Supersymmetry constraints on type IIB supergravity

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Supersymmetry is used to derive conditions on higher derivative terms in the effective action of type IIB supergravity. Using these conditions, we are able to prove earlier conjectures that certain modular invariant interactions of order $(\alpha')^3$ relative to the Einstein-Hilbert term are proportional to eigenfunctions of the Laplace operator on the fundamental domain of SL(2,$\mathbb{Z}$). We also discuss how these arguments generalize to terms of higher order in α', as well as to compactifications of supergravity. [S0556-2821(99)05102-4]

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I. INTRODUCTION

Despite the intense interest in the structure of M theory, the general constraints imposed by supersymmetry have not been systematically investigated. At low energies in eleven dimensions, M theory should be well approximated by eleven-dimensional supergravity [1]. However, eleven-dimensional supergravity is not a consistent quantum theory and new ingredients are needed which modify the ultraviolet properties of the theory. While eventually we hope to have a microscopic formulation of M theory, it is interesting to unravel the extent to which its structure is constrained simply by general symmetry principles. For example, the cancellation of chiral gauge and gravitational anomalies induced on the five-brane leads immediately to a term in the effective action of the form

$$\int C^{(3)}\wedge X_8(R), \qquad (1.1)$$

where X_8 is an eightform constructed from curvatures, and $C^{(3)}$ is the three form tensor field [2,3]. This term is eighth order in derivatives compared to classical terms in the effective action which are second order. As usual, the order in a momentum expansion counts the number of derivatives plus twice the number of fermions. Clearly, we can generate many more terms needed for a supersymmetric effective action by acting with the lowest order supersymmetry transformations on these higher derivative terms. Some of these terms have been deduced from duality arguments. Furthermore, as soon as there are eight derivative terms in the effective action, there will be sixth order modifications to the classical supersymmetry transformations. The action is then no longer invariant under supersymmetry unless we add yet higher order terms to the effective action.

Ideally, it would be possible to describe the theory in a manner that is independent of the background. The moduli of an arbitrary compactification would then emerge from components of the tensor bosonic fields. However, in practice it is only feasible to define the effective action with respect to a given moduli space. The simplest example with substantial structure is the Poincaré invariant ten-dimensional background appropriate to the type IIB superstring, which has moduli space SL(2,$\mathbb{R}$)/$U(1)$. The type IIB string arises by compactifying M theory on a T^2, where the volume of the torus is taken to zero [4]. The complex structure of the torus becomes the complex coupling $\tau=C^{(0)}+ie^{-\phi}$ of the IIB theory, where $C^{(0)}$ is the Ramond-Ramond (R⊗R) scalar and ϕ is the dilaton.

The type IIB effective action is expressed as an expansion in powers of α' with the classical theory defined by an 'action' $S^{(0)}$ of order $(\alpha')^{-4}$ [5–8]. It is well-known that the self-duality constraint on the five-form field strength in the type IIB theory presents an obstacle to actually writing a globally defined covariant action[1] $S^{(0)}$. However, the analysis in our paper will actually only involve the field equations. We will use terminology appropriate for a theory with an action, but merely as a shorthand method of packaging these equations. We could avoid this problem by compactifying the type IIB theory on a circle. The classical moduli space would then be SL(2,$\mathbb{R}$)/$U(1)\times\mathbb{R}$.

The supersymmetry transformations on an arbitrary field Ψ will be expressed as the series

$$\delta_\epsilon\Psi=(\delta^{(0)}+\alpha'\delta^{(1)}+\cdots+(\alpha')^n\delta^{(n)}+\cdots)\Psi, \qquad (1.2)$$

while the effective action has the following expansion:

$$S=S^{(0)}+\alpha' S^{(1)}+\cdots+(\alpha')^n S^{(n)}+\cdots. \qquad (1.3)$$

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[1]This issue has been thoroughly discussed in Ref. [9].

A factor of $(\alpha')^{-4}$ has been absorbed into the definition of S and $S^{(n)}$.

In principle, the action can be constructed by a Noether procedure which imposes the conditions

$$\left(\sum_{m=0}^{r}(\alpha')^m\delta^{(m)}\right)\sum_{n=0}^{r}(\alpha')^n S^{(n)}=0, \tag{1.4}$$

order by order in α'. There are no $n=1$ or $n=2$ terms at tree-level or one-loop, and these terms are not expected to appear at all in Eq. (1.3). Therefore, the first corrections are the terms of order $(\alpha')^3$ relative to $S^{(0)}$. These terms are eighth order in derivatives.

In practice, building the complete effective action from scratch using this Noether method is extremely complicated even for a low number of derivatives. However, we will show in this paper that the exact form of special classes of M theory or type IIB interactions can be uniquely determined in this manner. A similar analysis has recently been used to obtain powerful constraints on maximally supersymmetric Yang-Mills theories [10]. The lesson to be drawn from that analysis is that the constraints imposed by supersymmetry are most easily exhibited by studying the variation of terms in the effective action with the maximal number of fermionic fields.

Among other issues, one of our aims will be to establish the validity of some conjectured higher derivative interactions in the effective action. An example of such a term is the interaction, $\int\sqrt{g}f^{(0,0)}(\tau,\bar{\tau})\mathcal{R}^4$, where $\mathcal{R}^4$ is a particular contraction of four Weyl curvatures [11]. The SL(2,$\mathbb{Z}$) symmetry of the IIB theory requires that $f^{(0,0)}(\tau,\bar{\tau})$ be a modular function of the complex scalar field τ and its complex conjugate, $\bar{\tau}$. It was noted in Ref. [11] that $f^{(0,0)}$ is an eigenfunction of the Laplace operator on the SL(2,$\mathbb{Z}$) moduli space with eigenvalue $\frac{3}{4}$,

$$\nabla^2 f^{(0,0)}\equiv 4\tau_2^2\frac{\partial}{\partial\tau}\frac{\partial}{\partial\bar{\tau}}f^{(0,0)}=\frac{3}{4}f^{(0,0)}. \tag{1.5}$$

This equation has the solution (see, for example, Ref. [12]),

$$f^{(0,0)}=\sum_{(m,n)\neq(0,0)}\frac{\tau_2^{3/2}}{|m+n\tau|^3}, \tag{1.6}$$

which is the unique solution, up to an arbitrary overall constant factor, for a choice of asymptotic behavior near the boundary $\tau_2\to\infty$ of the fundamental domain of SL(2,$\mathbb{Z}$). The asymptotic behavior is determined by the weak coupling expansion of $f^{(0,0)}$, where $\tau_2=e^{-\phi}$ is large, which possesses a tree-level and one-loop term but no other perturbative corrections. In addition, there are an infinite number of D-instanton corrections.[2] Another term of the same order is the sixteen dilatino term, $f^{(12,-12)}\lambda^{16}$, where the dilatino λ^a $(a=1,\ldots,16)$ is a complex SO(9,1) Weyl spinor. This term was discussed in Ref. [15] where it was argued that

$$f^{(12,-12)}(\tau,\bar{\tau})=D^{12}f^{(0,0)}. \tag{1.7}$$

The modular covariant derivative D will be defined in the next section. This means that $f^{(12,-12)}$ should also be an eigenfunction of the Laplace operator, but it now transforms with the nontrivial holomorphic and anti-holomorphic modular weights indicated by the superscripts.

More generally, there are many other terms in $S^{(3)}$ that are related to the $\mathcal{R}^4$ term by supersymmetry at the linearized level [15,16]. The moduli dependence of these terms is packaged into a variety of modular forms, $f^{(w,-w)}(\tau,\bar{\tau})$. In Sec. II, we will review how linearized supersymmetry leads to the existence of all the terms in $S^{(3)}$ once the presence of the $\mathcal{R}^4$ term is assumed. However, linearized supersymmetry is certainly not powerful enough to determine the moduli dependent coefficients $f^{(w,-w)}$.

In Sec. III, we will use the full nonlinear supersymmetry to determine the nonholomorphic modular forms. This requires a detailed analysis of the lowest order supersymmetry transformations which generally mix all the terms in $S^{(3)}$. We will make a judicious choice of terms to consider in order to encounter minimal complications. Not surprisingly as in the cases of Ref. [10], it turns out that the terms with the maximal number of fermions are the appropriate ones for this purpose. The particular terms we will consider are $f^{(12,-12)}\lambda^{16}$ and $f^{(11,-11)}\lambda^{15}\psi^*_\mu$, where the latter is a piece of the $\lambda^{14}\hat{G}$ term. Our notation and conventions are explained in Appendix A—a hat on a field strength indicates that it includes certain fermion bilinears in its definition in order to make it "supercovariant."

In addition, we will be forced to consider terms arising from $O[(\alpha')^3]$ supersymmetry transformations acting on the classical action. These terms from $S^{(0)}$ mix under a supersymmetry variation with the relevant terms in $S^{(3)}$. For our particular purpose, it will be important to consider a $\lambda^2\lambda^{*2}$ term in the IIB action that has not to our knowledge been given explicitly in the literature. The form of this term, including its precise normalization, is determined by supersymmetry in Appendix B. By requiring invariance of the action at order $(\alpha')^3$ together with closure of the supersymmetry algebra, we will be able to determine certain modifications to the supersymmetry transformations, encoded in $\delta^{(3)}$, as well as the precise coefficients of the terms in $S^{(3)}$ under investigation. As usual, the supersymmetry algebra only closes with the use of the equations of motion.

In particular, we will find that the coefficients $f^{(11,-11)}$ and $f^{(12,-12)}$ do indeed satisfy the appropriate Laplace equations, proving the earlier conjectures about these modular forms. Furthermore, once these functions have been determined the other terms in $S^{(3)}$ that are related to these by linearized supersymmetry, including the $\mathcal{R}^4$ term, follow without the need for detailed analysis.

There have also been generalizations of the $\mathcal{R}^4$ conjecture to an infinite series of higher order terms in the type IIB effective action [17,18]. In Sec. IV we outline how our tech-

[2]Some supplementary evidence for the expression (1.6), based on linearized supersymmetry, is given in Refs. [13,14].

nique can be extended to determine the coefficients of some of these higher derivative interactions. We demonstrate how the constraints imposed by supersymmetry on these higher derivative interactions can be obtained but we do not carry through the detailed calculation, which would be reasonably complicated. It would be very interesting to extend this analysis to compactified supergravity to prove and generalize, for example, conjectures similar to those in Ref. [19].

