

Cosmologies with varying light speed

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We analyze a generalization of general relativity that incorporates a cosmic time variation of the velocity of light in vacuum, c , and the Newtonian gravitation “constant” G proposed by Albrecht and Magueijo. We find exact solutions for Friedmann universes and determine the rate of variation of c required to solve the flatness and classical cosmological constant problems. Potential problems with this approach to the resolution of the flatness and classical cosmological constant problems are highlighted. Reformulations are suggested which give the theory a more desirable limit as a theory of varying G in the limit of constant c and its relationship to theories with varying electron charge and constant c are discussed. [S0556-2821(99)03404-9]

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I. INTRODUCTION

Albrecht and Magueijo [1] have recently drawn attention to the possible cosmological consequences of time variations in c , the velocity of light in vacuum. In particular, there are mathematical formulations of this possibility which offer new ways of resolving the horizon and flatness problems of cosmology, distinct from their resolutions in the context of the standard inflationary universe theory [2] or the pre-big-bang scenario of low-energy string theory [3]. Moreover, in contrast with the case of the inflationary universe, varying c may provide an explanation for the relative smallness of the cosmological constant today. The study presented in Ref. [1] envisages a sudden fall in the speed of light, precipitated by a phase transition or some shift in the values of the fundamental constants, and explores the general consequences that might follow from a sufficiently large change.

In this paper we explore the formulation of Ref. [1] in further detail, showing that one can solve the cosmological field equations that define it for general power-law variations of c and G . This allows us to determine the rate and sense of the changes required in c if the flatness, horizon, and cosmological constant problems are to be solved. We find that these questions have simple answers that generalize the conditions for an inflationary resolution of the flatness and horizon problems in standard generalized inflation. We will also explore some of the consequences of the varying c model for other cosmological problems, like isotropy, and its implications for the evolution of black hole horizon. We will then look more critically at the formulation of a varying- c theory of gravitation that is proposed in Ref. [1], and suggest some ways in which it might be improved. In particular, since the proposed formulation also includes the possibility of varying the gravitation “constant” $G(t)$, we consider the relation between this part of the theory and other well-defined scalar-tensor gravity theories which incorporate varying G .

There have been several high-precision tests of any possible space and time variations of the fine structure constant at different times in the past [4,5]. Although any theory which admits c variation allows us to compare its predictions with the observational constraints on time variations of the fine structure constant, these observational limits arise from relatively low-redshift observations or from primordial nu-

cleosynthesis and are not necessarily of relevance to the varying- c theory under examination here. The period of c -variation is expected to be confined to the very early universe and its observational consequences are likely to be manifested most sensitively through the spectrum of any microwave background fluctuations that are created.

II. THE ALBRECHT-MAGUEIJO MODEL

If we begin with the field equations of general relativity

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (1)$$

then we cannot simply “write in” variations of constants such as G or c as we could in a non-geometrical theory like Newtonian gravity [6]. For example, if we wished to allow G , say, to vary in time then we encounter a consistency problem if the energy-momentum tensor T_{ab} and its vanishing covariant divergence retains its usual form and physical meaning as conservation of energy and momentum. The covariant divergence of the left-hand side of Eq. (1) vanishes and if the conservation of energy and momentum is expressed by the vanishing divergence of T_{ab} in the usual way then G must be constant. Thus scalar-tensor gravity theories of the Jordan-Brans-Dicke (JBD) sort [7] must derive the variation of G from that of a dynamical scalar field which contributes its own energy-momentum tensor to the right-hand side of the field equations. An alternative way of incorporating a varying G or c with a minimum of change to the underlying theory of gravity is to allow the conventional energy-momentum tensor to have a non-vanishing divergence. Thus the usual energy and momentum will not be conserved. Equivalently, one might interpret this as simply changing the definition of the conserved quantity. Thus, since the divergence left-hand side of Eq. (1) must vanish we require that the divergence of $G(\mathbf{x})c(\mathbf{x})^{-4}T_{ab}$ vanishes.

