Gravitational wave radiation from compact binary systems in the Jordan-Brans-Dicke theory

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In this paper we analyze the signal emitted by a compact binary system in the Jordan-Brans-Dicke theory. We compute the scalar and tensor components of the power radiated by the source and study the scalar wave form. Eventually we consider the detectability of the scalar component of the radiation by interferometers and resonant-mass detectors. [S0556-2821(99)01202-3]

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I. INTRODUCTION

The detection of gravitational waves (GWs) is a field of active research from the point of view of both the development of suitable detectors and of the study of possible sources and signal analysis. The detectors now operating as GW observatories are of the resonant-mass type and have a sensitivity to typical millisecond GW bursts of $h \approx 6 \times 10^{-19}$ (h is the wave amplitude) or, in spectral units, 10^{-21} $(\mathrm{Hz})^{-1/2}$ over a bandwidth of a few Hz around 1 kHz [1]. The first bound is appropriate for describing the sensitivity to gravitational collapses while the square of the second bound represents the input GW spectrum that would produce a signal equal to the noise spectrum actually observed at the output of the detector. With this sensitivity it is possible to monitor the strongest potential sources of GWs in our galaxy and in the local group (distances of ≈ 1 Mpc). In order to improve the sensitivity of these instruments, more advanced transducers and amplifiers are under development as well as new resonant-mass detectors of spherical shape. Furthermore a huge effort is under way to build large laser interferometers. It is widely believed that in the near future, sensitivities of the order of 10^{-23} $(\mathrm{Hz})^{-1/2}$ over a bandwidth of several hundred Hz will be attained allowing the observation of GW sources up to distances of the order of 100 Mpc [1]. It thus seems that the detection of GWs is highly probable at the beginning of the new millennium. In addition to information of astrophysical interest, the detection of GWs gives an opportunity to test the content of the theory of gravity. In fact, it has been shown that a single spherical resonant-mass detector [2], or an array of interferometers [3], have the capability to probe the spin content of the incoming GWs.

One of the most intensively studied GW sources is the inspiralling compact binary system [4] made of neutron stars or black holes. In the Newtonian regime, the system has a clean analytic behavior and emits a wave-form of increasing amplitude and frequency that can sweep up to the kHz range of frequencies. In this paper we study the radiation emitted by this source in the framework of the Jordan-Brans-Dicke (JBD) theory. We consider this theory to be of particular interest, since the coupling between the scalar field and the metric has the same form of that of string theory, which is widely believed to give a consistent quantum extension of classical gravity. Our main motivation then comes from the attempt to explore a possible experimental signature of string theory as already discussed in Ref. [2]. Furthermore the results obtained here generalize to any theory with a JBD type coupling between matter and gravitation.

There has been much work in this domain in the past years. Before going to the plan of the paper, we shortly review it. In Ref. [5] binary systems were first proposed as possible sources from which extract more stringent bounds on ω_{BD} [see Eq. (2.1) for its definition] than those obtained from solar system data. An analysis of spherically symmetric collapse of inhomogeneous dust was carried on in Ref. [3] and later confirmed in Ref. [6]. The case of homogeneous dust was treated in Ref. [7]. In Ref. [8] a test particle around a Kerr black hole was studied and results very similar to those of our Sec. V A for interferometers were found. In Refs. [9,10] it was pointed out that deviations from general relativity can be much different in strong and weak gravity. In Ref. [10] these deviations were parametrized in a two-dimensional space and exclusion plots were drawn out of the available data. Finally in Ref. [11] spherical collapses were studied in a formalism which kept in account strong gravity effects.

The plan of the paper is the following. In Sec. II we describe the scalar and tensor GW solutions of the JBD theory. In Sec. III we compute the power emitted in tensor and scalar GWs by a binary system. In Sec. IV we concentrate on the scalar wave form. In Sec. V, we study the interaction between the scalar wave form and two types of earth-based detectors: interferometers and spherical resonant-mass detectors, giving limits for the detectability of the signals coming from typical binary sources. Eventually, in Sec. VI, we draw some conclusions.

II. SCALAR AND TENSOR GWs IN THE JBD THEORY

In the Jordan-Fierz frame, in which the scalar field mixes with the metric but decouples from matter, the action reads [12]

$$\begin{aligned} S &= S_{\mathrm{grav}}[\phi, g_{\mu\nu}] + S_m[\psi_m, g_{\mu\nu}] \\ &= \frac{c^3}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega_{\mathrm{BD}}}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] \\ &\quad + \frac{1}{c}\int d^4x L_m[\psi_m, g_{\mu\nu}], \end{aligned} \tag{2.1}$$

where $\omega_{\rm BD}$ is a dimensionless constant, whose lower bound is fixed to be $\omega_{\rm BD}\approx 600$ by experimental data [13], $g_{\mu\nu}$ is the metric tensor, ϕ is a scalar field, and ψ_m collectively denotes the matter fields of the theory.

As a preliminary analysis, we perform a weak field approximation around the background given by a Minkowskian metric and a constant expectation value for the scalar field

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},$$

$$\varphi=\varphi_0+\xi. \tag{2.2}$$

The standard parametrization $\varphi_0=2(\omega_{\rm BD}+2)/G(2\omega_{\rm BD}+3)$, with G the Newton constant, reproduces GR in the limit $\omega_{\rm BD}\to\infty$, which implies $\varphi_0\to 1/G$. Defining the new field

$$\theta_{\mu\nu}=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h-\eta_{\mu\nu}\frac{\xi}{\varphi_0}, \tag{2.3}$$

where h is the trace of the fluctuation $h_{\mu\nu}$, and choosing the gauge

$$\partial_\mu\theta^{\mu\nu}=0 \tag{2.4}$$

one can write the field equations in the following form:

$$\partial_\alpha\partial^\alpha\theta_{\mu\nu}=-\frac{16\pi}{\varphi_0}\tau_{\mu\nu}, \tag{2.5}$$

$$\partial_\alpha\partial^\alpha\xi=\frac{8\pi}{2\omega_{\rm BD}+3}S, \tag{2.6}$$

where

$$\tau_{\mu\nu}=\frac{1}{\varphi_0}(T_{\mu\nu}+t_{\mu\nu}), \tag{2.7}$$

$$S=-\frac{T}{2(2\omega_{\rm BD}+3)}\left(1-\frac{1}{2}\theta-2\frac{\xi}{\varphi_0}\right)-\frac{1}{16\pi}\left[\frac{1}{2}\partial_\alpha(\theta\partial^\alpha\xi)+\frac{2}{\varphi_0}\partial_a(\xi\partial^\alpha\xi)\right]. \tag{2.8}$$

