Observational constraints on power-law cosmologies

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In a class of models designed to solve the cosmological constant problem by coupling scalar or tensor classical fields to the space-time curvature, the universal scale factor grows as a power law in the age, $a \propto t^{\alpha}$, regardless of the matter content or cosmological epoch. We investigate constraints on such ''power-law cosmologies'' from the present age of the Universe, the magnitude-redshift relation, and from primordial nucleosynthesis. Constraints from the current age of the Universe and from the high-redshift supernovae data require ''large'' α (≈ 1), while consistency with the inferred primordial abundances of deuterium and helium-4 forces α to lie in a very narrow range around a lower value (≈ 0.55). Inconsistency between these independent cosmological constraints suggests that such power-law cosmologies are not viable.
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I. INTRODUCTION

According to general relativity all mass or energy gravitates, including the energy density of the vacuum. In modern quantum field theory the vacuum is the lowest energy—but not necessarily the zero energy—state. From this perspective a cosmological constant (Λ) may be associated with the vacuum energy density, $\rho_{\rm vac} = \Lambda/8\pi G \equiv \Omega_{\Lambda}\rho_{\rm c}$, where $\rho_{\rm c} \equiv 3{\rm H_0}^2/8\pi G \sim 10^{-48}$ GeV^4 is the critical density. Although some recent data favor a non-zero value of Λ [1], observations do limit $\Omega_{\Lambda} \lesssim 1$ [2] corresponding to a vacuum energy density that is very small when compared to that expected from physics at the Planck scale ($\sim 10^{19}$ GeV). This is because although we may wish to set $\Lambda = 0$ in the Einstein equations, quantum fluctuations in the fields present in the Universe can establish a non-zero vacuum energy and, hence, a non-zero effective cosmological constant. We may associate the vacuum energy density with an energy scale M which might be the scale associated with the spontaneous symmetry breaking from one vacuum state to another, $\rho_{\rm vac} \sim M^4$. In some sense the only ''natural'' scale in cosmology is the Planck scale, $M \sim 10^{19}$ GeV. In this case the observations require that the present vacuum energy density is some 120 orders of magnitude smaller than its ''natural'' value. The smallness of $\rho_{\rm vac}$ is a key problem in modern cosmology: the ''Λ'' or ''cosmological constant problem'' [3].

One class of attempts to solve the Λ problem considers the evolution of classical fields which are coupled to the curvature of the space-time background in such a way that their contribution to the energy density self-adjusts to cancel the vacuum energy [4]. Although the dynamical framework in these approaches is well defined, the addition of the special fields is unmotivated but for solving the cosmological constant problem. The common result of these approaches is that the vacuum energy may be nearly cancelled and the expansion of the Universe is governed by the uncompensated vacuum energy density. In such models the expansion is a power-law in time, independent of the matter content or cosmological epoch (see Ford [4]). That is, in such models the scale factor varies according to $a(t) \propto t^{\alpha}$, where α is determined solely by the parameters of the model and can be anywhere in the range $0 \leqslant \alpha \leqslant \infty$. In addition, there are models designed to solve other cosmological fine-tuning problems (e.g., flatness [5]) which also result in power-law cosmologies.

In this paper we explore the constraints on α from the age-expansion rate data, from the magnitude-redshift relation of type Ia supernovae (SN Ia) at redshifts 0.4–0.8, and from the requirement that primordial nucleosynthesis produce deuterium and helium-4 in abundances consistent with those inferred from observational data.

II. CONSTRAINTS FROM THE AGE-EXPANSION RATE

In power-law cosmologies the scale factor $a(t)$, the redshift z, and the cosmic microwave background (CMB) temperature $T(t)$ are related to their present values (labelled by the subscript ''0'') by