When discussing any theory in which there is a varying of some traditional “constant” of nature, it is important to recognize that invariant operational meaning can only be attached to space-time variations of *dimensionless* constants. Variations in dimensional constants can always be transformed away by a suitable choice of coordinate frame

(change of units); this is discussed in detail in Ref. [13]. We will be interested in theories in which a time variation of the fine structure constant is represented through a variation in c . An example (not unique) of a representation as a theory with varying electron charge, e , but constant c has been given by Bekenstein [5], and another by Barrow and Magueijo [13]. In varying speed of light (VSL) theories a varying α is interpreted as $c \propto \hbar \propto \alpha^{-1/2}$ and e is constant, Lorentz invariance is broken, and so by construction there is a preferred frame for the formulation of the physical laws. In the minimally coupled theory one then simply replaces c by a field in this preferred frame. Hence, the action remains as [13]

$$S = \int dx^4 \left(\sqrt{-g} \left(\frac{\psi(R+2\Lambda)}{16\pi G} + \mathcal{L}_M \right) + \mathcal{L}_\psi \right) \quad (2)$$

with $\psi(x^\mu) = c^4$. The dynamical variables are the metric $g_{\mu\nu}$, any matter field variables contained in the matter Lagrangian \mathcal{L}_M , and the scalar field ψ itself. The Riemann tensor (and the Ricci scalar) is to be computed from $g_{\mu\nu}$ at constant ψ in the usual way. This can only be true in one frame: additional terms in $\partial_\mu \psi$ must be present in other frames; see Refs. [1–13] for more detailed discussion.

Varying the action with respect to the metric and ignoring surface terms leads to

$$G_{\mu\nu} - g_{\mu\nu} \Lambda = \frac{8\pi G}{\psi} T_{\mu\nu}. \quad (3)$$

Therefore, Einstein's equations do not acquire new terms in the preferred frame. Minimal coupling at the level of Einstein's equations is at the heart of the model's ability to solve the cosmological problems. It requires of any action-principle formulation that the contribution \mathcal{L}_ψ must not contain the metric explicitly, and so does not contribute to the energy-momentum tensor.

Albrecht and Magueijo (AM) propose that a time-variable c should not introduce changes in the curvature terms in Einstein's equations in the cosmological frame and that Einstein's equations must still hold. Thus, c changes in the local Lorentzian frames associated with the cosmological expansion and is a special-relativistic effect. The resulting theory is clearly not covariant and so implementation of this idea requires a specific choice of time coordinate. Choosing that specific time to be comoving proper time, and assuming the universe is spatially homogeneous and isotropic, so that there are no spatial variations in c or G , leads to the requirement that the Friedmann equations still retain their form with $c(t)$ and $G(t)$ varying. Thus the expansion scale factor obeys the equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)\rho}{3} - \frac{Kc^2(t)}{a^2} \quad (4)$$

$$\ddot{a} = -\frac{4\pi G(t)}{3} \left(\rho + \frac{3p}{c^2(t)} \right) a \quad (5)$$

where p and ρ are the density and pressure of the matter, respectively, and K is the metric curvature parameter. By

differentiating Eq. (4) with respect to time and substituting in Eq. (5), we find the generalized conservation equation incorporating possible time variations in $c(t)$ and $G(t)$,

$$\dot{\rho} + 3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = -\rho \frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G a^2}. \quad (6)$$

The cosmological constant is a further feature of interest in these theories. In general relativity we know that there are two equivalent ways of introducing a cosmological constant into the field equations and their cosmological solutions. An explicit constant Λ term can be added to the Einstein-Hilbert gravitational Lagrangian. Alternatively, a matter field can be introduced which describes a perfect fluid with a stress satisfying $p = -\rho c^2$. In theories other than general relativity these two prescriptions need not lead to the same contribution to the field equations and their solutions. For example, in Brans-Dicke theory or other scalar-tensor theories one can see explicitly that they contribute different terms to the cosmological evolution equations (see Barrow and Maeda [11]). The matter Lagrangian term \mathcal{L}_M will in general also contain large quantum contributions, so $\mathcal{L}_M \equiv \mathcal{L}_M[\text{class}] + \mathcal{L}_M[\text{quantum}]$.

If we examine the action given in Eq. (2) then we see that there are two ways in which a cosmological constant can arise in a VSL theory. In our discussion we shall model the explicit Λ term of geometrical origin in Eq. (2), or Eq. (3). This can still be interpreted as a matter field obeying the equation of state $p_\Lambda = -\rho_\Lambda c^2$, as would be contributed by any slowly changing self-interacting scalar field. We do not address the quantum gravitational or quantum field theoretic versions of the problem discussed in Refs. [12,8] or solve the problem associated with the presence of quantum contributions described by any constant term in $\mathcal{L}_M[\text{quantum}]$.

