

Can induced gravity isotropize Bianchi type I, V, or IX universes?

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We analyze if Bianchi type I, V, and IX models in the induced gravity (IG) theory can evolve to a Friedmann-Roberson-Walker (FRW) expansion due to the nonminimal coupling of gravity and the scalar field. The analytical results that we find for the Brans-Dicke (BD) theory are now applied to the IG theory which has $\omega \ll 1$ (ω being the square ratio of the Higgs boson to Planck mass) in a cosmological era in which the IG potential is not significant. We find that the isotropization mechanism crucially depends on the value of ω . Its smallness also permits inflationary solutions. For the Bianchi type V model, inflation due to the Higgs potential takes place afterwards, and subsequently spontaneous symmetry breaking ends with an effective FRW evolution. The ordinary tests of successful cosmology are well satisfied. [S0556-2821(98)06624-7]

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I. INTRODUCTION

One of the main problems of modern cosmology is to find a satisfactory explanation to both the small-scale inhomogeneity of matter distribution and the large-scale degree of isotropy measured in the cosmic microwave background radiation (CMBR) by the Cosmic Background Explorer (COBE) satellite [1,2]. Accordingly, one desires to construct a general model that explains both, antagonistic, properties of our Universe. In order to find such a solution, it is interesting to investigate if homogeneous, anisotropic (Bianchi) models can predict the level of isotropy detected by the COBE satellite and, at the same time, reproduce the local character of our Universe. There have been various attempts to solve this problem (for instance, see Ref. [3]), but inflationary cosmologies are still the most appealing since they provide explanations to some other problems, as well. Accordingly, if at its outset the Universe was neither homogeneous nor isotropic, then because of a de Sitter stage, it would tend to homogeneity and isotropy. Inflationary models, however, still assume some fine-tuned initial conditions and most of them have assumed a Friedmann-Robertson-Walker (FRW) symmetry from the outset: This is the first fine-tuning one invokes in doing cosmology, since from all possible sets of initial conditions, a FRW universe selects a very special set of homogeneous and isotropic space-time geometries. Then, by considering more general space-time symmetries the question arises as to whether inhomogeneous and/or anisotropic cosmological models help to understand the naturalness of inflation. In general relativity (GR) within anisotropic, Bianchi-type models it is claimed that a positive cosmological constant provides an effective means of isotropizing homogeneous universes [4]. The idea behind is that of the cosmic no-hair conjecture which states that in the pres-

ence of a cosmological constant the universe evolves into a de Sitter space-time [5], at least locally [6]. The no-hair property ensures that all inhomogeneities will be smoothed out in a region of the event horizon. The conjecture has been proved for a number of models [7], where it was realized that it is highly related to the homogenization and isotropization of cosmological models. The initial conditions for inflation have been reviewed in Ref. [8] and the situation is that inhomogeneous, anisotropic models with negative curvature satisfy the conjecture as well, but big initial inhomogeneities lead to the formation of black holes in some regions [9]; however, in other regions inflation is possible, achieving a physical scenario in which inflating regions are surrounded by black holes [10]. This resembles the chaotic scenario of initial conditions [11].

In general, it is suggested that a patch of the Universe should be at some level homogeneous to consider it as a right model where inflation can take place; otherwise, inflation can be prevented. Accordingly, before one regards inflation one should analyze the properties of the Universe, and see if some set of initial conditions will bring our Universe to a sufficient smooth patch to start the inflationary expansion. This has motivated us to analyze if homogeneous, anisotropic models with a nonminimal coupling tend to lose its hairs. Most of the results above apply to GR with perfect fluids minimally coupled (for nonperfect fluids see Ref. [12]), and for nonminimally coupled fields see Refs. [13–15], where the no-hair conjecture has been proved for some scalar tensor theories, including some particular potentials and various nonminimal couplings. By trying to tackle this problem, we have shown that Bianchi-type models (I, V, and IX) in Brans-Dicke (BD) theory [16] tend to isotropize as time goes on [17]. These models show an asymptotic FRW behavior, but only few with an inflationary stage. However, inflation turns out to be an important, desirable feature to solve the above-mentioned problems. Thus, in order to obtain an inflationary model with a graceful exit, one usually introduces a potential term. Therefore, in the present inves-

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tigation we consider an induced gravity (IG) theory that includes a nonminimal coupling with gravity (in the manner of BD) and a potential associated with the scalar field, which in our case is identified with a Higgs field. Within this theory our cosmological scenario begins with an anisotropic expansion of Bianchi type I, V, or IX, but only type V evolution to a FRW model is consistent with imposed restrictions. In this way one achieves a sufficiently smooth patch in the Universe, preparing the ‘‘system’’ to be able to inflate; i.e., all physical fields present isotropize. The isotropization mechanism occurs in a similar way as in BD theory [17] where no potential exists. However, in the present theory the isotropization process is inflationary, whereas in BD theory this is not necessarily the case; see Ref. [18]. The effective equivalence of both theories is possible because, during the time of isotropization, the potential term in the IG theory will not contribute significantly as a stress energy to the dynamical equations, in our scenario.¹ Afterwards, because of the Higgs potential of the theory, during the spontaneous symmetry breaking (SSB) of the Higgs field, a second inflationary era follows. Finally, after inflation a FRW behavior is dominant.

This work is organized as follows: In Sec. II we present the IG theory, pointing out some differences among it and BD and GR theories. In Sec. III the cosmological field equations are analyzed in view of the isotropization of the solutions, employing some results for BD theory that turn out to be valid for this theory, as well. In Sec. IV we present the above-mentioned physical scenario. Finally, the conclusions are written in Sec. V.

II. IG, BD, AND GR THEORIES

We have investigated two viable IG models of inflation, one using the SU(5) Higgs field [19] and, the other, the SU(2) Higgs field [20], both coupled nonminimally to gravity. The Lagrangian for both theories has the same mathematical form, and therefore, the qualitative behaviors of the cosmological models are very similar. In the present work, we consider the specific physical scenario using the SU(5) Higgs-field–gravity theory, fully discussed in Refs. [19,21]; however, most of the conclusions are valid also for the SU(2) Higgs-field–gravity model. The Lagrangian of the IG theory is, with signature (+,−,−,−),

$$\mathcal{L} = \left[\frac{1}{8\omega} \text{tr} \Phi^\dagger \Phi R + \frac{1}{2} \text{tr} D_\mu \Phi^\dagger D^\mu \Phi - V(\text{tr} \Phi^\dagger \Phi) + L_M \right] \sqrt{-g}, \quad (1)$$

where greek indices denote space-time components, R is the Ricci scalar, and Φ is the SU(5) isotensorial Higgs field. The symbol D_μ means the covariant gauge derivative with respect to all gauged groups: $D_\mu \Phi = \Phi_{|\mu} + i g_5 [A_\mu, \Phi]$, where

$A_\mu = A_\mu^a \tau_a$ are the gauge fields of the inner symmetry group, τ_a are its generators, and g_5 is the coupling constant of the gauge group ($|\mu$ means the usual partial derivative). ω is a dimensionless, coupling constant parameter that regulates the strength of gravitation and L_M contains only the fermionic and massless bosonic fields, which belong to the inner gauge group SU(5), and the Higgs potential is given by

$$\begin{aligned} V(\text{tr} \Phi^\dagger \Phi) &= \frac{\mu^2}{2} \text{tr} \Phi^\dagger \Phi + \frac{\lambda}{4!} (\text{tr} \Phi^\dagger \Phi)^2 + \frac{3}{2} \frac{\mu^4}{\lambda} \\ &= \frac{\lambda}{24} \left(\text{tr} \Phi^\dagger \Phi + 6 \frac{\mu^2}{\lambda} \right)^2, \end{aligned} \quad (2)$$

where we added a constant term to prevent a negative cosmological constant after the SSB.² Because of the presence of mass terms in Eq. (2), the Lagrangian (1) is not conformally invariant with GR (cf. [23]); this is important to mention because there are a number of results using conformal transformations among different theories demanding the same physics, but in our case such transformations are not conformally invariant.

