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Elastic form factors and charge radii of π and K mesons

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The elastic form factors of the π and K mesons are evaluated in a field-theoretic framework based on the relativistic independent quark model with a scalar-vector harmonic potential. The predictions on the form factors are compared with the results of different relativistic approaches and existing experimental data. The results for the mean-square charge radii $\langle r^2_{\pi^+}\rangle = 0.47$ fm^2; $\langle r^2_{K^+}\rangle = 0.33$ fm^2; $\langle r^2_{K^0}\rangle = -0.078$ fm^2 are found to be consistent with those of several other calculations as well as experimental data. [S0556-2821(98)03323-2]

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The electromagnetic structure of hadrons, especially the pion, has been the topic of overwhelming interest for more than three decades [1]. The interaction of a charged pion with the electromagnetic field is described in terms of a single structure function namely the pion form factor $f_\pi(q^2)$, which effectively provides the deviation from a pion with pointlike electric charge and depends upon the four-momentum-transfer squared: $q^2 = -Q^2$. The estimation of the form factor $f_\pi(q^2)$ is significant because the cross section, which is an experimentally measurable and invariant physical quantity, is directly proportional to $|f_\pi(q^2)|^2$.

The elastic form factor of the π meson is measured by e^+e^- colliding beam experiments in a broad range of a timelike region $(Q^2<0)$. However, its measurement in a large spacelike region $(Q^2>0)$ is achieved only through an indirect approach based on the available reaction of pion electroproduction from the nucleons: $(ep \to e\pi^+ n$ and $en \to e\pi^- p)$. In this reaction the presence of the nucleons and their structure complicate theoretical models used to extract $f_\pi(q^2)$ from the measurement. Thus an analysis of the form factor in the spacelike region becomes predominantly model dependent. The experimental investigations spread over the last two decades on the pion and kaon form factors [2–17] have yielded enough data to guide theoretical models. The recent plan for the measurement at the Continuous Electron Beam Accelerator Facility (CEBAF) [18] has, in fact, aroused renewed interest in the theoretical evaluation of electromagnetic properties of π and K mesons.

Theoretically quantum chromodynamics (QCD) [19] provides an economical and successful description of the electromagnetic structure of hadrons [20], although the traditional approach based on vector meson dominance (VMD) and its extended version (EVMD) still provide a useful complementary framework to describe timelike as well as spacelike form factors [21], especially in the long distance domain where perturbative QCD breaks down. It has been argued [22,23] that the perturbative QCD contribution is too insignificant compared to that of soft nonperturbative QCD. Moreover, free-quark-model calculations based on perturbative QCD involve extra parameters to describe bound-state effects, which actully control the theoretical predictions in any bound-state problem. There is, indeed, little justification for continued application of perturbative QCD to exclusive processes involving the bound-state hadrons. On the other hand, the form factor data are easily reproduced by considering the important nonperturbative QCD effects[24]. However, owing to the complicacies inherent in the nonperturbative QCD, it is not straightforward to analyze the elastic form factors and their q^2 dependence on first principles QCD application. Therefore a large number of phenomenological models have been extended to analyze pion and kaon form factors with varying degrees of success. Some of them include the bag model [25], the QCD-motivated quark potential model [26], and the light cone relativistic constituent quark models (RCQM) [27–30], etc. As an alternative approach, we have developed a relativistic independent quark model based on an average confining potential in the scalar-vector harmonic form [31–39]: $U(r) = \frac{1}{2}(1+\gamma^0)(ar^2 + V_0)$, $a>0$, where (a, V_0) are the potential parameters. The predictive power of such a model has been demonstrated in wide ranging hadronic phenomena such as the static hadronic properties [31,32], radiative [33], weak radiative [34], leptonic [35], weak leptonic [36,37], semileptonic [38], and rare radiative [39] decays of light and heavy mesons. The aim of this paper is to study the applicability of the model in comparison with other model predictions as well as the experimental data for pion and kaon form factors and their corresponding charge radii. The matrix element for π^+ coupling to the electromagnetic field has the covariant expansion in terms of two form factors: $f_\pi(q^2)$ and $g_\pi(q^2)$ as $\langle \pi^+(\vec{k})|J^{em}_\mu|\pi^+(\vec{k}')\rangle = (k'+k)_\mu f_\pi(q^2) + (k'-k)_\mu g_\pi(q^2)$, where $q \equiv (k'-k)$ is the four momentum transfer. The current conservation condition applied to the $\pi^+ \to \pi^+$ transition-matrix element, requires $g_\pi(q^2)=0$. The only nonvanishing form factor $f_\pi(q^2)$ in the rest frame of the initial π^+ meson, is obtained in the form $f_\pi(q^2) = [1/(E_\pi + m_\pi)] \times \langle \pi^+(\vec{k})|J^{em}_0|\pi^+(\vec{k}'=0)\rangle$. The energy momentum associated with the final π^+ meson is given by $E_\pi = m_\pi + -q^2/2m_\pi$; $|\vec{k}| = \sqrt{(E^2_\pi - m^2_\pi)}$. A similar expression for the charged and neutral kaon form factors is identically found to be