II. HIGHER ORDER TERMS IN THE TYPE IIB EFFECTIVE ACTION

A. Linearized supersymmetry and terms of order $(\alpha')^3$

The existence of a large number of interactions in the IIB theory that are related to the $\mathcal{R}^4$ interaction can be motivated very simply by using linearized supersymmetry. This can be implemented by packaging the physical fields or their field strengths into a constrained superfield $\Phi(x,\theta)$, where θ^a ($a=1,\ldots,16$) is a complex Grassmann coordinate that transforms as a Weyl spinor of SO(9,1). This superfield satisfies the constraints

$$\bar{D}\Phi=0, \quad \bar{D}^4\bar{\Phi}=0=D^4\Phi, \tag{2.1}$$

where the first constraint is a chirality condition that ensures that Φ is independent of θ^*. The last two constraints imply that the components of Φ satisfy the free field equations. The superfield terminates after the θ^8 term and has a component expansion that takes the form[3]

$$\begin{aligned}\Phi=&\tau+i\overline{\theta^*}\lambda+\hat{G}_{\mu\nu\rho}\overline{\theta^*}\gamma^{\mu\nu\rho}\theta+\cdots\\&+\mathcal{R}_{\mu\sigma\nu\tau}\overline{\theta^*}\gamma^{\mu\nu\rho}\theta\overline{\theta^*}\gamma^{\sigma\tau}{}_{\rho}\theta\\&+\partial_\mu\hat{F}_{5\nu\rho\sigma\tau\omega}\overline{\theta^*}\gamma^{\mu\nu\rho}\theta\overline{\theta^*}\gamma^{\sigma\tau\omega}\theta+\cdots+\theta^8\partial^4\overline{\tau}.\end{aligned} \tag{2.2}$$

The symbol $\hat{G}_{\mu\nu\rho}$, $\mu,\nu,\rho=0,\ldots,9$, denotes the "supercovariant" combination of G and fermion bilinears defined in Appendix A, where $G_{\mu\nu\rho}$ and $G^*_{\mu\nu\rho}$ are complex combinations of the field strengths of the R⊗R (Ramond-Ramond) and NS⊗NS (Neveu-Schwarz–Neveu-Schwarz) two-form potentials. The four-theta terms are $\mathcal{R}$, the Weyl curvature, and $F_{5_{\rho_1\cdots\rho_5}}$, which is the field strength of the fourth-rank R⊗R potential. The gamma matrices with world indices are defined by $\gamma^\mu=e^\mu_m\gamma^m$, where $m=0,\ldots,9$ is the SO(9,1) tangent-space index and e^μ_m is the inverse zehnbein. A barred Weyl spinor, such as $\bar{\theta}$, is defined by

$$\bar{\theta}_a\equiv\theta^*_b(\gamma^0)_{ba}. \tag{2.3}$$

[3]We are using the usual convention that $\gamma^{\mu_1\cdots\mu_p}$ is the antisymmetrized product of p gamma matrices, normalized so that $\gamma^{\mu_1\cdots\mu_p}\equiv\gamma^{\mu_1}\cdots\gamma^{\mu_p}$ when $\mu_1\neq\cdots\neq\mu_p$.

We have been sketchy about the precise coefficients in Eq. (2.2) since their values will not concern us. The interactions that will be of interest in the next section are those that arise by integrating a function of Φ over the sixteen components of θ. In Einstein frame, this leads to interaction terms of the following form:

$$\begin{aligned}S^{(3)}&=(\alpha')^3\int d^{10}x d^{16}\theta \det eF[\Phi]+\text{c.c.}\\&=(\alpha')^3\int d^{10}x \det e(f^{(12,-12)}\lambda^{16}+f^{(11,-11)}\hat{G}\lambda^{14}+\cdots\\&\quad+f^{(8,-8)}\hat{G}^8+\cdots+f^{(0,0)}\mathcal{R}^4+\cdots+f^{(-12,12)}\lambda^{*16}),\end{aligned} \tag{2.4}$$

where $\det e=\det e^m_\mu$ is the determinant of the zehnbein. The SL(2,$\mathbb{Z}$) symmetry of the IIB theory requires that all the functions, $f^{(w,-w)}(\tau,\bar{\tau})$, are modular forms with holomorphic and antiholomorphic weights as indicated in the superscripts. Many terms have been hidden in the ellipsis in Eq. (2.4).

We will mainly consider the first two terms in parentheses on the right-hand-side of Eq. (2.4) where we are using the precise notation

$$(\lambda^r)_{a_{r+1}\cdots a_{16}}\equiv\frac{1}{r!}\epsilon_{a_1\cdots a_{16}}\lambda^{a_1}\cdots\lambda^{a_r}, \tag{2.5}$$

so that

$$\lambda^{16}=\frac{1}{16!}\epsilon_{a_1\cdots a_{16}}\lambda^{a_1}\cdots\lambda^{a_{16}}, \tag{2.6}$$

and

$$\begin{aligned}\hat{G}\lambda^{14}&\equiv\hat{G}_{\mu\nu\rho}(\gamma^{\mu\nu\rho}\gamma^0)_{a_{15}a_{16}}(\lambda^{14})_{a_{15}a_{16}},\\&=\frac{1}{14!}\hat{G}_{\mu\nu\rho}(\gamma^{\mu\nu\rho}\gamma^0)_{a_{15}a_{16}}\epsilon_{a_1\cdots a_{16}}\lambda^{a_1}\cdots\lambda_{a_{14}}.\end{aligned} \tag{2.7}$$

Later we will make use of the simple identities

$$\begin{aligned}(\lambda^{14})_{ab}\lambda^c&=(\lambda^{15})_b\delta^c_a-(\lambda^{15})_a\delta^c_b,\\(\lambda^{15})_a\lambda^b&=\delta^b_a\lambda^{16},\\(\lambda)^{15}_a\lambda^a&=16\lambda^{16},\end{aligned} \tag{2.8}$$

and

$$(\lambda^{14})_{ab}\lambda_c\lambda_d=\lambda^{16}(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc}). \tag{2.9}$$

B. Modular covariance

The various coefficient functions in the effective action are $(w,\hat{w})$ forms, where w refers to the holomorphic modular weight and $\hat{w}$ to the anti-holomorphic modular weight. A nonholomorphic modular form $F^{(w,\hat{w})}$ transforms as

$$F^{(w,\hat{w})} \to F^{(w,\hat{w})}(c\tau+d)^{w}(c\bar{\tau}+d)^{\hat{w}}, \tag{2.10}$$

under the SL(2,$\mathbb{Z}$) transformation taking

$$\tau \to \frac{a\tau+b}{c\tau+d}. \tag{2.11}$$

Equation (2.10) describes a $U(1)$ transformation when $\hat{w}=-w$.

The modular covariant derivative

$$\mathcal{D}_w = i\left(\frac{\partial}{\partial\tau} - i\frac{w}{2\tau_2}\right) \tag{2.12}$$

maps $F^{(w,\hat{w}}$ into $F^{(w+2,\hat{w})}$ while the antiholomorphic covariant derivative $\bar{\mathcal{D}}_{\hat{w}} = \bar{\mathcal{D}}^{*}_{\hat{w}}$ maps $F^{(w,\hat{w})}$ into $F^{(w,\hat{w}+2)}$. It is more convenient for our purposes to define the covariant derivatives

$$D_w = \tau_2\mathcal{D} = i\left(\tau_2\frac{\partial}{\partial\tau} - i\frac{w}{2}\right), \quad \bar{D}_{\hat{w}} = \tau_2\bar{\mathcal{D}} = -i\left(\tau_2\frac{\partial}{\partial\bar{\tau}} + i\frac{\hat{w}}{2}\right), \tag{2.13}$$

which change the U(1) charge of F by two units

$$D_w F^{(w,\hat{w})} = F^{(w+1,\hat{w}-1)}, \quad \bar{D}_{\hat{w}} F^{(w,\hat{w})} = F^{(w-1,\hat{w}+1)}. \tag{2.14}$$

The Laplace operator on the fundamental domain of SL(2,$\mathbb{Z}$) is defined to be

$$\nabla_0^2 \equiv \nabla^2 = 4\tau_2^2\frac{\partial}{\partial\tau}\frac{\partial}{\partial\bar{\tau}}, \tag{2.15}$$

when acting on (0,0) forms. More generally, we shall be interested in the Laplacian acting on $(w,-w)$ forms. There are two such Laplacians which are defined by

$$\begin{aligned}\nabla^2_{(-)w} &= 4D_{w-1}\bar{D}_{-w}\\ &= 4\tau_2^2\frac{\partial}{\partial\tau}\frac{\partial}{\partial\bar{\tau}} - 2iw\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\bar{\tau}}\right) - w(w-1),\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}\nabla^2_{(+)w} &= 4\bar{D}_{-w-1}D_w\\ &= 4\tau_2^2\frac{\partial}{\partial\tau}\frac{\partial}{\partial\bar{\tau}} - 2iw\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\bar{\tau}}\right) - w(w+1),\\ &= \nabla^2_{(-)w} - 2w.\end{aligned} \tag{2.17}$$

Now consider a $(w,-w)$ form that is an eigenfunction of the Laplace operator $\nabla^2_{(-)w}$ with eigenvalue σ_w,

$$\nabla^2_{(-)w}F^{(w,-w)} = 4D_{w-1}\bar{D}_{-w}F^{(w,-w)} = \sigma_w F^{(w,-w)}. \tag{2.18}$$

Applying $\bar{D}_{-w}$ to this equation gives

$$\nabla^2_{(+)w-1}F^{(w-1,-w+1)} = \sigma_w F^{(w-1,-w+1)}. \tag{2.19}$$

It is also easy to see that

$$\begin{aligned}&\nabla^2_{(-)w-1}F^{(w-1,-w+1)}\\ &\quad = 4D_{w-2}\bar{D}_{-w+1}F^{(w-1,-w+1)},\\ &\quad = (\sigma_w + 2w - 2)F^{(w-1,-w+1)},\end{aligned} \tag{2.20}$$

where $F^{(w-1,-w+1)} = \bar{D}_{-w}F^{(w,-w)}$. Repeating this for m steps gives

$$\begin{aligned}&\nabla^2_{(-)w-m}F^{(w-m,-w+m)}\\ &\quad = 4D_{w-m-1}\bar{D}_{-w+m+1}F^{(w-m,-w+m)},\\ &\quad = (\sigma_w + 2mw - m^2 - m)F^{(w-m,-w+m)}.\end{aligned} \tag{2.21}$$

Similarly,

$$\begin{aligned}&\nabla^2_{(+)w-m}F^{(w-m,-w+m)}\\ &\quad = (\sigma_w + 2mw - 2w - m^2 + m)F^{(w-m,-w+m)}.\end{aligned} \tag{2.22}$$

This relation between eigenvalue equations will be useful in analyzing the equations that are satisfied by the modular forms that enter in $S^{(3)}$.