If we wish to incorporate such a cosmological constant term, Λ , (which we shall assume to be constant) then we can define a vacuum stress obeying an equation of state

$$p_\Lambda = -\rho_\Lambda c^2, \quad (7)$$

where

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}. \quad (8)$$

Then, replacing ρ by $\rho + \rho_\Lambda$ in Eq. (6), we have the generalization

$$\dot{\rho} + 3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) + \dot{\rho}_\Lambda = -\rho \frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G a^2}. \quad (9)$$

We shall assume that the remaining matter obeys an equation of state of the form

$$p = (\gamma - 1)\rho c^2(t) \quad (10)$$

where γ is a constant.

III. THE FLATNESS AND LAMBDA PROBLEMS

If we specialize the standard situation in general relativity in which we set $\dot{G} \equiv 0 \equiv \dot{c}$ then the conservation equation, Eq. (9), gives $\rho \propto a^{-3\gamma}$ and the $8\pi G\rho/3$ term will dominate the curvature term, Kc^2a^{-2} at large a so long as the matter stress obeys $\rho + 3p/c^2 < 0$, $\rho + p/c^2 \geq 0$, that is, if

$$0 \leq \gamma < \frac{2}{3}. \quad (11)$$

This is what we shall mean by the *flatness problem*. Since the scale factor then evolves as $a(t) \propto t^{2/3\gamma}$ if $\gamma > 0$ or $\exp\{H_0 t\}$, H_0 constant if $\gamma = 0$, we see that it also grows faster than the proper size of the particle horizon scale ($\propto t$) so long as $\gamma < 2/3$. Thus, a sufficiently long period of evolution in the early universe during which the expansion is dominated by a gravitationally repulsive stress with $\rho + 3p/c^2 < 0$ can solve the flatness and horizon problems. Such a period of accelerated expansion is called ‘‘inflation.’’ However, we notice that if a constant vacuum stress, ρ_Λ , associated with a non-zero cosmological constant is added to the right-hand side of Eq. (9) then, in order to explain why it does not totally dominate the $8\pi G\rho/3$ term at large $a(t)$, we would need a period during which the early universe was dominated by an extreme fluid with $\gamma < 0$, that is $\rho + p/c^2 < 0$. This is problematic because it leads to contraction, $\dot{a} < 0$, and apparent instabilities of the vacuum and of flat space-time. This is what we shall mean by the *cosmological constant (or lambda) problem*. Other quantum cosmological attempts to solve this cosmological constant problem have so far proved unsuccessful [8].

We see that the inflationary solution of the flatness problem relies on a change in the evolutionary behavior of the matter term in the Friedmann equation which allows it to dominate the curvature term at large $a(t)$ and transform the zero-curvature solution into the late-time attractor. In contrast, the varying- c model of AM will provide a solution of the flatness and lambda problems by introducing a variation that increases the rate of fall-off of the curvature (Kc^2a^{-2}) and cosmological constant (ρ_Λ) terms in the Friedmann equation with respect to the $8\pi G\rho/3$ term at large $a(t)$. This is possible for all values of γ and does not necessarily require the existence of a period during which $\rho + 3p/c^2$ is negative.

IV. EXACT SOLUTIONS WITH VARYING $G(t)$ AND $c(t)$

We need to obtain solutions of Eqs. (6) and (9) in order to evaluate the effects of varying G and c on the expansion dynamics. Let us first consider the implications for the flatness and horizon problems alone by setting $\Lambda = 0$.

A. The flatness problem

In order to solve Eq. (6) we assume that the rate of variation of c is proportional to the expansion rate of the universe; that is,

$$c(t) = c_0 a^n; c_0, n \text{ const.} \quad (12)$$

We shall see that we do not need to prescribe a form for the time variation of $G(t)$.