$$a/a_0 = 1/(1+z) = T_0/\beta T = (t/t_0)^{\alpha}, \tag{1}$$

where β accounts for any non-adiabatic expansion due to entropy production [e.g., in standard cosmology $\beta = 1$ for $T < m_e$ and $\beta = (11/4)^{1/3}$ for $T > m_e$ accounting for the heating due to $e^{\pm}$ annihilation assuming instantaneous annihilation at $T = m_e$]. All models (all choices of α) are ''normalized'' by requiring that they have the current temperature, $T_0 = 2.728$ K [6], at present (t_0). For the present age of the Universe we adopt $t_0 = 14 \pm 2$ Gyr [7]; we explore the small effect on our constraints of this choice for t_0. The Hubble parameter, $H = \dot{a}/a$ provides a measure of the expansion rate. For power-law cosmologies, $Ht = \alpha$, so that at present $H_0 t_0 = \alpha$. If we adopt a central estimate and allow for a generous uncertainty in the Hubble parameter $H_0 = 70 \pm 10$ km s^{-1} Mpc^{-1} [8], we limit α:

$$\alpha = H_0 t_0 = 1.0 \pm 0.2. \quad (2)$$

Consistency with the present age of the Universe suggests that $\alpha \gtrsim 0.6$. In order for α to be as small as 0.5, $H_0 \sim 50$ km s^{-1} Mpc^{-1} and $t_0 \sim 10$ Gyr.

III. CONSTRAINTS FROM THE MAGNITUDE-REDSHIFT RELATION

The expansion of the Universe in power-law cosmologies is completely described by the Hubble parameter and the deceleration parameter. In these models the deceleration parameter is

$$q(t) = -H^{-2}(\ddot{a}/a) = q_0 = \frac{1}{\alpha} - 1, \quad (3)$$

so that for $\alpha \gtrsim 1/2$, $q_0 \lesssim 1$. The larger α, the smaller q_0 and vice versa. As $\alpha (= H_0 t_0)$ increases from 1/2 to 1, q_0 decreases from 1 to 0; negative values of q_0 require $\alpha > 1$.

For spatially flat power-law cosmologies the luminosity distance and/or angular-diameter distance redshift relations assume a very simple form. The luminosity distance (in units of the Hubble distance c/H_0) as a function of redshift is

$$d_L(z) = q_0^{-1}[(1+z) - (1+z)^{(1-q_0)}]. \quad (4)$$

Note that for $\alpha = 1/2$ $(q_0 = 1)$, $d_L(z) = z$ while for $\alpha = 1$ $(q_0 = 0)$, $d_L(z) = z(1+z)$. Recently two independent groups [1] have been accumulating observations of possible ''standard candles,'' SN Ia, at relatively high redshifts ($z \sim 0.4 - 0.8$). The difference in apparent magnitudes of objects with the same intrinsic luminosity but at different redshifts provides a valuable, classical cosmological test. The figure of merit for power-law cosmologies is the expected difference in apparent magnitudes, $\Delta m \equiv 5 \log[d_L(z_2)/d_L(z_1)]$, for $z_1 = 0.4$ and $z_2 = 0.8$ as a function of α. As α increases from 1/2 to 1, Δm increases from 1.5 (magnitudes) to 1.8. For comparison, the recent discovery of a SN Ia at $z = 0.83$ by Perlmutter *et al.* [1] suggests that $\Delta m \approx 2.0 \pm 0.2$, favoring a small (or even negative) q_0, corresponding to a ''large'' value of $\alpha \gtrsim 1$. The case for negative q_0 is strengthened with additional data from the High-Z Supernova Search Team [1]: $q_0 < 0$ at the 3-σ level, corresponding to $\alpha > 1$ with similar statistical confidence. Primordial nucleosythesis provides a powerful constraint on such models.

IV. CONSTRAINTS FROM PRIMORDIAL NUCLEOSYNTHESIS

As we have seen above, observations of the recent Universe ($z \lesssim 1$) favor values of α close to unity. Such power-law models are a disaster for primordial nucleosynthesis. For example, suppose that $\alpha = 1$, as results in some Λ-regulating models [4] or as proposed in Allen [5], and ask how old was the Universe when nucleosynthesis began at a temperature of order 80 keV? From Eq. (1) the answer is 45 years. At this stage any neutrons have long since decayed and there can be no nucleosynthesis.[1] This simple example helps to focus the physical origin of the big bang nucleosynthesis (BBN) challenge to power-law cosmologies. Unless a suitable early time-temperature relation exists, neither helium-4 nor deuterium will be produced primordially in amounts comparable to those inferred from the observational data.