An indication of why this is so comes from various duality arguments that relate type II string theories and M theory. Firstly, it was argued in Refs. [11,20] that the function $f^{(0,0)}$ should satisfy (1.5), in which case it should be an eigenfunction of the ∇_0^2 on the fundamental domain of $SL(2,\mathbb{Z})$ with eigenvalue 3/4. Furthermore, in Ref. [15] it was argued that the nonholomorphic modular forms that arise as coefficients in $S^{(3)}$ are related to each other by applying covariant derivatives. For example, it was suggested that[4]

[4]We are using a more uniform notation for the modular forms here than in Ref. [11].

$$f^{(12,-12)}=(\tau_2\mathcal{D})^{12}f^{(0,0)}=D^{12}f^{(0,0)}\equiv D_{11}\cdots D_1D_0f^{(0,0)}. \tag{2.23}$$

Using this relation and Eq. (2.21) for the case $m=w$, and assuming that $f^{(0,0)}$ indeed satisfies Eq. (1.5), leads to the eigenvalue equation that $f^{(12,-12)}$ is expected to satisfy

$$\nabla^2_{(-)12}f^{(12,-12)}=\left(-132+\frac{3}{4}\right)f^{(12,-12)}. \tag{2.24}$$

In the next section we will prove, using supersymmetry alone, that $f^{(12,-12)}$ does satisfy this equation.

The solution to the Laplace equation (1.5) with eigenvalue $\sigma=\frac{3}{4}$ is unique if we assume that $f^{(0,0)}$ has a power law behavior near the boundary of the fundamental domain of SL(2,$\mathbb{Z}$) which agrees with the known tree-level and one-loop contributions. More generally, let us denote a solution of the scalar Laplace equation with eigenvalue $\sigma=s(s-1)>\frac{1}{4}$ by $E_s(\tau)$ [12],

$$\nabla^2E_s=s(s-1)E_s. \tag{2.25}$$

We can express $E_s(\tau)$ in terms of the nonholomorphic Eisenstein series

$$E_s(\tau)=\frac{1}{2}\tau_2^s\sum_{(m,n)=1}|m\tau+n|^{-2s}, \tag{2.26}$$

where (m,n) denotes the greatest common divisor of m and n. The eigenfunctions $E_s(\tau)$ are singled out by their power law behavior near the boundary of the moduli space, which agrees with the known tree-level and perturbative contributions to the interactions that we are considering. It follows from Eq. (2.23) that $f^{(12,-12)}$ is also determined uniquely by its Laplace equation (2.24) if the presence of a tree-level term is assumed. While the arguments leading to Eq. (2.24) were motivated in prior work rather indirectly by various dualities, our purpose is to prove that Eq. (2.24) and related conditions follow directly from a rather simple application of supersymmetry.

III. DETERMINING TERMS IN $S^{(3)}$ USING SUPERSYMMETRY

We now proceed to a precise determination of the modular forms that enter into Eq. (2.4). This starts by selecting two specific terms in the effective Lagrangian at order $(\alpha')^3$,

$$L_1^{(3)}=\det e(f^{(12,-12)}(\tau,\bar{\tau})\lambda^{16}+f^{(11,-11)}(\tau,\bar{\tau})\hat{G}\lambda^{14}). \tag{3.1}$$

Our notation is chosen so that $(\alpha')^{n-4}\int d^{10}xL^{(n)}=S^{(n)}$. We will show that these are related by a subset of the supersymmetry transformations that do not mix with any of the other terms at this order. We will also take into account terms from the variation of the lowest order action $S^{(0)}$ that can mix with the variations of Eq. (3.1).

We will only need to consider those terms in $\hat{G}$ that are bilinear in the fermions. After using the identity

$$\begin{aligned}(\gamma^{\mu\nu\rho}\gamma^0)_{ab}(\lambda)^{14}_{ab}(\bar{\psi}_\mu\gamma_{\nu\rho}\lambda)&=-144\bar{\psi}_\mu\gamma^\mu\gamma^0\lambda^{15}\\&=144\lambda^{15}\gamma^\mu\psi^*_\mu,\end{aligned} \tag{3.2}$$

where we have used the fact that $\gamma_{\nu\rho}\gamma^{\mu\nu\rho}=-72\gamma^\mu$, the relevant terms in $L_1^{(3)}$ can be expressed as

$$L_1^{(3)}=\det e(f^{(12,-12)}\lambda^{16}-3\cdot 144f^{(11,-11)}(\lambda^{15}\gamma^\mu\psi^*_\mu)+\cdots). \tag{3.3}$$

The ellipsis represents other terms in $\hat{G}$ which do not affect the subsequent argument.

First consider the lowest order supersymmetry transformation of Eq. (3.3) into $\det e\lambda^{16}\psi^*_\mu\epsilon$. From Eq. (3.3) we have

$$\begin{aligned}\delta_1^{(0)}L_1^{(3)}&=\delta^{(0)}(\det e)f^{(12,-12)}\lambda^{16}+\det e f^{(12,-12)}\delta^{(0)}(\lambda^{16})\\&\quad-3\times 144\det e\left(\frac{\partial}{\partial\tau}f^{(11,-11)}\delta^{(0)}\tau(\lambda^{15}\gamma^\mu\psi^*_\mu)+f^{(11,-11)}\delta^{(0)}(\lambda^{15}\gamma^\mu\psi^*_\mu)\right)\\&=i\det e\left(\bar{\epsilon}^*\gamma^\mu\psi^*_\mu f^{(12,-12)}\lambda^{16}+\frac{1}{8}(\lambda^{15})_a(\gamma^{\mu\nu\rho}\epsilon)_a\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda f^{(12,-12)}+6\times 144iD_{11}f^{(11,-11)}(\lambda^{15})_a(\gamma^\mu\psi^*_\mu)_a\bar{\epsilon}^*\lambda\right)\\&=-i\det e(\bar{\epsilon}^*\gamma^\mu\psi^*_\mu)\lambda^{16}(8f^{(12,-12)}+6\times 144D_{11}f^{(11,-11)}),\end{aligned} \tag{3.4}$$

where we have only kept terms proportional to $\lambda^{16}\psi^*_\mu\epsilon$. In passing from the first to the second line in this equation, we have made use of the standard $\delta^{(0)}$ supersymmetry transformations summarized in Appendix A. It is important to check whether there could also be a contribution of the same form as Eq. (3.4) arising from a $(\alpha')^3\delta^{(3)}$ variation of the fields in the lowest order action $S^{(0)}$. However, it is easy to see by inspection that no term with $\lambda^{16}\psi^*_\mu$ can arise from the variation of any term[5] in $S^{(0)}$. This means that we must require $\delta_1^{(0)}L_1^{(3)}=0$, which implies that

$$D_{11}f^{(11,-11)}=-\frac{4}{3\times 144}f^{(12,-12)}. \tag{3.5}$$

This condition is consistent with the modular weights assigned to the functions $f^{(w,-w)}$.

In order to find another condition relating $f^{(12,-12)}$ and $f^{(11,-11)}$, we now consider the term in the variation of Eq. (3.3) that is proportional to $\det e\lambda^{16}\lambda^*\epsilon^*$. This term is

$$\begin{aligned}\delta_2^{(0)}L_1^{(3)}&=\det e\left(\frac{\partial f^{(12,-12)}}{\partial\bar\tau}\delta^{(0)}\bar\tau\lambda^{16}+f^{(12,-12)}\lambda^{15}\delta^{(0)}\lambda-3\times 144 f^{(11,-11)}\lambda^{15}\delta^{(0)}(\gamma^\mu\psi_\mu^*)\right)\\&=-2i\det e\lambda^{16}(\bar\epsilon\lambda^*)\left[-i\left(\tau_2\frac{\partial}{\partial\bar\tau}-6i\right)f^{(12,-12)}+3\times 144\times\frac{15}{2}f^{(11,-11)}\right]+\cdots\\&=-2i\det e\lambda^{16}(\bar\epsilon\lambda^*)\left(\bar D_{-12}f^{(12,-12)}+3\times 144\times\frac{15}{2}f^{(11,-11)}\right)+\cdots,\end{aligned} \tag{3.6}$$

where we have made explicit only the terms containing $\lambda^{16}\lambda^*\epsilon^*$ in the second line.

In this case, there is another contribution of the same form as $\delta_2^{(0)}L_1^{(3)}$ that arises from the $(\alpha')^3\delta^{(3)}$ variation of terms in the lowest order IIB Lagrangian $L^{(0)}$ (recall that we are really using the action as a shorthand for the IIB equations of motion). Even though the complete set of interactions in the classical theory is not tabulated explicitly in the literature (it is implicit in the superspace formulation [8]), it is easy to convince oneself that the only possible term that can vary into $\delta_2^{(0)}L_1^{(3)}$ is a term of the form

$$L_1^{(0)}=-\frac{c}{6}\det e\bar\lambda\gamma^{\mu\nu\rho}\lambda^*\bar\lambda^*\gamma_{\mu\nu\rho}\lambda, \tag{3.7}$$

which is the unique tensor structure containing $\lambda^2\lambda^{*2}$. The coefficient c has been left free in this formula, but it is determined by the lowest order supersymmetry transformations. It is determined in Appendix B by considering the mixing of $L_1^{(0)}$ with

$$L_2^{(0)}=\frac{3}{2}i\det e\bar\lambda\gamma^\mu\lambda Q_\mu \tag{3.8}$$

and

$$L_3^{(0)}=i\det e\bar\lambda\gamma^\mu\gamma^\omega\psi_\mu^*P_\omega. \tag{3.9}$$

The term $L_2^{(0)}$ is the connection part of the Dirac action for the dilatino

$$i\int d^{10}x\det e\bar\lambda\gamma D\lambda, \tag{3.10}$$

and its coefficient is determined by the U(1) charge for λ. The normalization of the second term can be extracted from the gravitino field equation (Eq. (4.12) of Ref. [7]), but is also determined by the supersymmetry considerations in Appendix B. The value of c deduced in Appendix B is

$$c=-\frac{3}{128}. \tag{3.11}$$

Of course, the arbitrary Newton coupling has been set equal to a particular value in defining the absolute normalization of the action, but this value cancels out of all that follows.