In Eq. (2.7), $T_{\mu\nu}$ is the matter stress-energy tensor and $t_{\mu\nu}$ is the gravitational stress-energy pseudo-tensor, that is a function of quadratic order in the weak gravitational fields $\theta_{\mu\nu}$ and ξ. The reason why we have written the field equations at the quadratic order in $\theta_{\mu\nu}$ and ξ is that in this way, as we will see later, the expressions for $\theta_{\mu\nu}$ and ξ include all the terms of order $(v/c)^2$, where v is the typical velocity of the source (Newtonian approximation).

Let us now compute τ^{00} and S at the order $(v/c)^2$. Introducing the Newtonian potential U produced by the rest-mass density ρ,

$$U(\vec{x},t)=\int\frac{\rho(\vec{x}\,',t)}{|\vec{x}-\vec{x}\,'|}d^3x', \tag{2.9}$$

the total pressure p and the specific energy density Π (that is the ratio of energy density to rest-mass density) we get (for a more detailed derivation, see Ref. [14])

$$\tau^{00}=\frac{1}{\varphi_0}\rho, \tag{2.10}$$

$$S\simeq-\frac{T}{2(2\omega_{\rm BD}+3)}\left(1-\frac{1}{2}\theta-2\frac{\xi}{\varphi_0}\right)=\frac{\rho}{2(2\omega_{\rm BD}+3)}\left(1+\Pi-3\frac{p}{\rho}+\frac{2\omega_{\rm BD}+1}{\omega_{\rm BD}+2}U\right). \tag{2.11}$$

Far from the source, Eqs. (2.5) and (2.6) admit wavelike solutions, which are superpositions of terms of the form

$$\theta_{\mu\nu}(x)=A_{\mu\nu}(\vec{x},\omega)\exp(ik^\alpha x_\alpha)+\text{c.c.}, \tag{2.12}$$

$$\xi(x)=B(\vec{x},\omega)\exp(ik^\alpha x_\alpha)+\text{c.c.}. \tag{2.13}$$

Without affecting the gauge condition (2.4), one can impose $h=-2\xi/\varphi_0$ (so that $\theta_{\mu\nu}=h_{\mu\nu}$). Gauging away the superfluous components, one can write the amplitude $A_{\mu\nu}$ in terms of the three degrees of freedom corresponding to states with helicities ± 2 and 0 [15]. For a wave traveling in the z direction, one thus obtains

$$A_{\mu\nu}=\begin{pmatrix}0&0&0&0\\0&e_{11}-b&e_{12}&0\\0&e_{12}&-e_{11}-b&0\\0&0&0&0\end{pmatrix}, \tag{2.14}$$

where $b=B/\varphi_0$.

III. POWER EMITTED IN GWs

The power emitted by a source in GWs depends on the stress-energy pseudo-tensor $t^{\mu\nu}$ according to the following expression:

$$P_{\rm GW}=r^2\int\Phi d\Omega=r^2\int\langle t^{0k}\rangle\hat{x}_k d\Omega, \tag{3.1}$$

where r is the radius of a sphere which contains the source, Ω is the solid angle, Φ is the energy flux, and the angular brackets imply an average over a region of size much larger than the wavelength of the GW. At the quadratic order in the weak fields we find

$$\langle t_{0z}\rangle=-\hat{z}\frac{\varphi_0c^4}{32\pi}\left[\frac{4(\omega_{\rm BD}+1)}{\varphi_0^2}\langle(\partial_0\xi)(\partial_0\xi)\rangle+\langle(\partial_0h_{\alpha\beta})(\partial_0h^{\alpha\beta})\rangle\right]. \tag{3.2}$$

Substituting Eqs. (2.12), (2.13) into Eq. (3.2), one gets

$$\langle t_{0z}\rangle = -\hat{z}\frac{\varphi_0 c^4\omega^2}{16\pi}\left[\frac{2(2\omega_{\rm BD}+3)}{\varphi_0^2}|B|^2 + A^{\alpha\beta *}A_{\alpha\beta} - \frac{1}{2}|A^{\alpha}{}_{\alpha}|^2\right], \quad (3.3)$$

and, using Eq. (2.14),

$$\langle t_{0z}\rangle = -\hat{z}\frac{\varphi_0 c^4\omega^2}{8\pi}\,[|e_{11}|^2+|e_{12}|^2+(2\omega_{\rm BD}+3)|b|^2]. \quad (3.4)$$

From Eq. (3.4) we see that the purely scalar contribution, associated to b, and the traceless tensorial contribution, associated to $e_{\mu\nu}$, are completely decoupled and can thus be treated independently.

A. Power emitted in tensor GWs

Equation (2.7) differs from the corresponding tensor field in GR only by a multiplicative factor. Then we can directly write the final result using the well-known expression for the power emitted by a system of binary stars in GR:

$$(P_{\rm ten})_n = \frac{1}{G\varphi_0}(P_{\rm GR})_n\,. \quad (3.5)$$

If we take $\omega_{\rm BD}=600$ in Eq. (3.5), the multiplicative factor $1/G\varphi_0$ differs from one for one part in 10^3.