In our discussion we concentrate on the abundances of ^{4}He and D for the following reasons. Observations reveal that the most metal-poor stars and/or H II regions have a *minimum*, non-zero abundance of ^{4}He; for this primordial mass fraction we adopt the generous range $0.22 \leqslant Y_P \leqslant 0.26$ [10]. Any viable cosmological model must account for this much ^{4}He. Similarly, the observation of significant abundances of deuterium requires primordial production [11]. Here, too, we adopt a generous range $1 \times 10^{-5} \leqslant D/H \leqslant 2 \times 10^{-4}$ [12]. Any model producing too much or too little deuterium is excluded. Note that if/when we identify a viable power-law model consistent with these D and ^{4}He constraints, we do check the consistency of the predicted abundances of ^{3}He and ^{7}Li.

To understand BBN in power-law cosmologies it is helpful to briefly review primordial nucleosynthesis in the standard model (SBBN). At high temperature ($\gtrsim 1$ MeV) charged-current weak interactions among neutrons, protons, electrons, positrons, and neutrinos maintain neutron-proton equilibrium: $n/p = \exp(-Q/T)$ (where $Q = 1.29$ MeV is the neutron-proton mass difference). In SBBN the weak interaction rates interconverting neutrons and protons become slower than the universal expansion rate for $T \lesssim 1$ MeV (when the Universe is of order 1 s old) and the n/p ratio ''freezes out,'' decreasing only very slowly due to out-of-equilibrium weak interactions and free neutron decay (with a lifetime of 887 s). All the while neutrons and protons are colliding to form deuterium which is rapidly photodissociated by the cosmic background photons (gamma rays at this epoch). The very low abundance of D removes the platform for building heavier nuclei; nucleosynthesis is delayed by this ''photodissociation bottleneck.'' When the temperature drops below ~ 80 keV (the Universe is ~ 3 minutes old) the deuterium bottleneck is broken and nuclear reactions quickly burn the remaining free neutrons ($n/p \approx 1/7$) into ^{4}He ($Y_P \approx 0.25$), leaving behind trace amounts of D, ^{3}He, and ^{7}Li [13]. If the light elements are to be properly synthesized during BBN, the above scenario must be mimicked in a viable power-law cosmology.

First, let us consider the photodissociation bottleneck and free neutron decay. To ensure that *some* primordial nucleosynthesis will occur neutrons must have not decayed before the deuterium bottleneck is broken. Thus, we require that $t \lesssim 887$ s when $T \approx 80$ keV. This leads immediately to a constraint on α (which depends only logarithmically on our choice of $t_0 = 14$ Gyr); from Eq. (1), $\alpha \lesssim 0.58$.

Power-law models which succeed in having **any** *BBN are*

[1] We verify in our numerical results presented below that pp or $p + e^- + p$ (pep) reactions are inadequate to compensate for the absence of neutrons.

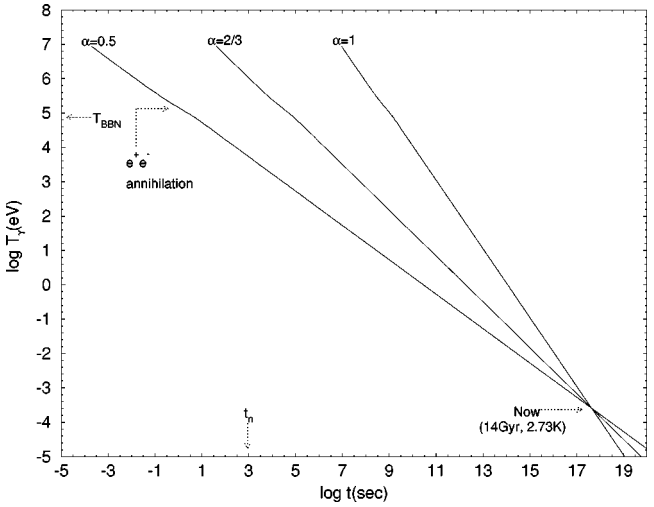

FIG. 1. The age-temperature relation for three power-law cosmologies ($\alpha=1/2,\ 2/3$, and 1). Time is measured in seconds and temperature in eV. $T_{BBN}\approx 80$ keV is the temperature at which nucleosynthesis begins; $t_n=887$ s is the neutron lifetime; $t_0=14$ Gyr is the present age of the Universe (where $T_0=2.73$ K). Note the slight ''kink'' due to entropy production at $e^{\pm}$ annihilation.

