Quantum evolution of Schwarzschild–de Sitter (Nariai) black holes

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We calculate the one-loop effective action for conformal matter (scalars, spinors, and vectors) on spherically symmetric background. Such an effective action (in the large N approximation and expansion on curvature) is used to study quantum aspects of Schwarzschild–de Sitter (SdS) black holes (BHs) in the nearly degenerated limit (Nariai BH). We show that for all types of the above matter SdS BHs may evaporate or antievaporate in accordance with a recent observation by Bousso and Hawking for minimal scalars. Some remarks about the energy flow for SdS BHs in the regime of evaporation or antievaporation are also made. The study of the no boundary condition shows that this condition supports antievaporation for nucleated BHs (at least in the frame of our approximation). That indicates the possibility that some pair created cosmological BHs may not only evaporate but also antievaporate. Hence cosmological primordial BHs may survive much longer than expected. [S0556-2821(99)03602-4]

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I. INTRODUCTION

In the absence of consistent quantum gravity, the natural way to take into account quantum effects in the early Universe or in black holes (BHs) is to consider matter quantum field theory [say, some grand unified theory (GUT)] in a curved background. The study of quantum GUTs in curved space-time (see [1] for a review) shows the existence of a beautiful phenomenon: asymptotic conformal invariance (see [2,1] for a review). According to it, there exists a large class of asymptotically free GUTs that tend to conformally invariant free theory at high curvature or at high energies (i.e., in the vicinity of BHs or in the early Universe). Hence, for the above background one can describe GUT as the collection of free conformal fields. If one knows the effective action of such a system one can apply it to the investigation of the quantum evolution of strongly gravitating objects.

In recent work [3] the quantum evolution of Schwarzschild–de Sitter (Nariai) BHs has been studied for Einstein gravity with N minimal quantum scalars. The large N and s-wave approximation has been used in such an investigation. The possibility of quantum antievaporation of such BHs (in addition to well-known evaporation process [4]) has been discovered. In [5] another model (of quantum conformal scalars with Einstein gravity) has been considered in a better approach to the effective action (the large N approximation, partial expansion on curvature, and partial s-wave reduction). The possibility of Schwarzschild–de Sitter (SdS) BH antievaporation has been confirmed as well in the model of Ref. [5].

Having in mind the above remarks on the representation of some GUT in the vicinity of BHs as a collection of free conformal fields, we continue to study the quantum dynamics of SdS BHs. We start from Einstein gravity with quantum conformal matter (N scalars, N_1 vectors, and $N_{1/2}$ fermions). Working in the large N approximation (where only matter quantum effects are dominant) we also use the partial derivative expansion of effective action (EA) (without s-wave reduction). As a main qualitative result we find that extreme SdS (Nariai) BHs may indeed evaporate as well as antievaporate. We also try to answer the question: Can the no-boundary Hartle-Hawking condition be consistent with antievaporation? This question may be really important for the estimation of primordial BH creation (see [6] and references therein) (and their existence in the present Universe) as SdS BHs actually may appear through such a process.

II. EFFECTIVE ACTION FOR CONFORMAL MATTER

We first derive the effective action for conformally invariant matter (for a general review of effective action in curved space see [1]). Let us start from Einstein gravity with N conformal scalars χ_i, N_1 vectors A_μ, and $N_{1/2}$ Dirac spinors ψ_i:

$$S=-\frac{1}{16\pi G}\int d^4x\sqrt{-g_{(4)}}\{R^{(4)}-2\Lambda\}+\int d^4x\sqrt{-g_{(4)}}\times\left\{\frac{1}{2}\sum_{i=1}^{N}\left(g_{(4)}^{\alpha\beta}\partial_\alpha\chi_i\partial_\beta\chi_i+\frac{1}{6}R^{(4)}\chi_i^2\right)-\frac{1}{4}\sum_{j=1}^{N_1}F_{j\mu\nu}F_j^{\mu\nu}+\sum_{k=1}^{N_{1/2}}\bar{\psi}_k D\!\!\!\!/\,\psi_k\right\}. \tag{1}$$

The convenient choice for the spherically symmetric space-time is

$$ds^2=f(\phi)[f^{-1}(\phi)g_{\mu\nu}dx^\mu dx^\nu+r_0^2d\Omega], \tag{2}$$

where $\mu,\nu=0,1$, $g_{\mu\nu}$ and $f(\phi)$ depend only on x^0 and x^1, and r_0^2 is a constant.

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Let us start the calculation of the effective action due to conformal matter on the background (2). In the calculation of the effective action, we present the effective action as $\Gamma = \Gamma_{ind} + \Gamma[1, g_{\mu\nu}^{(4)}]$, where $\Gamma_{ind} = \Gamma[f, g_{\mu\nu}^{(4)}] - \Gamma[1, g_{\mu\nu}^{(4)}]$ is the conformal anomaly induced action, which is quite well known [7] and $g_{\mu\nu}^{(4)}$ is the metric (2) without a multiplier in front of it, i.e., $g_{\mu\nu}^{(4)}$ corresponds to

$$ds^2 = [\tilde{g}_{\mu\nu} dx^\mu dx^\nu + r_0^2 d\Omega], \quad \tilde{g}_{\mu\nu} \equiv f^{-1}(\phi) g_{\mu\nu}. \tag{3}$$

The conformal anomaly for the above matter is well known:

$$T = b\left(F + \frac{2}{3}\Box R\right) + b' G + b'' \Box R, \tag{4}$$

where $b = (N + 6N_{1/2} + 12N_1)/120(4\pi)^2$, $b' = -(N + 11N_{1/2} + 62N_1)/360(4\pi)^2$, and $b'' = 0$, but, in principle, b'' may be changed by the finite renormalization of the local counterterm in the gravitational effective action, F is the square of the Weyl tensor, and G is Gauss-Bonnet invariant.

