{\tilde{\epsilon}}\gamma^a \tilde{\theta}), \tag{4.21}$$

$$\delta z_a = -\frac{1}{2}(\bar{\epsilon}\gamma^a\theta) + \frac{1}{2}(\bar{\tilde{\epsilon}}\gamma^a\tilde{\theta}), \tag{4.22}$$

$$\delta\zeta_\alpha = -2z_a(\bar{\epsilon}\gamma^a)_\alpha - 2x^a(\bar{\epsilon}\gamma_a)_\alpha + \frac{2}{3}(\bar{\epsilon}\gamma_a\theta)(\bar{\theta}\gamma^a)_\alpha, \tag{4.23}$$

$$\delta\zeta_{\tilde{\alpha}} = -2z_a(\bar{\tilde{\epsilon}}\gamma^a)_{\tilde{\alpha}} + 2x^a(\bar{\tilde{\epsilon}}\gamma_a)_{\tilde{\alpha}} - \frac{2}{3}(\bar{\tilde{\epsilon}}\gamma_a\tilde{\theta})(\bar{\tilde{\theta}}\gamma^a)_{\tilde{\alpha}}, \tag{4.24}$$

$$\delta y_a = -(\bar{\tilde{\epsilon}}\gamma_a\theta), \tag{4.25}$$

$$\delta\phi_\alpha = -2x^a(\bar{\tilde{\epsilon}}\gamma_a)_\alpha - 2y_a(\bar{\epsilon}\gamma^a)_\alpha + (\bar{\tilde{\epsilon}}\gamma^a\theta)(\bar{\theta}\gamma_a)_\alpha + \frac{1}{3}(\bar{\tilde{\epsilon}}\gamma^a\tilde{\theta})(\bar{\tilde{\theta}}\gamma_a)_\alpha, \tag{4.26}$$

$$\delta\phi_{\tilde{\alpha}} = -2x^a(\bar{\epsilon}\gamma_a)_{\tilde{\alpha}} - 2y_a(\bar{\tilde{\epsilon}}\gamma^a)_{\tilde{\alpha}} + (\bar{\tilde{\epsilon}}\gamma^a\tilde{\theta})(\bar{\theta}\gamma_a)_{\tilde{\alpha}} + \frac{1}{3}(\bar{\epsilon}\gamma^a\theta)(\bar{\theta}\gamma_a)_{\tilde{\alpha}}. \tag{4.27}$$

We find that the type-IIB superstring action is constructed as

$$I = \int d^2\xi\left[-\frac{1}{2}\sqrt{-g}g^{ij}L_i{}^aL_j{}^b\eta_{ab} - \frac{1}{2}\epsilon^{ij}\left(L_i^aL_{ja} + \frac{1}{4}L_i^\alpha L_{j\alpha} + \frac{1}{4}L_i{}^{\tilde{\alpha}}L_{j\tilde{\alpha}}\right)\right], \tag{4.28}$$

where g_{ij} and η_{ab} are the world sheet and the spacetime metric, respectively, and $g=\det g_{ij}$. The last three terms in the action constitute a manifestly supersymmetric form of the Wess-Zumino term. The two-form b is defined in terms of the Wess-Zumino term in the action as

$$b = \frac{1}{2}\left(L^a\wedge L_a + \frac{1}{4}L^\alpha\wedge L_\alpha + \frac{1}{4}L^{\tilde{\alpha}}\wedge L_{\tilde{\alpha}}\right), \tag{4.29}$$

and the three-form $h=db$ is calculated as

$$h = -\frac{1}{2}L^a\wedge L^\alpha\wedge L^\beta(\gamma_a)_{\alpha\beta} + \frac{1}{2}L^a\wedge L^{\tilde{\alpha}}\wedge L^{\tilde{\beta}}(\gamma_a)_{\tilde{\alpha}\tilde{\beta}}. \tag{4.30}$$

Note that all of the dependence on the new fermionic coordinates has dropped out from the expression of h. In fact, the antisymmetric part of the action is the well-known Wess-Zumino term of the type-IIB superstring up to total derivative terms:

$$\int \epsilon^{ij}b_{ij} = \int \frac{1}{2}\epsilon^{ij}\left[\partial_i x^a((\bar{\theta}\gamma_a\partial_j\theta) - (\bar{\tilde{\theta}}\gamma_a\partial_j\tilde{\theta})) + \frac{1}{2}(\bar{\tilde{\theta}}\gamma_a\partial_i\tilde{\theta})(\bar{\theta}\gamma^a\partial_j\theta)\right]. \tag{4.31}$$

The coefficient of the Wess-Zumino term is chosen so that the action is invariant under the κ-symmetry transformations

$$\delta_\kappa\theta^\alpha = (1+\Gamma)^\alpha{}_\beta\kappa^\beta, \quad \delta_\kappa\theta^{\tilde{\alpha}} = (1-\Gamma)^{\tilde{\alpha}}{}_{\tilde{\beta}}\kappa^{\tilde{\beta}},$$
$$\delta_\kappa x^a = -\frac{1}{2}(\bar{\tilde{\theta}}\gamma^a\delta_\kappa\tilde{\theta}) - \frac{1}{2}(\bar{\theta}\gamma^a\delta_\kappa\theta), \tag{4.32}$$

where $\Gamma = [1/(2\sqrt{-g})]\epsilon^{ij}L_i{}^aL_j{}^b\gamma_{ab}$.