From Eqs. (6) and (10), we have

$$\frac{(\rho a^{3\gamma})'}{\rho a^{3\gamma}} = -\frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G\rho a^2},$$

hence

$$\frac{(G\rho a^{3\gamma})'}{G\rho a^{3\gamma}} = \frac{3Kc\dot{c}}{4\pi G\rho a^2}$$

and so, using Eq. (12), we obtain

$$(G\rho a^{3\gamma})' = \frac{3Kc_0^2 n a^{2n+3\gamma-3} \dot{a}}{4\pi}. \quad (13)$$

Integrating, we have the exact solutions

$$G(t)\rho a^{3\gamma} = \frac{3Kc_0^2 n a^{2n+3\gamma-2}}{4\pi(2n+3\gamma-2)} + B, \quad \text{if } 2n+3\gamma \neq 2, \quad (14)$$

$$G(t)\rho a^{3\gamma} = \frac{3Kc_0^2 n \ln a}{4\pi} + B, \quad \text{if } 2n+3\gamma = 2, \quad (15)$$

with a constant of integration, $B > 0$. This holds for arbitrary $G(t)$ variations. Analogous solutions can be found from Eq. (13) for other possible variations of c with respect to $a(t)$ or t but we shall confine our attention to the simple case of Eq. (12).

Returning to the Friedmann equation (4), we see that the condition for the solution of the flatness problem is the same as that which would hold if G were constant. The only impact of $G(t)$ is to multiply ρ , so we have, in Eq. (4) (writing only the $2n+3\gamma \neq 2$ case explicitly),

$$\frac{\dot{a}^2}{a^2} = \frac{2Kc_0^2 n a^{2n-2}}{(2n+3\gamma-2)} + B' a^{-3\gamma} - Kc_0^2 a^{2n-2}$$

$$\frac{\dot{a}^2}{a^2} = B' a^{-3\gamma} + \frac{Kc_0^2 a^{2n-2}(2-3\gamma)}{(2n+3\gamma-2)}, \quad B' \text{ const,} \quad (16)$$

and the B' term dominates the curvature (K) terms at large a , so the flatness problem can be solved so long as

$$n \leq \frac{1}{2}(2-3\gamma) \quad (17)$$

[where we have noted from Eq. (15) that the flatness problem is also resolved in the $2n+3\gamma=2$ case]. We see that this condition is a straightforward generalization of the inflationary condition (11) that obtains when c is constant. We confirm that the variation in G does not enter this condition in a significant way. Also, the sign of $3\gamma-2$ determines the overall sign of the curvature term in the Friedmann equation. However, unlike in the constant c case, it is possible to resolve the flatness problem without appeal to a matter source

with $3\gamma - 2 < 0$. In particular, we see that in a radiation-dominated universe ($\gamma = 4/3$), the flatness problem is soluble if $c(t)$ falls sufficiently rapidly as the universe expands, with

$$n \leq -1, \quad (18)$$

while for dust ($\gamma = 1$) we require a slower decrease, with

$$n \leq -\frac{1}{2}. \quad (19)$$

B. The horizon problem

The variation of the expansion scale factor at large $a(t)$ approaches

$$a(t) \propto t^{2/3\gamma} \quad (20)$$

when the curvature term becomes negligible, as in the situation when c is constant. The proper distance to the horizon increases as

$$d_h \propto c t \propto a^n t \propto t^{(3\gamma + 2n)/3\gamma} \quad (21)$$

and so we see that the scale factor can grow as fast, or faster, than d_h as t increases if

$$2 \geq 2n + 3\gamma.$$

This is the condition to solve the horizon problem and is identical to that for the solution of the flatness problem, just as in the situation with constant c .

C. The Lambda problem

If we consider the case with a non-zero Λ term, that is contributed by a stress obeying $p_\Lambda = -\rho_\Lambda c^2$, then we need to solve Eq. (9) with $c(t)$ varying as Eq. (12). In this case we need to assume a form for the variation of G with a or t , and we shall again assume that its rate of variation is proportional to the expansion rate of the universe, with

$$G(t) = G_0 a^q \quad (22)$$

where G_0 and q are constants. We assume that Λ is constant. From Eq. (8) we have

$$\dot{\rho}_\Lambda = \frac{\Lambda c_0^2 (2n - q) a^{2n - q - 1} \dot{a}}{8\pi G_0}.$$

Now we look for solutions of Eq. (9) with

$$\rho a^{3\gamma} = D a^{-f} + E a^{-g} \quad (23)$$

where B , D , and E are constants.