The conformal anomaly induced effective action Γ_{ind} may be written as [7]:

$$\begin{aligned} W = {} & b\int d^4x \sqrt{-g} F\sigma + b' \int d^4x \sqrt{-g} \\ & \times \left\{ \sigma\left[2\Box^2 + 4R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{4}{3}R\Box + \frac{2}{3}(\nabla^\mu R)\nabla_\mu\right]\sigma \right. \\ & \left. + \left(G - \frac{2}{3}\Box R\right)\sigma\right\} - \frac{1}{12}\left(b'' + \frac{2}{3}(b + b')\right) \\ & \times \int d^4x \sqrt{-g}[R - 6\Box\sigma - 6(\nabla\sigma)(\nabla\sigma)]^2, \end{aligned} \tag{5}$$

where $\sigma = \frac{1}{2}\ln f(\phi)$ and σ-independent terms are dropped. All four-dimensional quantities (curvatures and covariant derivatives) in Eq. (5) should be calculated on the metric (3). [We did not write the subscript (4) for them.] Note that after calculation of Eq. (5) on the metric (3), we will get effectively two-dimensional gravitational theory.

In the next step we calculate $\Gamma[1, g_{\mu\nu}^{(4)}]$. This term corresponds to the conformally invariant part of effective action. In this calculation we may apply a Schwinger–DeWitt (SDW) type of expansion of effective action with ζ regularization [8] (or other ultraviolet regularization). This expansion represents the expansion on powers of curvature invariants. Note that we add such EA to the Einstein gravity action. Hence the first two terms of the SDW expansion (the cosmological and linear curvature terms) may be dropped as they only lead to finite renormalization of the Hilbert-Einstein action (redefinition of cosmological and gravitational coupling constants). Then the leading (curvature quadratic term of this expansion) may be read (see [1])

$$\begin{aligned} \Gamma[1, g_{\mu\nu}^{(4)}] = {} & \int d^4x \sqrt{-g}\left\{\left[bF + b'G + \frac{2b}{3}\Box R\right]\ln\frac{R}{\mu^2}\right\} \\ & + O(R^3), \end{aligned} \tag{6}$$

where μ is mass-dimensional constant parameter and all the quantities are calculated on the background (3). The condition of the application of the above expansion is $|R| < R^2$ (the curvature is nearly constant). In this case we may be limited to only the first few terms.

III. QUANTUM DYNAMICS ON SPHERICAL BACKGROUND

We now solve the equations of motion obtained from the above effective Lagrangians $S + \Gamma$. In the following we use $\tilde{g}_{\mu\nu}$ and σ as a set of independent variables and we write $\tilde{g}_{\mu\nu}$ as $g_{\mu\nu}$ if there is no confusion.

Γ_{ind} [W in Eq. (5)] is rewritten after the reduction to two dimensions as

$$\begin{aligned} \frac{\Gamma_{ind}}{4\pi} = {} & \frac{br_0^2}{3}\int d^2x \sqrt{-g}\left((R^{(2)} + R_\Omega)^2 + \frac{2}{3}R_\Omega R^{(2)} + \frac{1}{3}R_\Omega^2\right)\sigma + b' r_0^2 \int d^2x \sqrt{-g}\left\{\sigma\left(2\Box^2 + 4R^{(2)\mu\nu}\nabla_\mu\nabla_\nu - \frac{4}{3}(R^{(2)} + R_\Omega)\Box\right.\right. \\ & \left.\left. + \frac{2}{3}(\nabla^\mu R^{(2)})\nabla_\mu\right)\sigma + \left(2R_\Omega R^{(2)} - \frac{2}{3}\Box R^{(2)}\right)\sigma\right\} - \frac{1}{12}\left\{b'' + \frac{2}{3}(b + b')\right\} r_0^2 \\ & \times \int d^2x \sqrt{-g}\{(R^{(2)} + R_\Omega - 6\Box\sigma - 6\nabla^\mu\sigma\nabla_\mu\sigma)^2 - (R^{(2)} + R_\Omega)^2\}. \end{aligned} \tag{7}$$

Here $R_\Omega = 2/r_0^2$ is the scalar curvature of S^2 with the unit radius. The superscript (2) expresses the quantity in two dimensions, but we abbreviate it if there is no confusion. We also note that in two dimensions the Riemann tensor $R_{\mu\nu\sigma\rho}$ and $R_{\mu\nu}$ are expressed via the scalar curvature R and the metric tensor $g_{\mu\nu}$ as $R_{\mu\nu\sigma\rho} = \frac{1}{2}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})R$ and $R_{\mu\nu} = \frac{1}{2}g_{\mu\nu}R$.

Let us derive the equations of motion taking into account quantum corrections from the above effective action. In the following we work in the conformal gauge: $g_{\pm\mp} = -\frac{1}{2}e^{2\rho}$ and $g_{\pm\pm} = 0$ after considering the variation of the effective action $\Gamma + S$ with respect to $g_{\mu\nu}$ and σ. Note that the tensor $g_{\mu\nu}$ under consideration is the product of the original metric tensor and the σ function $e^{-2\sigma}$ and the equations given by the variations of $g_{\mu\nu}$ are the combinations of the equations given by the variation of the original metric and σ equation.

Often we can drop the terms linear in σ in Eq. (7). In particular, one can redefine the corresponding source term as it is in the case of the infrared sector of four-dimensional (4D) quantum gravity [9]. In the following, we consider only this case. Then the variation of $S+\Gamma_{ind}+\Gamma[1,g^{(4)}_{\mu\nu}]$ with respect to $g^{\pm\pm}$ is given by