$$f_K(q^2) = [1/(E_K + m_K)]\ \langle K(\vec{k})|J^{em}_0|K(\vec{k}'=0)\rangle. \tag{1}$$

The meson state in this model is represented by an appropriate momentum wave packet in the form [33,36,38,39]

$$|M(\vec{k},S_V)\rangle = \frac{1}{\sqrt{N(\vec{k})}} \Sigma_{\lambda_1,\lambda_2}\zeta^M_{q_1,q_2}(\lambda_1,\lambda_2) \times\int d\vec{p}_1 d\vec{p}_2 \delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{k}) \times G_M(\vec{p}_1,\vec{p}_2)\hat{b}^\dagger_{q_1}(\vec{p}_1,\lambda_1)\hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_2,\lambda_2)|0\rangle. \quad (2)$$

With the normalization condition taken as $\langle M(\vec{k})|M(\vec{k}')\rangle = (2\pi)^3 2E_M\delta^{(3)}(\vec{k}-\vec{k}')$, the overall normalization factor of the wave packet is found to be

$$N(\vec{k}) = \frac{1}{(2\pi)^3 2E_M}\int d\vec{p}|G_M(\vec{p},\vec{k}-\vec{p})|^2 = \frac{\overline{N}(\vec{k})}{(2\pi)^3 2E_M}. \quad (3)$$

The effective momentum distribution function $G_M(\vec{p}_1,\vec{p}_2)$ for the constituent quark-antiquark inside the meson is taken in the model [33,36,38,39] in the form $G_M(\vec{p}_1,\vec{p}_2) = \sqrt{g_{q_1}(\vec{p}_1)\tilde{g}_{q_2}(\vec{p}_2)}$, where the factors $g_{q_1}(\vec{p}_1)$ and $\tilde{g}_{q_2}(\vec{p}_2)$ represent the momentum probability amplitude of the quark and antiquark with momentum $\vec{p}_1$ and $\vec{p}_2$, respectively, and are derivable from the model dynamics by suitable momentum-space projections of the corresponding bound-state orbitals of the model. Using appropriate meson states for the initial and final mesons in the form as in Eq. (2), and taking SU(2)-flavor symmetry $(m_u=m_d=m)$, $f_\pi(q^2)$ can be derived as

$$f_\pi(q^2) = \frac{(e_u-e_d)}{(E_\pi+m_\pi)}\sqrt{\frac{E_\pi m_\pi}{\overline{N}(0)\overline{N}(\vec{k})}}\int d\vec{p}\,\frac{G_\pi(\vec{p},-\vec{p})G_\pi(\vec{k}+\vec{p},-\vec{p})[(E_p+m)(E_{k+p}+m)+\vec{p}^2]}{\sqrt{E_pE_{k+p}(E_p+m)(E_{k+p}+m)}}. \quad (4)$$