In this way, we construct the type-IIB superstring action from the type-IIB superalgebra. Note that by constructing the type-IIB superstring action, the *D*-string charges Σ^A play no role. This is consistent with Σ^A representing *D*-string charges. In this sense, we refer to the algebra corresponding to the Maurer-Cartan equations (4.2)–(4.5) as the type-IIB superstring algebra.

V. TYPE-IIB *D*-STRING

The *D*-string action characterized by left-invariant vielbeins correspond to *D*-string charges cannot be constructed by means of the type-IIB superalgebra. We show that the Wess-Zumino term and the modified two-form field strength $\mathcal{F}=F-b$, where F is the two-form field strength and b is the pullback to the world sheet of the $R\otimes R$ two-form gauge potential, can be constructed in terms of the left-invariant vielbeins corresponding to *D*-string charges if we start with a superalgebra obtained by *modifying* the type-IIB superalgebra.

We start with the *modified* type-IIB superalgebra: Eqs. (4.2)–(4.6) and

$$de'_\alpha = -e^\beta\wedge e^a(\gamma_a)_{\alpha\beta} - e^{\tilde{\beta}}\wedge e'_a(\gamma^a)_{\alpha\tilde{\beta}}, \tag{5.1}$$

$$de'_{\tilde{\alpha}} = -e^{\tilde{\beta}}\wedge e^a(\gamma_a)_{\tilde{\alpha}\tilde{\beta}} - e^\beta\wedge e'_a(\gamma^a)_{\tilde{\alpha}\beta}, \tag{5.2}$$

which is closed due to the well-known identity $(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)}=0$. As in Sec. IV A, if one puts tildes on the spinor indices of the γ matrices, one obtains the following identities:

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)}=0, \quad (\gamma_a)_{\alpha(\beta}(\gamma^a)_{\tilde{\gamma}\tilde{\delta})}=0, \tag{5.3}$$

and those in which the tilded spinor indices are exchanged for the untilded spinor indices. The numbers of tildes in these identities are 0, 4, 2 in contrast to the identities in the type-IIB superalgebra, where the numbers of tildes were 0, 4, 1, 3. The difference in the number of tildes tells us that the modified type-IIB superalgebra is not related to the type-IIA superalgebra by the *T*-duality transformation, as shown in Sec. VI. The modification is simply to interchange Σ^α with $\Sigma^{\tilde{\alpha}}$. This is trivial from the type-IIB perspective, because the generators Σ^α and $\Sigma^{\tilde{\alpha}}$ are not distinguished inherently. From the type-IIA perspective, however, this is nontrivial, since a spinor index with a tilde corresponds to one with a different chirality under *T* duality.

The left-invariant vielbeins are found to be Eqs. (4.12)–(4.17) and

$$L'_{i\alpha}=\partial_i\phi_\alpha+2\partial_i x^a(\bar{\theta}\gamma_a)_\alpha+2\partial_i y_a(\bar{\tilde{\theta}}\gamma^a)_\alpha+(\bar{\tilde{\theta}}\gamma^a\partial_i\theta)(\bar{\tilde{\theta}}\gamma_a)_\alpha+\frac{1}{3}(\bar{\theta}\gamma^a\partial_i\theta)(\bar{\theta}\gamma_a)_\alpha, \quad (5.4)$$

$$L'_{i\tilde{\alpha}}=\partial_i\phi_{\tilde{\alpha}}+2\partial_i x^a(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}}+2\partial_i y_a(\bar{\theta}\gamma^a)_{\tilde{\alpha}}+(\bar{\theta}\gamma^a\partial_i\theta)(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}}+\frac{1}{3}(\bar{\tilde{\theta}}\gamma^a\partial_i\tilde{\theta})(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}}. \quad (5.5)$$

The supersymmetry transformations are obtained as Eqs. (4.20)–(4.25) and

$$\delta\phi_\alpha=-2x^a(\bar{\epsilon}\gamma_a)_\alpha-2y_a(\bar{\tilde{\epsilon}}\gamma^a)_\alpha+(\bar{\tilde{\epsilon}}\gamma^a\tilde{\theta})(\bar{\theta}\gamma_a)_\alpha+\frac{1}{3}(\bar{\epsilon}\gamma^a\theta)(\bar{\theta}\gamma_a)_\alpha, \quad (5.6)$$

$$\delta\phi_{\tilde{\alpha}}=-2x^a(\bar{\tilde{\epsilon}}\gamma_a)_{\tilde{\alpha}}-2y_a(\bar{\epsilon}\gamma^a)_{\tilde{\alpha}}+(\bar{\tilde{\epsilon}}\gamma^a\theta)(\bar{\theta}\gamma_a)_{\tilde{\alpha}}+\frac{1}{3}(\bar{\tilde{\epsilon}}\gamma^a\tilde{\theta})(\bar{\tilde{\theta}}\gamma_a)_{\tilde{\alpha}}. \quad (5.7)$$