$$\begin{aligned}0=&\frac{1}{4\pi}\frac{\delta(S+\Gamma_{ind}+\Gamma[1,g^{(4)}_{\mu\nu}])}{\delta g^{\pm\pm}}=-\frac{r_0^2}{16\pi G}e^{2\rho+2\sigma}[(\partial_\pm\sigma)^2-\partial_\pm^2\sigma+2\partial_\pm\sigma\partial_\pm\rho]+b'r_0^2\Bigg[8e^{2\rho}\partial_\pm\sigma\partial_\pm(e^{-2\rho}\partial_+\partial_-\sigma)\\
&-8\sigma\partial_\pm^2\sigma\partial_+\partial_-\rho+\frac{2}{3}e^{2\rho}\partial_\pm\sigma\partial_\pm\{R_4\sigma\}+\frac{8}{3}e^{2\rho}\sigma\partial_\pm R_4\partial_\pm\sigma\Bigg]-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2[4e^{2\rho}\partial_\pm\sigma\partial_\pm R_4-4(\partial_\pm\sigma)^2\partial_+\partial_-\rho\\
&+12e^{2\rho}\partial_\pm\sigma\partial_\pm\{e^{-2\rho}(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\}-12(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)(\partial_\pm\sigma)^2]+\Bigg\{[-\partial_\pm^2\rho-2(\partial_\pm\rho)^2]\\
&-\frac{1}{4}\partial_\pm^2+\frac{3}{2}\partial_\pm^2\rho+\frac{3}{2}\partial_\pm\rho\partial_\pm\Bigg\}\left[\frac{16}{3}b'r_0^2(-\sigma\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\right]\\
&+r_0^2\Bigg[-\frac{1}{3}be^{2\rho}\partial_\pm\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\partial_\pm R_4+4\left\{[-\partial_\pm^2\rho-2(\partial_\pm\rho)^2]-\frac{1}{4}\partial_\pm^2+\frac{3}{2}\partial_\pm^2\rho+\frac{3}{2}\partial_\pm\rho\partial_\pm\right\}\Bigg(\frac{8}{3}b\partial_+\partial_-\rho\ln\left(\frac{R_4}{\mu^2}\right)\\
&+b\partial_+\partial_-\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}+\left\{b\left[\frac{32}{3}e^{-2\rho}(\partial_+\partial_-\rho)^2+\frac{2e^{2\rho}}{3r_0^2}-\frac{4}{3}\partial_+\partial_-R_4+\frac{16}{r_0^2}\left(\frac{b}{3}+b'\right)\right]\right\}\frac{1}{R_4}\Bigg)\Bigg].\end{aligned}\tag{8}$$

Here $R_4\equiv 8e^{-2\rho}\partial_+\partial_-\rho+2/r_0^2$. Usually the equation given by g^{++} or g^{--} can be regarded as the constraint equation with respect to the initial or boundary conditions. The equations obtained here, however, are combinations of the constraint and σ equation of the motion since the tensor $g_{\mu\nu}$ under consideration is the product of the original metric tensor and the σ function $e^{-2\sigma}$.

The variations with respect to ρ are given by

$$\begin{aligned}0=&\frac{1}{4\pi}\frac{\delta(S+\Gamma_{ind}+\Gamma[1,g^{(4)}_{\mu\nu}])}{\delta\rho}=-\frac{r_0^2}{16\pi G}\left[4\partial_+\partial_-e^{2\sigma}-2e^{2\rho+4\sigma}\Lambda+\frac{4}{r_0^2}e^{2\rho+2\sigma}\right]+b'r_0^2\Bigg\{-32(\partial_+\partial_-\sigma)^2e^{-2\rho}\\
&-\frac{128}{3}e^{-2\rho}\partial_+\partial_-\rho(\sigma\partial_+\partial_-\sigma)+\frac{64}{3}\partial_+\partial_-(e^{-2\rho}\sigma\partial_+\partial_-\sigma)\Bigg\}-\frac{16}{3}\{-2\partial_+\sigma\partial_-\sigma e^{-2\rho}\partial_+\partial_-\rho+\partial_+\partial_-(\partial_+\sigma\partial_-\sigma e^{-2\rho})\}\\
&-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2[16\partial_+\partial_-\{e^{-2\rho}(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\}-48e^{-2\rho}(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)^2]\\
&+r_0^2\Bigg(-\frac{64}{3}be^{-2\rho}(\partial_+\partial_-\rho)^2\ln\left(\frac{R_4}{\mu^2}\right)+\frac{64}{3}b\partial_+\partial_-\left\{e^{-2\rho}\partial_+\partial_-\rho\ln\left(\frac{R_4}{\mu^2}\right)\right\}+\frac{4be^{2\rho}}{3r_0^2}\ln\left(\frac{R_4}{\mu^2}\right)+\frac{16}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_+\partial_-\ln\left(\frac{R_4}{\mu^2}\right)\\
&+\frac{64}{3}be^{-2\rho}\partial_+\partial_-\rho\partial_+\partial_-\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}-\frac{64}{3}b\partial_+\partial_-\left[e^{-2\rho}\partial_+\partial_-\left\{\ln\left(\frac{R_4}{\mu^2}\right)\right\}\right]-\frac{16e^{-2\rho}\partial_+\partial_-\rho}{R_4}\Bigg\{\frac{32}{3}be^{-2\rho}(\partial_+\partial_-\rho)^2\\
&+\frac{2be^{2\rho}}{3r_0^2}-\frac{4}{3}b\partial_+\partial_-R_4+\frac{16}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_+\partial_-\rho\Bigg\}-\partial_+\partial_-\Bigg[\frac{8e^{-2\rho}}{R_4}\Bigg\{\frac{32}{3}be^{-2\rho}(\partial_+\partial_-\rho)^2+\frac{2be^{2\rho}}{3r_0^2}-\frac{4}{3}b\partial_+\partial_-R_4\\
&+\frac{16}{r_0^2}\left(\frac{b}{3}+b'\right)\partial_+\partial_-\rho\Bigg\}\Bigg]\Bigg).\end{aligned}\tag{9}$$

The variations with respect to σ may be found as

$$0=\frac{1}{4\pi}\frac{\delta(S+\Gamma_{ind}+\Gamma[1,g_{\mu\nu}^{(4)}])}{\delta\sigma}=-\frac{r_0^2}{16\pi G}\left[8e^{2\sigma}\{3\partial_+\partial_-\sigma+3\partial_+\sigma\partial_-\sigma+\partial_+\partial_-\rho\}-4e^{2\rho+4\sigma}\Lambda+\frac{4}{r_0^2}e^{2\rho+2\sigma}\right]$$
$$+b'r_0^2\left[32\partial_+\partial_-(e^{-2\rho}\partial_+\partial_-\sigma)+\frac{64}{3}[e^{-2\rho}\partial_+\partial_-\rho\partial_+\partial_-\sigma+\partial_+\partial_-(\sigma e^{-2\rho}\partial_+\partial_-\rho)]+\frac{32}{3r_0^2}\partial_+\partial_-\sigma\right.$$
$$\left.+\frac{2}{3}\{\partial_+R_4\partial_-\sigma+\partial_-R_4\partial_+\sigma\}\right]-\left\{b''+\frac{2}{3}(b+b')\right\}r_0^2[2\partial_+\partial_-R_4-2\{\partial_+R_4\partial_-\sigma+\partial_-R_4\partial_+\sigma\}+48\partial_+\partial_-$$
$$\times\{e^{-2\rho}(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\}-48\partial_+\{e^{-2\rho}\partial_-\sigma(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\}-48\partial_-\{e^{-2\rho}\partial_+\sigma(\partial_+\partial_-\sigma+\partial_+\sigma\partial_-\sigma)\}]. \tag{10}$$

Note that now the real four-dimensional metric is given by $ds^2=-e^{2\sigma+2\rho}dx^+dx^-+r_0^2e^{2\sigma}d\Omega$. The above equations give the complete system of quantum corrected equations of motion for the system under discussion.