In (3.5) $(P_{\rm GR})_n$ is the power emitted at frequency $n\omega_0$ (where ω_0 is the orbital frequency) by a system of binary stars according to GR [16,17], averaged over one period of the elliptical motion and calculated in the Newtonian approximation

$$(P_{\rm GR})_n = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2 m_2^2 m}{a^5}g(n;e), \quad (3.6)$$

where m_1 and m_2 are the masses of the two stars, m is the total mass $m=m_1+m_2$, a is the major semiaxis, and e is the eccentrity of the ellipse. The function $g(n;e)$ depends on the Bessel functions $J_k(ne)$:

$$\begin{aligned} g(n;e) = \frac{n^4}{8}\,[&J_n^2(ne)(e^2-2)^2/(n^2e^4)+4J_n^2(ne)(1-e^2)^3/e^4 \\ &+J_n^2(ne)/(3n^2)+4J_n(ne)J_n'(ne)(e^2-2) \\ &\times(1-e^2)/(ne^3)-8J_n(ne)J_n'(ne) \\ &\times(1-e^2)^2/(ne^3)]+4J_n'^2(ne)(1-e^2)^2/e^2 \\ &+4J_n'^2(ne)(1-e^2)/(n^2e^2). \end{aligned} \quad (3.7)$$

In the last phases of the binary system evolution, the orbit becomes more and more circular, because the bodies radiate the most at their closest approach [16]. In the case of null eccentricity $e=0$, the function $g(n;e)$ reduces to a Kronecker delta, $g(n;e=0)=\delta_{n2}$, and the tensor GW frequency is twice the orbital frequency. Summing over all the harmonics n [16], one obtains

$$P_{\rm ten} = \sum_{n=1}^{\infty}(P_{\rm ten})_n = \frac{32}{5}\frac{G^3}{\varphi_0 c^5}\frac{m_1^2 m_2^2 m}{a^5}f(e), \quad (3.8)$$

where

$$f(e) = \frac{1}{(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right). \quad (3.9)$$

Equations (3.6) and (3.8) are obtained in the approximation of pointlike masses (weak self-gravity). For compact binary systems such as PSR 1913 + 16, they can be used upon replacing the masses, m_1, m_2, by the Schwarzschild masses of the stars [18].

B. Power emitted in scalar GWs

We now rewrite the scalar wave solution (2.13) in the following way:

$$\xi(\vec{x},t) = \xi(\vec{x},\omega)e^{-i\omega t} + \text{c.c.} \quad (3.10)$$

In vacuo, the spatial part of the previous solution (3.10) satisfies the Helmholtz equation

$$(\nabla^2+\omega^2)\xi(\vec{x},\omega)=0. \quad (3.11)$$

The solution of Eq. (3.11) can be written as

$$\xi(\vec{x},\omega) = \sum_{jm} X_{jm}h_j^{(1)}(\omega r)Y_{jm}(\theta,\varphi), \quad (3.12)$$

where $h_j^{(1)}(x)$ are the spherical Hankel functions of the first kind, r is the distance of the source from the observer, $Y_{jm}(\theta,\varphi)$ are the scalar spherical harmonics, and the coefficients X_{jm} give the amplitudes of the various multipoles which are present in the scalar radiation field. Solving the inhomogeneous wave equation (2.6), we find

$$X_{jm} = 16\pi i\omega\int_V j_l(\omega r')Y_{lm}^*(\theta,\varphi)S(\vec{x},\omega)dV, \quad (3.13)$$

where $j_l(x)$ are the spherical Bessel functions and r' is a radial coordinate which assumes its values in the volume V occupied by the source.

Substituting Eq. (3.2) in Eq. (3.1), considering the expressions (3.10) and (3.12), and averaging over time, one finally obtains

$$P_{\rm scal} = \frac{(2\omega_{\rm BD}+3)c^4}{8\pi\varphi_0}\sum_{jm}|X_{jm}|^2. \quad (3.14)$$

To compute the power radiated in scalar GWs, one has to determine the coefficients X_{jm}, defined in Eq. (3.13). The detailed calculations can be found in Appendix A, while here we only give the final results. Introducing the reduced mass

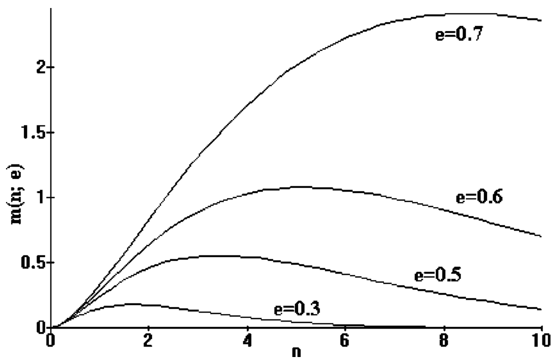

FIG. 1. Monopole function against the index n for different values of the eccentricity e.

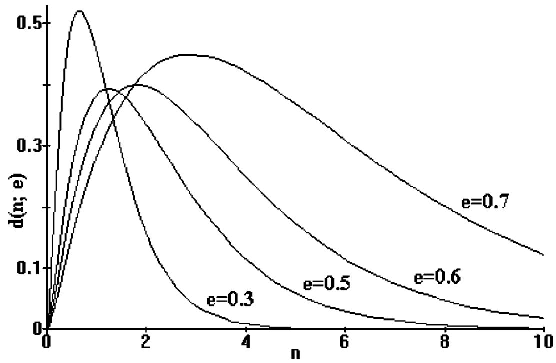

FIG. 2. Dipole function against the index n for different values of the eccentricity e.