The field $(2\pi/\omega) \text{tr} \Phi^\dagger \Phi$ plays the role of the inverse of Newton’s gravitational constant (G^{-1}), and after a SSB process, when the Φ field becomes a constant, $\text{tr} \Phi^\dagger \Phi = -6\mu^2/\lambda$, the potential vanishes. In this way, after the SSB this theory becomes *effectively* GR. Further, some fermions and boson fields that become massive after the breaking appear as a source’s contribution to the right hand side of Einstein equations; for details see Refs. [20,21].

From Eq. (1), the gravity field equations are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} &= - \frac{4\omega}{\text{tr} \Phi^\dagger \Phi} [T_{\mu\nu} + V(\text{tr} \Phi^\dagger \Phi) g_{\mu\nu}] \\ &\quad - \frac{4\omega}{\text{tr} \Phi^\dagger \Phi} \left[\text{tr} D_{(\mu} \Phi^\dagger D_{\nu)} \Phi \right. \\ &\quad \left. - \frac{1}{2} \text{tr} D_\lambda \Phi^\dagger D^\lambda \Phi g_{\mu\nu} \right] \\ &\quad - \frac{1}{\text{tr} \Phi^\dagger \Phi} [(\text{tr} \Phi^\dagger \Phi)_{|\mu|}{}_{|\nu} \\ &\quad - (\text{tr} \Phi^\dagger \Phi)^{|\lambda}{}_{|\lambda} g_{\mu\nu}], \end{aligned} \quad (3)$$

¹One can alternatively prepare the ‘‘system’’ in such a way that the potential term is at the very beginning the dominant contribution to the dynamical equations and, therefore, inflation occurs directly. This is the ordinary scenario within which inflation takes automatically place and the no-hair conjecture is satisfied.

²From the particle physics point of view, it is not suggested to add a cosmological constant, but neither is this forbidden [22]; this constant is $\Lambda = 12\pi G \mu^4/\lambda \sim 10^{21} \text{ GeV}^2$. However, in cosmology fitting the dynamics of cluster of galaxies suggests (in GR) that $\Omega_\Lambda \sim \mathcal{O}(1)$ [2], that is, $\Lambda \sim 10^{-83} \text{ GeV}^2$; this is the heart of the cosmological constant problem.

where $T_{\mu\nu}$ is the energy-momentum tensor belonging to $L_M\sqrt{-g}$ in Eq. (1) alone, $\parallel\mu$ is the usual covariant derivative, and the Higgs field equation is

$$(D^\lambda\Phi)_{\parallel\lambda} + \frac{\delta V}{\delta\Phi^\dagger} - \frac{1}{4\omega}R\Phi = 2\frac{\delta L_M}{\delta\Phi^\dagger} = 0, \quad (4)$$

where the contraction of the double covariant derivative is $(D^\lambda\Phi)_{\parallel\lambda} = D_\lambda D^\lambda\Phi + \Gamma_{\mu\lambda}^\lambda D^\mu\Phi$. If there were any Yukawa couplings, this equation would not be equal to zero (this is actually the case for the standard model of particle physics; see Ref. [20]).

Mathematically, IG and BD theories are equal except for the potential and the meaning of covariant derivatives. This can be seen by identifying $\phi = (2\pi/\omega)\text{tr}\Phi^\dagger\Phi$, where ϕ is the BD field. Indeed, BD and IG theories are related. The idea to induce gravity by a Higgs field has been already discussed elsewhere [24], and the motivation for us is that the field coupled to the matter content of the Universe, in the manner of Brans and Dicke [16], is the same that produces their masses, i.e., a Higgs field. Then, the identification of both scalar fields is very appealing; see Refs. [25,19]. Though this identification is quite simple, the resulting IG theory presented above is more intricate than BD theory. Accordingly, in the IG theory the Φ field is a Higgs field with its associated potential. Then, a matter content appears explicitly with its corresponding energy scales. In fact, there are three energy scales to deal with: the Planck, the Higgs boson, and the X boson masses. The Higgs mass energy is given through Eq. (4) [see also Eq. (6) below], $M_H = -[4\omega/(3+2\omega)]\mu^2$, and it determines the dynamical behavior of the Φ field once the SSB begins to occur. A second energy scale is given by the X (the same as the Y) boson mass, $M_X = \sqrt{10\pi g_5\mu/\sqrt{\lambda}} \approx 10^{15}$ GeV. Finally, the Planck energy scale is given through $M_{Pl} \equiv \sqrt{2G}$. After the SSB $(2\pi/\omega)\text{tr}\Phi^\dagger\Phi = 1/G$, implying that the coupling constant must be $\omega = (-6\pi/\lambda)(\mu/M_{Pl})^2 \approx 10^{-6}$, which is very small because one is forcing two energies scales given by the Planck and boson masses through the nonminimal coupling in Eq. (1) to match.

In contrast to what happens in the IG theory, in BD theory there is no potential and nor is GR induced after a SSB process. Therefore, the ϕ field in BD theory should be nowadays a cosmic, scalar function, and to fit well the theory with the experimental data the coupling constant must have a great value [26], $\omega > 500$, making BD and GR theories very similar.

In Sec. IV, we will present a cosmological scenario in the IG theory, yet employing some results found for BD theory which turn out to be also valid for the IG theory. As pointed out above, both theories have the same mathematical form when the potential term plays no significant role in the field equations. Therefore, in order to translate analytic BD results to the IG theory and to clarify when this situation is correct, we put the above equations in terms of the BD field $\phi = (2\pi/\omega)\text{tr}\Phi^\dagger\Phi$, which in our case represents the excited Higgs field. Then, the IG gravity equations are now

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\frac{8\pi}{\phi}[\hat{T}_{\mu\nu} + V(\phi)g_{\mu\nu}] - \frac{\omega}{\phi^2}\left[\phi_{\parallel\mu}\phi_{\parallel\nu} - \frac{1}{2}\phi_{\parallel\lambda}\phi^{\parallel\lambda}g_{\mu\nu}\right] - \frac{1}{\phi}[\phi_{\parallel\mu\parallel\nu} - \phi^{\parallel\lambda}_{\parallel\lambda}g_{\mu\nu}], \quad (5)$$

and the Higgs field equation is

$$\phi^{\parallel\lambda}_{\parallel\lambda} - \frac{4\omega}{3+2\omega}(\phi - G^{-1}) = \frac{8\pi}{3+2\omega}\hat{T}, \quad (6)$$

where \hat{T} is the trace of the *effective* energy-momentum tensor, $\hat{T}_{\mu\nu}$, given by

$$\hat{T}_{\mu\nu} = T_{\mu\nu} + \frac{G}{4\pi}\phi M_{ab}^2\left(A_\mu^a A_\nu^b - \frac{1}{2}g_{\mu\nu}A_\lambda^a A^{\lambda b}\right), \quad (7)$$

where M_{ab}^2 is the gauge boson mass square matrix, stemming from the covariant gauge derivatives [see discussion after Eq. (1)].

The continuity equation (energy-momentum conservation law) reads

$$\hat{T}_{\mu}{}^\nu{}_{\parallel\nu} = 0, \quad (8)$$

and in the present particle physics theory, SU(5) grand unified theory (GUT), all the fermions remain massless after the first symmetry breaking and no baryonic matter is originated in this way. This is the reason to have Eq. (4) equal to zero, too.