IV. EVOLUTION OF SCHWARZSCHILD–DE SITTER BLACK HOLES DUE TO QUANTUM CONFORMAL MATTER BACK REACTION

We now consider the Schwarzschild–de Sitter family of black holes and its nearly degenerated case, the so-called Nariai solution [10]. We follow here the work of Bousso and Hawking [3]. The Schwarzschild type of black hole solution in de Sitter space has two horizons: One is the usual event horizon and the other is the cosmological horizon, which is the proper one in de Sitter space. The Nariai solution is given by a limit of the Schwarzschild–de Sitter black hole where two horizons coincide with each other. In the limit, the two horizons have the same temperature since the temperature is proportional to the inverse root of the horizon area. Therefore, two horizons are in thermal equilibrium in the limit. We are now interested in the instability of the Nariai limit. Near the limit the temperature of the event horizon is higher than that of the cosmological one since the area of the event horizon is smaller than that of the cosmological horizon. This implies that there would be a thermal flow from the event horizon to the cosmological one. This means that the system would become unstable and the black hole would evaporate. We also have to note that the above cosmological black holes may naturally appear through quantum pair creation [11,6], which may occur in the inflationary universe [12].

In the Nariai limit, the space-time has the topology of $S^1\times S^2$ and the metric is given by

$$ds^2=\frac{1}{\Lambda}(\sin^2\chi d\psi^2-d\chi^2-d\Omega). \tag{11}$$

Here the coordinate χ has period π. If we change the coordinates variables by

$$r=\ln\tan\frac{\chi}{2},\quad t=\frac{\psi}{4}, \tag{12}$$

we obtain

$$ds^2=\frac{1}{\Lambda\cosh^2 r}(-dt^2+dr^2)+\frac{1}{\Lambda}d\Omega. \tag{13}$$

This form corresponds to the conformal gauge in two dimensions. Note that the transformation (12) has a one to one correspondence between (ψ,χ) and (t,r) if we restrict χ to $0\leqslant\chi<\pi$ (r runs from $-\infty$ to $+\infty$).

Now we solve the equations of motion. Since the Nariai solution is characterized by the constant ϕ (or σ), we now assume that σ is a constant even when including the quantum correction

$$\sigma=\sigma_0\ \ (\text{const}). \tag{14}$$

We also first consider static solutions and replace $\partial_\pm$ by $\pm\frac{1}{2}\partial_r$. Then we find that the total constraint equation obtained by Eq. (8) is trivially satisfied.

Assuming that a solution is given by a constant two-dimensional scalar curvature

$$R=-2e^{-2\rho}\partial_r^2\rho=R_0\ \ (\text{const}), \tag{15}$$

the equations of motions given by Eqs. (9) (variation over ρ) and (10) (variation over σ) become the two algebraic equations

$$0=-\frac{r_0^2}{8\pi G}\left(-\Lambda e^{4\sigma_0}+\frac{2}{r_0^2}e^{2\sigma_0}\right)$$
$$+r_0^2\left\{b\left(-\frac{R_0^2}{3}+\frac{4}{3r_0^4}\right)\ln\left(\frac{R_0+\frac{2}{r_0^2}}{\mu^2}\right)\right.$$
$$\left.-\left\{b\left(\frac{R_0^2}{3}+\frac{4}{3r_0^4}\right)+\frac{8}{r_0^2}\left(\frac{b}{3}+b'\right)R_0\right\}\frac{R_0}{R_0+\frac{2}{r_0^2}}\right\}, \tag{16}$$

$$0=R_0-4\Lambda e^{2\sigma_0}+\frac{4}{r_0^2}. \tag{17}$$

The above equations can be solved with respect to σ_0 and R_0 in general, although it is difficult to get the explicit expression.

Equation (15) can be integrated to be

$$e^{2\rho}=e^{2\rho_0}\equiv\frac{2C}{R_0}\frac{1}{\cosh^2(r\sqrt{C})}. \tag{18}$$

Here C is a constant of integration.

We now consider the perturbation around the Nariai type of solution (14) and (18),

$$\rho=\rho_0+\epsilon R(t,r),\quad \sigma=\sigma_0+\epsilon S(t,r). \tag{19}$$

Here ϵ is an infinitesimally small parameter. Then we obtain the linearized ρ and σ equations

$$\begin{aligned}0=&4\pi b' r_0^2\left\{-\frac{16}{3}R_0\sigma_0\Delta S+\frac{32}{3}\frac{R_0\sigma_0}{C}\Delta(\Delta S)\right\}-4\pi(b+b')r_0^2\frac{16}{3}\frac{R_0}{C}\Delta(\Delta S)-\frac{r_0^2}{16\pi G}e^{2\sigma_0}\left[8\Delta S-\frac{8\Lambda C}{R_0}e^{2\sigma_0}(R+2S)\right.\\&\left.+\frac{16C}{r_0^2R_0}(R+S)\right]+4\pi r_0^2\left[\frac{2C}{R_0}\left\{-\frac{2bR_0}{3}\ln\left(\frac{R_4}{\mu^2}\right)+\frac{b}{R_4}\left(-R_0^2+\frac{4}{3r_0^4}\right)+b\left(\frac{R_0^2}{3}+\frac{4}{3r_0^4}\right)\frac{2}{r_0^2R_4^2}-\frac{8}{r_0^2}\left(\frac{b}{3}+b'\right)\left\{\frac{4R_0}{R_4}-\frac{2R_0^2}{R_0^2}\right\}\right\}\right.\\&\times\left(-2R_0R+\frac{4R_0}{C}\Delta R\right)+\left\{\frac{8b}{3}\ln\left(\frac{R_4}{\mu^2}\right)+\frac{32b}{3}\frac{R_0}{R_4}+\frac{4b}{R_4^2}\left(\frac{R_0^2}{3}+\frac{4}{3r_0^4}\right)\right\}\Delta\left(-2R_0R+\frac{4R_0}{C}\Delta R\right)\\&\left.-\frac{32b}{3}\frac{R_0}{CR_4}\Delta\left[\Delta\left(-2R_0R+\frac{4R_0}{C}\Delta R\right)\right]+\left\{b\left(-\frac{4R_0}{3}+\frac{16}{3r_0^4R_0}\right)\ln\left(\frac{R_4}{\mu^2}\right)-\frac{4b}{3}\frac{R_0^2}{R_4}-\frac{8b}{3r_0^2}\frac{2R_0}{R_4}+\frac{32}{r_0^2}\left(\frac{b}{3}+b'\right)\frac{2R_0^2}{R_4}\right\}CR_0\right],\end{aligned} \tag{20}$$