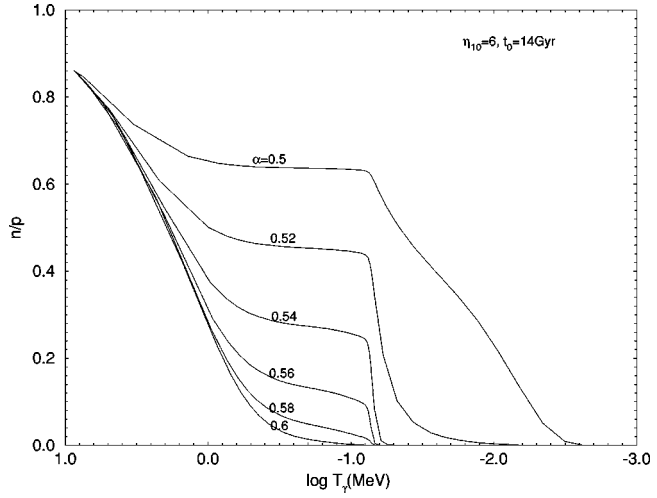

FIG. 2. The neutron-to-proton ratio as a function of temperature for several choices of α. $T_{BBN}\approx 80$ keV is the temperature at which nucleosynthesis begins.

in conflict with the constraints from the present age/Hubble parameter and the SN Ia magnitude-redshift relations discussed above.

To explore BBN in power-law cosmologies in more detail it is important to understand how the time-temperature relation in these models changes with α. In Fig. 1 the time-temperature relation is shown for several choices of α. The larger α, the faster the Universe expands. For example, for a fixed time early on (say, $t=t_n=887$ s) the higher α, the higher the temperature. Similarly, if we fix on a definite temperature in the early Universe, the higher α the older the Universe. This may seem counterintuitive because, although the Universe (with higher α) is indeed expanding faster, it is *younger* at a fixed temperature for *lower* α since all models (choices of α) are constrained to have $T_0=2.7$ K at 14 Gyr. Thus the requirement that BBN occur before the free neutrons decay bounds α from above.

There also exists a *lower* bound to α since for low α (young Universe) the Universe will have too little time for nuclear reactions to build up any significant abundances of the light elements. As α decreases, the weak interactions decouple at higher temperatures and neutrons have less time to decay thus leading to a larger neutron fraction at the time of nucleosynthesis. Provided nuclear reactions are efficient, this increased neutron fraction results in more ^{4}He. However, once α becomes sufficiently small, nuclear reactions become inefficient and no nucleosynthesis occurs. For small enough α,^{4}He should decrease with decreasing α. The critical α delineating these regimes depends on the nucleon density since a young age can be compensated by having a higher nucleon density leading to faster nuclear reaction rates. We have, therefore, explored BBN numerically for a wide range of choices of α and of η, the universal nucleon-to-photon ratio ($\eta\equiv n_N/n_\gamma$; $\eta_{10}\equiv 10^{10}\eta$).

In Fig. 2 the evolution of the neutron-to-proton ratio as a function of temperature is shown for several choices of α. For $T\gtrsim 80$ keV, the decline in n/p reflects neutron decay; the larger α, the older the Universe (for fixed T), and the more neutrons have decayed. The precipitous decline in n/p for $T\lesssim 80$ keV is due mainly to nuclear reactions incorporating free neutrons in the light nuclides. As expected from our semi-analytic argument above, if α is too large, too few neutrons remain when BBN can begin. Note that the smaller α the larger the freeze-out abundance of neutrons (at ~ 1 MeV) and the smaller the effects of neutron decay. If α is too small, nuclear reactions are inefficient at forming heavier nuclei and the decline after 80 keV is not as severe.