The source term in Eq. (6) is important for reheating, since the Higgs field remains coupled to \hat{T} , i.e., to gauge boson fields. In Ref. [27] is claimed that there are no couplings between the Higgs field and other fermionic or bosonic fields, but in our induced gravity approach there indeed exist bosonic field couplings.

Equations (5),(6),(7),(8) are the field equations for the IG theory written in terms of the BD field. These equations would describe BD theory if the potential in Eq. (5) vanishes, implying that the second term on the left hand side of Eq. (6) vanishes, and if the second term on the right hand side of Eq. (7) vanishes, as well. Then, by bringing BD analytic results to the IG theory, one has to be sure that these conditions apply.

The above equations reduce to the GR equations once the SSB takes place, when the ϕ field becomes a constant and the potential vanishes.

Next, we consider anisotropic universes and study their asymptotic behavior.

III. ANISOTROPIC MODELS AND ASYMPTOTIC BEHAVIOR

We consider homogeneous, anisotropic Bianchi type models that could experience, at least in principle, an isotropization mechanism evolving to a FRW model. Therefore,

we study the dynamics of Bianchi type I, V, and IX space-time symmetries in a synchronous coordinate frame; a general discussion of Bianchi models is found in Ref. [28].

In order to translate the results of Ref. [17] to IG theory, we will put the cosmological field equations in terms of the following scaled variables and definitions: the scaled Higgs field $\psi \equiv \phi a^{3(1-\nu)}$, a new cosmic time parameter $d\eta \equiv a^{-3\nu} dt$, $(\)' \equiv d/d\eta$, the ‘‘volume’’ $a^3 \equiv a_1 a_2 a_3$, and the Hubble parameters $H_i \equiv a_i'/a_i$ corresponding to the scale factors $a_i = a_i(\eta)$ for $i = 1, 2, 3$. One can assume a barotropic equation of state for the perfect fluid represented by $\hat{T}_{\mu\nu}$, $p = \nu\rho$, with ν a constant. Using these definitions and the above-mentioned metrics, one obtains the cosmological equations from Eqs. (5)–(8):

$$(\psi H_i)' - \psi a^{6\nu} C_{ij} = \frac{8\pi a^{3(1+\nu)}}{3+2\omega} \left[[1+(1-\nu)\omega]\rho + (3+2\omega)V + \frac{\delta V}{G} \right] \quad \text{for } i = 1, 2, 3, \quad (9)$$

$$\begin{aligned} & H_1 H_2 + H_1 H_3 + H_2 H_3 + [1+(1-\nu)\omega](H_1 + H_2 + H_3) \frac{\psi'}{\psi} \\ & - (1-\nu)[1+\omega(1-\nu)/2](H_1 + H_2 + H_3)^2 \\ & - \frac{\omega}{2} \left(\frac{\psi'}{\psi} \right)^2 - \frac{C_j}{2} a^{6\nu} = 8\pi \frac{a^{3(1+\nu)}}{\psi} [\rho + V], \quad (10) \\ & \psi'' + (\nu-1)a^{6\nu} C_j \psi \\ & = \frac{8\pi a^{3(1+\nu)}}{3+2\omega} \left[2(2-3\nu) + 3(1-\nu)^2 \omega \right] \rho \\ & + 3(1-\nu)(3+2\omega)V + (1-3\nu) \frac{\delta V}{G}, \quad (11) \end{aligned}$$

and

$$\rho a^{3(1+\nu)} = \text{const} \equiv M_\nu, \quad (12)$$

where $\delta V \equiv (\partial V/\partial \psi) \partial \psi/\partial \phi$ and $C_j \equiv \sum_i C_{ij}$ is the curvature corresponding to different j -Bianchi models ($j = \text{I, V, or IX}$). The subscript $i = 1, 2, 3$ refers to the three scale factors. Accordingly, one has that

	I	V	IX
$C_{ij} \equiv$	0	$\frac{2}{a_1^2}$	$\frac{a_1^4 - a_2^4 - a_3^4 + 2a_2^2 a_3^2}{-2a^6}$
	0	$\frac{2}{a_1^2}$	$\frac{a_2^4 - a_3^4 - a_1^4 + 2a_1^2 a_3^2}{-2a^6}$
	0	$\frac{2}{a_1^2}$	$\frac{a_3^4 - a_1^4 - a_2^4 + 2a_1^2 a_2^2}{-2a^6}$

(13)

Equations (9), (10), (11), (12) form the complete set of equations to be integrated. For the Bianchi type V model there is additionally the following constriction:

$$H_2 + H_3 = 2H_1, \quad (14)$$

implying that a_2 and a_3 are inverse proportional functions, $a_2 a_3 = a_1^2$.

We study in the following only the anisotropic character of the solutions, and not the influence of the potential of the theory. Otherwise, the potential term will automatically produce an inflationary stage from its outset (see footnote I; cf. Ref. [4]), and what we desire is to have a model in which inflation takes place only after the isotropization process has almost concluded, up to some extent at least. This would guarantee that anisotropic stresses decrease with time, as is the case in GR [4,7]. We want to investigate the dynamics before inflation occurs to see if the model dynamically tends to a FRW model. If this were the case, any physical perturbation (hairs) present will experience an isotropization mechanism, resulting in the smoothing of any patch of the Universe. For instance, let us assume there exist additionally other fields (dilaton, matter fields, etc.), whose stress energies do not contribute significantly to the dynamical processes, at least for some time interval.³ Then, the Universe dynamics governed by the ϕ field will isotropize all these extra fields. Thus, thinking in a chaotic scenario where the initial conditions for inflation imply that the region, and the inflaton field itself, should be sufficiently homogeneous and isotropic, then, after such an isotropization process it is more likely that inflation takes place successfully. Therefore, an isotropization mechanism can be important in pre-inflationary dynamics.

Inflation is a nice feature to solve the problems of standard model of cosmology, but most realistic models of inflation [2,29,30] are fine-tuned. For instance, a fact that is usually omitted is that to achieve enough e -foldings of expansion new inflationary models demand the initial inflaton field (say, φ) to have very small values, about $\varphi_0 < 10^{-5} v$, where v is the true vacuum value of the φ field. This fact can be understood with the help of the slow roll-over conditions: $-V'' < 9H^2$ and $(V'/V)^2 < 48\pi G$, which in turn imply, respectively, that $v \geq M_{Pl}$ and $\varphi/v < v/M_{Pl}$. Typically, GUT theories have $v \sim 10^{14-15}$ GeV; then, the second condition implies very small initial values for φ , whereas the first condition is a severe impediment (or inconsistency with realistic particle physics) to have enough e -foldings of expansion and, hence, to solve the horizon and flatness problems of cosmology. Another fine-tuning aspect or, to say precisely, inconsistency relies on the fact that $\lambda < 10^{-12}$ [λ coming from a potential similar to Eq. (2)] to fit well the temperature fluctuations measured by the COBE satellite [1]. Yet from particle physics one expects that $\lambda \sim 1$ (in any case not that small as required above). Further, such smallness of λ works in an opposite sense as for pre-

³Remember that any field, governed by its field equation, has an inherent typical time determined by its mass scale or by some constant of nature involved in its field equation. Normally, if there are many fields present, one expects every field to be significant for the evolution in some characteristic time scale.

ducing a high reheating temperature (T_{RH}) after inflation, since typically $T_{RH} \sim \lambda^{1/4} v$. Then, the baryon asymmetry could not be attained, unless very fine-tuning initial conditions are chosen. Therefore, it is interesting to investigate models that achieve a successful inflationary stage when they do not start with standard, inflationary initial conditions. Accordingly, our motivation is to study cosmological scenarios in scalar tensor theories with a particle physics content, and to consider more general initial conditions to understand some of the *ad hoc* assumptions or problems of inflationary cosmologies. An important issue is naturally the study of cosmological isotropization processes.

