Composite model with large mixing of neutrinos

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We suggest a simple composite model that induces the large flavor mixing of a neutrino in the supersymmetric theory. This model has only one hyper-color in addition to the standard gauge group, which makes composite states of preons. In this model, the **10** and **1** representations in SU(5) grand unified theory are composite states and produce the mass hierarchy. This explains why large mixing is realized in the lepton sector, while small mixing is realized in the quark sector. This model can naturally solve the atmospheric neutrino problem. We can also solve the solar neutrino problem by improving the model.
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I. INTRODUCTION

Recent experiments of Super-Kamiokande suggest large mixing between ν_μ and other neutrinos [1,2]. As for the solar neutrino problem [3], there is the large angle solution of the matter induced resonant Mikheyev-Smirnov-Wolfenstein (MSW) oscillation [4] between ν_e and other neutrinos as well as the small angle MSW solution [5]. The vacuum oscillation solution [6] also suggests the large mixing of neutrinos.[1]

These experimental results seem to suggest the possibility that large flavor mixing is realized in the lepton sector.[2] If it is true, one has to explain why large mixing is realized in the lepton sector, while small mixing is realized in the quark sector. The possibility of neutrino large mixing has been studied from the viewpoint of the mass matrix texture [11,12], the seesaw enhancement mechanism [13], the analysis of the renormalization group equation [14], the grand unified theories [15,16], the pseudo Dirac neutrino mass [17], the singular seesaw mechanism [18], the radiative induced neutrino mass [19], and so on.

In this paper we suggest a simple model that naturally induces large mixing of neutrinos in the supersymmetric gauge theory. This model has only one hypercolor in addition to the standard gauge group, which makes composite states of preons. If **10** and **1** representations in SU(5) grand unified theory (GUT) are composite states, they produce mass hierarchy which induces large mixing in the lepton sector and small mixing in the quark sector. This model can naturally solve the atmospheric neutrino problem. We can also solve the solar neutrino problem by improving the model.

In Sec. II, we suggest a composite model. In Sec. III, we try to improve the model in order to solve the solar neutrino problem. Section IV gives summary and discussions.

II. A COMPOSITE MODEL

In the supersymmetric gauge theory, various composite models have been built in the *s*-confinement theory [20,21]. One example of the *s*-confinement theory is the $Sp(2N)$ gauge theory with one antisymmetric tensor A and six fundamentals Qs [22]. Kaplan, Lepeintre, and Schmaltz have built composite models by using this theory [23]. In this paper, we try to build the composite model of SU(5) GUT by using this theory.

We consider SU(5) GUT with three right-handed neutrinos $\overline{N_R^c}$s. Quarks and leptons are represented by $\mathbf{10}_i$, $\overline{\mathbf{5}_i}$, and $\mathbf{1}_i$ representations of SU(5) as

$$\begin{aligned} \mathbf{10}_i &= (Q_L, \overline{U_R^c}, \overline{E_R^c})_i, \\ \overline{\mathbf{5}_i} &= (\overline{D_R^c}, L_L)_i, \\ \mathbf{1}_i &= (\overline{N_R^c})_i, \end{aligned} \tag{1}$$

where the index i $(i=1,2,3)$ stands for the generation number. Q_L, L_L, $\overline{U_R^c}$, $\overline{D_R^c}$, and $\overline{E_R^c}$ express quark doublet, lepton doublet, right-handed up-sector, right-handed down-sector, and right-handed charged lepton fields, respectively.

The Sp$(2N)$ theory with one A and six Qs has the composite states

$$\begin{aligned} &\mathrm{Tr}\, A^m, m=2,3,\dots,N, \\ &QA^nQ, n=0,1,\dots,N-1. \end{aligned} \tag{2}$$

We consider the case of $N=3$. We assume that the field Q transforms under the SU(5) gauge symmetry as well as Sp(6) gauge symmetry, and the gauge coupling of SU(5) is much weaker than that of Sp(6). In our model we introduce preon fields as Table I.