The two-form b is defined as

$$b=-\frac{1}{4}(L^\alpha\wedge L'_\alpha-L^{\tilde{\alpha}}\wedge L'_{\tilde{\alpha}}). \quad (5.8)$$

The primed vielbeins correspond to the D-string charges Σ^A, and the action constructed with this Wess-Zumino term can be regarded as a ''gauge-fixed'' D-string action. In fact, the three-form $h=db$ turns out to be Eq. (4.30) obtained for the type-IIB superstring. All of the dependence on the new fermionic coordinates has dropped out from the expression of h.

Can the total (gauge-unfixed) D-string action be constructed? To this end, we must first determine a superinvariant modified two-form field strength $\mathcal{F}=dA-b$, where A is a U(1) gauge field on the world sheet. The b is the conventional two-form which is read off from the integrand of the right-hand side (RHS) of Eq. (4.31). If we obtain the two-form $\mathcal{F}$, the D-string action can be constructed as in Ref. [10] or [11]. We observe that

$$\frac{1}{2}\epsilon^{ij}(L_i{}^\alpha L'_{j\alpha}-L_i{}^{\tilde{\alpha}}L'_{j\tilde{\alpha}})=-2\epsilon^{ij}(F_{ij}-b_{ij}), \quad (5.9)$$

where we introduce F_{ij} as

$$\epsilon^{ij}F_{ij}=-\frac{1}{2}\epsilon^{ij}\left[\partial_i y_a\partial_j(\bar{\tilde{\theta}}\gamma^a\theta)+\frac{1}{2}\partial_i\phi_\alpha\partial_j\theta^\alpha+\frac{1}{2}\partial_i\phi_{\tilde{\alpha}}\partial_j\theta^{\tilde{\alpha}}\right]. \quad (5.10)$$

The RHS of Eq. (5.10) is a total derivative term, and then we can regard F_{ij} as the field strength of a U(1) gauge field A_i:

$$A_i=-\frac{1}{2}\left[y_a\partial_i(\bar{\tilde{\theta}}\gamma^a\theta)+\frac{1}{2}\phi_\alpha\partial_i\theta^\alpha-\frac{1}{2}\phi_{\tilde{\alpha}}\partial_i\theta^{\tilde{\alpha}}\right]. \quad (5.11)$$

Note that this is parametrized by D-string coordinates associated with D-string charges Σ^A. The supersymmetry transformation is found to be

$$\delta A_i=-\frac{1}{2}\left[-x^a(\bar{\epsilon}\gamma_a\partial_i\theta)+\frac{1}{6}(\bar{\epsilon}\gamma_a\theta)(\bar{\theta}\gamma^a\partial_i\theta)-(\theta\rightarrow\tilde{\theta},\epsilon\rightarrow\tilde{\epsilon})\right]-\frac{1}{4}\partial_i((\bar{\tilde{\epsilon}}\gamma_a\theta)(\bar{\theta}\gamma^a\tilde{\theta})). \quad (5.12)$$

In order to see the relation to the well-known supersymmetry transformation of the U(1) gauge field in Ref. [10], we consider a U(1) gauge transformation and obtain an alternative form

$$A_i=\frac{1}{2}\left[\partial_i y_a(\bar{\tilde{\theta}}\gamma^a\theta)+\frac{1}{2}\partial_i\phi_\alpha\theta^\alpha-\frac{1}{2}\partial_i\phi_{\tilde{\alpha}}\theta^{\tilde{\alpha}}\right]. \quad (5.13)$$

This transforms under the supersymmetry transformation as

$$\delta A_i=-\frac{1}{2}\left[\partial_i x^a(\bar{\epsilon}\gamma_a\theta)+\frac{1}{6}(\bar{\epsilon}\gamma_a\theta)(\bar{\theta}\gamma^a\partial_i\theta)-(\theta\rightarrow\tilde{\theta},\epsilon\rightarrow\tilde{\epsilon})\right]-\frac{1}{4}(\partial_i\phi_\alpha\epsilon^\alpha-\partial_i\phi_{\tilde{\alpha}}\epsilon^{\tilde{\alpha}}), \quad (5.14)$$

which is similar to the well-known form [10] except for the last two terms. These two terms are the total derivative terms, and the supersymmetry transformation δF_{ij} is nothing but the one obtained there. We conclude that the U(1) gauge field on the world sheet can be constructed in this way. It is interesting that the U(1) gauge field is constructed explicitly in terms of the D-string charges. Using the modified field strength $\mathcal{F}$, one constructs the D-string action, as in Ref. [10] or [11]. In this sense, we refer to the superalgebra corresponding to the Maurer-Cartan equations (4.2), (4.6), (5.1), and (5.2) as D-string superalgebra. Hence the modified type-IIB superalgebra describes type-IIB superstrings and D-strings on an equal footing.