We return to our model in which the isotropization mechanism occurs before inflation. Accordingly, one must guarantee that potential terms in Eqs. (9),(10),(11) are less significant than the perfect fluid term (given through ρ). These conditions imply respectively that

$$\begin{aligned} & [1 + (1 - \nu)\omega]\rho \\ & > \frac{3 + 2\omega}{16\pi} M_H^2 M_{Pl}^2 [(3 + 2\omega)(\phi G - 1)^2 + 2(\phi G - 1)], \end{aligned} \quad (15)$$

$$\rho > V(\phi) = \frac{3 + 2\omega}{16\pi} M_H^2 M_{Pl}^2 (\phi G - 1)^2,$$

$$(1 - 3\nu)\rho > -\frac{3 + 2\omega}{4\pi} M_H^2 M_{Pl}^2 (\phi G - 1).$$

The first condition is the most restrictive, but it suffices to have $\rho \geq M_H^2 M_{Pl}^2$ for⁴ $\phi G > 1$, which is not a severe condition at all. Under these assumptions the IG cosmological equations are *effectively* equivalent to the BD cosmological equations. Therefore, we are able to employ the analytic solutions found for the Bianchi type I, V, and IX models in BD theory [17] on the IG theory. These solutions are valid during the time interval the above inequalities apply, say, from the initial time η_0 to η_1 .

In order to analyze the anisotropic character of the solutions, we have constructed the following ‘‘constraint’’ equation [17] using BD equations analogous to Eqs. (9),(10), (11), valid from the time η_0 to η_1 :

$$\begin{aligned} \sigma(\eta) \equiv & -(H_1 - H_2)^2 - (H_2 - H_3)^2 - (H_3 - H_1)^2 = \frac{3}{2(1 - \nu)} \left(\frac{\psi''}{\psi} \right) - \frac{1}{(1 - \nu)^2} \left(\frac{\psi'}{\psi} \right)^2 - \frac{(1 - 3\nu)}{(1 - \nu)^2} \left(\frac{(1 - 3\nu)m_\nu \eta + \eta_0}{\psi} \right) \left(\frac{\psi'}{\psi} \right) \\ & + \frac{\left[2 - 3\nu + \frac{3}{2}\omega(1 - \nu)^2 \right]}{(1 - \nu)^2} \left(\frac{(1 - 3\nu)m_\nu \eta + \eta_0}{\psi} \right)^2 + \frac{3[2 + \omega(1 - \nu)(1 + 3\nu)]m_\nu}{2(1 - \nu)\psi}. \end{aligned} \quad (16)$$

σ is the anisotropic shear. $\sigma = 0$ is a necessary condition to obtain a FRW cosmology since it implies $H_1 = H_2 = H_3$; cf. Ref. [31]. If the sum of the squared differences of the Hubble expansion rates tends to zero, it would mean that the anisotropic scale factors tend to a single function of time which is, certainly, the scale factor of the FRW models.

We have shown elsewhere [17] that $\psi = A_j \eta^2 + B_j \eta + C_j$ is a solution for the homogeneous, anisotropic models, where A_j , B_j , and C_j are some constants depending on the j -Bianchi type. The Hubble expansion rates are given through

$$H_1 + H_2 + H_3 = \frac{1}{(1 - \nu)} \frac{\left[2A_j - \frac{8\pi M_\nu}{3 + 2\omega} (1 - 3\nu) \right] \eta + B_j - \eta_0}{\psi},$$

$$H_i = \frac{1}{3} (H_1 + H_2 + H_3) + \frac{h_i}{\psi}, \quad (17)$$

where the h_i 's are functions that determine the anisotropic character of the solutions and are intimately related to the constants A_j , B_j , and C_j as follows [33,17]:

Bianchi type I:

$$A_I = \left[2 - 3\nu + \frac{3}{2}\omega(1 - \nu)^2 \right] m_\nu, \quad (18)$$

⁴Note that $\phi G > 1$ is equivalent to $\text{tr } \Phi^\dagger \Phi > -6\mu^2/\lambda$.

$$C_I = \frac{-3(1-\nu)^2(h_1^2+h_2^2+h_3^2)/2+(1-3\nu)\eta_0 B_I + B_I^2 - \left[2-3\nu + \frac{3}{2}\omega(1-\nu)^2\right]\eta_0^2}{3m_\nu(1-\nu)^2(3+2\omega)},$$

where the h_i are constants and B_I remains a free parameter.
Bianchi type V:

$$A_V = -\frac{(1-3\nu)^2}{1+3\nu}m_\nu,$$

$$B_V = -2\frac{1-3\nu}{1+3\nu}\eta_0,$$

$$C_V = \frac{-1}{m_\nu(1+3\nu)} \times \left[\frac{(1+3\nu)^2(h_1^2+h_2^2+h_3^2)}{18\nu+\omega(1+3\nu)^2} + \eta_0^2 \right],$$

where $h_1=0$ in accordance with Eq. (14), and $h_2 (= -h_3)$ is a constant.

Bianchi type IX: In this case the h_i are functions, $h_i = h_i(\eta)$, obeying the equation

$$h'_i = a^{6\beta}\psi C_{iIX} + \frac{2A_{IX} - [2(2-3\beta) + 3(1-\beta)^2\omega]m_\beta}{3(1-\beta)}, \quad i=1,2,3, \quad (19)$$

subject to the condition

$$h_1^2 + h_2^2 + h_3^2 \equiv K^2 = -\frac{\omega^3}{2(1-\beta)^2} [P\eta^2 + Q\eta + S], \quad (20)$$

where the constants P , Q , and S , given in terms of A_{IX} , B_{IX} , and C_{IX} , stand for

$$P = XA_{IX} - [4A_{IX} - Y](1-3\beta)^2m_\beta,$$

$$Q = XB_{IX} - [4A_{IX}\eta_0 - 2Ym_\beta\eta_0 + 2(1-3\beta)B_{IX}] \times (1-3\beta)m_\beta,$$

$$S = XC_{IX} - [2\Delta + 2(1-3\beta)m_\beta\eta_0 B_{IX} - Ym_\beta^2\eta_0^2],$$

$$X \equiv 3(1+3\beta)(1-\beta)^2\omega m_\beta + 6(1-\beta)m_\beta - 2(1+3\beta)A_{IX},$$

$$Y \equiv 2(2-3\beta) + 3(1-\beta)^2\omega. \quad (21)$$

The h_i 's can be further given as

$$h_1 = -\left[\frac{\kappa^2 + 4\kappa + 1}{3(\kappa^2 + \kappa + 1)} \right] K, \quad (22)$$

$$h_2 = \left[\frac{-\kappa^2 + 2\kappa + 2}{3(\kappa^2 + \kappa + 1)} \right] K, \quad (23)$$

and

$$h_3 = \left[\frac{2\kappa^2 + 2\kappa - 1}{3(\kappa^2 + \kappa + 1)} \right] K, \quad (24)$$

where κ is an unknown function of η . Unfortunately, we have not achieved yet to obtain the explicit functional dependence of $\kappa = \kappa(\eta)$. The axisymmetric case ($a_1 = a_2 \neq a_3$), assuming a quadratic function for ψ , gives the closed FRW solution, implying that $B_{IX} = C_{IX} = 0$.

The above-presented Bianchi models obey the condition

$$h_1 + h_2 + h_3 = 0; \quad (25)$$

then, the shear, Eq. (16), becomes

$$\sigma(\eta) = -\frac{3(h_1^2 + h_2^2 + h_3^2)}{\psi^2}. \quad (26)$$

This equation admits solutions such that $\sigma \rightarrow 0$ as $\eta \rightarrow \infty$ (or $t \rightarrow \infty$); that is, one has time asymptotic isotropization solutions, similar to the solutions found for Bianchi models in GR (see Ref. [32]). In fact, one does not need to impose an asymptotic, infinity condition, but just that $\eta \gg \eta_*$, where η_* is yet some arbitrary value, to warrant that σ can be bounded from above. For the Bianchi type IX model, $h_1^2 + h_2^2 + h_3^2$, given by Eq. (20), is not a constant but a quadratic function of η ; however, the denominator of Eq. (26) is a quartic polynomial in η , and therefore, an asymptotic isotropic behavior, similar to the other models, is also expected.