A similar expression for charged and neutral kaon form factor is, in general, found to be

$$f_K(q^2) = \frac{1}{(E_K+m_K)}\sqrt{\frac{E_K m_K}{\overline{N}(0)\overline{N}(\vec{k})}}\left[e_q\int d\vec{p}_q\,\frac{G_K(\vec{p}_q,-\vec{p}_q)G_K(\vec{k}+\vec{p}_q,-\vec{p}_q)[(E_{p_q}+m_q)(E_{k+p_q}+m_q)+\vec{p}_q^2]}{\sqrt{E_{p_q}E_{k+p_q}(E_{p_q}+m_q)(E_{k+p_q}+m_q)}} - e_s\int d\vec{p}_s\,\frac{G_K(-\vec{p}_s,\vec{p}_s)G_K(-\vec{p}_s,\vec{k}+\vec{p}_s)[(E_{p_s}+m_s)(E_{k+p_s}+m_s)+\vec{p}_s^2]}{\sqrt{E_{p_s}E_{k+p_s}(E_{p_s}+m_s)(E_{k+p_s}+m_s)}}\right]. \quad (5)$$

Here we take the subscript $q\to u$ and $q\to d$ for the charged and neutral kaon form factor, respectively. To a reasonably good approximation the energy of the recoil quark-antiquark is taken as $E_{\vec{k}+\vec{p}} = \sqrt{(\vec{k}+\vec{p})^2+m_q^2} \simeq \sqrt{\vec{k}^2+\vec{p}^2+m_q^2}$.

For an estimation of the form factors we take the potential parameters (a,V_0), quark mass m_q, and the corresponding quark binding energy E_q as those used in previous applications of the model [31–39]. Accordingly, we use here

$$(a;V_0) \equiv (0.017\,166\ \text{GeV}^3; -0.1375\ \text{GeV})$$
$$m_u = m_d = 0.078\,75\ \text{GeV}; \quad E_u = E_d = 0.471\,25\ \text{GeV}$$
$$m_s = 0.315\,75\ \text{GeV}; \quad E_s = 0.591\ \text{GeV}. \quad (6)$$

The meson masses appearing in the form factor expression are taken to be the observed masses [40]. With these parameters already fixed, we perform almost a parameter-free calculation. Our prediction on the form factor $f_\pi(Q^2)$ is depicted in Fig. 1 which provides a reasonable agreement with the data [2–6]. Most of the quark model calculations, more or less, provide overall agreement in the lower Q^2 range ($Q^2 < 2\ \text{GeV}^2$); but fail to do so in the higher range ($Q^2 > 2\ \text{GeV}^2$). In such calculations [41], for example, the elastic form factor of the pion was rather seriously underestimated in the high-Q^2 range. This discrepancy was argued to be due to the factor $\sqrt{m_\pi E_\pi}/(E_\pi+m_\pi)$. In order to bring their theoretical curve in line with the data, such a factor was normalized to the unit at the zero recoil point which could effect only a marginal improvement in the result. Our prediction, on the other hand, though it remains slightly below the reported data in the low-Q^2 range ($Q^2 < 2\ \text{GeV}^2$) as depicted in Figs. 1, 2, and 5, agrees remarkably well in the high-Q^2

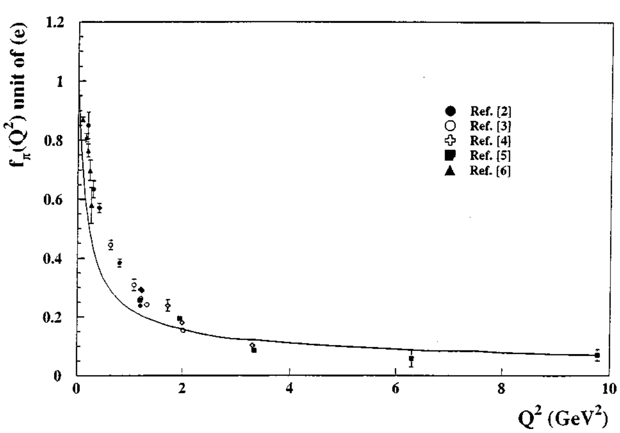

FIG. 1. The present model prediction on the charged pion form factor $f_{\pi^+}(Q^2)$ compared to the experimental data [2–6].