We now comment on the existence of a U(1) gauge field in type-II superstrings. We observe that the modified field strength for type-IIB superstrings is constructed by observing that

$$\epsilon^{ij}\left(L_i{}^aL_{ja}+\frac{1}{4}L_i{}^\alpha L_{j\alpha}+\frac{1}{4}L_i{}^{\tilde{\alpha}}L_{j\tilde{\alpha}}\right)=-2\epsilon^{ij}(F_{ij}-b_{ij}), \quad (5.15)$$

where F_{ij} is introduced as

$$\epsilon^{ij}F_{ij}=-\frac{1}{2}\epsilon^{ij}\left[\partial_i x^a\partial_i z_a+\frac{1}{4}\partial_i\zeta_\alpha\partial_j\theta^\alpha+\frac{1}{4}\partial_i\zeta_{\tilde{\alpha}}\partial_j\theta^{\tilde{\alpha}}\right] \quad (5.16)$$

and the b_{ij} is the conventional two-form defined by the integrand of the RHS of Eq. (4.31). Thus the U(1) gauge field can be defined as

$$A_i=\frac{1}{2}\left[z_a\partial_i x^a-\frac{1}{4}\zeta_\alpha\partial_i\theta^\alpha-\frac{1}{4}\zeta_{\tilde{\alpha}}\partial_i\theta^{\tilde{\alpha}}\right] \tag{5.17}$$

and is parametrized by coordinates corresponding to type-IIB superstring charges. The supersymmetry transformation is found to be

$$\delta A_i=-\frac{1}{2}\left[\partial_i x^a(\bar{\epsilon}\gamma_a\theta)+\frac{1}{6}(\bar{\epsilon}\gamma_a\theta)(\bar{\theta}\gamma^a\partial_i\theta)-\frac{1}{2}\partial_i(x^a(\bar{\epsilon}\gamma_a\theta))-(\theta\to\tilde{\theta},\epsilon\to\tilde{\epsilon})\right], \tag{5.18}$$

which is identical to Eq. (5.14) up to a total derivative term. In this way, we can construct a U(1) gauge field for type-IIB superstrings. For type-IIA superstrings, one can construct a U(1) gauge field parametrized by coordinates corresponding to type-IIA superstring charges in a similar way. The fact that a U(1) gauge field can be constructed in type-II superstring theories is consistent with the spacetime scale-invariant formulation of p-branes and type-II superstrings [11,12]. For p-brane theories, the same procedure presented above will produce not only the supersymmetry transformation of the p-form gauge field, but also the explicit form of the p-form gauge field in terms of the p-brane charges.

VI. *T*-DUALITY AND IDENTITIES

The modified type-IIB superalgebra will not be related by the T duality to the type-IIA superalgebra. This is seen by recognizing the fact that identities characterizing the type-IIA superalgebra cannot be written in a covariant form after the T-duality transformation.

For completeness, we start with identities in the M algebra and reduce to ones in the type-IIA superalgebra. Then, performing the T-duality transformation, we determine whether the resulting identities are rearranged in a covariant form or not.

One of the two identities characterizing the M algebra is Eq. (3.7):

$$(\gamma_\mu)_{(\alpha\beta}(\gamma^{\mu\nu})_{\gamma\delta)}=0. \tag{6.1}$$

We reduce this identity to ones of the type-IIA superalgebra as follows. For $\nu=b\neq\natural$, one obtains the identities

$$(1)_{\dot{\alpha}(\beta}(\gamma^a)_{\gamma\delta)}+(\gamma^a{}_b)_{\dot{\alpha}(\beta}(\gamma^b)_{\gamma\delta)}=0, \tag{6.2}$$

$$(1)_{\alpha(\dot{\beta}}(\gamma^a)_{\dot{\gamma}\dot{\delta})}+(\gamma^a{}_b)_{\alpha(\dot{\beta}}(\gamma^b)_{\dot{\gamma}\dot{\delta})}=0, \tag{6.3}$$

which are the characteristic identities in the presence of $D0$- and $D2$-branes. As for $\nu=\natural$, the well-known identities

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)}=0,\quad(\gamma_a)_{\dot{\alpha}(\dot{\beta}}(\gamma^a)_{\dot{\gamma}\dot{\delta})}=0 \tag{6.4}$$

are obtained. We next consider the T-duality transformation. By the procedure explained in Sec. IV A, we obtain the T dual of the identity (6.2) for $a=i\neq\sharp$:

$$(\gamma^\sharp)_{\tilde{\alpha}(\beta}(\gamma^i)_{\gamma\delta)}-(\gamma^i)_{\tilde{\alpha}(\beta}(\gamma^\sharp)_{\gamma\delta)}+(\gamma^{\sharp i}{}_j)_{\tilde{\alpha}(\beta}(\gamma^j)_{\gamma\delta)}=0, \tag{6.5}$$

which is rewritten in a covariant form as

$$(\gamma^{[a})_{\tilde{\alpha}(\beta}(\gamma^{b]})_{\gamma\delta)}+\frac{1}{2}(\gamma^{ab}{}_c)_{\tilde{\alpha}(\beta}(\gamma^c)_{\gamma\delta)}=0. \tag{6.6}$$