$$\begin{aligned}0=&4\pi b' r_0^2\left\{\frac{16}{3}R_0\Delta S+16\frac{R_0}{C}\Delta(\Delta S)+\frac{32}{3r_0^2}\Delta S+\frac{8}{3}\sigma_0\Delta\left(-2R_0R+\frac{4R_0}{C}\Delta R\right)\right\}-4\pi\times16(b+b')r_0^2\frac{R_0}{C}\Delta(\Delta S)\\&-\frac{r_0^2}{16\pi G}e^{2\sigma_0}\left\{8\Delta S+\Delta R+\frac{C}{2}S-\frac{16\Lambda C}{R_0}e^{2\sigma_0}(R+2S)+\frac{16C}{r_0^2R_0}(R+S)\right\}.\end{aligned} \tag{21}$$

Here

$$\Delta=\cosh^2(r\sqrt{C})\partial_+\partial_- \tag{22}$$

and R_4 becomes constant $R_4=R_0+2/r_0^2$. Equations (20) and (21) can be solved by assuming that R and S are given by the eigenfunctions of Δ:

$$R(t,r)=Pf_A(t,r),\quad S(t,r)=Qf_A(t,r),\quad \Delta f_A(t,r)=Af_A(t,r). \tag{23}$$

Note that Δ can be regarded as the Laplacian on the two-dimensional hyperboloid and the explicit expression for the eigenfunctions is given later. Using Eq. (23), we can rewrite Eqs. (20) and (21) [using Eq. (17)] as

$$\begin{aligned}0=&\left\{\frac{64\pi b'\sigma_0R_0}{3}\left(-A+2\frac{A^2}{C}\right)-\frac{64\pi(b+b')R_0}{3}\frac{A^2}{C}-\frac{1}{4\pi G}e^{2\sigma_0}(2A-C)\right\}Q+\left[4\pi\left(\frac{2C}{R_0}\left[-\frac{2R_0}{3}\ln\left(\frac{R_4}{\mu^2}\right)+\frac{b}{R_4}\left(-R_0^2+\frac{4}{3r_0^4}\right)\frac{1}{R_4}\right.\right.\right.\\&\left.+\frac{b}{R_4^2}\left(\frac{R_0^2}{3}+\frac{4}{3r_0^4}\right)\frac{2}{r_0^2}-\frac{2}{r_0^2}\left(\frac{b}{3}+b'\right)\left\{\frac{4R_0}{R_4}-\frac{2R_0^2}{R_4^2}\right\}\right]\left(-2R_0+\frac{4R_0}{C}A\right)+\left\{\frac{8b}{3}\ln\left(\frac{R_4}{\mu^2}\right)+\frac{32b}{3}\frac{R_0}{R_4}+\left(\frac{R_0^2}{360}+\frac{1}{90r_0^4}\right)\frac{4}{R_4^2}\right.\\&\left.+\frac{16}{r_0^2R_4}\left(\frac{b}{3}+b'\right)\right\}\left(-2R_0A+\frac{4R_0}{C}A^2\right)-\frac{32bR_0}{3CR_4}\left(-2R_0A^2+\frac{4R_0}{C}A^3\right)+\left\{b\left(-\frac{4R_0}{3}+\frac{16}{3r_0^4R_0}\right)\ln\left(\frac{R_4}{\mu^2}\right)-\frac{4bR_0^2}{3R4}\right.\\&\left.\left.\left.-\frac{8b}{3r_0^2}\frac{2}{R_4}-\frac{8}{r_0^2}\left(\frac{b}{3}+b'\right)\frac{2R_0^2}{R_4}\right\}C\right)-\frac{C}{8\pi G}e^{2\sigma_0}\left(-1+\frac{4}{r_0^2R_0}\right)\right]P\\&\equiv M_Q(A)Q+M_P(A)P,\end{aligned}$$

$$0=\left\{64\pi b' R_0\left(\frac{1}{3}A+\frac{1}{C}A^2\right)+\frac{128\pi}{3r_0^2}b'A-64\pi(b+b')\frac{R_0}{C}A^2-\frac{e^{2\sigma_0}}{4\pi G}\left(6A-C-\frac{4C}{r_0^2R_0}\right)\right\}Q$$
$$+\left\{\frac{64\pi b'\sigma_0 R_0}{3}\left(-A+\frac{2}{C}A^2\right)-\frac{e^{2\sigma_0}}{4\pi G}(2A-C)\right\}P\equiv N_Q(A)Q+N_P(A)P. \tag{24}$$

In order for the above two algebraic equations to have nontrivial solutions for P and Q, A should satisfy

$$0=F(A)\equiv M_Q(A)N_P(A)-M_P(A)N_Q(A). \tag{25}$$

Before analyzing Eqs. (16), (17), and (25), we now briefly discuss how (anti)evaporation is described. As in [5], we consider the following function as an eigenfunction of Δ in Eq. (23):

$$f_A(t,r)=\cosh t\alpha\sqrt{C}\cosh^{\alpha} r\sqrt{C},\quad A\equiv\frac{\alpha(\alpha-1)C}{4}. \tag{26}$$

Note that there is a one to one correspondence between A and α if they are restricted to $A>0$ and $\alpha<0$. Any linear combination of two solutions is a solution. The perturbative equations of motion (24) are always linear differential equations. The horizon is given by the condition

$$\nabla\sigma\cdot\nabla\sigma=0. \tag{27}$$

Substituting Eq. (26) into Eq. (27), we find that the horizon is given by $r=\alpha t$. Therefore, on the horizon, we obtain $S(t,r(t))=Q\cosh^{1+\alpha}t\alpha\sqrt{C}$. This tells us that the system is unstable if there is a solution $0>\alpha>-1$, i.e., $0<A<C/2$. On the other hand, the perturbation becomes stable if there is a solution where $\alpha<-1$, i.e., $A>C/2$. The radius of the horizon r_h is given by $r_h=e^{\sigma}=e^{\sigma_0+\epsilon S(t,r(t))}$. Let the initial perturbation be negative $Q<0$. Then the radius shrinks monotonically, i.e., the black hole evaporates in the case of $0>\alpha>-1$. On the other hand, the radius increases in time and approaches the Nariai limit asymptotically $S(t,r(t))\to Qe^{(1+\alpha)t|\alpha|\sqrt{C}}$ in the case of $\alpha<-1$. The latter case corresponds to the antievaporation of the black hole observed by Bousso and Hawking [3].