This requires

$$f = q - 2n - 3\gamma \quad (24)$$

$$g = 2 + q - 2n - 3\gamma = 2 + f \quad (25)$$

$$D = \frac{\Lambda c_0^2 (q - 2n)}{8\pi G_0 (2n + 3\gamma)} \quad (26)$$

$$E = \frac{3Kc_0^2 n}{4\pi G_0 (2n + 3\gamma - 2)} \quad (27)$$

where we only write down the solution for the cases where $2n + 3\gamma \neq 0$ or 2 . Thus, the density is

$$8\pi G(t)\rho = \frac{\Lambda c_0^2 (q - 2n) a^{2n}}{2n + 3\gamma} + \frac{6Kc_0^2 n a^{2n-2}}{2n + 3\gamma - 2}. \quad (28)$$

If we substitute in the Friedmann equation containing ρ_Λ ,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} (\rho + \rho_\Lambda) - \frac{Kc^2}{a^2} \quad (29)$$

then we have

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c_0^2 a^{2n} (q + 3\gamma)}{3(2n + 3\gamma)} + \frac{Kc_0^2 a^{2n-2} (2 - 3\gamma)}{2n + 3\gamma - 2}. \quad (30)$$

We see that the Λ term on the right-hand side of Eq. (30) will fall off faster than the curvature and matter density terms so long as $c(t)$ falls off fast enough, with

$$n < -\frac{3\gamma}{2}. \quad (31)$$

As expected, this requires a more rapid increase in $c(t)$ than is required to solve the flatness or horizon problems. In particular, in the radiation and dust cases it requires c to fall as the Universe expands, with $n < -2$ and $n < -3/2$, respectively.

V. BETTER FORMULATIONS

The most challenging problem besetting any attempt to examine the cosmological consequences of varying c is the formulation of a self-consistent theory which incorporates such a variation. The AM approach is not the only way of proceeding and so we should look more critically at its relation to other things that we know. The most interesting feature in this respect is the fact that it permits a description of varying G when c is kept constant. However, it is clear that in this case the resulting cosmological equations (4), (5) and (6) are not the equations describing the evolution of the Friedmann universe in a scalar-tensor gravity theory, like JBD. The reasons are clear. JBD theory meets the covariant divergence constraints imposed by the Bianchi identities by including the energy density contributed by the scalar field that acts as the source of the variations in G . These contributions are not included in the AM formulation. In order to improve upon it we could take a scalar-tensor gravity theory and carry out the same procedure used to derive Eq. (6) and check that the time variation of c is still permitted. This means that we assume that the JBD equations hold for the Friedmann model, with c allowed to be a time variable, and

then derive the conservation equation from them. For JBD theory we have [9,11],

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\rho}{3\phi} - \frac{\dot{\phi}\dot{a}}{\phi a} + \frac{\omega\dot{\phi}^2}{6\phi^2} - \frac{Kc^2(t)}{a^2}, \quad (32)$$

$$\frac{3\ddot{a}}{a} = -\frac{8\pi}{(3+2\omega)\phi} \left[(2+\omega)\rho + \frac{3(1+\omega)p}{c^2(t)} \right] - \frac{\ddot{\phi}}{\phi} - \frac{\omega\dot{\phi}^2}{\phi^2}, \quad (33)$$

$$\ddot{\phi} + \frac{3\dot{a}\dot{\phi}}{a} = \frac{8\pi}{3+2\omega} \left(\rho - \frac{3p}{c^2(t)} \right) = \frac{8\pi\rho(4-3\gamma)}{3+2\omega}. \quad (34)$$

From these three equations we obtain the generalized conservation equation. Since after we impose the equation of state, the varying c term only appears in the first of these equations, the only new contribution to $2\dot{a}\ddot{a}$ is from the $-2Kc\dot{c}$ term. Hence, we have [10]

$$\dot{\rho} + 3\frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) = \frac{3Kc\dot{c}\phi}{4\pi a^2}. \quad (35)$$

It is instructive to compare this to the AM equation (6). We see that the incorporation of G variation differs. The $\rho\dot{G}/G$ term of Eq. (35) is absent but the K term has the same form with $G(t)$ replaced by $\phi^{-1}(t)$ as usual in JBD theory. However, the new system is more constrained because solutions to Eqs. (34) and (35) must be consistent.