This is a characteristic identity in the presence of $D3$-branes. In turn, for $a=\sharp$, one finds the identity

$$(\gamma_a)_{\tilde{\alpha}(\beta}(\gamma^a)_{\gamma\delta)}=0, \tag{6.7}$$

which is used in satisfying the Jacobi identities for the type-IIB superalgebra. The T dual of the identity (6.3) is found to be identities (6.6) and (6.7) with exchanging the tilded spinor indices for the untilded spinor indices. Identities (6.4) are transformed into

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\gamma\delta)}=0,\quad(\gamma_a)_{\tilde{\alpha}(\tilde{\beta}}(\gamma^a)_{\tilde{\gamma}\tilde{\delta})}=0, \tag{6.8}$$

respectively.

The second identity of the M algebra is Eq. (3.8):

$$(\gamma_\mu)_{(\alpha\beta}(\gamma^\mu)_{\gamma\delta)}+\frac{1}{10}(\gamma_{\mu\nu})_{(\alpha\beta}(\gamma^{\mu\nu})_{\gamma\delta)}=0, \tag{6.9}$$

which reduces to the identity in the type-IIA superalgebra,

$$(\bar{\phi}\dot{\phi})(\bar{\phi}\dot{\phi})+\frac{2}{5}(\bar{\phi}\gamma_a\phi)(\bar{\dot{\phi}}\gamma^a\dot{\phi})+\frac{1}{10}(\bar{\phi}\gamma_{ab}\dot{\phi})(\bar{\phi}\gamma^{ab}\dot{\phi})=0, \tag{6.10}$$

where ϕ and $\dot{\phi}$ are Grassmann-even spinors with opposite chirality each other. This is characteristic identity for the Maurer-Cartan equations (3.22) and (3.23) for primed dual forms. Under the T-duality transformation, this transforms into an identity which is not rewritten in a covariant form:

$$(\bar{\phi}\gamma^\sharp\tilde{\phi})(\bar{\phi}\gamma^\sharp\tilde{\phi})+\frac{2}{5}(\bar{\phi}\gamma_i\phi)(\bar{\tilde{\phi}}\gamma^i\tilde{\phi})-\frac{2}{5}(\tilde{\phi}\gamma^\sharp\phi)(\bar{\tilde{\phi}}\gamma^\sharp\tilde{\phi})+\frac{1}{10}(\tilde{\phi}\gamma_{ij\sharp}\tilde{\phi})(\bar{\phi}\gamma^{ij\sharp}\tilde{\phi})+\frac{1}{5}(\bar{\phi}\gamma_i\tilde{\phi})(\bar{\phi}\gamma^i\tilde{\phi})=0. \tag{6.11}$$

This was the reason why the ''superstring'' charges in the type-IIA superalgebra are discarded in performing the T-duality transformation in Sec. IV A.

Let us consider, conversely, the identity in the modified type-IIB superalgebra:

$$(\gamma_a)_{\alpha(\beta}(\gamma^a)_{\tilde{\gamma}\tilde{\delta})}=0. \tag{6.12}$$

This is transformed into an identity

$$(\gamma_i)_{\alpha\beta}(\gamma^i)_{\dot{\gamma}\dot{\delta}}-(\gamma_\sharp)_{\alpha\beta}(\gamma^\sharp)_{\dot{\gamma}\dot{\delta}}+(\gamma_{i\sharp})_{\alpha\dot{\gamma}}(\gamma^{i\sharp})_{\dot{\delta}\beta}+(\gamma_{i\sharp})_{\alpha\dot{\delta}}(\gamma^{i\sharp})_{\beta\dot{\gamma}}+(1)_{\alpha\dot{\gamma}}(1)_{\beta\dot{\delta}}+(1)_{\alpha\dot{\delta}}(1)_{\beta\dot{\gamma}}=0, \tag{6.13}$$

which is not rewritten in a covariant form.

From these observation, the modified type-IIB superalgebra is not rewritten as a T dual of the type-IIA superalgebra. In the next section, we try to relate the type-IIA superalgebra and the modified type-IIB superalgebra.

VII. UNIFICATION OF TYPE-II SUPERALGEBRAS

We encountered two type-IIB superalgebras. One is the type-IIB superalgebra, which is the T dual of the type-IIA superalgebra; the other is the modified type-IIB superalgebra, which describes the type-IIB superstring and D-string on an equal footing. The type-IIA superalgebra is related to the type-IIB superalgebra by the T duality, but not to the modified type-IIB superalgebra.