$\overline{Q_i}$ is the matter field of $\overline{\mathbf{5}_i}$ in Eq. (1). $\overline{\chi_j}$ $(j=1-3)$ is introduced in order to cancel the gauge anomaly, where the index j has no relation to the generation number. H and $\overline{H}$ are Higgs fields which are singlet under Sp(6) gauge symmetry. We introduce Z_2 discrete symmetry in order to distinguish

[1]Recent Super-Kamiokande data of the electron energy spectrum suggest the vacuum oscillation solution with maximal mixing is favored [7,8].

[2]Although the Liquid Scintillation Neutrino Detectors (LSND) results suggest the small mixing between $\overline{\nu_\mu}$ and $\overline{\nu_e}$ [9], confirmation of the LSND results still awaits future experiments. Recent measurements in the KARMEN detector exclude part of the LSND allowed region [10].

TABLE I. Quantum numbers of preon chiral superfields in the model.

Preon	Sp(6)	SU(5)	$[U(1)_w]$	Z_2
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1		
Q	$\Box$	$\Box$		
Q'	$\Box$	1		$-$
$\overline{Q_i}$	1	$\overline{\Box}$		
$\overline{\chi_j}$	1	$\overline{\Box}$		$-$
H	1	$\Box$	$(+1)$	
$\overline{H}$	1	$\overline{\Box}$		
N_3	1	1		
S	1	1	(-1)	

TABLE II. Quantum numbers of chiral superfields after Sp(6) confines.

	SU(5)	$[U(1)_w]$	Z_2
$N_1 = A^3$	$\mathbf{1}$		
$N_2 = A^2$	$\mathbf{1}$		
N_3	$\mathbf{1}$		
$\mathbf{10}_1 = QA^2Q$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$		
$\mathbf{10}_2 = QAQ$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$		
$\mathbf{10}_3 = Q^2$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$		
$\chi_1 = QQ'$	$\Box$		$-$
$\chi_2 = QAQ'$	$\Box$		$-$
$\chi_3 = QA^2Q'$	$\Box$		$-$
$\overline{\mathbf{5}}_i = \overline{Q_i}$	$\overline{\Box}$		
$\overline{\chi_j}$	$\overline{\Box}$		$-$
H	$\Box$	$(+1)$	
$\overline{H}$	$\overline{\Box}$		
S	$\mathbf{1}$	(-1)	

Higgs fields H, $\overline{H}$ and matter field $\overline{Q_i}$ from extra fields. N_3 is the right-handed neutrino of the third generation. S is singlet under both Sp(6) and SU(5) gauge symmetries. In addition to Sp(6) and SU(5) gauge symmetries, we introduce anomalous $U(1)_w$ gauge symmetry,[3] which induces a Fayet-Iliopoulos term $\xi^2 \sim (g_S^2/192\pi^2)\mathrm{Tr}\, q_w M_p^2$ from the string loop corrections [25], where g_S and q_w are the string coupling and the charge of the anomalous $U(1)_w$ gauge symmetry, respectively. S and H have charges of $U(1)_w$ as Table I. We assume that many extra fields Xs which have plus $U(1)_w$ charges induce a Fayet-Iliopoulos term ξ of the order of the Planck scale. Then we can expect that S obtains the vacuum expectation value (VEV) of the order of the Planck scale $M_p \simeq O(10^{18})$ GeV. Extra fields Xs do not contribute to the low energy phenomenology and mass matrices of quark and lepton. We will see the reason why the field S and $U(1)_w$ gauge symmetry are introduced later.

We consider the situation that Sp(6) dynamical scale Λ satisfies $M_{GUT} < \Lambda < M_p$, where M_{GUT} is the SU(5) GUT scale of $O(10^{16})$ GeV. This condition is required by the proton stability. If $\Lambda < M_{GUT}$, D-term interactions in Kähler potential which are suppressed by $(1/\Lambda)$ might induce too rapid proton decay.[4]

Below the scale of Λ, this theory is described by the Sp(6) singlet states. We regard these Sp(6) singlet states as quark, lepton, Higgs field, and extra fields as Table II.