When the matter content is radiation the conservation equation (34) has the particular solution

$$\phi = G^{-1} = \text{const}$$

and then Eq. (35) is identical to the AM equation (6) in the case where G is constant but c varies with time. The conditions for resolving the flatness and cosmological constant problems then become the same as given for the AM formulation above. For other equations of state the situation is more complicated. Rearranging the equations, we need to solve

$$(\rho a^{3\gamma})' = \frac{3Kc\dot{c}\phi a^{3\gamma-2}}{4\pi}$$

$$(\dot{\phi} a^3)' = \frac{8\pi\rho a^3(4-3\gamma)}{3+2\omega}$$

together with Eq. (32). When $K=0$ they are solved by the usual exact solutions of JBD theory. When the universe is radiation dominated these equations have particular solutions with ϕ constant which are identical to those derived in the last section for the case of varying c and constant G . These equations will be studied in more detail elsewhere [14].

The JBD theory is not the most general scalar-tensor gravity theory. It is a particular case of a general class in which the constant BD parameter, ω , is a function of the

scalar field ϕ [11]. For these theories, the Friedmann universe (with c constant) is governed by the equations (in the $\Lambda=0$ case)

$$2\left(\frac{\dot{a}}{a}\right)' + \left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{Kc^2}{a^2} = -\frac{\omega(\phi)\dot{\phi}^2}{2\phi^2} - \frac{8\pi p}{\phi c^2} - \frac{\ddot{\phi}}{\phi},$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi\rho}{3\phi} - \frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\omega(\phi)\dot{\phi}^2}{6\phi^2} - \frac{Kc^2}{a^2}, \quad (36)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\omega'\dot{\phi}^2}{2\omega+3} = \frac{8\pi\rho(4-3\gamma)}{2\omega+3}. \quad (37)$$

From these equations we obtain the same generalization of the mass-energy conservation as in JBD [which is the case $\omega(\phi)=\text{constant}$], given by Eq. (35). Again, radiation-dominated solutions (with $\phi=G^{-1}$ const) and varying c of the form found in the last section are always exact particular solutions of these equations.

VI. SOME PROBLEMS

A. The velocity problem

If c varies there may be a problem with the perturbations to the isotropic expansion of the universe which are powers of v/c . For example, for rotational velocity perturbations in matter obeying $p=(\gamma-1)\rho c^2$, the conservation of angular momentum gives $\rho a^4 v = \text{constant}$. Although the original formulation described above does not allow us to impose energy conservation in the usual way ($E \propto c^2$), we shall assume that angular momentum conservation is preserved since it does not incorporate direct c dependence. Hence, we have

$$v \propto a^{3\gamma-4}.$$

Now, if

$$c(t) = c_0 a^n$$

we have

$$\frac{v}{c} \propto a^{3\gamma-4-n}.$$

If the flatness problem is to be solved, we need $2-2n > 3\gamma$ so we will have $3\gamma-4-n < -2-3n$. For the Λ problem to be solved we need $3\gamma+2n < 0$. Hence, in the radiation era we need $n < -1$ to solve the flatness problem and $n < -2$ to solve the Λ problem. Therefore, when $\gamma=4/3$, we have

$$\frac{v}{c} \propto a^{-n}$$

we see that v/c grows in time in both cases as $a \rightarrow \infty$. During the dust era, $\gamma=1$, and so

$$\frac{v}{c} \propto a^{-1-n}$$

where $n < -1/2$ solves flatness (allowing v/c to decay in $-1 < n < -1/2$) and $n < -3/2$ solves Λ (which would always require v/c to grow).

In general, the condition for solution of the flatness problem has v/c growing as a power of a equal to $3\gamma - 4 - n > 9\gamma/2 - 5$, while a solution of the Λ problem has it growing as $3\gamma - 4 - n > 9\gamma/2 - 4$. So we can only *avoid* blow-up of the velocity perturbations at large a if $\gamma < 10/9$ (if we also want to solve flatness) or $\gamma < 8/9$ (if we also want to solve Λ).