As a trial to relate the type-IIA superalgebra to the modified type-IIB superalgebra, we examine a unification in a $(10+1)$-dimensional $N=2$ superalgebra of the modified type-IIB superalgebra and the M algebra (hence, the type-IIA superalgebra). This is motivated by the fact that the identity, in $N=2D=10+1$,

$$(\gamma_\mu)_{(\alpha\beta}(\gamma^{\mu\nu})_{\tilde{\gamma}\tilde{\delta})}=0, \tag{7.1}$$

is projected into the identity (6.12) in the modified type-IIB superalgebra.

We first present the relevant part of $(10+1)$-dimensional $N=2$ superalgebra and then consider the relations to the M algebra and the modified type-IIB superalgebra. We begin with the $N=2$ $(10+1)$-dimensional superalgebra generated by bosonic charges:

$$de^\mu=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma^\mu)_{\alpha\beta}-\frac{1}{2}e^{\tilde{\alpha}}\wedge e^{\tilde{\beta}}(\gamma^\mu)_{\tilde{\alpha}\tilde{\beta}}, \tag{7.2}$$

$$de_\mu=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma_\mu)_{\alpha\beta}+\frac{1}{2}e^{\tilde{\alpha}}\wedge e^{\tilde{\beta}}(\gamma_\mu)_{\tilde{\alpha}\tilde{\beta}}, \tag{7.3}$$

$$de_{\mu\nu}=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma_{\mu\nu})_{\alpha\beta}-\frac{1}{2}e^{\tilde{\alpha}}\wedge e^{\tilde{\beta}}(\gamma_{\mu\nu})_{\tilde{\alpha}\tilde{\beta}}, \tag{7.4}$$

$$de^{\mu\nu}=-\frac{1}{2}e^\alpha\wedge e^\beta(\gamma^{\mu\nu})_{\alpha\beta}+\frac{1}{2}e^{\tilde{\alpha}}\wedge e^{\tilde{\beta}}(\gamma^{\mu\nu})_{\tilde{\alpha}\tilde{\beta}}, \tag{7.5}$$

$$de'_{\mu\nu}=-e^\alpha\wedge e^{\tilde{\beta}}(\gamma_{\mu\nu})_{\alpha\tilde{\beta}}, \tag{7.6}$$

$$de'_\mu=-e^\alpha\wedge e^{\tilde{\beta}}(\gamma_\mu)_{\alpha\tilde{\beta}}. \tag{7.7}$$

In addition to these equations we consider the following equations:

$$de_\alpha=-e^\beta\wedge e^\mu(\gamma_\mu)_{\alpha\beta}+(1-\lambda)e^\beta\wedge e_\mu(\gamma^\mu)_{\alpha\beta}-\frac{1}{10}e^\beta\wedge e_{\mu\nu}(\gamma^{\mu\nu})_{\alpha\beta}$$
$$+\frac{1}{10}(1-\lambda)e^\beta\wedge e^{\mu\nu}(\gamma_{\mu\nu})_{\alpha\beta}-\frac{1}{10}(2-\lambda)e^{\tilde{\beta}}\wedge e'_{\mu\nu}(\gamma^{\mu\nu})_{\alpha\tilde{\beta}}-(2-\lambda)e^{\tilde{\beta}}\wedge e'_\mu(\gamma^\mu)_{\alpha\tilde{\beta}}, \tag{7.8}$$

$$de_{\tilde{\alpha}}=-e^{\tilde{\beta}}\wedge e^\mu(\gamma_\mu)_{\tilde{\alpha}\tilde{\beta}}+e^{\tilde{\beta}}\wedge e_\mu(\gamma^\mu)_{\tilde{\alpha}\tilde{\beta}}+\frac{1}{10}e^{\tilde{\beta}}\wedge e_{\mu\nu}(\gamma^{\mu\nu})_{\tilde{\alpha}\tilde{\beta}}-\frac{1}{10}e^{\tilde{\beta}}\wedge e^{\mu\nu}(\gamma_{\mu\nu})_{\tilde{\alpha}\tilde{\beta}}, \tag{7.9}$$

$$de_{\mu\alpha}=-e^\beta\wedge e^\nu(\gamma_{\mu\nu})_{\alpha\beta}-e^\beta\wedge e_{\mu\nu}(\gamma^\nu)_{\alpha\beta}-e^{\tilde{\beta}}\wedge e'_{\mu\nu}(\gamma^\nu)_{\alpha\tilde{\beta}}-e^{\tilde{\beta}}\wedge e'_\nu(\gamma_\mu{}^\nu)_{\alpha\tilde{\beta}}$$
$$-e^{\tilde{\beta}}\wedge e^\nu(\gamma_{\mu\nu})_{\alpha\tilde{\beta}}-e^{\tilde{\beta}}\wedge e_{\mu\nu}(\gamma^\nu)_{\alpha\tilde{\beta}}-e^\beta\wedge e'_{\mu\nu}(\gamma^\nu)_{\alpha\beta}-e^\beta\wedge e'_\nu(\gamma_\mu{}^\nu)_{\alpha\beta}, \tag{7.10}$$