We can now see that $L_1^{(0)}$ can vary into the same form as $\delta_2^{(0)}L_1^{(3)}$ if we assume a variation of λ^* of the form

$$\delta^{(3)}\lambda_a^*=-\frac{1}{6}ig(\tau,\bar\tau)(\lambda^{14})_{cd}(\gamma^{\mu\nu\rho}\gamma^0)_{dc}(\gamma_{\mu\nu\rho}\epsilon^*)_a, \tag{3.12}$$

where $g(\tau,\bar\tau)$ is a function to be determined. We will show momentarily that there must be such a term in the variation of λ^* if the supersymmetry algebra is to close. Substituting in Eq. (3.7) gives a contribution

$$\begin{aligned}\delta^{(3)}L_1^{(0)}&=\frac{2c}{36}i\det eg(\tau,\bar\tau)\bar\lambda\gamma^{\mu\nu\rho}\gamma_{\rho_1\rho_2\rho_3}\\&\quad\times\epsilon^*(\lambda^{14})_{cd}(\gamma^{\rho_1\rho_2\rho_3}\gamma^0)_{dc}\bar\lambda^*\gamma_{\mu\nu\rho}\lambda\\&=-480\times 16ic\det eg(\tau,\bar\tau)\lambda^{16}(\bar\epsilon\lambda^*).\end{aligned} \tag{3.13}$$

Some of these manipulations make use of the gamma matrix identities listed in Appendix A 1. Comparing with Eq. (3.6) we see that in order for the total contribution to δL_1 to vanish at order $(\alpha')^3$, there must be a linear relation between

[5] The only relevant terms are those involving only fermionic fields since bosonic fields vary into derivatives. The only fermion interactions that could vary into the required form would be terms such as $\lambda^2\lambda^*\psi_\mu^*,\lambda^3\psi_\mu^*,\ldots,$ which are excluded from the classical theory since they violate U(1) charge conservation.

the function g and the functions $f^{(11,-11)}$ and $\bar{D}_{-12}f^{(12,-12)}$,

$$\bar{D}_{-12}f^{(12,-12)}+3\times 144\times\frac{15}{2}f^{(11,-11)}+240\times 16cg=0. \tag{3.14}$$

A further constraint on these functions is obtained by requiring the supersymmetry algebra to close which requires the use of the fermionic equations of motion as well as the equation for F_5. In fact, these equations of motion were determined in the classical theory by requiring closure of the superalgebra for the low-energy type IIB theory in Ref. [7]. Another important feature of supergravity theories such as this is that the algebra need only close up to a field-dependent local symmetry transformation. In the case at hand, the important fact is that the commutator of two supersymmetry transformations $[\delta_1,\delta_2]$ gives the usual transport term $\overline{\epsilon_2}\gamma^\mu\epsilon_2 D_\mu$ together with a supersymmetry transformation $\delta_{\hat\epsilon}$ and terms that vanish by the equations of motion. The supersymmetry parameter $\hat\epsilon$ is field dependent.

We will consider closure of the supersymmetry transformations on the field λ^*. First, keeping only the terms linear in λ derivatives, we find (as in Eq. (4.5) of Ref. [7])

$$(\delta_1^{(0)}\delta_2^{(0)}-\delta_2^{(0)}\delta_1^{(0)})\lambda^* =\xi^\mu D_\mu\lambda^*-\frac{3}{8}i[\overline{\epsilon_2}\gamma^\rho\epsilon_1-(1\leftrightarrow 2)]\gamma_\rho\gamma^\mu D_\mu\lambda^* -\frac{1}{96}i[\overline{\epsilon_2}\gamma^{\rho_1\rho_2\rho_3}\epsilon_1-(1\leftrightarrow 2)]\gamma_{\rho_1\rho_2\rho_3}\gamma^\mu D_\mu\lambda^*, \tag{3.15}$$

where

$$\xi^\mu=-2\,\mathrm{Im}\,\overline{\epsilon_2}\gamma^\mu\epsilon_1. \tag{3.16}$$

The first term on the right-hand-side is of the form expected for the commutator of two supersymmetry transformations. The remaining terms are proportional to the lowest-order term in the λ^* equation of motion. Many other terms that we will not need also contribute to the full commutator to complete the low-energy λ^* field equation on the right-hand side, as well as generating local transformations of λ^*.

The higher order terms in $L^{(3)}$ modify the equations of motion and this should also be apparent by considering the closure of the algebra. Therefore, we now consider terms that enter at order $(\alpha')^3$ from the commutator of a $\delta^{(0)}$ with a $\delta^{(3)}$. More precisely, we shall consider terms in the commutator involving only ϵ_2^* and ϵ_1,

$$\begin{aligned}(\epsilon_{\epsilon_1}^{(0)}\delta_{\epsilon_2^*}^{(3)}-\delta_{\epsilon_2^*}^{(3)}\delta_{\epsilon_1}^{(0)})\lambda_a^* &=-\frac{1}{3}\left(\tau_2\frac{\partial}{\partial\tau}-i\frac{45}{8}\right)ig(\overline{\epsilon_1^*}\lambda)(\lambda^{14})_{cd}(\gamma^{\mu\nu\rho}\gamma^0)_{dc}(\gamma_{\mu\nu\rho}\epsilon_2^*)_a\\ &=\frac{2}{48}\frac{8}{3}288\lambda_b^{15}i\left[\frac{3}{8}\overline{\epsilon_2}\gamma^\mu\epsilon_1(\gamma_\mu)_{ba}+\frac{1}{96}\overline{\epsilon_2}\gamma^{\mu\nu\rho}\epsilon_1(\gamma_{\mu\nu\rho})_{ba}\right]\left(\tau_2\frac{\partial}{\partial\tau}-i\frac{45}{8}\right)g\\ &=32D_{11}g\lambda_b^{15}\left[\frac{3}{8}\overline{\epsilon_2}\gamma^\mu\epsilon_1(\gamma_\mu)_{ba}+\frac{1}{96}\overline{\epsilon_2}\gamma^{\mu\nu\rho}\epsilon_1(\gamma_{\mu\nu\rho})_{ba}\right]+\delta_{\hat\epsilon}\lambda^*.\end{aligned} \tag{3.17}$$

In passing from the first to the second equation, we have used once more the Fierz identity and various gamma matrix identities given in Appendix A 1. In the last line, we have separated a term

$$\delta_{\hat\epsilon}\lambda^*=-i\frac{1}{24}g(\overline{\epsilon_1^*}\lambda)(\lambda^{14})_{cd}(\gamma^{\mu\nu\rho}\gamma^0)_{dc}(\gamma_{\mu\nu\rho}\epsilon_2^*)_a, \tag{3.18}$$

which is to be identified with a supersymmetry transformation of the form (3.12) with a particular field-dependent coefficient

$$\hat\epsilon=\frac{i}{4}\epsilon_2^*(\overline{\epsilon_1^*}\lambda). \tag{3.19}$$

This is unambiguously identified by the fact that it is needed in order to change the $\frac{45}{8}$ in the previous lines to the $\frac{44}{8}$ which is contained in D_{11}. This is correlated with the fact that the function g transforms with weight (11, −11).

In writing Eq. (3.17), we have taken pains to express the right-hand side as a sum of precisely the same tensor structures that appear on the right-hand side of Eq. (3.15). Combining Eqs. (3.15) and (3.17) (including the powers of α') we see that in order for the right-hand side of the commutator to vanish the λ^* field equation must be of the form

$$i\gamma^\mu D_\mu\lambda^*-(\alpha')^3 32D_{11}g\lambda^{15}+\cdots=0, \tag{3.20}$$

where the ellipsis indicates terms with different structure that we have not considered. This equation has to be identified with the appropriate sum of terms in the λ^* equation of motion that is obtained by varying the action with respect to λ. At the same order in α' this is given by

$$i\gamma^\mu D_\mu\lambda^*-(\alpha')^3 f^{(12,-12)}\lambda^{15}+\cdots=0, \tag{3.21}$$

where we have only made explicit the term that is proportional to λ^{15}. Comparing Eqs. (3.20) and (3.21) gives the relation

$$32D_{11}g=f^{(12,-12)}. \tag{3.22}$$

Substituting Eq. (3.22) into Eq. (3.5) gives

$$g=-\frac{3\times 144}{128}f^{(11,-11)}. \tag{3.23}$$

There is no ambiguity in this relation between g and $f^{(11,-11)}$ because there is no solution to $D_{11}g=0$. Substituting Eq. (3.23) into Eq. (3.14) using the value $c=-\frac{3}{128}$ gives

$$\bar{D}_{-12}f^{(12,-12)}=3\times 144\left(-\frac{15}{2}+\frac{45}{64}\right)f^{(11,-11)}. \tag{3.24}$$

The two simultaneous first-order differential equations (3.24) and (3.5) are simply reduced to the independent second-order equations

$$\nabla^2_{(-)12}f^{(12,-12)}=4D_{11}\bar{D}_{-12}f^{(12,-12)}=\left(-132+\frac{3}{4}\right)f^{(12,-12)}, \tag{3.25}$$

$$\nabla^2_{(+)11}f^{(11,-11)}\equiv 4\bar{D}_{-12}D_{11}f^{(11,-11)}=\left(-132+\frac{3}{4}\right)f^{(11,-11)}.$$

The first of these equations is the same as Eq. (2.24). Therefore, the modular form $f^{(12,-12)}$ is uniquely determined to be the function suggested in Ref. [15] if we assume that $f^{(12,-12)}$ has a tree-level and one-loop contribution at weak coupling. This function can be expressed as $D^{12}f^{(0,0)}$ where $f^{(0,0)}$ satisfies the Laplace equation (2.24) with eigenvalue $\frac{3}{4}$ (the proof that this function is actually the coefficient of the $\mathcal{R}^4$ term will follow from the argument in the next paragraph). Similarly, the second equation in Eq. (3.25) gives a unique expression for the modular form $f^{(11,-11)}$.

Having determined $f^{(12,-12)}$ and $f^{(11,-11)}$ we would now like to determine the remaining terms in Eq. (2.4) of the same order but lower U(1) charge, such as $\mathcal{R}^4$. A simple way to determine these terms is to consider the constraints on the coefficient functions that follow from linearized supersymmetry and then to impose the requirement that the effective action be SL(2,$\mathbb{Z}$) invariant. Linearized supersymmetry, described in Sec. II, is valid to leading order in $(\tau_2)^{-1}$. We saw that in that approximation the terms in Eq. (2.4) are expressed as an integral of a function of the superfield $F[\Phi]$ over one-half of superspace. Furthermore, it was argued in Ref. [15] that the linearized approximation is exact for the leading charge K D-instanton contributions to the coefficient functions $f^{(p,-p)}$. These can be extracted by choosing $F[\Phi]=e^{2\pi iK\Phi}$ and agree with the expectation that the coefficients are related by

$$f^{(p,-p)}=D_{p-1}\cdots D_0 f^{(0,0)}. \tag{3.26}$$

Only the Abelian pieces of the covariant derivatives affect the argument to leading order in $(\tau_2)^{-1}$ which does not build in the required modular invariance. The modular covariant expressions are reproduced by using the fully modular covariant derivatives in Eq. (3.26).