We should be more careful in the case of $A=C/2$. When $A=C/2$, $f_A(r,t)$ is, in general, given by

$$f_A(r,t)=\left\{\frac{\cosh[(t+a)\sqrt{C}]}{\cosh(r\sqrt{C})}+\sinh(b\sqrt{C})\tanh(r\sqrt{C})\right\}. \tag{28}$$

Then the condition (27) gives $t+a=\mp(r-b)$. Therefore, on the horizon, we obtain $S(t,r(t))=Q\cosh b$. This is a constant, that is, evaporation or antievaporation does not occur. The radius of the horizon does not develop in time. In the classical case, Eq. (25) has the form

$$0=\left\{\frac{1}{4\pi G}e^{2\sigma_0}(2A-C)\right\}^2-\frac{C}{8\pi G}e^{2\sigma_0}\frac{e^{2\sigma_0}}{4\pi G}\left(6A-C-\frac{4C}{r_0^2R_0}\right). \tag{29}$$

Since $R_0=4r_0^2$ in the classical case, the solution of Eq. (29) is given by $A=C/2$. Therefore, the horizon does not develop in time and the black hole does not evaporate or antievaporate. The result is, of course, consistent with that of [3].

In general, it is very difficult to analyze Eqs. (16), (17), and (25). The equations, however, become simple in the limit of $r_0\to\infty$. Note that r_0, the radius of S_2, is a free parameter and the SDW type of expansion in Eq. (6) becomes exact in the limit since $R\sim O(r_0^{-2})$. In order to consider the limit, we now redefine R_0 and σ_0 as

$$R_0=\frac{1}{r_0^2}H_0,\quad \sigma_0=-\ln(\mu r_0)+s_0. \tag{30}$$

Then Eq. (16) is rewritten as

$$0=H_0-\frac{4\Lambda}{\mu^2}e^{2s_0}+4. \tag{31}$$

Substituting Eq. (31) into Eq. (17), we obtain

$$(-H_0^2+4)\ln(\mu r_0)+O(1)=0. \tag{32}$$

This tells us that

$$H_0=\pm 2+O([\ln(\mu r_0)]^{-1}). \tag{33}$$

Since we can expect that $H_0\sim 2$ would correspond to the classical limit where $H_0=4$, we consider only the case of $H_0\sim 2$. Then using Eq. (31), we find

$$e^{2s_0}=\frac{3\mu^2}{2\Lambda}. \tag{34}$$

Then the metric in the quantum Nariai type of solution has the form

$$ds^2=\frac{3C}{2\Lambda}\frac{1}{\cosh^2(r\sqrt{C})}(-dt^2+dr^2)+\frac{3C}{2\Lambda}d\Omega. \tag{35}$$

Substituting Eqs. (33) and (34) into Eq. (25), we obtain

$$0=F(A)=\frac{1}{r_0^4}\left\{[\ln(\mu r_0)]^2\left(\frac{128\pi b'}{3}\right)^2\right.$$
$$\left.\times\left(-A+\frac{2A^2}{C}\right)^2+O(1)\right\}. \tag{36}$$

Equation (36) tells us that $A=0,C/2$. When $A=0$, S is constant and evaporation or antievaporation does not occur. As discussed before, the horizon does not develop in time when $A=C/2$, either. The result, however, might be an artifact in the limit of $r_0\to\infty$. In order to consider physically reliable results, we start from the next order of $[\ln(\mu r_0)]^{-1}$. Then using Eqs. (16) and (17), we find

$$H_0=2+[\ln(\mu r_0)]^{-1}\left(\frac{2b+3b'}{b}+\frac{9}{512\pi^2 bG\Lambda}\right), \tag{37}$$

$$\sigma_0=-\ln(\mu r_0)+\frac{1}{2}\ln\left(\frac{3\mu^2}{2\Lambda}\right)+[\ln(\mu r_0)]^{-1}$$
$$\times\frac{\mu^2}{8\Lambda}\left(\frac{2b+3b'}{b}+\frac{9}{512\pi^2 bG\Lambda}\right).$$

Since we are now interested in the problem of antievaporation, we consider the solution of $A\sim C/2$. The solution $A\sim 0$ would correspond to usual evaporation. Assuming $A=C/2+[\ln(\mu r_0)]^{-1}a_1$ and substituting Eq. (37) into Eq. (25), we find

$$a_1=0,\quad a_1=-\frac{(b+b')C}{8b'}. \tag{38}$$

In the first solution, the horizon does not develop in time again and we would need the analysis of the higher order of $[\ln(\mu r_0)]^{-1}$. An important point is that the second solution is positive when $2N+7N_{1/2}>26N_1$. When a_1 is positive, $A>C/2$, i.e., antievaporation occurs. Let us consider the SU(5) group with N_s scalar multiplets and N_f fermion multiplets in the adjoint representation of the gauge group. Then, the above relation looks like $2N_s+7N_f>26$. We see that for SU(5) GUT with three spinor multiplets and three scalar multiplets antievaporation is expected for SdS BHs. Similarly, one can estimate the chances for antievaporation in the arbitrary GUT under discussion. On the other hand, when $2N+7N_{1/2}<26N_1$, evaporation occurs. For example, for the above GUT with two spinor multiplets and two scalar multiplets we expect that matter quantum effects induce the evaporation of SdS BHs. This result would be nonperturbative and exact in the leading order of the $1/N$ expansion. Of course, A goes to $C/2$ in the limit of $r_0\to+\infty$, when the SDW type of expansion becomes exact, and the black hole does not evaporate or antievaporate in the limit. If there is some external perturbation, which gives effectively finite r_0, however, antievaporation may occur.