$$de_{\mu\tilde{\alpha}}=-e^{\tilde{\beta}}\wedge e^\nu(\gamma_{\mu\nu})_{\tilde{\alpha}\tilde{\beta}}-e^{\tilde{\beta}}\wedge e_{\mu\nu}(\gamma^\nu)_{\tilde{\alpha}\tilde{\beta}}-e^\beta\wedge e'_{\mu\nu}(\gamma^\nu)_{\tilde{\alpha}\beta}-e^\beta\wedge e'_\nu(\gamma_\mu{}^\nu)_{\tilde{\alpha}\beta}$$
$$-e^\beta\wedge e^\nu(\gamma_{\mu\nu})_{\tilde{\alpha}\beta}-e^\beta\wedge e_{\mu\nu}(\gamma^\nu)_{\tilde{\alpha}\beta}-e^{\tilde{\beta}}\wedge e'_{\mu\nu}(\gamma^\nu)_{\tilde{\alpha}\tilde{\beta}}-e^{\tilde{\beta}}\wedge e'_\nu(\gamma_\mu{}^\nu)_{\tilde{\alpha}\tilde{\beta}}, \tag{7.11}$$

$$de_{\alpha\beta}=\frac{1}{2}e^\mu\wedge e^\nu(\gamma_{\mu\nu})_{\alpha\beta}+\frac{1}{2}e^\mu\wedge e_{\mu\nu}(\gamma^\nu)_{\alpha\beta}-2e^\gamma\wedge e_{\mu\alpha}(\gamma^\mu)_{\beta\gamma}-\frac{1}{4}e^\gamma\wedge e_{\mu\gamma}(\gamma^\mu)_{\alpha\beta}$$
$$-2e^{\tilde{\gamma}}\wedge e_{\mu\alpha}(\gamma^\mu)_{\beta\tilde{\gamma}}-\frac{1}{4}e^{\tilde{\gamma}}\wedge e_{\mu\tilde{\gamma}}(\gamma^\mu)_{\alpha\beta}+\frac{1}{2}e'_\mu\wedge e'_\nu(\gamma^{\mu\nu})_{\alpha\beta}+e^\mu\wedge e'_\nu(\gamma_\mu{}^\nu)_{\alpha\beta}$$
$$+\frac{1}{2}e^\mu\wedge e'_{\mu\nu}(\gamma^\nu)_{\alpha\beta}+\frac{1}{2}e'_\mu\wedge e'_{\nu\lambda}(\gamma^\lambda)_{\alpha\beta}\eta^{\mu\nu}+\frac{1}{2}e'_\mu\wedge e_{\nu\lambda}(\gamma^\lambda)_{\alpha\beta}\eta^{\mu\nu}. \tag{7.12}$$

The Jacobi identity for the dual form $de_{\mu\alpha}$ is satisfied such that the exterior derivatives of the first and second lines in Eq. (7.10) vanish separately. However, the Jacobi identity for the dual form $de_{\alpha\beta}$ requires both lines in Eq. (7.10).

Let us consider the relation to the M algebra. We discard the dual forms with spinor indices with a tilde, since we need only $N=1$ supersymmetry. In addition, we discard the primed dual forms. These procedures cause the following to occur: Eqs. (7.9) and (7.11) decouple from our analysis; Eq. (7.8), with identifying $e^{\mu\nu}$ with $e_{\mu\nu}$, turns out to be Eq. (3.3); Eqs. (7.10) and (7.12) result in Eqs. (3.5) and (3.6), respectively. We thus find that the $N=2$ superalgebra includes the M algebra in this way.

We next consider the relation to the modified type-IIB superalgebra. We project the spinor indices to ones with the same chirality and perform a dimensional reduction of the 11th dimension $x^{\natural}$. The dual forms $e_{\natural a}$, $e^{\natural a}$, and $e'_{\natural a}$ are identified with e^a, e_a, and e'_a, respectively. We find that if we set $\lambda=2$, Eq. (7.8) turns into Eq. (4.4) after a trivial overall scaling. Equation (7.9) is found to become Eq. (4.5). Equations (7.10) and (7.11) with $\mu=\natural$, after a trivial overall rescaling, turn out to be

$$de'_{\alpha}=-e^{\tilde{\beta}}\wedge e^a(\gamma_a)_{\alpha\tilde{\beta}}-e^{\beta}\wedge e'_a(\gamma^a)_{\alpha\beta}-e^{\beta}\wedge e^a(\gamma_a)_{\alpha\beta}-e^{\tilde{\beta}}\wedge e'_a(\gamma^a)_{\alpha\tilde{\beta}}, \tag{7.13}$$

$$de'_{\tilde{\alpha}}=-e^{\beta}\wedge e^a(\gamma_a)_{\tilde{\alpha}\beta}-e^{\tilde{\beta}}\wedge e'_a(\gamma^a)_{\tilde{\alpha}\tilde{\beta}}-e^{\tilde{\beta}}\wedge e^a(\gamma_a)_{\tilde{\alpha}\tilde{\beta}}-e^{\beta}\wedge e'_a(\gamma^a)_{\tilde{\alpha}\beta}. \tag{7.14}$$

Note that the RHSs of these equations are constructed by adding the RHSs of Eqs. (4.7) and (4.8) of the type-IIB superalgebra and the RHSs of Eqs. (5.1) and (5.2) of the modified type-IIB superalgebra. The rest of the equations will be a part of $D3$-brane charges.