The analytic flat, open and closed FRW solutions are obtained if $h_i = 0$, for the Bianchi type I, V and IX models, respectively. In this case, it implies that $B_j = C_j = 0$ for all the Bianchi models considered here.

IV. PHYSICAL SCENARIO

We consider in the following a scenario in which the isotropization process can occur from the very beginning, η_0 , until the time η_* , corresponding to a time scale before inflation happens. That is, one has that $\eta_* \leq \eta_1$, where η_1 is the time when inflation starts because the potential stress energy begins to be the major contribution to Eqs. (9),(10), (11). In this way, the isotropization of hairs is guaranteed indeed before the de Sitter stage occurs.

The integration of Eq. (17) to get explicitly the scale factor functions is straightforward, and was reported in Refs. [33,17]. The solutions are characterized by the sign of the discriminant, $\Delta_j \equiv B_j^2 - 4A_jC_j$, which implies two different

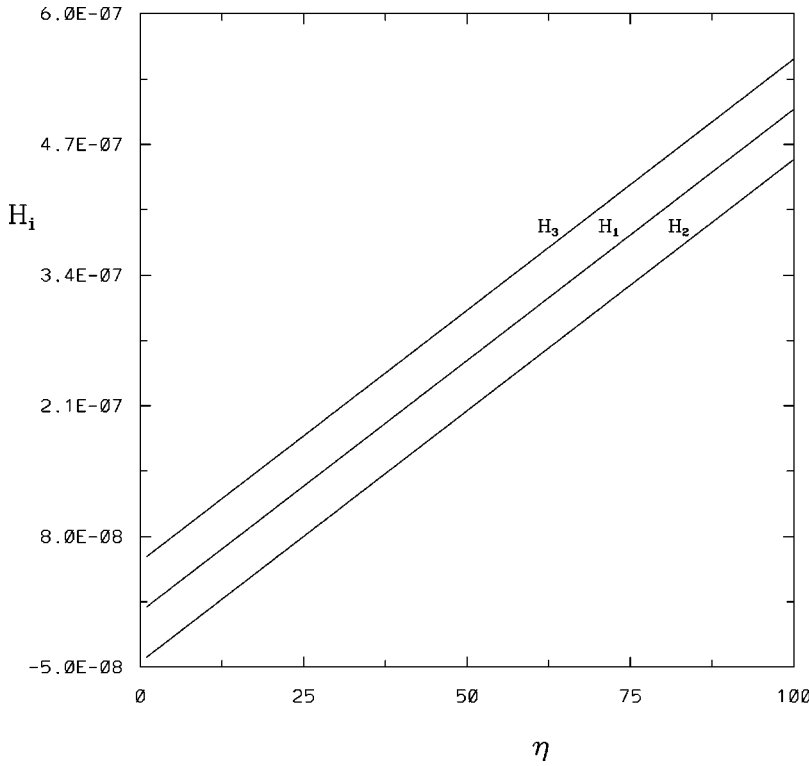


FIG. 1. The Hubble parameters as a function of the time η . For these plots we have taken $\nu = 0$, $\omega = 10^{-6}$, and $h_2 = \eta_0 = m_0 = 1$, where the choice of the latter parameters is arbitrary; they are related to the initial conditions. In this case, one has that $H_{10} > 0$, $H_{30} > 0$, and $H_{20} < 0$. The latter condition implies an initial contracting scale factor (a_2); however, after some evolution it expands.

behaviors depending on it being positive or not. The solutions $\Delta > 0$ are restricted to be valid in a specific time interval⁵ and, additionally, they are asymptotically anisotropic. The solutions $\Delta \leq 0$ are valid during the whole cosmic time (η) interval. Independent of the initial value the Hubble parameters may have (including $H_i < 0$), because of Eq. (26), as $\eta \rightarrow \eta_*$, $\sigma \rightarrow \sigma_{\min} \approx 0$, where the value of η_* is fixed by the degree of isotropy (σ_{\min}) at that time in each Bianchi model.

We analyze the conditions for this scenario to be viable on the Bianchi type I, V, and IX models.

Bianchi I: The discriminant of this model is given by [33]

$$\begin{aligned} \Delta_I = B_I^2 - 4A_I C_I = B_I^2 - \frac{4 \left[2 - 3\nu + \frac{3}{2} \omega (1 - \nu)^2 \right]}{3(1 - \nu)^2 (3 + 2\omega)} \\ \times \left[B_I^2 + (1 - 3\nu) \eta_0 B_I - \frac{3}{2} (1 - \nu)^2 (h_1^2 + h_2^2 + h_3^2) \right. \\ \left. - \left(2 - 3\nu + \frac{3}{2} \omega (1 - \nu)^2 \right) \eta_0^2 \right]. \end{aligned} \quad (27)$$

In our case, $\omega \ll 1$, and Δ_I is always positive. Therefore, here the isotropization is not possible, and the physical scenario fails.

⁵The solutions with $\Delta > 0$ are qualitatively $a_i \sim e^{\arctanh \eta}$ valid for $-1 < \eta < 1$, whereas the solutions with $\Delta < 0$ are $a_i \sim e^{\arctan \eta}$ valid for $-\infty < \eta < +\infty$.

Bianchi V: The discriminant of this model is given by [17]

$$\Delta_V = B_V^2 - 4A_V C_V = \frac{-8(1 - 3\nu)^2}{18\nu + (1 + 3\nu)^2 \omega} h_2^2, \quad (28)$$

which implies $\Delta_V < 0$. In this case, the solutions isotropize and the scenario is successfully achieved.

We plot the Hubble parameters as a function of the time η for $\omega = 10^{-6}$ and $\nu = 0$ (corresponding to a dust gas of bosons). Figure 1 shows how the anisotropic Hubble parameters evolve with the same slope. This is because the smallness of ω causes ψ to be initially almost a constant, and the H_i solutions are linear, with different anisotropic parameters (h_i) contributing as different initial ordinates; see Eqs. (17). As time elapses the anisotropy becomes almost unobservable because of the scale (see Fig. 2), making the difference among the three rates of expansion always smaller; that is, after some time η_* the solutions become indistinguishable from the open FRW solution, which is given by H_1 (see Ref. [17]). Then, as mentioned in the previous section, the anisotropy is bounded from above. In Ref. [3] is claimed that the remaining anisotropy is under the limits imposed by the COBE satellite on the temperature fluctuations observed in the CMBR [1].