V. ENERGY FLOW AND NO BOUNDARY CONDITION

We now briefly discuss Hawking radiation. Since the space-time that we are now considering is not asymptotically Minkowski, we will consider the radial component in the flow of the energy $T_{tr}=T_{++}-T_{--}$.

Usually the Einstein equation can be written as

$$\frac{1}{16\pi G}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=T^c_{\mu\nu}+T^q_{\mu\nu}. \tag{39}$$

Here $T^c_{\mu\nu}$ is the classical part of the matter energy momentum tensor, which vanishes in the case under consideration, and $T^q_{\mu\nu}$ is the quantum part, which we are now interested in. Comparing Eq. (39) with Eq. (8), we find

$$T^q_{\pm\pm}=\frac{r_0^2}{32\pi G}e^{2\sigma}\{(\partial_\pm\sigma)^2-\partial_\pm^2\sigma+2\partial_\pm\sigma\partial_\pm\rho\}. \tag{40}$$

Substituting the solution of the perturbation (26), we find

$$T^q_{tr}=-\frac{3\alpha(\alpha+1)\epsilon}{32\pi G\Lambda}\sinh(t\alpha\sqrt{C})$$
$$\times\sinh(r\sqrt{C})\cosh^{\alpha-1}(r\sqrt{C})+O(\epsilon^2). \tag{41}$$

T^q_{tr} is positive when $0>\alpha>-1$ and negative when $\alpha<-1$, i.e., there is a flow from the event horizon to the cosmological horizon when $0>\alpha>-1$. The direction of the flow changes when $\alpha<-1$. It exactly corresponds to evaporation for $0>\alpha>-1$ and antievaporation for $\alpha<-1$.

Let us turn now to the study of no boundary condition in the evolution of black holes. As is known, the cosmological BH does not appear after the gravitational collapse of a star since the background space-time is not de Sitter space but flat Minkowski space. Bousso and Hawking, however, conjectured that the cosmological black holes could be pair created by the quantum process in an inflationary universe since the universe is similar to de Sitter space. They have also shown that no boundary condition [13] determines the fate of the black holes and they should always evaporate (for minimal scalars). Now we make a similar analysis for conformal matter. In our case, the analytic continuation to Euclidean space-time is given by replacing $t=i\tau$. By the further changing the variables τ and r by

$$v=\tau\sqrt{C},\quad \sin u=\frac{1}{\cosh r\sqrt{C}}, \tag{42}$$

the metric in the quantum Nariai type of solution corresponding to Eq. (35) has the form

$$ds^2=\frac{3}{2\Lambda}(du^2+\sin^2 u dv^2+d\Omega). \tag{43}$$

The metric (43) tells us that four-dimensional Euclidean space-time can be regarded as the product of two two-spheres $S^2\times S^2$. In the Euclidean signature, the operator $(4/C)\Delta$ becomes the Laplacian on the unit two-sphere S^2. The nucleation of the black hole is described by cutting the two-sphere at $u=\pi/2$ and joining to it a Lorentzian $(1+1)$-dimensional de Sitter hyperbolid [3] by analytically continuing u by $u=\pi/2+i\hat{t}$ and regarding $\hat{t}$ as the time coordinate. In the Euclidean signature, the eigenfunction $f_A(t,r)$ in Eq. (26) is not single valued unless α is an integer. Therefore, $f_A(t,r)$ is not adequate when we discuss the nucleation of the black holes. Instead of $f_A(t,r)$, we consider the following eigengfunction $\tilde{f}_A(t,r)$ in the Euclidean signature:

$$\tilde{f}_A(u,v)=f_0\cos(v)P^1_\nu(\cos u),\quad A=\frac{\nu(\nu+1)}{4}C. \tag{44}$$

Here f_0 is a constant and $P^1_\nu(x)$ is given by the associated Legendre function

$$P^1_\nu(x)=\sin\,\pi\nu(1-x^2)^{1/2}\times\sum_{n=0}^{\infty}\frac{\Gamma(1-\nu+n)\Gamma(2+\nu+n)}{(n+1)!n!}\left(\frac{1-x}{2}\right)^n. \tag{45}$$

We can assume $\nu\geqslant 0$ without any loss of generality. Note that $\tilde{f}(u,v)$ vanishes at the south pole $u=0$ since $P^1_\nu(x=\cos u=1)=0$. Therefore, $\tilde{f}(u,v)$ is single valued on the hemisphere and does not conflict with the no boundary condition [13]. We should also note that $\tilde{f}(u,v)$ is not real when analytically continuing u by $u=\pi/2+i\hat{t}$ into the Lorentzian signature but, as in [3], we can make $\tilde{f}(u,v)$ real in the late Lorentzian time (large $\hat{t}$) by the suitable choice of the constant f_0 since

$$P^1_\nu(\cos u)=P^1_\nu(-i\sinh\hat{t})\sim\frac{\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma(\nu)}(-i)^\nu e^{\nu\hat{t}}\quad\text{when }\hat{t}\to+\infty. \tag{46}$$

Since $A>C/2$ when $\nu>1$, we can expect that antievaporation would occur in the pair created black holes. In order to confirm it, we consider the behavior of the horizon in the late Lorentzian time (large $\hat{t}$). By using Eq. (46) we find that the condition of the horizon (27) gives $\cos v\propto e^{-\hat{t}}$. Therefore, we find on the horizon $\hat{f}(v(\hat{t}),\hat{t})\propto e^{(\nu-1)\hat{t}}$. This tells us, as in the case of Sec. IV, that the perturbation is stable when $A>C/2$ $(\nu>1)$ and unstable when $A<C/2$ $(0<\nu<1)$. The previous analysis in (38) implies that there is a solution of $A>C/2$. Therefore, the antievaporation (stable mode) can occur even in the nucleated black holes and some black holes do not evaporate and can survive. This result is different from that of Bousso and Hawking, who claimed that the pair created cosmological black holes most probably evaporate. (Note, however, that they considered another type of matter-minimal scalar, while we deal with conformal matter.) In any case, this question deserves further investigation.