of the binary system $\mu = m_1 m_2/m$ and the gravitational self-energy for the body a (with $a=1,2$)

$$\Omega_a = -\frac{1}{2}\int_{V_a} \frac{\rho(\vec{x})\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} d^3x d^3x' \tag{3.15}$$

one can write the Fourier components with frequency $n\omega_0$ in the Newtonian approximation as [see Eqs. (A15), (A23), (A31), (A32)]

$$(X_{00})_n = -\frac{16\sqrt{2\pi}}{3}\frac{i\omega_0\varphi_0}{\omega_{\rm BD}+2}\frac{m\mu}{a} nJ_n(ne), \tag{3.16}$$

$$(X_{1\pm1})_n = -\sqrt{\frac{2\pi}{3}}\frac{2i\omega_0^2\varphi_0}{\omega_{\rm BD}+2}\left(\frac{\Omega_2}{m_2}-\frac{\Omega_1}{m_1}\right)\mu a \times\left[\pm J_n'(ne) - \frac{1}{e}(1-e^2)^{1/2}J_n(ne)\right], \tag{3.17}$$

$$(X_{20})_n = \frac{2}{3}\sqrt{\frac{\pi}{5}}\frac{i\omega_0^3\varphi_0}{\omega_{\rm BD}+2}\mu a^2 nJ_n(ne), \tag{3.18}$$

$$(X_{2\pm2})_n = \mp 2\sqrt{\frac{\pi}{30}}\frac{i\omega_0^3\varphi_0}{\omega_{\rm BD}+2}\mu a^2\frac{1}{n}\{(e^2-2)J_n(ne)/(ne^2) + 2(1-e^2)J_n'(ne)/e \mp 2(1-e^2)^{1/2} \times[(1-e^2)J_n(ne)/e^2 - J_n'(ne)/(ne)]\}. \tag{3.19}$$

Substituting these expressions in Eq. (3.14), leads to the power radiated in scalar GWs in the nth harmonic

$$(P_{\rm scal})_n = P_n^{j=0} + P_n^{j=1} + P_n^{j=2}, \tag{3.20}$$

where the monopole, dipole, and quadrupole terms are, respectively,

$$P_n^{j=0} = \frac{64}{9(\omega_{\rm BD}+2)}\frac{m^3\mu^2G^4}{a^5c^5}n^2J_n^2(ne) = \frac{64}{9(\omega_{\rm BD}+2)}\frac{m^3\mu^2G^4}{a^5c^5}m(n;e), \tag{3.21}$$

$$P_n^{j=1} = \frac{4}{3(\omega_{\rm BD}+2)}\frac{m^2\mu^2G^3}{a^4c^3}\times\left(\frac{\Omega_2}{m_2}-\frac{\Omega_1}{m_1}\right)^2 n^2\left[J_n'^2(ne) + \frac{1}{e^2}(1-e^2)J_n^2(ne)\right] = \frac{4}{3(\omega_{\rm BD}+2)}\frac{m^2\mu^2G^3}{a^4c^3}\left(\frac{\Omega_2}{m_2}-\frac{\Omega_1}{m_1}\right)^2 d(n;e), \tag{3.22}$$

$$P_n^{j=2} = \frac{8}{15(\omega_{\rm BD}+2)}\frac{m^3\mu^2G^4}{a^5c^5}g(n;e). \tag{3.23}$$

In Figs. 1–3 we plot the monopole $m(n;e)$, dipole $d(n;e)$, and quadrupole $g(n;e)$ functions against the index n, for different values of the eccentricity e.

From the figures one can infer the dominant harmonics in the scalar GW radiation. In the case of circular orbit, the dipole function $d(n;e)$ reduces to a Kronecker delta $d(n;e=0)=\delta_{n1}$, while the monopole function $m(n;e)$ goes to zero.

The total power radiated in scalar GWs by a binary system is the sum of three terms

$$P_{\rm scal} = P^{j=0} + P^{j=1} + P^{j=2}, \tag{3.24}$$

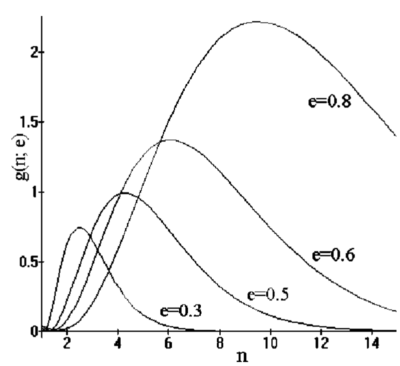

FIG. 3. Quadrupole function against the index n for different values of the eccentricity e.

where

$$P^{j=0}=\frac{16}{9(\omega_{\mathrm{BD}}+2)}\frac{G^4}{c^5}\frac{m_1^2m_2^2m}{a^5}\frac{e^2}{(1-e^2)^{7/2}}\left(1+\frac{e^2}{4}\right), \tag{3.25}$$

$$P^{j=1}=\frac{2}{\omega_{\mathrm{BD}}+2}\left(\frac{\Omega_2}{m_2}-\frac{\Omega_1}{m_1}\right)^2 \times\frac{G^3}{c^3}\frac{m_1^2m_2^2}{a^4}\frac{1}{(1-e^2)^{5/2}}\left(1+\frac{e^2}{2}\right), \tag{3.26}$$

$$P^{j=2}=\frac{8}{15(\omega_{\mathrm{BD}}+2)}\frac{G^4}{c^5}\frac{m_1^2m_2^2m}{a^5} \times\frac{1}{(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right). \tag{3.27}$$

Note that $P^{j=0},P^{j=1},P^{j=2}$ all go to zero in the limit $\omega_{\mathrm{BD}}\to\infty$.

IV. SCALAR GWs

We now give the explicit form of the scalar GWs radiated by a binary system. To this end, note that the major semiaxis a is related to the total energy E of the system through the following equation:

$$a=-\frac{Gm_1m_2}{2E}. \tag{4.1}$$

Let us consider the case of a circular orbit, remembering that in the last phase of evolution of a binary system this condition is usually satisfied. Furthermore we will also assume $m_1=m_2$. With these positions only the quadrupole term (3.23) of the gravitational radiation is different from zero. The total power radiated in GWs, averaged over time, is then given by Eq. (3.8), (3.25)–(3.27)

$$P=\frac{8}{15(\omega_{\mathrm{BD}}+2)}\frac{G^4}{c^5}\frac{m_1^2m_2^2m}{d^5}[6(2\omega_{\mathrm{BD}}+3)+1], \tag{4.2}$$

where d is the relative distance between the two stars. The time variation of d in one orbital period is

$$\dot{d}=-\frac{Gm_1m_2}{2E^2}P. \tag{4.3}$$

Finally, substituting Eqs. (4.1), (4.2) in Eq. (4.3) and integrating over time, one obtains

$$d=2\left(\frac{2}{15}\frac{12\omega_{\mathrm{BD}}+19}{\omega_{\mathrm{BD}}+2}\frac{G^3m_1m_2m}{c^5}\right)^{1/4}\tau^4, \tag{4.4}$$

where we have defined $\tau=t_c-t$, t_c being the time of the collapse between the two bodies.