The mass term of Higgs particles H and $\overline{H}$ is induced from the operator $[\langle S\rangle H\overline{H}] \sim M_p H\overline{H}$. We assume that triplet-doublet splitting is realized in another mechanism, and Higgs doublets obtain suitable weak scale vacuum expectation values (VEVs) as $\langle H\rangle = v$ and $\langle \overline{H}\rangle = \overline{v}$, where $v^2 + \overline{v}^2 = (174\ \mathrm{GeV})^2$, according to the supersymmetry (SUSY) breaking effects.[5]

We assume χ_j and $\overline{\chi_j}$ do not take VEVs. Z_2 symmetry distinguishes χ_j from H, and $\overline{\chi_j}$ from $\overline{H}$, and $\overline{Q_i}$. At the scale of Λ, the mass matrix of χ_j and $\overline{\chi_j}$ is given by

$$(\overline{\chi_3}\ \overline{\chi_2}\ \overline{\chi_1})\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon & \epsilon^2 & \epsilon^3 \end{pmatrix} M_p \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix},$$

where $\epsilon \equiv \Lambda/M_p$. For example, the mass of χ_1 and $\overline{\chi_1}$ is induced as $[(QQ')\overline{\chi_1}] \sim \Lambda \chi_1 \overline{\chi_1}$. The dimensionless parameter ϵ is very important which will express the mass hierarchy of quark and lepton later. The above mass matrix has $O(1)$ coefficients, and we do not consider the case of zero determinant. Then three mass eigenvalues of χ_j and $\overline{\chi_j}$ are of $O(\epsilon M_p)$, $O(\epsilon^2 M_p)$, and $O(\epsilon^3 M_p)$. They are heavy enough not to affect the low energy phenomenology.

The Sp(6) strong dynamics induces the nonperturbative superpotential W_{dyn} [22], which is written as

$$\begin{aligned} W_{dyn} \simeq {} & \frac{g^3}{\Lambda^2} N_2^2 \mathbf{10}_3^2 \chi_1 + \frac{g^2}{\Lambda} N_1 [\mathbf{10}_2 \mathbf{10}_3 \chi_1 + \chi_2 \mathbf{10}_3 \mathbf{10}_3] \\ & + \frac{g^2}{\Lambda} N_2 [\mathbf{10}_3 \mathbf{10}_3 \chi_3 + \chi_1 \mathbf{10}_1 \mathbf{10}_3] \\ & + g[\mathbf{10}_3 \mathbf{10}_1 \chi_3 + \mathbf{10}_1 \mathbf{10}_1 \chi_1] \\ & + g[\mathbf{10}_1 \mathbf{10}_2 \chi_2 + \mathbf{10}_2 \mathbf{10}_2 \chi_3] \end{aligned} \qquad (3)$$

according to the field assignment of Table II. Here the factor $g \simeq 4\pi$ follows the power counting arguments in Ref. [26].

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[3]We expect the $U(1)_w$ anomaly is canceled by the Green-Schwarz mechanism [24].

[4]I would like to thank Professor T. Yanagida for teaching me this process.

[5]In this paper we assume that the SUSY is broken at the low energy, whose effects are negligible at the scale of M_{GUT}.

Since $\chi_j s$ have masses around grand unified theory (GUT) scale and do not take VEVs, the dynamically generated interactions of Eq. (3) have nothing to do with the low energy phenomenology.