VI. DISCUSSION

In summary, we studied the large N effective action for conformal matter on a spherically symmetric background. The application of this effective action to the investigation of quantum evolution of SdS BHs shows that such BHs may evaporate or antievaporate in the nearly degenerate limit. No boundary condition is shown to be consistent with the antievaporation of SdS BHs. Other boundary conditions may be discussed in the same way as a generalization of this work. Some remarks about energy flow in the regime of evaporation or antievaporation are also given.

Let us now compare our results with these of other papers [3,5], where similar questions have been investigated. Bousso and Hawking treated the quantum effects of 4D minimal scalars using s-wave and large N approximations. In our other paper [5], the scale (σ) dependent part of the effective action is given by the large N trace anomaly induced effective action (without using the s-wave approximation), but the scale independent part is determined using the s-wave approximation, i.e., spherical reduction. That work [5] deals with conformal quantum scalars only. In the present work we used the effective action whose scale dependent part is given by a large N trace anomaly as in the previous paper [5] but whose scale independent part is given by a Schwinger–De Witt type of expansion, which is essentially the power series expansion on the curvature invariants corresponding to the rescaled metric (3). Since the curvature in the Nariai limit and the perturbation around it is almost constant, the rescaled scalar curvature is always $O(r_0^{-2})$. This tells us that the Schwinger–De Witt type of expansion given in this paper would become exact in the limit of $r_0\to+\infty$. Therefore, the analysis given here using the $r_0\to+\infty$ limit would also be exact. Moreover, the results of the present work are given for arbitarary conformal matter (scalars, spinors, and vectors). In all these works, the same qualitative result is found: the possibility that SdS (Nariai) BHs may antievaporate. As we see from the estimation above, such antievaporation may be quite general for many GUTs. Moreover, pair created (primordial) BHs may antievaporate due to conformal quantum matter effects when applying no boundary condition.

As a very interesting generalization of the above work, it could be helpful to understand whether antievaporation is a specific feature of SdS BHs or it may be realized also for other BHs with multiply horizons. In order to clarify this issue we studied Reissner-Nordstrøm–de Sitter BHs where a preliminary investigation shows also the possibility of antievaporation due to quantum effects. We hope to report on this in the future.

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APPENDIX

String presentation of 4D Einstein scalar theory

Starting from Einstein gravity with N minimal scalars,

$$S=-\frac{1}{16\pi G}\int d^4x\sqrt{-g_{(4)}}\Bigg[(R^{(4)}-2\Lambda)-\frac{16\pi G}{2}\sum_{a=1}^{N}g^{\alpha\beta}_{(4)}\partial_\alpha\chi_a\partial_\beta\chi_a\Bigg], \tag{A1}$$

one can consider the spherically symmetric space-time

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu+f(\phi)d\Omega. \tag{A2}$$

Reducing the action (A1) for the metric (A2), we get

$$\frac{S}{4\pi}=-\frac{1}{16\pi G}\int d^2x\sqrt{-g}\Big\{f(R-2\Lambda)+2 +2(\nabla^\mu f^{1/2})(\nabla_\mu f^{1/2})-\frac{16\pi G}{2}\sum_{a=1}^{N}f(\phi)\nabla^\alpha\chi_a\nabla_\alpha\chi_a\Big\}. \tag{A3}$$

The reduced action belongs to the class of actions described by

$$S=\int d^2x\sqrt{-g}\Big\{C(\phi)R+V(\phi)+\frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi -\frac{1}{2}\tilde{f}(\phi)\sum_{a=1}^{N}\nabla^\alpha\chi_a\nabla_\alpha\chi_a\Big\}, \tag{A4}$$

where from Eq. (A3) we get

$$C(\phi)=-\frac{f(\phi)}{4G},\quad V(\phi)=-\frac{1}{4G}[2-2\Lambda f(\phi)], \tag{A5}$$

$$Z(\phi)=-\frac{1}{4G}\frac{f'^2}{f},\quad \tilde{f}(\phi)=-4\pi f(\phi).$$

Working in the conformal gauge $g_{\mu\nu}=e^{2\sigma}\bar{g}_{\mu\nu}$, we may present the action (A4) as a σ model

$$S=\int d^2x\sqrt{-\bar{g}}\Big[\frac{1}{2}G_{ij}(X)\bar{g}^{\mu\nu}\partial_\mu X^i\partial_\nu X^j+\bar{R}\Phi(X)+T(X)\Big], \tag{A6}$$

where

$$X^i=\{\phi,\sigma,\chi_a\},\quad \Phi(X)=C(\phi),\quad T(X)=V(\phi)e^{2\sigma},$$

$$G_{ij}=\begin{pmatrix} Z(\phi) & 2C'(\phi) & 0\\ 2C'(\phi) & 0 & 0\\ 0 & 0 & -\tilde{f}(\phi)\end{pmatrix}. \tag{A7}$$

Thus we presented reduced 4D Einstein scalar theory as a σ model. Similarly, one can present 4D Einstein conformal scalar reduced theory

$$S=\int d^2x\sqrt{-g}\Big\{C(\phi)R+V(\phi)+\frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi -\frac{1}{2}\tilde{f}(\phi)\sum_{a=1}^{N}\Big(\nabla^\alpha\chi_a\nabla_\alpha\chi_a+\frac{1}{6}\chi_a^2\Big)\Big\} \tag{A8}$$

in a form like Eq. (A6) with slightly changed metric G_{ij} and Φ,T:

$$\Phi(X)=C(\phi)-\frac{1}{2}\tilde{f}(\phi),\quad T(X)=V(\phi)e^{2\sigma},$$

$$G_{ij}=\begin{pmatrix} Z(\phi) & 2C'(\phi) & -\frac{1}{3}\tilde{f}'(\phi)\sum_{a=1}^{N}\chi_a^2\\ 2C'(\phi) & 0 & -\frac{2}{3}\tilde{f}(\phi)\chi_a\\ -\frac{1}{3}\tilde{f}'(\phi)\sum_{a=1}^{N}\chi_a^2 & -\frac{2}{3}\tilde{f}(\phi)\chi_a & -\tilde{f}(\phi)\end{pmatrix}. \tag{A9}$$

One can show (see [14]) that the off-shell effective action in the stringy parametrization (A6) is different from the one calculated in dilatonic gravity (A4) in a covariant gauge. However, on shell all such effective actions coincide, as they should (see [14]). The main qualitative result of this appendix is that one can study quantum evolution of black holes using also the σ model approach.

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