In Figs. 3 and 4 we concentrate on the *interesting* range of α and show the predicted abundances of ^{4}He (Fig. 3) and of D (Fig. 4) for a variety of η values covering more than an order of magnitude. In Fig. 5 we show iso-abundance con-

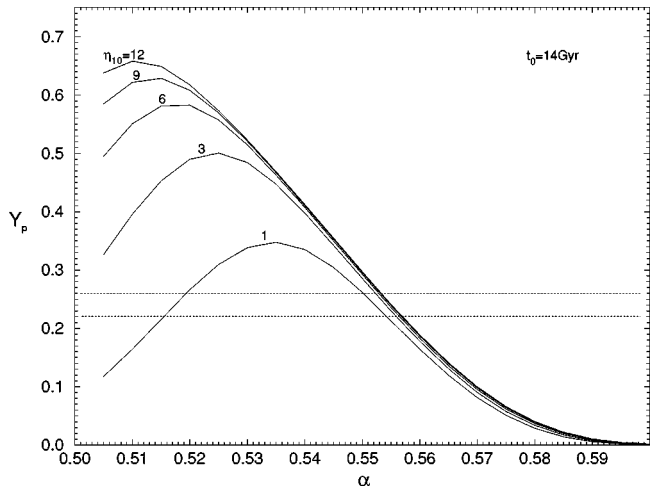

FIG. 3. The primordial ^{4}He mass fraction, Y_P, as a function of α for several choices of the nucleon-to-photon ratio, $\eta_{10}\equiv 10^{10}n_N/n_\gamma$. The horizontal dotted lines delimit the adopted range of the primordial abundance.

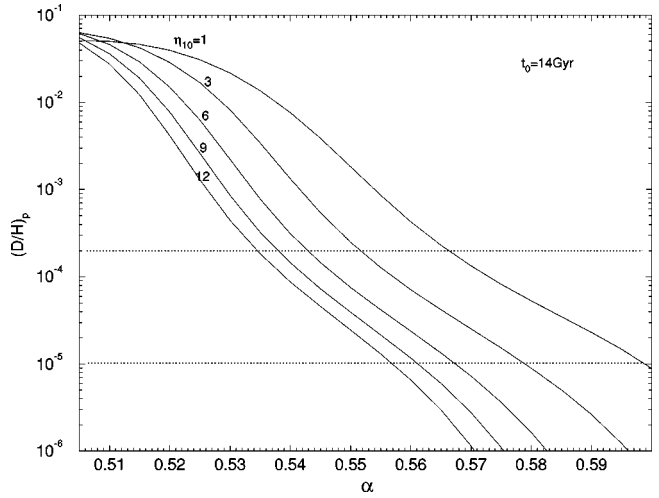

FIG. 4. As for Fig. 3 for the primordial deuterium abundance, D/H.

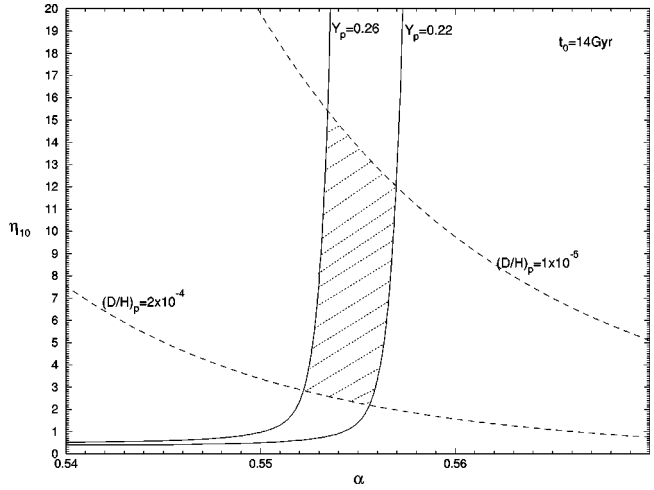

FIG. 5. Iso-abundance contours in the $\eta_{10}-\alpha$ plane corresponding to the observational bounds on D and ^{4}He.

tours in the $\eta-\alpha$ plane corresponding to the observed ranges for D and ^{4}He adopted above. As anticipated, there is a very narrow range of α (0.552–0.557) which results in an acceptable yield of primordial ^{4}He and D for $2\lesssim\eta_{10}\lesssim 15$. If we impose a further constraint from ^{7}Li [14], the upper bound on η_{10} is reduced to 12.