From Eqs. (3.12), (3.16)–(3.19) and (B3) one can deduce the form of the scalar field (see Appendix B for details) which, for equal masses, is

$$\xi(t)=-\frac{2\mu}{r(2\omega_{\mathrm{BD}}+3)}\left[v^2+\frac{m}{d}-(\hat{n}\cdot\vec{v})^2+\frac{m}{d^3}(\hat{n}\cdot\vec{d})\right], \tag{4.5}$$

where r is the distance of the source from the observer, and $\hat{n}$ is the versor of the line of sight from the observer to the binary system center of mass. Indicating with γ the inclination angle, that is the angle between the orbital plane and the reference plane (defined to be a plane perpendicular to the line of sight), and with ψ the true anomaly, that is the angle between d and the x axis in the orbital plane x-y, yields $\hat{n}\cdot\vec{d}=d\sin\gamma\sin\psi$. Then from Eq. (4.5) one obtains

$$\xi(t)=\frac{2G\mu m}{(2\omega_{\mathrm{BD}}+3)c^4dr}\sin^2\gamma\ \cos[2\psi(t)], \tag{4.6}$$

which can also be written as

$$\xi(\tau)=\xi_0(\tau)\sin[\chi(\tau)+\overline{\chi}], \tag{4.7}$$

where $\overline{\chi}$ is an arbitrary phase and the amplitude $\xi_0(\tau)$ is given by

$$\xi_0(\tau)=\frac{2G\mu m}{(2\omega_{\mathrm{BD}}+3)c^4dr}\sin^2\gamma =\frac{1}{2(2\omega_{\mathrm{BD}}+3)r}\left(\frac{\omega_{\mathrm{BD}}+2}{12\omega_{\mathrm{BD}}+19}\right)^{1/4}\left(\frac{15G}{2c^{11}}\right)^{1/4}\frac{M_c^{5/4}}{\tau^{1/4}}\sin^2\gamma. \tag{4.8}$$

In the last expression, we have introduced the definition of the chirp mass $M_c=(m_1m_2)^{3/5}/m^{1/5}$.

V. DETECTABILITY OF THE SCALAR GWs

Let us now study the interaction of the scalar GWs with two types of GW detectors.

As usual, we characterize the sensitivity of the detector by the spectral density of strain $S_h(f)[\mathrm{Hz}]^{-1}$. The optimum performance of a detector is obtained by filtering the output with a filter matched to the signal. The energy signal-to-noise ratio SNR of the filter output is given by the well-known formula:

$$\mathrm{SNR}=\int_{-\infty}^{+\infty}\frac{|H(f)|^2}{S_h(f)}df, \tag{5.1}$$

where, in our case, $H(f)$ is the Fourier transform of the scalar gravitational wave form $h_s(t)=G\xi_0(t)$.

We must now take into account the astrophysical restrictions on the validity of the wave form (4.7) which is obtained in the Newtonian approximation for pointlike masses. In the following, we will take the point of view that this approximation breaks down when there are five cycles remaining to collapse [24,25].

The five-cycles limit will be used to restrict the range of M_c over which our analysis will be performed. From Eq. (4.4), one can obtain

$$\omega_g(\tau)=2\omega_0=2\sqrt{\frac{Gm}{d^3}}$$
$$=2\left(\frac{15c^5}{64G^{5/3}}\right)^{3/8}\left(\frac{\omega_{BD}+2}{12\omega_{BD}+19}\right)^{3/8}\frac{1}{M_c^{5/8}}\tau^{3/8}. \tag{5.2}$$

Integrating Eq. (5.2) yields the amount of phase until coalescence

$$\chi(\tau)=\frac{16}{5}\left(\frac{15c^5}{64G^{5/3}}\right)^{3/8}\left(\frac{\omega_{BD}+2}{12\omega_{BD}+19}\right)^{3/8}\left(\frac{\tau}{M_c}\right)^{5/8}. \tag{5.3}$$

Setting Eq. (5.3) equal to the limit period, $T_{5\text{ cycles}}=5(2\pi)$, solving for τ, and using Eq. (5.2) leads to

$$\omega_{5\text{ cycles}}=2\pi(6870\ \text{Hz})\left(\frac{\omega_{BD}+2}{12\omega_{BD}+19}\right)^{3/5}\frac{M_\odot}{M_c}. \tag{5.4}$$

Taking $\omega_{BD}=600$, the previous limit reads

$$\omega_{5\text{ cycles}}=2\pi(1547\ \text{Hz})\frac{M_\odot}{M_c}. \tag{5.5}$$

A. Interferometers

An interferometric detector measures the relative displacements between the mirrored faces of test masses arranged in the L-shaped configuration of a Michelson interferometer. The directivity antenna pattern for a tensorial wave is such that the maximum detector output is obtained for a wave inpinging perpendicularly with respect to the plane defined by the interferometer arms. On the contrary a scalar JBD wave (which is also transverse) inpinging in the same direction will give a null effect. In the case of a scalar wave, the maximum effect will be obtained for a wave propagating along one interferometer arm. Assuming such a direction and setting $\sin\gamma=1$ in Eq. (4.8), we can, for instance, evaluate the SNR for the VIRGO interferometer, presently under construction. We use for $S_h(f)$ the VIRGO noise spectrum as modeled in Ref. [19], which is the sum of three main components: thermal noise in the pendola, thermal noise in the mirrors, shot noise at high frequency:

$$S_h(f)=10^{-47}\left[\alpha_1\left(\frac{f}{100\ \text{Hz}}\right)^{-5}+\alpha_2\left(\frac{f}{100\ \text{Hz}}\right)^{-1}+\alpha_3\left(\frac{f}{100\ \text{Hz}}\right)^{2}\right]\text{Hz}^{-1}, \tag{5.6}$$

where $\alpha_1=2.0$, $\alpha_2=91.8$, and $\alpha_3=1.23$.