B. Shear evolution

We can extend the AM model to include the effects of simple anisotropies with isotropic curvatures. This would correspond to generalizations of the Bianchi type I and type V universes in the flat and open cases, respectively. In the spirit of Eqs. (4), (5) the required generalization which includes the shear anisotropy scalar, σ , is

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)\rho}{3} + \frac{\sigma^2}{3} - \frac{Kc^2(t)}{a^2} \quad (38)$$

$$\frac{\dot{\sigma}}{\sigma} = -\frac{4\pi G(t)}{3} \left(\rho + \frac{3p}{c^2(t)} \right) a - \frac{2\sigma^2}{3} \quad (39)$$

where $a(t)$ is now the geometric-mean expansion scale factor. From these equations we may derive the analogue of the matter conservation equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) + \frac{\sigma}{4\pi G} \left(\dot{\sigma} + 3\frac{\dot{a}}{a} \sigma \right) = -\rho \frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G a^2}. \quad (40)$$

This equation resembles the conservation equation for two non-interacting perfect fluids, one of which (the ‘‘anisotropy’’ energy density) has the equation of state $p = \rho c^2$. However, we see that the original AM prescription leaves the material density coupled to the shear evolution. If we add the shear evolution equation from the standard (constant c) case then we have

$$\dot{\sigma} + 3\frac{\dot{a}}{a} \sigma = 0 \quad (41)$$

and the density evolution obeys the same equation as that for the isotropic model ($\sigma = 0$).

C. Black holes

It is interesting to note that if a black hole forms with radius

$$R_g = \frac{2GM}{c^2} \quad (42)$$

then, from Eq. (42)

$$\frac{\dot{R}_g}{R_g} = \frac{\dot{G}}{G} - 2\frac{\dot{c}}{c} = -\frac{\dot{\rho}_\Lambda}{\rho_\Lambda}.$$

This seems to imply that if G is constant and c is falling then primordial black holes would grow in size at the same rate as the universe is expanding. There will also be consequences for black hole evaporation. The Planck scale will grow rapidly $\propto c^{-5}$. It is possible to examine the changes to the laws of black hole mechanics using the results for the change in the characteristics of black body radiation thermodynamics but we do not digress to discuss this here.

D. Late-time fields

Another aspect of this model is worth commenting on. We have determined the rate of variation of $c(t)$ that would allow the flatness (or Λ) to be solved. However, we could also use these equations in the cases where the flatness problem was *not* solved in the same way as a description of a cosmological matter field that plays the role of a matter field which can dominate the expansion dynamics of the universe at late times. Of course, as with the possibility of present-day contributions to the dynamics by the curvature or Λ terms in the case where c is constant, this requires a special tuning of the initial sizes of these terms in the Friedmann equation with respect to the density term in order that their effects just start to become significant close to the present epoch.

An interesting point to notice about c -varying models is that if the universe were to contain a dynamically significant cosmological constant term at late times then it would naturally have negligible curvature term and so resemble a zero-curvature universe with subcritical total density, just like inflationary universes with a residual lambda term. This property allows natural almost flat asymptotes to arise, as pointed out in Ref. [14].

E. Fine structure constant variations

One interesting feature of any self-consistent theory of c variation is that it enables us to evaluate the consequences of time (and space) variations in dimensionless coupling constants like the fine structure constant. In an expanding universe with c varying we require $\rho c^2 \propto (k_B T)^4 / (\hbar c)^3$. Assuming particle wavelengths are unchanged in the absence of expansion and the masses of quantum particles are conserved so \hbar/c remains constant. As the universe expands the Planck spectrum remain Planckian but the temperature varies as [1]

$$\dot{T} + T \left(\frac{\dot{a}}{a} - 2\frac{\dot{c}}{c} \right) = 0.$$

Hence,

$$T \propto \frac{c^2}{a} \propto a^{2n-1}$$

for the power-law variation, Eq. (12), assumed above. Strong observational limits could be placed on any deviation of n from zero using data which establish bounds on the temperature of the microwave background at low redshifts.

The required variation of \hbar/c means that (for constant e^2) the fine structure constant varies as

$$\alpha = \frac{e^2}{\hbar c} \propto \frac{1}{c^2}.$$

If we assume a more modest variation in the evolution of $c(t) \propto (\ln t)^{-1}$ then it becomes possible to examine some possible late-time consequences of varying α using this model. A detailed particular theoretical formulation has been provided by Bekenstein [5]. Elsewhere, Barrow and Magueijo [13] have provided a Lagrangian-based theory with varying e which differs from Bekenstein’s (unlike Bekenstein’s theory it does not postulate that the gravitational field equations remain unchanged). This theory may be transformed by a suitable change of units into a theory with constant electron charge e and varying c like the one described above, or it may be presented as a theory with constant c and varying e . This features displays explicitly that invariant meaning can only be ascribed to variations of dimensionless “constants” (in this case the fine structure constant).