It should, of course, also be true that the expressions for all the coefficients $f^{(p,-p)}$ in Eq. (2.4) also emerge from a more detailed application of the Noether procedure that considers all the possible mixing of terms in $S^{(3)}$ with arbitrary U(1) charges.

IV. COMMENTS ON HIGHER DERIVATIVE INTERACTIONS

A. Some general comments

More speculative extensions of the $\mathcal{R}^4$ conjecture have been suggested in Refs. [18,17,21]. For example, interactions of the form

$$(\alpha')^{-4}\sum_{g,\hat{g}=1}^{\infty}\sum_{p=2-2g}^{2g-2}(\alpha')^{2g+2\hat{g}-1}\int d^{10}x\,\det e \\ \times F_5^{4\hat{g}-4}G^{2g-2+p}G^{*2g-2-p} \\ \times[f_{g+\hat{g}-1}^{(p,-p)}(\tau,\bar{\tau})\mathcal{R}^4+\cdots+f_{g+\hat{g}-1}^{(12+p,-12-p)}(\tau,\bar{\tau})\lambda^{16}], \tag{4.1}$$

arose[6] Refs. [17,21]. The case $g=\hat{g}=1$ corresponds to the terms that we considered in the earlier sections. The modular functions $f_g^{(q,-q)}$ are expected to be given by the generalized Eisenstein series

$$f_g^{(q,-q)}=\sum_{(m,n)\neq(0,0)}\frac{\tau_2^{g+1/2}}{(m+n\tau)^{g+1/2+q}(m+n\bar{\tau})^{g+1/2-q}}. \tag{4.2}$$

Note that for $q=0$, these coefficient functions are proportional to $E_{g+1/2}(\tau)$, where E_s was defined in Eq. (2.26). Expanding Eq. (4.2) for small coupling ($\tau_2\to\infty$) leads, as in the case $g=1$, to two power-behaved terms that are to be identified with perturbative terms in string theory. These correspond to a tree-level term and a g-loop term. In fact, the case $s=\frac{3}{2}$ is the physical lower bound on s since in that case the loop term is of the lowest possible genus $g=1$. The agreement of the perturbative behavior of Eq. (4.2) with the known perturbative contributions to Eq. (4.1) computed in Ref. [22] is a primary motivation for the form of these coefficient functions. The perturbative contributions were computed in a topological formalism further studied in Ref. [23]. As in the case $g=1$, there are no higher order perturbative corrections but there is an infinite series of D-instanton corrections. The conjectured functions $f_g^{(q,-q)}$ in Eq. (4.2) are again eigenfunctions of the Laplace operator acting on $(q,-q)$ forms, as in the $g=1$ case. Now, however, the eigenvalue depends on g. For example,

$$4\tau_2^2\partial_\tau\partial_{\bar{\tau}}f_g^{(0,0)}=\left(\frac{1}{4}+\frac{g}{2}\right)f_g^{(0,0)}. \tag{4.3}$$

[6] More precisely, the interactions suggested in Refs. [17,21] only included the $\mathcal{R}^4$ terms in this expression.

From the perspective of superspace, the status of terms with $g+\hat{g}>3$ is quite different from the terms we considered in Sec. III for which $g+\hat{g}=2$. Those terms could be written as integrals over $\frac{1}{2}$ the on-shell superspace, which is described in terms of a superspace with a single Weyl SO(9,1) spinor. For this reason, we could have anticipated the fact that they satisfied very constraining nonrenormalization conditions. Cases in which $g+\hat{g}=3$ [terms of order $(\alpha')^5$ relative to the Einstein-Hilbert term] appear to be similarly special since, by dimensional analysis, they correspond to integrals over $\frac{3}{4}$ of the on-shell superspace, i.e., over 24 Grassmann spinor components. Since there is no covariant description of SO(9,1) spinors with 24 components, there is no obviously simple superspace description of such terms. However, as we will see in the next subsection an analysis of the supersymmetry transformations similar to the preceding one is likely to determine the form of these $O[(\alpha')^5]$ terms and provide further motivation for the conjectured terms in Eq. (4.1) at this order.

B. An outline of how terms in $S^{(5)}$ are constrained

We will not present a detailed analysis of terms in $S^{(5)}$ but rather, we will give a schematic outline of how supersymmetry constrains at least some of these terms. Consequently, we will not be concerned about the exact normalizations or tensor structures that arise in the various terms.

We will consider interactions in $S^{(5)}$ with $\hat{g}=1$ and $g=2$, which are terms of order $(\alpha')^5$ relative to the Einstein-Hilbert term. An important consideration is that the absence of (α') and $(\alpha')^2$ corrections to the effective action (the absence of $S^{(1)}$ and $S^{(2)}$ terms) means that the supersymmetry transformations have modifications that begin with $(\alpha')^3\delta^{(3)}$. These transformations do not mix any of the lower order terms in $S^{(0)}+S^{(3)}$ with the terms in $S^{(5)}$. We therefore only need to consider $S^{(0)}+(\alpha')^5S^{(5)}$ and $\delta^{(0)}+(\alpha')^5\delta^{(5)}$.

In complete analogy to our earlier analysis, we will begin by considering the term in $L^{(5)}$ of modular weight (14,−14),

$$L_1^{(5)}=\det e\lambda^{16}\hat{G}^4 f_2^{(14,-14)}(\tau,\overline{\tau}), \tag{4.4}$$

recalling that $\hat{G}$ is the supercovariant extension of G containing fermion bilinears. The tensor structure is hidden in the abbreviation $\hat{G}^4$ which should read

$$t_{\mu_1\cdots\mu_{12}}\hat{G}^{\mu_1\mu_2\mu_3}\cdots\hat{G}^{\mu_{10}\mu_{11}\mu_{12}}, \tag{4.5}$$

for a tensor structure t which we will not specify here but would be determined in a more complete treatment.

As before, the first supersymmetry variation of Eq. (4.4) to consider is the one acting on $\overline{\tau}$ given in Eq. (A21),

$$\delta_1^{(0)}L_1^{(5)}=-2\det e\lambda^{16}(\overline{\epsilon\lambda}^*)\hat{G}^4\left(\tau_2\frac{\partial}{\partial\overline{\tau}}-7i\right)f_2^{(14,-14)}(\tau,\overline{\tau}). \tag{4.6}$$

In this case, there are two other terms in $S^{(5)}$ that can vary into Eq. (4.6). The first is similar in structure to the term that appeared in our earlier analysis

$$L_2^{(5)}=\det e\lambda^{15}\gamma^\mu\psi_\mu^*\hat{G}^4 f_2^{(13,-13)}(\tau,\overline{\tau}), \tag{4.7}$$

which is a piece of the supercovariant combination $\det e\lambda^{14}\hat{G}^5$. The relevant supersymmetry variation gives

$$\delta_1^{(0)}L_2^{(5)}=\det e\lambda^{15}\gamma^\mu\delta^{(0)}(\psi_\mu^*)\hat{G}^4 f^{(13,-13)}(\tau,\overline{\tau}), \tag{4.8}$$

where $\delta^{(0)}(\gamma^\mu\psi_\mu^*)$ is given in Appendix A.

The second term is a new possibility

$$L_3^{(5)}=\det e\lambda^{16}\hat{G}^3\hat{G}^*\tilde{f}_2^{(13,-13)}(\tau,\overline{\tau}). \tag{4.9}$$

The relevant part of this expression is the fermion bilinear in $\hat{G}^*$ proportional to $\psi\lambda^*$. Since $\delta_1^{(0)}\psi$ contains a $\hat{G}\epsilon^*$ piece, the variation

$$\delta_1^{(0)}L_3^{(5)}=\det e\lambda^{16}\hat{G}^3(\delta_1^{(0)}\hat{G}^*)\tilde{f}_2^{(13,-13)}(\tau,\overline{\tau}), \tag{4.10}$$

mixes with Eq. (4.6).

In addition, it is necessary to consider the mixing of these terms with terms of the classical action. The two terms that are relevant are $L_1^{(0)}$ given in Eq. (3.7) and $L_4^{(0)}$ given by

$$L_4^{(0)}=\psi_\mu\gamma_{\nu\rho}\overline{\lambda}G^{\mu\nu\rho}. \tag{4.11}$$

For these terms to mix with Eq. (4.6) there need to be modifications to the supersymmetry transformations that take the schematic form

$$\delta^{(5)}\lambda^*\sim g_1(\tau,\overline{\tau})\hat{G}^4(\lambda^{14})_{cd}(\gamma^{\mu\nu\rho}\gamma^0)_{dc}(\gamma_{\mu\nu\rho}\epsilon^*),$$

$$\delta^{(5)}\psi_\mu\sim g_2(\tau,\overline{\tau})\lambda^{16}(\hat{G}^3\epsilon^*)_\mu. \tag{4.12}$$

Invariance under supersymmetry then gives a linear relation between the functions

$$\overline{D}_{-14}f_2^{(14,-14)},\quad f_2^{(13,-13)},\quad \tilde{f}_2^{(13,-13)},\quad g_1,\quad g_2. \tag{4.13}$$

Additional constraints that relate $f_2^{(14,-14)}$ and $f_2^{(13,-13)}$ can be obtained by considering a second supersymmetry variation that mixes $L_1^{(5)}$ and $L_2^{(5)}$ and with no other terms at order $(\alpha')^5$. An appropriate transformation to consider is

$$\delta_2^{(0)}L_1^{(5)}=\delta_2^{(0)}(\det e\lambda^{16}\hat{G}^4)f_2^{(14,-14)}$$
$$\sim(\det e\lambda^{16}\hat{G}^4)\overline{\epsilon^*}\gamma^\mu\psi_\mu^* f_2^{(14,-14)}+\cdots \tag{4.14}$$

and

$$\delta_2^{(0)}L_2^{(5)}=2\det e\left(\tau_2\frac{\partial}{\partial\tau}+\frac{13}{2}i\right)f_2^{(13,-13)}\lambda^{16}\overline{\epsilon^*}\gamma^\mu\psi_\mu^*\hat{G}^4, \tag{4.15}$$

where we are using parts of $\delta^{(0)}\lambda$ from Eq. (A24), $\delta^{(0)}e_{\mu}^{m}$ from Eq. (A23) and $\delta^{(0)}\tau$ from Eq. (A21). In addition we must consider the variation of a term $L_5^{(0)}$ in $S^{(0)}$ where $L_5^{(0)}$ takes the form

$$L_5^{(0)}=\overline{\psi}_{\mu}\gamma_{\nu}\psi_{\rho}^{*}G^{\mu\nu\rho}. \tag{4.16}$$

A variation of this term which mixes with Eqs. (4.14) and (4.15) is induced by the new transformation

$$\delta^{(5)}\psi_{\mu}^{*}=g_3(\tau,\overline{\tau})\lambda^{16}\hat{G}^3_{\mu\nu\rho}\gamma^{\nu\rho}\epsilon, \tag{4.17}$$

where g_3 is another function that has to be determined. Invariance under supersymmetry then relates $D_{13}f_2^{(13,-13)}$, $f_2^{(14,-14)}$, and g_3.