Integrating Eq. (5.1) over the range 10–500 Hz, one obtains the following SNR:

$$\text{SNR}=7.7\times10^4\left(\frac{r}{\text{Mpc}}\right)^{-2}\left(\frac{M_c}{M_\odot}\right)^{5/3}\times\left(\frac{\omega_{BD}+2}{12\omega_{BD}+19}\right)^{1/2}\left[\frac{1}{2(2\omega_{BD}+3)}\right]^2. \tag{5.7}$$

For $\omega_{BD}=600$, we find

$$\text{SNR}=3.8\times10^{-3}\left(\frac{r}{\text{Mpc}}\right)^{-2}\left(\frac{M_c}{M_\odot}\right)^{5/3}. \tag{5.8}$$

The inspiralling of two neutron stars of 1.4 solar masses each will then give SNR $=1$ at a source distance $r\simeq70$ kpc. The inspiralling of two black holes of 10 solar masses each will give SNR$=1$ at a distance $r\simeq300$ kpc. To get this last limit we have integrated Eq. (5.1) up to $\omega_{5\text{ cycles}}=2\pi(178\ \text{Hz})$.

B. Spherical detectors

A GW excites those vibrational modes of a resonant body having the proper simmetry. Most people consider the next generation of resonant-mass detectors will be of spherical shape. In the framework of the JBD theory the spheroidal modes with $l=2$ and $l=0$ are sensitive to the incoming GWs [2,20]. Thanks to its multimode nature, a single sphere is capable of detecting GWs from all directions and polarizations. We evaluate the SNR of a resonant-mass detector of spherical shape for its quadrupole mode with $m=0$ and its monopole mode. In a resonant-mass detector, $S_h(f)$ is a resonant curve and can be characterized by its value at resonance $S_h(f_n)$ and by its half height width [21]. $S_h(f_n)$ can thus be written as

$$S_h(f_n)=\frac{G}{c^3}\frac{4kT}{\sigma_n Q_n f_n}. \tag{5.9}$$

Here σ_n is the cross section associated with the nth resonant mode, T is the thermodynamic temperature of the detector, and Q_n is the quality factor of the mode.

The half height width of $S_h(f)$ gives the bandwidth of the resonant mode

$$\Delta f_n=\frac{f_n}{Q_n}\Gamma_n^{-1/2}. \tag{5.10}$$

Here, Γ_n is the ratio of the wideband noise in the nth resonance bandwidth to the narrowband noise.

From the resonant-mass detector viewpoint, the chirp signal can be treated as a transient GW, depositing energy in a time-scale short with respect to the detector damping time. We can then consider constant the Fourier transform of the wave form within the band of the detector and write [21]

$$\text{SNR}=\frac{2\pi\Delta f_n|H(f_n)|^2}{S_h(f_n)}. \tag{5.11}$$

The cross sections associated to the vibrational modes with $l=0$ and $l=2$, $m=0$ are, respectively [22],

$$\sigma_{(n0)}=H_n\frac{GMv_s^2}{c^3(\omega_{\rm BD}+2)}, \tag{5.12}$$

$$\sigma_{(n2)}=\frac{F_n}{6}\frac{GMv_s^2}{c^3(\omega_{\rm BD}+2)}. \tag{5.13}$$

All parameters entering the previous equation refer to the detector: M is its mass, v_s the sound velocity of the material and the constants H_n and F_n are given in Ref. [22]. The signal-to-noise ratio can be calculated analytically by approximating the wave form with a truncated Taylor expansion around $t=0$, where $\omega_g(t=0)=\omega_{nl}$ [23,24]

$$h_s(t)\approx G\xi_0(t=0)\sin\left[\omega_{nl}t+\frac{1}{2}\left(\frac{d\omega}{dt}\right)_{t=0}t^2\right]. \tag{5.14}$$

Using quantum limited readout systems, one finally obtains

$$({\rm SNR}_n)_{l=0}=\frac{5\times 2^{1/3}H_nG^{5/3}}{32(\omega_{\rm BD}+2)(12\omega_{\rm BD}+19)\hbar c^3}\frac{{M_c}^{5/3}Mv_s^2}{r^2{\omega_{n0}}^{4/3}}\sin^4\gamma \tag{5.15}$$

$$({\rm SNR}_n)_{l=2}=\frac{5\times 2^{1/3}F_nG^{5/3}}{192(\omega_{\rm BD}+2)(12\omega_{\rm BD}+19)\hbar c^3}\times\frac{{M_c}^{5/3}Mv_s^2}{r^2{\omega_{n0}}^{4/3}}\sin^4\gamma, \tag{5.16}$$

which are respectively the signal-to-noise ratio for the modes with $l=0$ and $l=2$, $m=0$ of a spherical detector.

It has been proposed to realize spherical detectors with $3m$ diameter, made of copper alloys, with mass of the order of 100 tons [26]. This proposed detector has resonant frequencies of $\omega_{12}=2\pi\times 807$ rad/s and $\omega_{10}=2\pi\times 1655$ rad/s. In the case of optimally oriented orbits (inclination angle $\gamma=\pi/2$) and $\omega_{\rm BD}=600$, the inspiralling of two compact objects of 1.4 solar masses each will then be detected with SNR $=1$ up to a source distance $r(\omega_{10})\simeq 30$ kpc and $r(\omega_{12})\simeq 30$ kpc.

VI. SUMMARY AND CONCLUSIONS

In this paper we have studied the wave forms emitted by a system of binary stars in the framework of the JBD theory and computed the power emitted in GWs for the tensor and scalar components. Eventually we derived limits for the detectability of such signals by interferometers and resonant mass detectors. In the former case we left aside the question of the detectability and discrimination of the scalar component of the GW [3] and we have concentrated on waves impinging from the most favourable direction. The detectability ranges obtained in Secs. V A and V B for the scalar component of the GWs emitted by a binary system, vary from few tens to few hundreds kpc for masses ranging from those of typical neutron stars ($1.4M_\odot$) to those of typical black holes ($10M_\odot$). We remind the reader that for the purely tensorial component (in this case the results obtained in the framework of the JBD theory are practically the same of those of GR) the detectability range (for $1.4M_\odot$) is $r\simeq 120$ Mpc for spherical detectors [25] and $r\simeq 300$ Mpc for interferometers [19]. The expected rate of coalescence events is of the order of 1 per year up to 100 Mpc [27]. We can thus conclude that binary systems look less promising than gravitational collapses [22] as sources of detectable scalar GWs from the next generation of earth-based detectors.