Under the above assumptions, masses of quark and lepton are produced from irrelevant operators suppressed by the Planck scale. Yukawa interactions which include composite states (**10** and **1**) are suppressed by the dimensionless parameter ϵ. The mass hierarchy is generated since quark and lepton are composite states. This model is one of the models of Froggatt-Nielsen mechanism [27]. In order to obtain the suitable mass hierarchy, we set $\epsilon \sim 1/10$, which suggests $g \sim 1/\epsilon$. We denote 3×3 flavor space mass matrices as m_{ij}, which represents $L_i m_{ij} R_j$, where L_i and R_j are left- and right-handed fermions, respectively. The mass matrix of up-sector m^u_{ij}, down-sector m^d_{ij}, charged lepton m^e_{ij}, and Dirac neutrino m^ν_{ij} are given by

$$m^u_{ij} \simeq \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon^2 \end{pmatrix} g^2 v \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} v, \tag{4}$$

$$m^d_{ij} \simeq \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \end{pmatrix} g\bar{v} \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix} \bar{v}, \tag{5}$$

$$m^e_{ij} \simeq \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix} g\bar{v} \sim \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \bar{v}, \tag{6}$$

$$m^\nu_{ij} \simeq \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} g^2 v, \tag{7}$$

respectively. For example, m^u_{11} is induced from the operator $[g^2/M_p^7 (QA^2Q)(QA^2Q)HS]$. The Dirac masses of m^u_{ij} and m^ν_{ij} have factor g^2 while m^d_{ij} and m^e_{ij} have factor g. It is because m^u_{ij} and m^ν_{ij} have additional factor $g\langle S\rangle/M_p \sim g$. That is why we introduce the field S and extra $\mathrm{U}(1)_w$ gauge symmetry.[6] The Majorana mass of right-handed neutrinos M_ν is given by

$$M_\nu \simeq \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} M_p. \tag{8}$$

Through the seesaw mechanism the mass matrix of three light neutrinos m_{ν_l} becomes

$$\begin{aligned} m_{\nu_l} &= -m_\nu M_\nu^{-1} m_\nu^T, \\ &\simeq -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (\epsilon^2 \;\; \epsilon \;\; 1) \begin{pmatrix} 1/\epsilon^4 & 1/\epsilon^3 & 1/\epsilon^2 \\ 1/\epsilon^3 & 1/\epsilon^2 & 1/\epsilon \\ 1/\epsilon^2 & 1/\epsilon & 1 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \epsilon^2 \\ \epsilon \\ 1 \end{pmatrix} (1 \;\; 1 \;\; 1) \frac{g^4 v^2}{M_p}, \\ &\simeq -\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{g^4 v^2}{M_p}. \end{aligned} \tag{9}$$

The above mass matrices show only order, and all elements have $O(1)$ coefficients. Here we consider the situation that all matrices have nonzero determinant.[7]

This model induces the following mass hierarchy:

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon^2 : \epsilon : 1,$$

$$m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1. \tag{10}$$

It suggests almost realistic mass hierarchy[8] with $\epsilon \sim 1/10$. This model suggests the large $\tan\beta (\equiv v/\bar{v})$.

Let us show that this model naturally induces large mixing in the lepton sector and small mixing in the quark sector. For the quark sector, mass matrices m^u_{ij} and m^d_{ij} in Eqs. (4) and (5) derive

$$V^{\text{quark}}_{KM\;ij} \equiv U^{u\dagger}_L U^d_L \simeq \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \tag{11}$$

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[6]Although S takes the VEV of $O(M_p)$, higher order operators in the superpotential $[(SH)^n (S^k X^l)^m]$ are negligible. It is because $\langle H\rangle \simeq 10^2$ GeV and $\langle X\rangle = 0$. These operators have nothing to do with the low energy phenomenology and mass matrices of quark and lepton.

[7]If the determinant of Majorana mass of the right-handed neutrino of Eq. (8) is zero, the inverse matrix M_ν^{-1} does not exist. However, it is interesting to consider such a case, because the singular seesaw mechanism can work [18].