In summary, we find that the $N=2$ superalgebra includes the M algebra and the free parameter λ in the M algebra has to be 2 in order for the type-IIB superstring algebra to be included. These imply that Eqs. (5.1) and (5.2) for the D-string superalgebra naturally arise as well as Eqs. (4.7) and (4.8) of the type-IIB superalgebra. Note that the commutators of the D-string superalgebra, which are not related to the type-IIA superalgebra by the T duality, emerge by considering the $(10+1)$-dimensional $N=2$ superalgebra. However, D-strings do not correspond to the superalgebra obtained from the $N=2$ superalgebra, since the pullback vielbeins $L'_{i\alpha}$ and $L'_{i\tilde{\alpha}}$ contain the RHSs of Eqs. (4.18) and (4.19). It is interesting to seek a unified superalgebra from which one can construct type-IIA superstrings, type-IIB superstrings, and D-strings.

VIII. SUMMARY AND DISCUSSION

We have presented a set of new spacetime superalgebras: the type-IIA superstring superalgebra, the type-IIB superstring superalgebra, and the D-string superalgebra. Using the new superalgebras, we have shown that Siegel's formulation generalizes to type-II superstrings and D-strings. Namely, we have constructed supercurrents on the supergroup manifolds corresponding to the superalgebras. We then wrote down the Wess-Zumino terms, which are second order in the supercurrents. The modified two-form field strength for D-strings was identified with a second-order expression of the supercurrents. From this expression, the U(1) gauge field on the world sheet of D-strings was obtained explicitly in terms of coordinates corresponding to D-string charges, including the new fermionic charges.

We succeeded in constructing the pullback to the D-string world sheet of the $R\otimes R$ two-form potential from the modified type-IIB superalgebra in Sec. V. We now comment on the relation to D-strings of the type-IIB superalgebra, which was obtained in Sec. IV by a T-duality transformation of the type-IIA superalgebra. We observe that using the pullbacks of left-invariant vielbeins (4.12)–(4.19) corresponding to the type-IIB superalgebra, the second-order expression $-\frac{1}{2}(L^a\wedge L'_a+\frac{1}{4}L^{\alpha}\wedge L'_{\alpha}+\frac{1}{4}L^{\tilde{\alpha}}\wedge L'_{\tilde{\alpha}})$ is calculated to be, up to total derivative terms,

$$-\frac{1}{2}\Bigg(dx^a\wedge[(\bar{\tilde{\theta}}\gamma_a d\theta)+(\bar{\theta}\gamma_a d\tilde{\theta})]+\frac{1}{3}(\bar{\theta}\gamma^a d\theta)\wedge(\bar{\theta}\gamma_a d\tilde{\theta})+\frac{1}{3}(\bar{\tilde{\theta}}\gamma^a d\tilde{\theta})\wedge(\bar{\tilde{\theta}}\gamma_a d\theta)\Bigg), \tag{8.1}$$

which corresponds to the pullback to the D-string world sheet of the NS$\otimes$NS two-form potential. Thus we find that interchanging new fermionic generators Σ^{α} and $\Sigma^{\tilde{\alpha}}$ results in exchanging $R\otimes R$ gauge potentials for the NS$\otimes$NS one, since the modification was simply interchanging Σ^{α} and $\Sigma^{\tilde{\alpha}}$.

In turn, the pullback to the F-string world sheet of the $R\otimes R$ two-form potential can be obtained from the pullback to the D-string world sheet of the $R\otimes R$ two-form potential. Using the resulting expressions, we can construct (p,q)-superstring actions in a manifestly supersymmetric form. We hope to report on this issue in the future [13]. In addition, it is interesting to see whether the formulation can be generalized to the other type-II-branes: NS5-branes and D p-branes ($p=$odd for the type-IIB superstring theory and $p=$even for the type-IIA superstring theory). Especially, now that we have the type-IIA superalgebra, including the $D2$-brane charges, the generalization to the $D2$-branes has to be examined. We leave this to the future.

We found a modified type-IIB superalgebra which includes the type-IIB superstring and D-string superalgebras as subalgebras and describes type-IIB superstrings and D-strings on an equal footing. However, this is not related by the T-duality transformation to the type-IIA superalgebra derived from the M algebra. In order to relate these superalgebras, we considered a unification in a $(10+1)$-dimensional $N=2$ superalgebra. The unification implies that the free parameter in the M algebra is fixed as $\lambda=2$. By considering the unification, the commutation relations of D-strings, which was not obtained by the T-duality transformation of the type-IIA superalgebra, are found to emerge. However, unnecessary relations are also generated in addition to the preferred *D*-string superalgebra, and the obtained superalgebra does not correspond to *D*-strings. It is interesting to consider the other unification which unifies the *M* algebra and the modified type-IIB superalgebra.

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