APPENDIX A:

In order to calculate the coefficients X_{jm} defined in Eq. (3.13), let us first express X_{jm} as a sum of Fourier components

$$X_{jm}(t)=\sum_{n=-\infty}^{+\infty}(X_{jm})_ne^{in\beta}, \tag{A1}$$

where β is the mean anomaly which, in terms of the orbital frequency ω_0 and of the time of periastron passage T_0 (or equivalently in terms of the eccentric anomaly α and the eccentricity), results

$$\beta=\omega_0(t-T_0)=\alpha-e\sin\alpha. \tag{A2}$$

In the so-called quadrupole approximation $y=\omega d/c\ll 1$, the spherical Bessel functions $j_l(y)$, which appear in Eq. (3.13), can be written as

$$j_l(y)=\sum_{k=0}^{\infty}\frac{(-1)^ky^{l+2k}}{2^kk!(2l+2k+1)!!}. \tag{A3}$$

Making use of the Newtonian approximation [including only two terms of the series (A3)], from (2.11) and (3.13) one can obtain X_{00} as

$$X_{00}=\frac{4\pi i\omega\varphi_0}{\omega_{\rm BD}+2}\int_V\left(1-\frac{(\omega r')^2}{6}\right)\times Y_{00}\rho\left(1+\Pi-3\frac{p}{\rho}+\frac{2\omega_{\rm BD}+1}{\omega_{\rm BD}+2}U\right)dV. \tag{A4}$$

To simplify Eq. (A4) we use the post-Newtonian expressions of the conserved quantities [14]

$$P^0=\int\rho\left(1+v^2+\frac{5\omega_{\rm BD}+4}{2(\omega_{\rm BD}+2)}U+\Pi\right)d\vec{x}, \tag{A5}$$

$$P^i=\int\rho\left(1+v^2+\frac{5\omega_{\rm BD}+4}{2(\omega_{\rm BD}+2)}U+\Pi+\frac{p}{\rho}\right)v^id\vec{x}-\frac{1}{2}\int\rho\left(1+\frac{v^2}{2}+\frac{3(\omega_{\rm BD}+1)}{\omega_{\rm BD}+2}U\right)W^id\vec{x}, \tag{A6}$$

where

$$W^i=\int \rho' \frac{[\vec{v}'\cdot|\vec{x}-\vec{x}'|](x-x')_i}{|\vec{x}-\vec{x}'|^3}d\vec{x}'. \qquad \text{(A7)}$$

To the required accuracy and modulo constants, one then obtains

$$X_{00}=-\frac{\sqrt{4\pi}i\omega\varphi_0\mu}{\omega_{\rm BD}+2}\left(v^2+\frac{(\omega d)^2}{6}+\frac{m}{r}\right). \qquad \text{(A8)}$$

In terms of the eccentric anomaly α, the above expression reads

$$X_{00}(t)=-\frac{\sqrt{4\pi}i\omega\varphi_0\mu m}{a(\omega_{\rm BD}+2)}G_{j=0}, \qquad \text{(A9)}$$

where

$$G_{j=0}=\frac{1+e\cos\alpha}{1-e\cos\alpha}+\frac{n^2}{6}(1-e\cos\alpha)^2+\frac{1}{1-e\cos\alpha}. \qquad \text{(A10)}$$

Let us express $G_{j=0}$ as a sum of Fourier components

$$G_{j=0}=\sum_{n=0}^{\infty}A_n\cos(n\beta), \qquad \text{(A11)}$$

where β is the mean anomaly defined in Eq. (A2). Using the integral expression of the Bessel functions

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\alpha-x\sin\alpha)d\alpha \qquad \text{(A12)}$$

and the recursion formula

$$J_{n+1}(x)+J_{n-1}(x)=\frac{2n}{x}J_n(x) \qquad \text{(A13)}$$

results in

$$A_n=\frac{2}{\pi}\int_0^{\pi}G_{j=0}\cos(n\beta)d\beta=\frac{16}{3}J_n(ne). \qquad \text{(A14)}$$

Then one finally has

$$(X_{00})_n=-\frac{16\sqrt{2\pi}}{3}\frac{i\omega_0\varphi_0}{\omega_{\rm BD}+2}\frac{m\mu}{a}nJ_n(ne). \qquad \text{(A15)}$$

Choosing the orbital plane as the x-y plane, yields $X_{10}=0$ and

$$X_{1\pm1}=\frac{4\pi i\omega\varphi_0}{\omega_{\rm BD}+2}\int_V\frac{\omega r'}{3}Y^*_{1\pm1}(\theta,\phi)\rho \times\left(1+\Pi-3\frac{p}{\rho}+\frac{2\omega_{\rm BD}+1}{\omega_{\rm BD}+2}U\right). \qquad \text{(A16)}$$