The final set of constraints follow from closure of the supersymmetry algebra on λ^*, ψ, and ψ^*. The part of the commutators

$$[\delta_{\epsilon_1},\delta_{\epsilon_2^*}]\lambda^*,\quad [\delta_{\epsilon_1},\delta_{\epsilon_2^*}]\psi_{\mu},\quad [\delta_{\epsilon_1^*},\delta_{\epsilon_2}]\psi_{\mu}^{*}, \tag{4.18}$$

proportional to $(\alpha')^5$ gives a sufficient number of relations to determine g_1, g_2, and g_3 in terms of the coefficient functions in $S^{(5)}$. For example, identifying the right-hand-side of the commutator

$$\begin{aligned}[\delta_{\epsilon_1},\delta_{\epsilon_2^*}]\lambda^*&\sim\delta_{\epsilon_1}^{(0)}(g_1(\tau,\overline{\tau})\hat{G}^4(\lambda^{14})_{cd})(\gamma^{\mu\nu\rho}\gamma^0)_{dc}(\gamma_{\mu\nu\rho}\epsilon_2^*)+\cdots,\\ &\sim D_{13}g_1\epsilon_1\lambda^{15}\hat{G}^4\epsilon_2^*+g_1\epsilon_1\hat{G}^*\lambda^{15}\hat{G}^3\epsilon_2^*\cdots,\end{aligned} \tag{4.19}$$

with the λ^* equation of motion will allow us to relate $D_{13}g_1$ and $f_2^{(14,-14)}$ as well as g_1 and $\tilde{f}_2^{(13,-13)}$, by analogy with the case we studied earlier. As with the earlier case, it is important to also subtract the variation in the reverse order $\delta^{(5)}_{\epsilon_2^*}\delta^{(0)}_{\epsilon_1}\lambda^*$. But we also need to add the variations $(\delta^{(5)}_{\epsilon_1}\delta^{(0)}_{\epsilon_2^*}-\delta^{(0)}_{\epsilon_2^*}\delta^{(5)}_{\epsilon_1})\lambda^*$, which give a nonvanishing contribution to Eq. (4.19) although there was no analogous contribution in the case considered in Sec. III. Such terms have been suppressed on the right-hand side of Eq. (4.19) but they will give additional contributions that must be taken into account. Likewise, the $(\alpha')^5$ part of the commutator

$$\begin{aligned}[\delta_{\epsilon_1},\delta_{\epsilon_2^*}]\psi_{\mu}&\sim g_2(\tau,\overline{\tau})\delta^{(0)}\lambda^{16}(\hat{G}^3\epsilon_2^*)_{\mu}+\cdots,\\ &\sim g_2(\epsilon_1\lambda^{15}\hat{G}^4\epsilon_2^*)_{\mu}+\cdots,\end{aligned} \tag{4.20}$$

determines the ψ_{μ} equation of motion and relates g_2 to $f_2^{(14,-14)}$. Lastly, g_3 is constrained by considering

$$\begin{aligned}[\delta_{\epsilon_1^*},\delta_{\epsilon_2}]\psi_{\mu}^{*}&\sim\delta^{(0)}[g_3(\tau,\overline{\tau})\lambda^{16}(\hat{G}^3\epsilon_2)_{\mu}]+\cdots,\\ &\sim(\overline{D}_{-13}g_3)\epsilon_1^*\lambda^*\lambda^{16}(\hat{G}^3\epsilon_2)_{\mu}+\cdots,\end{aligned} \tag{4.21}$$

which determines the ψ_{μ}^{*} equation of motion and relates g_3 and $\tilde{f}_2^{(13,-13)}$. In writing Eqs. (4.20) and (4.21) we have again been symbolic and suppressed the fact that it is essential to include all the terms involving products of $\delta^{(0)}$ with $\delta^{(5)}$ in the commutators, as with Eq. (4.19). The arguments of this subsection demonstrate how closure of the supersymmetry algebra together with a judicious choice of supersymmetry variations of the Lagrangian can completely determine the interactions in $S^{(5)}$.

C. Future directions

It is less clear how things might work for higher derivative terms in the string effective action. The most significant new feature, which follows simply from dimensional analysis, is that terms in Eq. (4.1) that contribute to $S^{(7)}$ can arise from integration over the whole of the superspace. We would not generally expect these terms to be protected. More pragmatically, at this order the Noether procedure escalates in complexity. This is largely because at order p, there are many possible terms $\delta^{(n)}S^{(m)}$ where $n+m=p$, that can mix under supersymmetry.

In the case of $p=7$, for example, $\delta^{(4)}S^{(3)}$ can mix with $\delta^{(7)}S^{(0)}$ and $\delta^{(0)}S^{(7)}$. This kind of mixing certainly complicates the systematics at higher orders. Nevertheless, it could still be the case that the conjectures in Refs. [18,17,21] are correct. At least the terms in Ref. [17] were special in perturbative string theory because of their relation to topological amplitudes, and this could be reflected in the systematics of the Noether construction. Should these conjectures prove true, they would point to some interesting and powerful implications of supersymmetry that would be satisfying to understand more deeply.

Another avenue that would be very fruitful to explore is the generalization of this analysis to compactified supergravity. The simplest example is the nine-dimensional theory with moduli space $SL(2,\mathbb{Z})\backslash SL(2,\mathbb{R})/O(2)\times\mathbb{R}$. This can be viewed as M theory on a two-torus where the $SL(2,\mathbb{Z})$ acts on the complex structure of the torus Ω and $\mathbb{R}$ is its volume V. The expected $\mathcal{R}^4$ term, given in Ref. [24], is of the form $[V^{-1/2}f^{(0,0)}(\Omega,\overline{\Omega})+2\pi^2/3V]\mathcal{R}^4$. New features enter the effective action in this case that are absent at the boundary of moduli space corresponding to ten-dimensional type IIB theory. Notably, the toroidal compactification of the eleven-form of Eq. (1.1) enters the action. An indirect argument given in Ref. [24] relates this by supersymmetry to the $\mathcal{R}^4$ term but it should now be possible to relate these terms directly. It has been suggested that in compactifications to lower dimensions, the appropriate modular functions are those associated with eigenfunctions of the Laplace operator on the U-duality moduli spaces [19]. These are cases that can certainly be analyzed with the tools that we have developed here. It would be extremely interesting to see what happens in low dimensions, where the U-duality group becomes exceptional, and for sufficiently low dimensions, infinite dimensional. These same techniques are also applicable to cases with less supersymmetry. For example, compactifications of M theory on hyperKähler spaces, and toroidal compactifications of the heterotic or type I strings. Undoubtedly, supersymmetry will continue to yield new insights about the nonperturbative structure of string theory and about M theory.

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APPENDIX A: TYPE IIB SUPERGRAVITY REVISITED

1. Some spinor and gamma matrix identities

The spinors that enter into the IIB theory are complex Weyl spinors. The gravitino and dilatino have opposite chiralities and the supersymmetry parameter has the same chirality as the gravitino. The complex conjugate of the product of a pair of spinors is defined by

$$(\lambda_a \rho_b)^* = -\lambda_a^* \rho_b^* . \tag{A1}$$

The conjugate of any spinor is defined by $\overline{\lambda} = \lambda^* \gamma^0$. We will choose our metric to be spacelike and the γ matrices to be real and satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{A2}$$

Nothing that

$$\gamma^0 \gamma^\mu = -(\gamma^\mu)^T \gamma^0, \tag{A3}$$

it follows that two complex chiral spinors of the same chirality, λ_1 and λ_2, satisfy the relations

$$\begin{aligned} \overline{\lambda_1} \gamma^\mu \lambda_2 &= -\overline{\lambda_2^*} \gamma^\mu \lambda_1^* , \\ \overline{\lambda_1} \gamma^{\mu\nu\rho} \lambda_2 &= \overline{\lambda_2^*} \gamma^{\mu\nu\rho} \lambda_1^* , \\ \overline{\lambda_1} \gamma^{\rho_1 \cdots \rho_5} \lambda_2 &= -\overline{\lambda_2^*} \gamma^{\rho_1 \cdots \rho_5} \lambda_1^* , \end{aligned} \tag{A4}$$

while two chiral spinors of opposite chiralities, λ and ϵ, satisfy

$$\begin{aligned} \overline{\lambda} \epsilon &= \overline{\epsilon^*} \lambda^*, \\ \overline{\lambda} \gamma^{\rho_1 \rho_2} \epsilon &= -\overline{\epsilon^*} \gamma^{\rho_1 \rho_2} \lambda^*, \\ \overline{\lambda} \gamma^{\rho_1 \rho_2 \rho_3 \rho_4} \epsilon &= \overline{\epsilon^*} \gamma^{\rho_1 \rho_2 \rho_3 \rho_4} \lambda^*. \end{aligned} \tag{A5}$$

The Fierz identity for ten-dimensional complex Weyl spinors can be expressed as

$$\begin{aligned} \lambda_1^a \overline{\lambda_2^b} = &-\frac{1}{16} \overline{\lambda_2} \gamma_\mu \lambda_1 \gamma_{ab}^\mu + \frac{1}{96} \overline{\lambda_2} \gamma_{\mu\nu\rho} \lambda_1 \gamma_{ab}^{\mu\nu\rho} \\ &- \frac{1}{3840} \overline{\lambda_2} \gamma_{\rho_1 \cdots \rho_5} \lambda_1 \gamma_{ab}^{\rho_1 \cdots \rho_5}, \end{aligned} \tag{A6}$$

where λ_1 and λ_2 are two chiral spinors of the same chirality.