Note that during the isotropization process, from η_0 to η_* , the solutions are of the form $H_i \approx D\eta + D_i$, where D and D_i are some constants. Not only do the Hubble parameters not diminish, but they increase with time. This implies for the scale factors a period of “strong” exponential expansion, $a_i \sim e^{D\eta^2/2 + D_i\eta}$, causing the isotropization of the model. Note that this solution is very peculiar and it has its

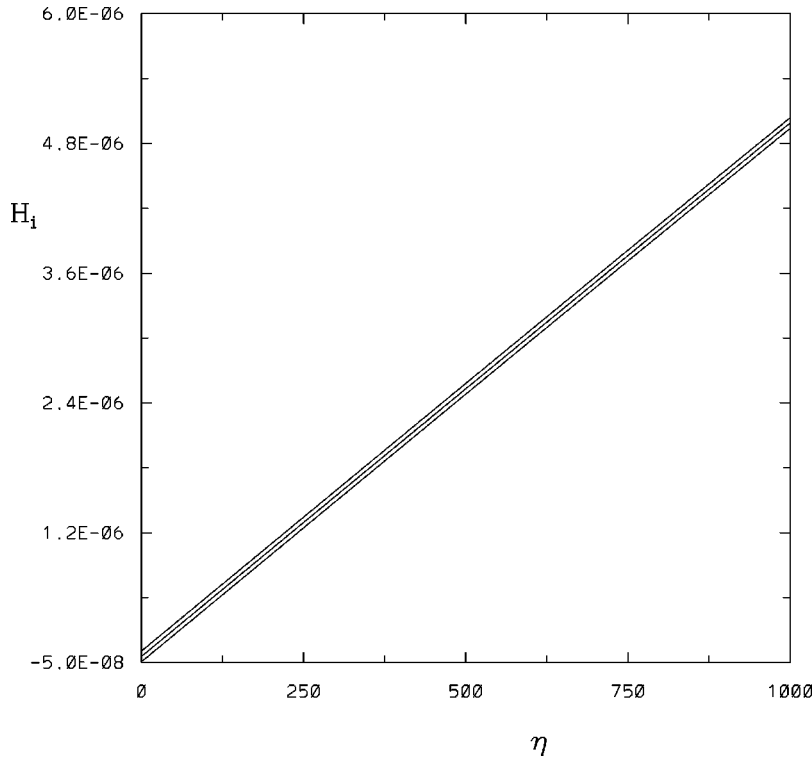


FIG. 2. This figure shows the same as Fig. 1, but now we plot until the time 1000, where one can already observe that the three scale factors become almost indistinguishable.

origin in the smallness of ω . If ω were not that small, a power law solution ($H_i \approx 1/\eta$) would be valid.⁶ It is curious that we initially aimed to have a model avoiding to begin with inflationary initial conditions, and because of the smallness of ω , we got an even stronger (than de Sitter) inflationary era caused by the Higgs field itself (kinetic+nonminimal coupling), but not by the potential of the theory. In this way, during this time the Universe begins to lose its *anisotropic* hairs because the strong exponential expansion dilutes any perturbation present in a distance smaller than its event horizon and the no-hair conjecture is then dynamically satisfied.

In GR the presence of a kinetic term associated to a field (say, ϕ), added minimally to the Lagrangian, induces the field to evolve like $\phi \sim 1/t^2$; see Ref. [30]. Here, we have the same behavior for ϕ , but this solution induces an exponential universe, regardless of the potential. In this case, one does not have a slow rollover dynamics, but the solution itself includes an exponential expansion with exit (no graceful exit problem) in a natural, evolutionary way tending to an effective FRW model. The reason for this is that after some elapsed time the variable ψ will be no longer effectively a constant and the quadratic term in η will dominate and, therefore, the solution behaves as shown in Fig. 3, that is, with $H_i \sim 1/\eta$ for $i=1,2,3$. Figure 4 shows the evolution of the physical Higgs field.

Afterwards, at $\eta = \eta_1 \geq \eta_*$ inflation due to the potential takes place. The initial conditions are such that when inflation is to start, the system is already almost isotropic. Then, inflation (due to the potential) begins after the model is to some degree an open FRW model. The dynamics of this stage is studied in Ref. [19].

Figures 1, 2, 3, and 4 describe the dynamics dominated by perfect fluid+kinetic terms in early stages, precisely when the potential stress energy does not play a significant role in the evolution. But after some elapsed time (at η_1) the potential enters into the “game” because the density diminishes rapidly as $\rho \sim 1/a^n$ with $n=4$ for radiation and $n=3$ for dust-like matter (boson fields), whereas the potential is only slowly varying. Thus, from the different energy stresses in Eqs. (9),(10),(11) the one which diminishes slower, as time elapses, is the potential term, in such a way that it will eventually be the major stress energy contribution to the dynamical equations. The potential dominance begins⁷ at the η_1 time such that $\eta_1 \geq \eta_*$ in order to guarantee some degree of isotropization of physical processes present before inflation takes place. Therefore, if any perturbation (*hairs*) is to be present before that time, it must have experienced some degree of isotropization, the same as the scale factors.

After inflation when the Higgs field approaches its symmetry breaking value, $\text{tr} \Phi^\dagger \Phi = -6\mu^2/\lambda$, the potential diminishes, and begins the high oscillation period of the Higgs field described by Eq. (6) or (11). The Higgs oscillations act

⁶This type of solution is also valid for some set of initial conditions including any arbitrary value of ω .

⁷In this paper we assume a chaotic scenario for the initial conditions ($\phi_0 > G^{-1}$; see footnote IV); see Refs. [19,20].

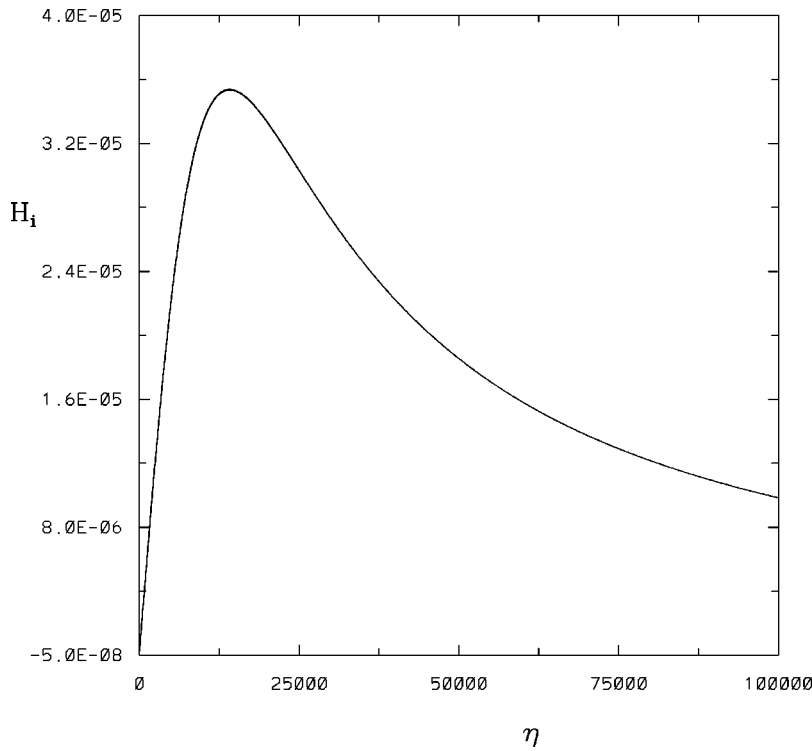


FIG. 3. The same as Fig. 1, but now until the time 10^5 . The three Hubble parameters, here superposed, evolve to an open FRW solution given by H_1 .

on the scale factors dynamics with a characteristic frequency given by M_H , which we have taken equal to 10^{14} GeV. This high oscillation period induces a FRW model ($a_i \sim t^{2/3}$) [34], as can be observed in Fig. 5. Then, inflation, caused by the potential, acts as a transient attractor, then graceful exiting. This result confirms the general theorems proved in Refs. [15] about the no-hair conjecture in scalar tensor theories. Naturally, one cannot expect this solution to

be ever trapped in the de Sitter attractor, since the Higgs field evolves to its SSB minimum, which is the state of lowest energy. Then, the no-hair conjecture fails during the high oscillation period of the Higgs field, as one expects.