Defining $\mathcal{G}=\Omega_2/m_2-\Omega_1/m_1$, one then obtains to the required order

$$X_{1\pm1}=\frac{4\pi i\omega^2\varphi_0\mu\mathcal{G}}{\sqrt{6\pi}(\omega_{\rm BD}+2)}d(\cos\psi\mp i\sin\psi), \qquad \text{(A17)}$$

where ψ is the true anomaly. In terms of the eccentric anomaly α, Eq. (A17) results in

$$X_{1\pm1}=-\sqrt{\frac{2\pi}{3}}\frac{2i\omega^2\varphi_0\mathcal{G}\mu a}{\omega_{\rm BD}+2}G_{j=1}, \qquad \text{(A18)}$$

where

$$G_{j=1}=\pm(\cos\alpha-e)-i(1-e^2)^{1/2}\sin\alpha. \qquad \text{(A19)}$$

The binary system center of mass calculated with respect to the gravitational self-energies Ω_a does not coincide with the center of mass with respect to the inertial masses m_a of the two bodies, if the masses are different (Nordtvedt effect): the resulting dipole moment is, as we have seen, a source of scalar radiation. If we express $G_{j=1}$ as a sum of Fourier components

$$G_{j=1}=B_0+\sum_{n=1}^{\infty}[B_n\cos(n\beta)+C_n\sin(n\beta)], \qquad \text{(A20)}$$

we obtain

$$B_n=\pm\frac{2}{n}J_n'(ne) \qquad \text{(A21)}$$

(where the prime indicates derivative with respect to the argument ne) and

$$C_n=\frac{2}{\pi}\int_0^{\pi}G_{j=1}\sin(n\beta)d\beta=-\frac{2i}{ne}(1-e^2)^{1/2}J_n(ne). \qquad \text{(A22)}$$

Then the nth component of the coefficient $X_{1\pm1}$ is

$$(X_{1\pm1})_n=-\sqrt{\frac{2\pi}{3}}\frac{2i\omega_0^2\varphi_0}{\omega_{\rm BD}+2}\mathcal{G}\mu a \times\left[\pm J_n'(ne)-\frac{1}{e}(1-e^2)^{1/2}J_n(ne)\right]. \qquad \text{(A23)}$$

Finally, in the case $j=2$ one obtains $X_{2\pm1}=0$ and

$$X_{20}=\frac{4\pi i\omega^3\varphi_0}{15(\omega_{\rm BD}+2)}\int_V r'^2Y^*_{20}(\theta,\phi)\rho dV, \qquad \text{(A24)}$$

$$X_{2\pm2}=\frac{4\pi i\omega^3\varphi_0}{15(\omega_{\rm BD}+2)}\int_V r'^2Y^*_{2\pm2}(\theta,\phi)\rho dV, \qquad \text{(A25)}$$

and in terms of α, using the Newtonian approximation,

$$X_{20}(t)=\sqrt{\frac{\pi}{5}}\frac{i\omega^3\varphi_0\mu a^2}{3(\omega_{\rm BD}+2)}(1-\cos\alpha)^2, \qquad \text{(A26)}$$

$$X_{2\pm2}=\mp\sqrt{\frac{\pi}{30}}\frac{i\omega^3\varphi_0\mu a^2}{(\omega_{\rm BD}+2)}G_{j=2}, \qquad \text{(A27)}$$

where

$$G_{j=2}=D_0+\sum_{n=1}^{\infty}[D_n\cos(n\beta)+E_n\sin(n\beta)]. \quad (A28)$$

Calculating D_n and E_n

$$D_n=-\frac{4}{n}\left[\frac{1}{ne^2}(2-e^2)J_n(ne)+\frac{2}{e}(e^2-1)J_n'(ne)\right], \quad (A29)$$

$$E_n=\mp\frac{8i}{n}\left[\frac{1-e^2}{e^2}J_n(ne)-\frac{1}{ne}J_n'(ne)\right], \quad (A30)$$

the nth components result in

$$(X_{20})_n=\frac{2}{3}\sqrt{\frac{\pi}{5}}\frac{i\omega_0^3\varphi_0}{\omega_{\rm BD}+2}\mu a^2 nJ_n(ne), \quad (A31)$$

$$(X_{2\pm2})_n=\mp2\sqrt{\frac{\pi}{30}}\frac{i\omega_0^3\varphi_0}{\omega_{\rm BD}+2}\mu a^2\frac{1}{n}\{(e^2-2)J_n(ne)/(ne^2)+2(1-e^2)J_n'(ne)/e\mp2(1-e^2)^{1/2}\times[(1-e^2)J_n(ne)/e^2-J_n'(ne)/(ne)]\}. \quad (A32)$$

APPENDIX B:

We want to determine explicitly the form of the scalar GWs radiated by a binary system. From Eqs. (3.12) and (A8), (A17), (A24), (A25), and taking into account that in the limit $r\to\infty$ the spherical Hankel functions become

$$h_l^{(1)}(\omega r)\sim\frac{e^{i[\omega r-(l+1)\pi/2]}}{\omega r}, \quad (B1)$$

one can easily obtain

$$\xi(\vec{x},\omega)=-\frac{2\mu}{(2\omega_{\rm BD}+3)Gr}e^{i\omega r}\{v^2+m/d+(\omega d)^2/6+2i\omega(\Omega_2/m_2-\Omega_1/m_1)\hat{n}\cdot\vec{d}-\omega^2d^2(3n_z^2-1)/12+\omega^2[(\hat{n}\cdot\vec{d})^2-(n_xd_y-n_yd_x)^2]/4\}_\omega, \quad (B2)$$

where the subscript ω indicates that all the quantities in the right member of the above expression (B2) are to be considered as Fourier components with frequency ω [for example, $v\to v(\omega)$].

The time dependent amplitude is [14]

$$\xi(\vec{x},t)=\xi(\vec{x},\omega)e^{-i\omega t}+\text{c.c.}=-\frac{2\mu}{(2\omega_{\rm BD}+3)Gr}\times\left[v^2+\frac{m}{d}-\frac{1}{6}\frac{d^2}{dt^2}(d_kd^k)\left(1-\frac{3n_z^2-1}{2}\right)-2\mathcal{G}\hat{n}\cdot\vec{v}-\frac{d^2}{dt^2}(\hat{n}\cdot\vec{d})^2-\frac{(n_xd_y-n_yd_x)^2}{4}\right]=-\frac{2\mu}{(2\omega_{\rm BD}+3)Gr}\left[v^2+\frac{m}{d}-2\mathcal{G}\hat{n}\cdot\vec{v}-(\hat{n}\cdot\vec{v})^2+\frac{m}{d^3}\hat{n}\cdot\vec{d}\right], \quad (B3)$$

where we have set $v=v(\omega)e^{-i\omega(t-r)}+\text{c.c.}$, etc.

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