An additional useful identity is

$$\gamma^{\rho_1 \cdots \rho_5} \lambda_1 \overline{\lambda_2} \gamma_{\rho_1 \cdots \rho_5} \lambda_3 = 0, \tag{A7}$$

where λ_1, λ_2, and λ_3 are three chiral spinors of the same chirality.

Some gamma matrix identities that are useful in proving the various relationships in the text are

$$\mathrm{tr}(\gamma_{\mu\nu\rho} \gamma^{\rho_1 \rho_2 \rho_3}) = -16(\delta_\mu^{\rho_1} \delta_\nu^{\rho_2} \delta_\rho^{\rho_3} - \delta_\mu^{\rho_2} \delta_\nu^{\rho_1} \delta_\rho^{\rho_3} + \delta_\mu^{\rho_2} \delta_\nu^{\rho_3} \delta_\rho^{\rho_1} - \delta_\mu^{\rho_3} \delta_\nu^{\rho_2} \delta_\rho^{\rho_1} + \delta_\mu^{\rho_3} \delta_\nu^{\rho_1} \delta_\rho^{\rho_2} - \delta_\mu^{\rho_1} \delta_\nu^{\rho_3} \delta_\rho^{\rho_2}) \gamma^\mu \gamma_\sigma \gamma_\mu = -8 \gamma_\sigma , \tag{A8}$$

$$\begin{aligned} \gamma^\mu \gamma_{\sigma_1 \sigma_2 \sigma_3} \gamma_\mu &= -4 \gamma_{\sigma_1 \sigma_2 \sigma_3}, \\ \gamma^\mu \gamma_{\sigma_1 \cdots \sigma_5} \gamma_\mu &= 0, \\ \gamma^{\mu\nu\rho} \gamma_\sigma \gamma_{\mu\nu\rho} &= -288 \gamma_\sigma , \\ \gamma^{\mu\nu\rho} \gamma_{\sigma_1 \sigma_2 \sigma_3} \gamma_{\mu\nu\rho} &= -48 \gamma_{\sigma_1 \sigma_2 \sigma_3}, \\ \gamma^{\mu\nu\rho} \gamma_{\sigma_1 \cdots \sigma_5} \gamma_{\mu\nu\rho} &= -14 \gamma_{\sigma_1 \cdots \sigma_5}. \end{aligned} \tag{A9}$$

2. The fields and their supersymmetry transformations

Here we will review various features of type IIB supergravity that are useful in the body of the paper. Most of this material can be found in Ref. [7] in a form that is adapted to the field definitions in which the global symmetry is SU(1,1) and the scalar fields parametrize the coset space SU(1,1)/U(1), which is the Poincaré disk. It is simple to transform this to our parametrization in which the global symmetry is SL(2,$\mathbb{R}$) and the scalars parametrize the coset space SL(2,$\mathbb{R}$)/U(1), or the upper half plane.

The theory is then defined in terms of the following fields: the scalar fields can be parametrized by the frame field

$$V \equiv \begin{pmatrix} V_-^1 & V_+^1 \\ V_-^2 & V_+^2 \end{pmatrix} = \frac{1}{\sqrt{-2i\tau_2}} \begin{pmatrix} \bar{\tau} e^{-i\phi} & \tau e^{i\phi} \\ e^{-i\phi} & e^{i\phi} \end{pmatrix}, \quad \text{(A10)}$$

where $V_\pm^\alpha$ ($\alpha=1,2$) is a SL(2,$\mathbb{R}$) matrix that transforms from the left by the global SL(2,$\mathbb{R}$) and from the right by the local U(1). Note that we are using a complex basis for convenience. A general transformation is then written as

$$(V_+^\alpha, V_-^\alpha) \to U_\beta^\alpha (V_+^\beta e^{i\Sigma}, V_-^\beta e^{-i\Sigma}), \quad \text{(A11)}$$

where U is a SL(2,$\mathbb{R}$) matrix and Σ is the U(1) phase. An appropriate choice of Σ fixes the gauge and eliminates the scalar field ϕ. We will make the gauge choice $\phi=0$. Since this gauge is not maintained by generic symmetry transformations, it is necessary to compensate a symmetry transformation with an appropriate local U(1) transformation to maintain the gauge. In particular, the local supersymmetry transformations require compensating local U(1) transformations. The supersymmetry and U(1) transformations of V_-^α are given by

$$\delta^{(0)} V_-^\alpha = i V_+^\alpha \bar{\epsilon} \lambda^* - i \Sigma V_-^\alpha. \quad \text{(A12)}$$

This choice ensures that the gauge $\phi=0$ is maintained if a local supersymmetry transformation is accompanied by a U(1) transformation with parameter

$$\Sigma = \frac{1}{2}(\bar{\epsilon}\lambda^* - \bar{\epsilon}^* \lambda). \quad \text{(A13)}$$

The SL(2,$\mathbb{R}$) singlet expression

$$Q_\mu = -i \epsilon_{\alpha\beta} V_+^\alpha \partial_\mu V_-^\beta, \quad \text{(A14)}$$

is the composite U(1) connection and transforms as $Q \to Q + \partial_\mu \Sigma$ under infinitesimal local U(1) transformations, while the SL(2,$\mathbb{R}$) singlet expression

$$P_\mu = -\epsilon_{\alpha\beta} V_+^\alpha \partial_\mu V_+^\beta, \quad \text{(A15)}$$

transforms with U(1) charge $q_P=2$. In the gauge $\phi=0$, the expression for P_μ takes the simple form

$$P_\mu = \frac{i}{2} \frac{\partial_\mu \tau}{\tau_2}. \quad \text{(A16)}$$

The fermions comprise the complex chiral gravitino, ψ_μ^a, which has U(1) charge $q_\psi = \frac{1}{2}$, and the dilatino λ^a with U(1) charge $q_\lambda = \frac{3}{2}$. These two fields have opposite chiralities. The graviton is a U(1) and SL(2,$\mathbb{R}$) singlet as is the antisymmetric fourth-rank potential $C^{(4)}$ which has a field strength $F_5 = dC^{(4)}$. As is well known, this field strength has an equation of motion that is expressed by the self-duality condition $F_5 = *F_5$, which cannot be obtained from a globally well-defined Lagrangian. For this reason, our considerations are restricted to statements concerning the on-shell properties of the theory where the fields satisfy the equations of motion.

The two antisymmetric second-rank potentials $B_{\mu\nu}$ and $C_{\mu\nu}^{(2)}$ have field strengths F^1 (NS⊗NS) and F^2 (R⊗R) that form an SL(2,$\mathbb{R}$) doublet F^α. It is very natural to package them into the SL(2,$\mathbb{R}$) singlet fields

$$G = -\epsilon_{\alpha\beta} V_+^\alpha F^\beta, \quad G^* = -\epsilon_{\alpha\beta} V_-^\alpha F^\beta, \quad \text{(A17)}$$

which carry U(1) charges $q_G = +1$ and $q_{G^*} = -1$, respectively.

In a fixed U(1) gauge, a global SL(2,$\mathbb{R}$) transformation which acts on τ by

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad \text{(A18)}$$

with $ad - bc = 1$, induces a U(1) transformation on the fields that depends on their charge. Thus, a field Φ with U(1) charge q_Φ transforms as

$$\Phi \to \Phi \left(\frac{c\bar{\tau} + d}{c\tau + d} \right)^{q_\Phi/2}. \quad \text{(A19)}$$

The higher derivative terms of interest to us only repect the SL(2,$\mathbb{Z}$) subgroup of SL(2,$\mathbb{R}$) for which *a,b,c,d* are integers and the continuous U(1) symmetry is broken.

The supersymmetry of the action is naturally described in terms of combinations of bosonic fields and fermion bilinears which are ‘‘supercovariant,’’ which means that they do not contain derivatives of the supersymmetry parameter ϵ in their transformations. These combinations are

$$\begin{aligned} \hat{G}_{\mu\nu\rho} &= G_{\mu\nu\rho} - 3\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\lambda - 6i\bar{\psi}^*_{[\mu}\gamma_\nu \psi_{\rho]}, \\ \hat{P}_\mu &= P_\mu - \bar{\psi}^* \lambda, \\ \hat{F}_{5\mu_1\cdots\mu_5} &= F_{5\mu_1\cdots\mu_5} - 5\bar{\psi}_{[\mu_1}\gamma_{\mu_2\mu_3\mu_4}\psi_{\mu_5]} \\ &\quad - \frac{1}{16}\bar{\lambda}\gamma_{\mu_1\cdots\mu_5}\lambda. \end{aligned} \quad \text{(A20)}$$

We will now present the lowest-order supersymmetry transformations, suitably adapted from those given in Ref. [7] to the SL(2,$\mathbb{R}$) parametrization. From Eqs. (A12) and (A13), it follows that

$$\delta^{(0)}\tau = 2\tau_2\overline{\epsilon^*}\lambda, \quad \delta^{(0)}\bar{\tau} = -2\tau_2\bar{\epsilon}\lambda^*. \tag{A21}$$

It follows from the definition of Q_μ and the transformations of τ and $\bar{\tau}$ that

$$\delta^{(0)}Q_\mu = -\bar{\epsilon}\lambda^* P_\mu + \text{c.c.} \tag{A22}$$

Also, the supersymmetry transformation of the zehnbein is given by

$$\delta^{(0)}e_\mu^m = i(\bar{\epsilon}\gamma^m\psi_\mu + \overline{\epsilon^*}\gamma^m\psi_\mu^*). \tag{A23}$$

The transformation of the dilatino is given, in the fixed U(1) gauge, by

$$\begin{aligned}\delta^{(0)}\lambda &= i\gamma^\mu\epsilon^*\hat{P}_\mu - \frac{1}{24}i\gamma^{\mu\nu\rho}\epsilon\hat{G}_{\mu\nu\rho} + \delta_\Sigma^{(0)}\lambda\\ &= i\gamma^\mu\epsilon^*\hat{P}_\mu + \frac{i}{8}\gamma^{\mu\nu\tau}\epsilon(\bar{\psi}_{[\mu}\gamma_{\nu\tau]}\lambda)\\ &\quad - i\gamma^\mu\epsilon^*(\overline{\psi_\mu^*}\lambda) + \delta_\Sigma^{(0)}\lambda + \cdots,\end{aligned} \tag{A24}$$

where we have only kept the terms that are needed in the body of this paper in the second line. The δ_Σ arises from the compensating U(1) gauge transformation

$$\delta_\Sigma^{(0)}\lambda_a = \frac{3}{2}i\Sigma\lambda_a = \frac{3}{4}i\lambda_a(\bar{\epsilon}\lambda^*) - \frac{3}{4}i\lambda_a(\overline{\epsilon^*}\lambda). \tag{A25}$$