A simple heuristic argument will serve to expose the physical origin of this concordant range for α. For these values of α, the power-law cosmologies are evolving very similarly to the standard model for 1 MeV $\gtrsim$ T $\gtrsim$ 30 keV. For α in this very narrow range the ages at fixed temperature are, within factors of order unity, the same as those in SBBN (i.e., for $\alpha\sim 0.55$, the temperature is around 1 MeV when the Universe is 1 second old and the temperature is around 100 keV when it is 1 minute old, ensuring that weak freeze out and the onset of nucleosynthesis work in concert as in SBBN). It is worth noting that this range for α is insensitive (logarithmically) to our choice of 14 Gyr for the present age of the Universe.

As Fig. 3 reveals, there is a second, lower range of values of α which, depending on η, might yield acceptable primordial helium. Although such models have very high neutron fractions when BBN commences (see Fig. 2), these models are so young the time for complete BBN is insufficient (unless the nucleon density is sufficiently high). This is shown dramatically in Fig. 4 where the very high deuterium abundances reflect the incomplete burning to ^{4}He. Note that the primordial yields in this low α limit are very sensitive to η since the yields are set by a competition between expansion and nuclear reaction rates. However, these lower values of α do not provide viable power law models from the BBN perspective since they cannot simultaneously produce the correct abundances of ^{4}He and D. In the limit of low α, D is always overproduced relative to ^{4}He since the Coulomb barriers involved in ^{4}He production inhibit the burning of D to ^{4}He.

There is one caveat we should note concerning our constraint on power-law cosmologies from BBN. Basically, we have seen that BBN requires that a potentially successful model have a time-temperature relation which crosses the lower left-hand region in Fig. 1 where t$\sim$887 s (the neutron lifetime) when T = 80 keV. This requirement has permitted the exclusion of larger values of α. If, however, entropy is released during or after BBN, a successful time-temperature relation may have existed during BBN for values of α larger than those allowed in the absence of such entropy production. For example, entropy may be released through the decay of a massive particle such as those associated with the moduli fields of supersymmetry (supergravity) [9] theory. Given the extra free parameters associated with this possibility [amount and timing of the entropy release; e.g., mass and lifetime of the decaying massive particle(s)], we have chosen to not explore here this option for avoiding our BBN constraints on power-law cosmologies.

V. DISCUSSION

Can the evolution of the Universe—from very early epochs to the present—be described by a simple power law relation between the age and the scale factor (temperature)? In standard cosmology the early Universe is radiation dominated (RD) and the expansion is a power law with $\alpha_{\rm RD}=1/2$. But, in standard cosmology the Universe switched from RD to matter dominated (MD) at a redshift between 10^3 and 10^4. Thereafter the Universe expanded (for a while at least) according to a power law with a different power: $\alpha_{\rm MD}=2/3$. If the present Universe has a low density (compared to the critical density) and lacks a significant cosmological constant, it is ''curvature'' dominated (CD) and its expansion may be well approximated by a power law with $\alpha_{\rm CD}=1$. Thus in standard cosmology, although power law expansion may provide a good description for some epochs, there is no single power which can describe the entire evolution from, for example, BBN to the present. The question then is, can a ''compromise'' α be found which is consistent with BBN as well as with observations of the present or recent Universe?

We have explored this question and answered it in the

negative. The present age-expansion rate (Hubble parameter) constraint $\alpha = H_0 t_0 = 1.0 \pm 0.2$ and the SN Ia magnitude-redshift relation require $\alpha \approx 1$ (or, $\alpha \gtrsim 0.6$), while production of primordial helium and deuterium force α to be smaller. The extreme sensitivity of the helium yield to α (see Fig. 3), precludes raising the upper bound on α from BBN. Unless the Universe is much younger ($\lesssim 10$ Gyr) and the Hubble parameter much smaller ($\lesssim 50$ km s^{-1} Mpc^{-1}) than currently believed and the SN Ia magnitude-redshift relation plagued by systematic errors, or there was substantial entropy release after BBN, power law cosmologies are not the solution to the cosmological constant problem.

ACKNOWLEDGMENTS

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