A more far-reaching aspect of c -variation is the fact that a fall in the value of c at very early times would lead to a strengthening of dimensionless gauge couplings unless other changes in the structure of physics could offset this. Thus, attempts to reconstruct the history of the very early universe could not count on the applicability of asymptotic freedom and the ideal gas condition at early times in the usual way.

VII. DISCUSSION

The contemplation of a variation in the speed of light in the very early universe presents a host of self-consistency problems and boundary effects with other parts of the core of modern physical theory. Unlike the case of varying G , it requires deep structural changes to many of the foundations of physics. In this paper we have considered a minimalist varying- c theory recently proposed by Albrecht and Magueijo [1] as a new way of solving the horizon and flatness problems. However, unlike inflation, it also offers a way of solving the cosmological constant problem. Whereas Albrecht and Magueijo modelled changes in c as a sudden fall in the universal value of c , occurring as if at a phase transition, we have considered the behavior of variations which vary as a power of the cosmological scale factor in order to determine the rates of change that are needed to solve different cosmological problems. Our main results are as follows:

(a) If the velocity of light varies with the cosmological scale factor, $a(t)$, as $c = c_0 a^n$ and the equation of state of matter is $p = (\gamma - 1)\rho c^2$, then the curvature term becomes negligible for the expansion of the Universe as $a \rightarrow \infty$ if c falls fast enough, with $n \leq \frac{1}{2}(2 - 3\gamma)$, regardless of the behavior of $G(t)$. The same condition allows for a resolution of the horizon problem. This generalizes the conditions for the inflationary resolution of the flatness and horizon problems which apply when c is constant ($n = 0$) and G is constant.

(b) If the velocity of light varies with the cosmological scale factor, $a(t)$, as $c = c_0 a^n$, the gravitation “constant” varies as $G(t) = G_0 a^q$, and the equation of state of matter is $p = (\gamma - 1)\rho c^2$, then the condition for a vacuum stress with

$p_\Lambda = -\rho_\Lambda c^2$, associated with a geometrical cosmological constant term to become negligible as $a \rightarrow \infty$ is that c fall at a rate such that $n < -3\gamma/2$.

(c) We have shown how it is possible to improve the formulation of the problem in order to recover a scalar-tensor theory describing G variation in the limit of constant c . This formulation only allows the solutions of the AM theory to persist in the case when the equation of state is that of black body radiation. This problem also suggests that we should attempt a fuller formulation of the varying- c theory by deriving the variation of c from some scalar field that contributes an energy density to the evolution of the Universe.

(d) We have highlighted some potential problems with the evolution of rotational velocities as the universe expands. It is also worth noting that the varying- c model does not solve the flatness and horizon problems by means of a period of evolution that is close to that described by a time-translation invariant de Sitter space-time. As a result there does not appear to be any distinctive set of fluctuations that should emerge from a period of cosmological evolution in which c changes.

(e) Possible variations in c lead to changes in the fine structure constant and to other gauge couplings in the very early universe if no other changes in physics exist. However, if the variation in c is confined to a very early period of evolution soon after the expansion commenced there need be no conflict with astronomical constraints on any time or space variation in the fine structure constant.

(f) A varying- c theory can be transformed in to a varying- e theory with constant c by a suitable transformation of units. Theories of this sort can be given a Lagrangian formulation if required.

We believe that these features of the naive varying- c model proposed by Albrecht and Magueijo are sufficiently interesting for it to be worthwhile exploring a fuller, more rigorous formulation of a varying- c gravity theory based upon an action principle which would enable its consequences for other aspects of physics and cosmology to be investigated more rigorously.

Note added. After this paper was completed, the author’s attention was drawn to two early papers by Moffat [15] which investigate the sudden variation in the value of the speed of light as a possible solution for the horizon and flatness problems which predate the study of Ref. [1] that motivated the discussion above. The reader is referred to these papers for further details of the model for discontinuous change in c that is proposed by Moffat.

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