The gravitino transformation is given by

$$\begin{aligned}\delta^{(0)}\psi_\mu &= D_\mu\epsilon + \frac{1}{480}i\gamma^{\rho_1\cdots\rho_5}\gamma_\mu\epsilon\hat{F}_{\rho_1\cdots\rho_5}\\ &\quad + \frac{1}{96}(\gamma_\mu^{\nu\rho\lambda}\hat{G}_{\nu\rho\lambda} - 9\gamma^{\rho\lambda}\hat{G}_{\mu\rho\lambda})\epsilon^*\\ &\quad - \frac{7}{16}\left(\gamma_\rho\lambda\bar{\psi}_\mu\gamma^\rho\epsilon^* - \frac{1}{1680}\gamma_{\rho_1\cdots\rho_5}\lambda\bar{\psi}_\mu\gamma^{\rho_1\cdots\rho_5}\epsilon^*\right)\\ &\quad + \frac{1}{32}i\left[\left(\frac{9}{4}\gamma_\mu\gamma^\rho + 3\gamma^\rho\gamma_\mu\right)\epsilon\bar{\lambda}\gamma_\rho\lambda\right.\\ &\quad - \left(\frac{1}{24}\gamma_\mu\gamma^{\rho_1\rho_2\rho_3} + \frac{1}{6}\gamma^{\rho_1\rho_2\rho_3}\gamma_\mu\right)\epsilon\bar{\lambda}\gamma_{\rho_1\rho_2\rho_3}\lambda\\ &\quad \left. + \frac{1}{960}\gamma_\mu\gamma^{\rho_1\cdots\rho_5}\epsilon\bar{\lambda}\gamma_{\rho_1\cdots\rho_5}\lambda\right] + \delta_\Sigma^{(0)}(\psi_\mu),\end{aligned} \tag{A26}$$

where the compensating U(1) transformation is given by

$$\delta_\Sigma^{(0)}\psi_\mu = \frac{1}{2}i\Sigma = \frac{1}{4}i\psi_\mu(\bar{\epsilon}\lambda^*) - \frac{1}{4}i\psi_\mu(\overline{\epsilon^*}\lambda). \tag{A27}$$

By using Eqs. (A6) and (A26) extensively we may manipulate the variation of $\gamma^\mu\psi_\mu^*$ into the form

$$\begin{aligned}\delta^{(0)}(\gamma^\mu\psi_\mu^*)_a &= -\frac{3}{4}i\lambda_a^*(\bar{\epsilon}\lambda)\\ &\quad + \frac{1}{1920}i(\gamma^{\rho_1\cdots\rho_5}\epsilon^*)_a(\bar{\lambda}\gamma_{\rho_1\cdots\rho_5}\lambda) + \cdots,\end{aligned} \tag{A28}$$

where we have only kept the terms bilinear in λ,λ^*. This implies the relation

$$(\lambda)_a^{15}\delta^{(0)}(\gamma^\mu\psi_\mu^*)_a = -15i\lambda^{16}(\bar{\lambda}\epsilon^*) + \cdots, \tag{A29}$$

which we use in the body of the text.

APPENDIX B: DETERMINATION OF THE COEFFICIENT *c*

To determine the coefficient c in $L_1^{(0)}$, we need to consider how this term mixes with other terms under supersymmetry transformations. We shall, in particular, consider the term in the dilatino transformation (A24),

$$\delta^{(0)}\lambda = i\gamma^\mu\epsilon^* P_\mu, \tag{B1}$$

which transforms $L_1^{(0)}$ into the form $\lambda\lambda^{*2}P_\mu\epsilon^*$.

There are two terms which mix with $L_1^{(0)}$ under this transformation. One of these, $L_2^{(0)}$, arises from the U(1) connection in the kinetic term $\bar{\lambda}\gamma^\mu D_\mu\lambda$,

$$L_2^{(0)} = \frac{3}{2}i\det e\bar{\lambda}\gamma^\mu\lambda Q_\mu. \tag{B2}$$

It follows from the transformation of Q_μ in Eq. (A22) that the relevant transformation of $L_2^{(0)}$ is

$$\delta^{(0)}L_2^{(0)} = -\frac{3}{2}i\det e\bar{\lambda}\gamma^\mu\lambda\bar{\epsilon}\lambda^* P_\mu. \tag{B3}$$

In addition to $L_2^{(0)}$, there is another term in the IIB action that can be deduced from the gravitino equation of motion [Eq. (4.12) of Ref. [7]],

$$L_3^{(0)} = i\det e\bar{\lambda}\gamma^\mu\gamma^\omega\psi_\mu^* P_\omega. \tag{B4}$$

The supersymmetry transformation of the gravitino (A26) gives the variation of $L_3^{(0)}$,

$$\delta^{(0)}L_3^{(0)}=\frac{1}{32}i\,\det e\,\overline{\lambda}\gamma^{\mu}\gamma^{\omega}\left[\left(\frac{9}{4}\gamma_{\mu}\gamma^{\rho}+3\gamma^{\rho}\gamma_{\mu}\right)\epsilon^*\overline{\lambda^*}\gamma_{\rho}\lambda^*\right.$$
$$-\left(\frac{1}{24}\gamma_{\mu}\gamma^{\rho_1\rho_2\rho_3}+\frac{1}{6}\gamma^{\rho_1\rho_2\rho_3}\gamma_{\mu}\right)\epsilon^*\overline{\lambda^*}\gamma_{\rho_1\rho_2\rho_3}\lambda^*+\left.\frac{1}{960}\gamma_{\mu}\gamma^{\rho_1\cdots\rho_5}\epsilon^*\overline{\lambda^*}\gamma_{\rho_1\cdots\rho_5}\lambda^*\right]P_{\omega}$$
$$=\frac{1}{32}i\,\det e\left[12\overline{\lambda}\epsilon^*\overline{\lambda^*}\gamma^{\omega}\lambda^*-\overline{\lambda}\left(\frac{1}{3}\gamma^{\omega}\gamma^{\rho_1\rho_2\rho_3}+\frac{1}{3}\gamma^{\rho_1\rho_2\rho_3}\gamma^{\omega}\right)\epsilon^*\overline{\lambda^*}\gamma_{\rho_1\rho_2\rho_3}\lambda^*\right.$$
$$\left.-\frac{1}{120}\overline{\lambda}\gamma^{\omega}\gamma^{\rho_1\cdots\rho_5}\epsilon^*\overline{\lambda^*}\gamma_{\rho_1\cdots\rho_5}\lambda^*\right]P_{\omega}. \tag{B5}$$

An important simplification occurs when the variations $\delta^{(0)}L_2$ and $\delta^{(0)}L_3$ are added together by adding Eqs. (B3) and (B5). To see this it is first useful to use the fundamental Fierz identity (A6), to write

$$A\equiv\overline{\lambda}\gamma^{\rho}\lambda\overline{\lambda}\epsilon^*=\frac{8}{9}\left[\frac{1}{16}\overline{\lambda}\gamma^{\mu}\gamma^{\omega}\epsilon^*\overline{\lambda}\gamma_{\mu}\lambda\right.$$
$$+\frac{1}{96}\overline{\lambda}\gamma^{\omega}\gamma^{\rho_1\rho_2\rho_3}\epsilon^*\overline{\lambda}\gamma_{\rho_1\rho_2\rho_3}\lambda$$
$$\left.-\frac{1}{240\times 16}\overline{\lambda}\gamma^{\omega}\gamma^{\rho_1\cdots\rho_5}\epsilon^*\overline{\lambda}\gamma_{\rho_1\cdots\rho_5}\lambda\right]P_{\omega}. \tag{B6}$$

The sum $\delta^{(0)}L_2+\delta^{(0)}L_3$ contains the terms $(-\frac{3}{2}+\frac{3}{8})iA=-9i/8A$. Substituting in Eqs. (B3) and (B5) gives

$$\delta^{(0)}L_2+\delta^{(0)}L_3=-\frac{i}{32}\det e\left[2\overline{\lambda}\gamma_{\mu}\gamma^{\omega}\epsilon^*\overline{\lambda}\gamma^{\mu}\lambda\right.$$
$$\left.+\frac{1}{3}\overline{\lambda}\gamma^{\rho_1\rho_2\rho_3}\gamma^{\omega}\epsilon^*\overline{\lambda^*}\gamma_{\rho_1\rho_2\rho_3}\lambda^*\right]P_{\omega}. \tag{B7}$$

This sum of the variations has to cancel the variation of the term $L_1^{(0)}$ using Eq. (3.7). To see this most clearly, it is useful to first manipulate $L_1^{(0)}$ using Eq. (A6) into the form

$$L_1^{(0)}=-\det e\frac{c}{6}\overline{\lambda^*}\gamma^{\mu\nu\rho}\lambda\overline{\lambda}\gamma_{\mu\nu\rho}\lambda^*$$
$$=\det e\frac{4c}{3}\left(\overline{\lambda}\gamma^{\mu}\lambda\overline{\lambda^*}\gamma_{\mu}\lambda^*\right.$$
$$\left.+\frac{1}{6}\overline{\lambda^*}\gamma^{\mu\nu\rho}\lambda^*\overline{\lambda^*}\gamma_{\mu\nu\rho}\lambda^*\right)+\cdots. \tag{B8}$$

Therefore the supersymmetry variation of $L_1^{(0)}$ may be expressed as

$$\delta^{(0)}L_1^{(0)}=\det e\frac{8c}{3}i\left(\overline{\lambda^*}\gamma^{\mu}\lambda^*\overline{\lambda}\gamma_{\mu}\gamma^{\omega}\epsilon^*\right.$$
$$\left.+\frac{1}{6}\overline{\lambda^*}\gamma^{\mu\nu\rho}\lambda^*\overline{\lambda}\gamma_{\mu\nu\rho}\gamma^{\omega}\epsilon^*\right)P_{\omega}+\cdots, \tag{B9}$$

which can be compared directly with Eq. (B7). In order for the sum of Eqs. (B7) and (B9) to vanish the coefficient c must have the value

$$c=-\frac{3}{128}. \tag{B10}$$

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