During inflation (due to the potential) perturbations of the Higgs field exit the Hubble horizon (H^{-1}), and they will later reenter to form the seeds of galaxy formation with a magnitude [27,35,19]

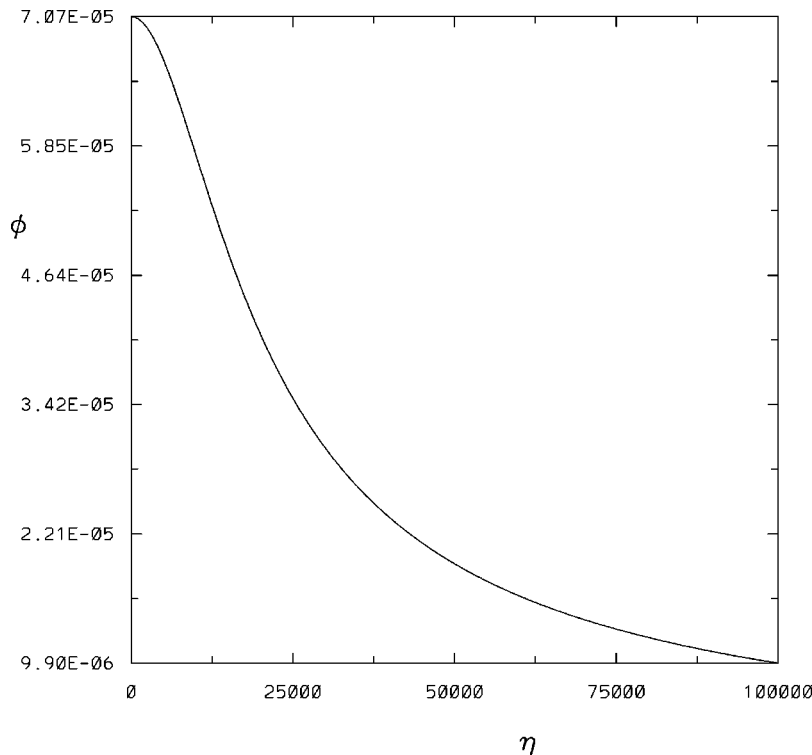


FIG. 4. The Higgs field ϕ as a function of time, for the same parameters as in Fig. 1.

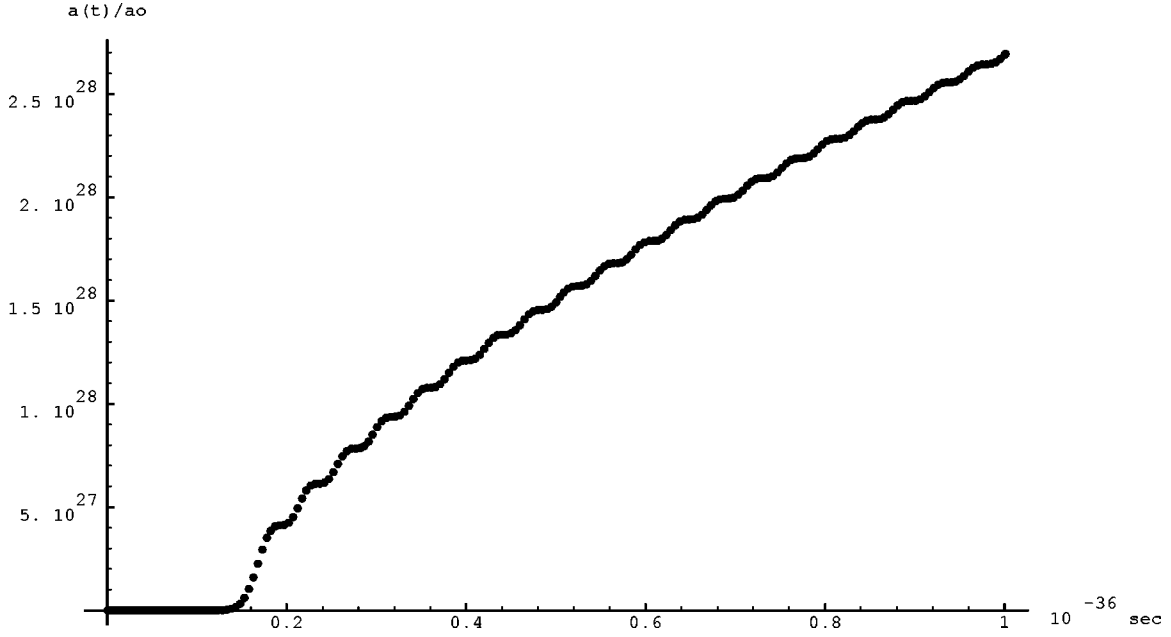


FIG. 5. The scale factor evolution during and after inflation until the time $t = 10^2 M_H^{-1}$, where M_H is the Higgs boson mass. One notes that the inflation time is approximately $t = 2 \times 10^{-37}$ s; later on, the Universe is “dark” matter dominated by ϕ bosons, perhaps until today, if reheating did not take away the coherent Higgs oscillations. The track imprinted by the Higgs coherent oscillations in the scale factor evolution at that time scale can be seen; afterwards, this influence will be imperceptible.

$$\begin{aligned} \left. \frac{\delta\rho}{\rho} \right|_{\eta_2} &\approx \frac{1}{\sqrt{1 + \frac{3\alpha}{4\pi}}} H \left. \frac{\delta\phi}{\phi} \right|_{\eta_2} = \sqrt{\frac{1}{6\pi}} \frac{M_H}{M_{Pl}} N(\eta_2) \\ &\approx 10 \frac{M_H}{M_{Pl}} < 10^{-5}, \end{aligned} \quad (29)$$

where η_2 is the time when the fluctuations of the scalar field leave H^{-1} during inflation and $N(\eta_2)$ is the number of e -folds of inflation at that time.

The gravitational wave (GW) perturbations considered normally should also be small [36],

$$h_{GW} \approx \left. \frac{H}{M_{Pl}} \right|_{\eta_2} \approx \left. \frac{M_H}{M_{Pl}} \frac{\sqrt{G\phi - 1}}{2} \right|_{\eta_2} \sim 10^{-5}, \quad (30)$$

which also lies within the experimental limits.

The spectral index of the scalar perturbations, n_s , serves as a test for models of the very early Universe, independently of the magnitude of the perturbations. It can be calculated using the slow roll approximation up to second order [37]. For $\omega \ll 1$, however, one can just take the first order to be sufficiently accurate [38]:

$$n_s = 1 - \frac{4}{2N + \omega} \approx 1 - \frac{2}{N}; \quad (31)$$

for $N = 65$, it implies $n_s \approx 0.97$ in accordance with the recent COBE DMR results [1].

Bianchi IX: For the Bianchi IX model the constants A_{IX}, B_{IX}, C_{IX} explicit values are unknown, except for the

value of A_{IX} of the isotropic model. To be physical this solution needs to have $\omega < -2$ [39]. Therefore, we expect to find isotropizing solutions only for $\omega < 0$, but our IG gravity requires $\omega > 0$ and the scenario is untenable.

V. CONCLUSIONS

Within an IG theory we have analyzed Bianchi type I, V, and IX models. Only the Bianchi type V model may isotropize by means of the nonminimal coupling of the theory, because the value of ω ($\ll 1$) is crucial for its isotropization. These results, extracted from the analytic solutions in BD theory [17], are here applied to the IG theory in an epoch when the potential stress energy is not significant for the evolution, that is, when both theories are mathematically equivalent. If this situation would have happened, initial anisotropies were washed out in a universe with Bianchi V type initial conditions. On the other side, if the potential dominates from the very beginning, inflation (because of the potential) occurs directly and it induces the same effect, as expected.