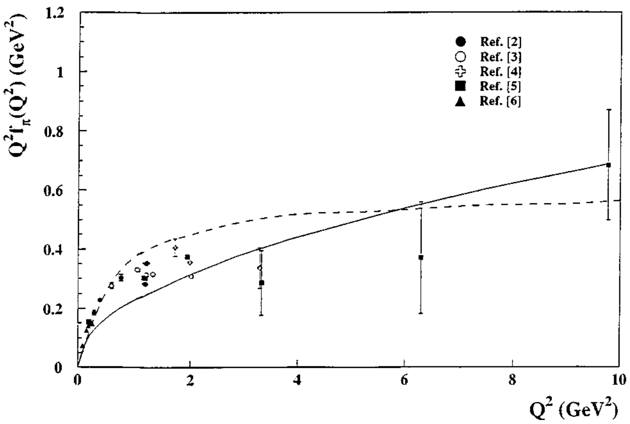

FIG. 2. The present model calculation of the charged pion form factor times Q^2 versus Q^2 compared to that of the vector meson dominance (VMD) model and the experimental data [2–6]. The solid line represents present prediction and the dotted line is the prediction of the VMD model with the ρ-meson pole only [i.e., $f_\pi(Q^2)=(1+Q^2/m_\rho^2)^{-1}$].

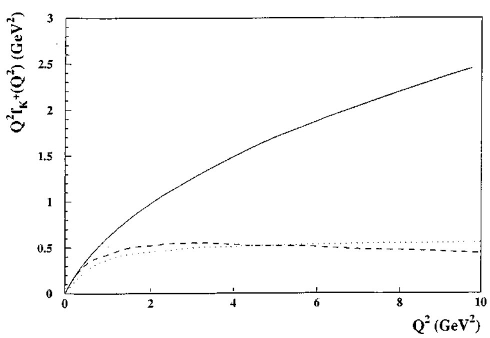

FIG. 3. The present model calculation of the charged kaon form factor times Q^2 versus Q^2 compared to that of the VMD model and the approach [42]. The solid line represents the present model prediction. The dotted and dashed line provides the prediction of VMD model including ρ-meson pole only and that of the calculation [42] based on the Bethe-Salpeter approach, respectively.

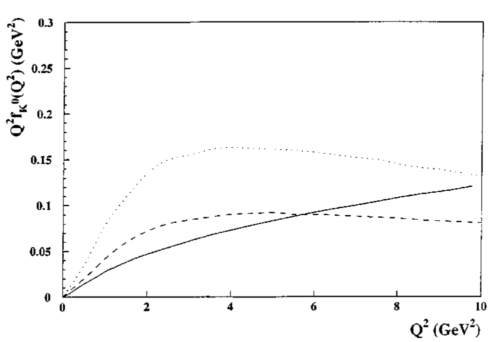

FIG. 4. The present model calculation of the neutral kaon form factor times Q^2 versus Q^2 compared to that of Refs. [30, 42]. The solid line represents the present model prediction. The dotted and dashed line provide the prediction of the relativistic constituent quark model (RCQM) [30] and that of the calculation based on Bethe-Salpeter approach [42], respectively.

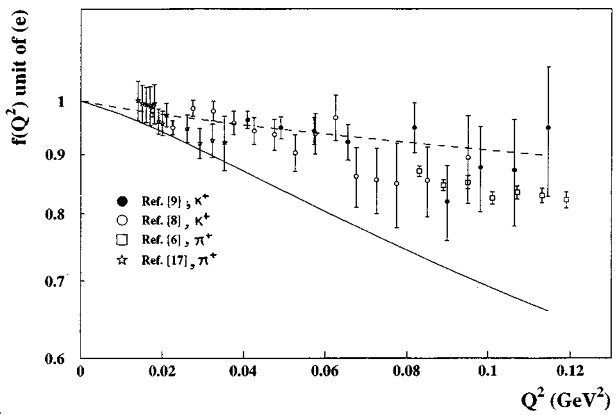

FIG. 5. The present model prediction on charged pion (solid line) and charged kaon (dashed line) form factors compared to the experimental data [6,8,9,17].