The isotropization mechanism in type V can be inflationary even before the potential plays a role or, otherwise, of the $H_i \sim 1/\eta$ type. In the former case, from η_0 to η_1 , the models experience a strong period of exponential expansion, while the Higgs field evolves as $\phi \sim 1/t^2$, achieving the isotropization of any hairs present (also possibly due to some other fields). After the isotropization mechanism has been concluded, because of Eq. (17), the solution turns away from an exponential behavior to become effectively a FRW model, where $H_i \approx H$ for $i = 1, 2, 3$. Afterwards, with the Universe “almost” isotropic the potential begins to dominate the dynamical equations, and therefore inflation due to the potential

occurs. Later on, as the Higgs field approaches its ground state value ($\text{tr } \Phi^\dagger \Phi \rightarrow -6\mu^2/\lambda$), inflation ends and an effective FRW dynamics is dominant. Thus, our scenario possesses two inflationary transient attractors, one produced by the nonminimal coupling and the other by the Higgs potential. The tests of inflation for this theory have been proved to be within the experimental limits in Refs. [19,20].

In the present work, we looked for conditions under which isotropization before inflation occurs in a theory, where the value of ω is strictly determined to be $\omega = 10^{-6}$. Otherwise, for arbitrary ω values, the mechanism of isotro-

pization can also develop before inflation. Here the Bianchi model I tends to the flat FRW solution, the Bianchi model V to the open FRW solution, and the Bianchi model IX to the closed FRW solution.

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- [1] G.F. Smoot *et al.*, *Astrophys. J.* **396**, L1 (1992); C.L. Bennett *et al.*, *ibid.* **436**, 423 (1994); E.L. Wright *et al.*, *ibid.* **436**, 443 (1994); A.J. Banday *et al.*, *ibid.* **436**, L99 (1994). Four years of COBE data have been reported recently by C.L. Bennett *et al.*, *ibid.* **464**, L1 (1996); K.M. G3rski *et al.*, *ibid.* **464**, L11 (1996); G. Hinshaw, *ibid.* **464**, L17 (1996).
- [2] E.W. Kolb and M.S. Turner, *The Early Universe*, ‘‘Frontiers in Physics’’ No. 69 (Addison-Wesley, Redwood, 1990).
- [3] J.D. Barrow, *Phys. Rev. D* **51**, 3113 (1995).
- [4] R.M. Wald, *Phys. Rev. D* **28**, 2118 (1983).
- [5] G.W. Gibbons and S.W. Hawking, *Phys. Rev. D* **15**, 2752 (1977); S.W. Hawking and I.G. Moss, *Phys. Lett.* **110B**, 35 (1982); W. Boucher and G.W. Gibbons, *The Very Early Universe*, edited by G.W. Gibbons and S.W. Hawking (Cambridge University Press, Cambridge, England, 1983), p. 273.
- [6] H.J. Schmidt, *Fortschr. Phys.* **41**, 179 (1993).
- [7] G. Steigman and M.S. Turner, *Phys. Lett.* **128B**, 295 (1983); T. Futamase, *Phys. Rev. D* **29**, 2783 (1984); \emptyset . Gr3n, *ibid.* **32**, 2522 (1985); I. Moss and V. Sahni, *Phys. Lett. B* **178**, 159 (1986); L.G. Jensen and J.A. Stein-Schabes, *Phys. Rev. D* **35**, 1146 (1987); A.K. Raychaudhuri and B. Modak, *Class. Quantum Grav.* **5**, 225 (1988); J.D. Barrow and G. G3tz, *Phys. Lett. B* **231**, 228 (1989); Y. Kitada and K. Maeda, *Phys. Rev. D* **45**, 1416 (1992); J. Ib3nez, R.J. van den Hoogen, and A.A. Coley, *ibid.* **51**, 928 (1995).
- [8] D.L. Goldwirth and T. Piran, *Phys. Rep.* **214**, 223 (1992).
- [9] T. Shiromizu, K. Nakao, H. Kodama, and K. Maeda, *Phys. Rev. D* **47**, R3099 (1993).
- [10] C. Chambers and I.G. Moss, *Phys. Rev. Lett.* **73**, 617 (1994).
- [11] A.D. Linde, *Phys. Lett.* **129B**, 177 (1983).
- [12] P. Anninos, R.A. Matzner, T. Rothman, and M.P. Ryan, Jr., *Phys. Rev. D* **43**, 3821 (1991).
- [13] L.O. Pimentel and J.A. Stein-Schabes, *Phys. Lett. B* **216**, 27 (1989).
- [14] T. Futamase, T. Rothman, and R.A. Matzner, *Phys. Rev. D* **39**, 405 (1989).
- [15] S. Capozziello, R. de Ritis, C. Rubano, and P. Scudellaro, *Phys. Lett. A* **201**, 145 (1995); S. Capozziello and R. de Ritis, *ibid.* **203**, 283 (1995).
- [16] C. Brans and R. Dicke, *Phys. Rev.* **124**, 925 (1961).
- [17] P. Chauvet and J.L. Cervantes-Cota, *Phys. Rev. D* **52**, 3416 (1995).
- [18] J.L. Cervantes-Cota, ‘‘Inflation in the Brans-Dicke theory?’’ (unpublished).
- [19] J.L. Cervantes-Cota and H. Dehnen, *Phys. Rev. D* **51**, 395 (1995).
- [20] J.L. Cervantes-Cota and H. Dehnen, *Nucl. Phys.* **B442**, 391 (1995).
- [21] H. Dehnen, H. Frommert, and F. Ghaboussi, *Int. J. Theor. Phys.* **31**, 109 (1992); H. Frommert, Ph.D. thesis, University of Konstanz, 1992.
- [22] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
- [23] N.D. Birrel and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1982), Chap. 3.
- [24] P. Minkowski, *Phys. Lett.* **71B**, 419 (1977); A. Zee, *Phys. Rev. Lett.* **42**, 417 (1979); S.L. Adler, *Rev. Mod. Phys.* **54**, 729 (1982).
- [25] H. Dehnen, H. Frommert, and F. Ghaboussi, *Int. J. Theor. Phys.* **29**, 537 (1990); H. Dehnen and H. Frommert, *ibid.* **30**, 985 (1991).
- [26] C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993), Chap. 8.
- [27] D.S. Salopek, J.R. Bond, and J.M. Bardeen, *Phys. Rev. D* **40**, 1753 (1989).
- [28] M.A.H. MacCallum, in *General Relativity. An Einstein Centenary Survey*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979), p. 533.
- [29] K.A. Olive, *Phys. Rep.* **190**, 307 (1990).
- [30] A.D. Linde, *Particle Physics and Inflationary Cosmology*, (Harwood Academic, Chur, Switzerland, 1990).
- [31] P. Chauvet, J. Cervantes-Cota, and H.N. N3ñez-Y3pez, in *Proceedings of the 7th Latin American Symposium on General Relativity and Gravitation, SILARG VII*, edited by J.C. D’Olivo *et al.* (World Scientific, Singapore, 1991), p. 487.
- [32] J.D. Barrow and D.H. Sonoda, *Phys. Rep.* **139**, 1 (1986).
- [33] V.A. Ruban and A.M. Finkelstein, *Gen. Relativ. Gravit.* **6**, 601 (1975).

- [34] M.S. Turner, Phys. Rev. D **28**, 1243 (1983).
- [35] R. Fakir and W.G. Unruh, Phys. Rev. D **41**, 1783 (1990); **41**, 1792 (1990); N. Makino and M. Sasaki, Prog. Theor. Phys. **86**, 103 (1991).
- [36] F.S. Accetta, D.J. Zoller, and M.S. Turner, Phys. Rev. D **31**, 3046 (1985).
- [37] E.W. Kolb and S.L. Vadas, Phys. Rev. D **50**, 2479 (1994); A.R. Liddle, P. Parsons, and J.D. Barrow, *ibid.* **50**, 7222 (1994).
- [38] D.I. Kaiser, Phys. Rev. D **52**, 4295 (1995).
- [39] P. Chauvet and O. Obregon, Astrophys. Space Sci. **66**, 515 (1979).