range up to 10 GeV^2. Figure 2 provides our prediction on $f_\pi(Q^2)$ times Q^2 in comparison with that of the vector meson dominance (VMD) model with the ρ-meson pole only taken as $f_\pi(Q^2)=(1+Q^2/m_\rho^2)^{-1}$ and the data [2–6]. From the model expression in Eq. (4), one can analytically extract out the Q^2 dependence of $Q^2 f_\pi(Q^2)$ in the limit $Q^2\to\infty$ to find an asymptotic behavior such as $\mathcal{L}t_{Q^2\to\infty}Q^2 f_\pi(Q^2)\simeq A\sqrt{Q^2}$. We evaluate the mean-square charge radius from the slope of $f_\pi(Q^2)$ at $Q^2\to 0$, i.e., $\langle r_{\pi^+}^2\rangle = -6(d/dQ^2)\,f_\pi(Q^2)|_{Q^2=0}$. Our result $\langle r_{\pi^+}^2\rangle=0.47$ fm^2 is consistent with the experimental results: (0.46 ± 0.011), 0.48 and (0.475 ± 0.025) fm^2 of Quenzer *et al.* [13], Heyn and Lang [14], and Geshkenbein *et al.* [15], respectively. It also compares well with those of several authors from fits to previous form factor data [16]. Figure 3 depicts our prediction for $Q^2 f_{K^+}(Q^2)$ which is found to be higher especially in the high-Q^2 range in comparision to that of the VMD model and the calculation [42] based on the covariant Bethe-Salpeter approach. However, our prediction for $Q^2 f_{K^0}(Q^2)$ as shown in Fig. 4, is more or less comparable with that of the analysis [42]. However, it is not so when compared with that of the relativistic constituent quark model (RCQM) [30], calculated using $\langle r^2\rangle_s=(0.25\text{ fm})^2$ and $\langle r^2\rangle_u=\langle r^2\rangle_d=(0.48\text{ fm})^2$ in the broken SU(3) symmetry. It is generally expected that the mean-square charge radius of a charged kaon is smaller than that of charged pion due to the presence of a comparatively heavier strange quark. Experimentally the pion charge radius is relatively well known since $f_\pi(q^2)$ is a well-measured quantity both in the spacelike and timelike region. In contrast the behavior of the kaon form factor is poorly constrained due to the absence of the electroproduction data, relatively high threshold ($q^2\simeq 4m_{K^+}^2$) for e^+e^- annihilation, and uncertainty in the isoscalar contribution. However, experimental and theoretical attempts over the last two decades [8–11,26,29,43,44] have provided a set of data in this sector. Our result $\langle r_{K^+}^2\rangle=0.33$ fm^2 is found to be consistent with that of Amendolia *et al.* [8], the timelike extrapolation of Baltnic *et al.* [11], the quark potential model of Godfrey and Isgur [26] and chiral perturbative-theory of Gasser and Leutwyler [43]. This also compares well with many other experimental and theoretical approaches [9,10,44]. We also find $\langle r^2_{K^+}\rangle$ less than $\langle r^2_{\pi^+}\rangle$ as expected. Our result $\langle r^2_{K^0}\rangle = -0.078\ \mathrm{fm}^2$ though it appears to be overestimated compared to that of Refs. [29, 44], certainly remains within the experimental limit: $(-0.054\pm 0.026)\ \mathrm{fm}^2$ [8].

The behavior of pion and kaon form factors in the region close to the zero recoil point ($Q^2\to 0$) is significant since at $Q^2\to 0$, the form factors are normalized to the unit and corresponding charge radii are precisely measured. Therefore, we depict in Fig. 5, the Q^2 dependence of $f_{\pi^+}(Q^2)$ and $f_{K^+}(Q^2)$ near the zero recoil point in the range $0\leqslant Q^2 < 0.12\ \mathrm{GeV}^2$. We find that our prediction for $f_{\pi^+}(Q^2)$ remains slightly below the data [6,17] and that for $f_{K^+}(Q^2)$ matches the available data [8,9]. Thus the predictions of this relativistic independent quark model for elastic form factors of π and K mesons and corresponding charge radii have been compared with those of several relativistic approaches and available experimental data; which provides more or less a reasonable agreement. The planned experiment at CEBAF for measuring independently the pion and kaon form factor at $Q^2 < 3\ \mathrm{GeV}^2$ could provide relevant information on the em structure of the light constituent quarks including the strange flavored one and could represent an interesting tool to discriminate among different models of the meson structure.

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