[8]The mass hierarchy of Eq. (10) is not precise for the first and the second generations of the down-quark and charged lepton sectors, since experiments suggest $m_d/m_s \sim m_e/m_\mu \sim O(10^{-2})$. In the next section, we will try to modify Eq. (10) by improving the model.

where U_L^u and U_L^d are unitary matrices which diagonalize m_{ij}^u and m_{ij}^d from the left-hand side, respectively. Then we can predict

$$V^{\text{quark}}_{KM\ ij} \simeq m_i^d / m_j^d \tag{12}$$

from Eqs. (10) and (11). If we input experimental values of masses in Eq. (12), we obtain

$$V_{us} \simeq \frac{m_d^{(\text{exp})}}{m_s^{(\text{exp})}} \sim 0.03-0.07, \quad V_{cb} \simeq \frac{m_s^{(\text{exp})}}{m_b^{(\text{exp})}} \simeq 0.02-0.04,$$

$$\frac{V_{ub}}{V_{cb}} \simeq \frac{m_d^{(\text{exp})}/m_b^{(\text{exp})}}{m_s^{(\text{exp})}/m_b^{(\text{exp})}} \sim 0.03-0.07, \tag{13}$$

where we use $m^{(\text{exp})}$ as the mass at 1 GeV [28], for one example.[9] This naive estimation derives too small V_{us} compared to the experimental value $V_{us}^{(\text{exp})} \simeq 0.22$. It is because we used wrong mass hierarchy $m_d/m_s \sim 10^{-1}$ of Eq. (10) when we estimate Eq. (13). Experiments suggest $m_d^{(\text{exp})}/m_s^{(\text{exp})} \sim 10^{-2}$ as Eq. (13). On the other hand, V_{cb} and V_{ub}/V_{cb} are roughly consistent with experimental values of $V_{cb}^{(\text{exp})} \simeq 0.036-0.046$ and $V_{ub}^{(\text{exp})}/V_{cb}^{(\text{exp})} \simeq 0.06-0.10$.

How about lepton flavor mixing? Equations (6) and (9) suggest that unitary matrices U_L^e and U^ν, which diagonalize m_{ij}^e and m_ν from the left-hand side,[10] respectively, both have the same form as

$$U_L^e \sim U^\nu \simeq \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}. \tag{14}$$

Thus the neutrino mixing matrix $V^{\text{lepton}} \equiv U_L^{e\dagger} U^\nu$ suggests the large mixing of $O(1)$, as long as the accidental cancellation occurs between U_L^e and U_ν. Equation (9) suggests mass squared differences are of order $\delta m^2 \simeq \mu^2 \simeq 10^{-2}$ eV^2 ($\mu \equiv g^4 v^2/M_p$), which can be the solution of the atmospheric neutrino problem with large mixing between ν_μ and ν_τ of $\delta m_{23}^2 \simeq (4-6) \times 10^{-3}$ eV^2 [1].[11] From Eq. (9) we can see that large mixing has nothing to do with the explicit form of Majorana mass at all. The Dirac mass structures of Eqs. (6) and (7) are crucial for large mixing, because the mass hierarchy of charged lepton and Dirac neutrino are produced only by right-handed fields. Flavor mixing is determined by the unitary matrices which diagonalize mass matrices from the left-hand side.

In this model, the origin of the mass hierarchy exists in the ''compositeness.'' This model naturally explains why large flavor mixing is realized in the lepton sector while small mixing is realized in the quark sector. This miracle comes from the field contents of SU(5) in Eq. (1). When $\mathbf{10}_i$ and $\mathbf{1}_i$ of SU(5) produce the mass hierarchy, large (small) mixing is realized in the lepton (quark) sector.

Now we discuss whether this model can solve the solar neutrino problem, or not. We can see that three mass eigenvalues of light neutrinos are all of order μ from Eq. (9). Then, two mass squared differences are naively of order $\delta m^2 \simeq \mu^2 \simeq 10^{-2}$ eV^2. Thus, we must introduce small parameters in the coefficients in order to obtain the mass squared difference of $O(10^{-5})$ eV^2 for the MSW solution, or $O(10^{-10})$ eV^2 for the vacuum oscillation solution. Here we assume that the neutrino mass matrix Eq. (9) has rank one,[12] which is the so-called ''democratic type'' mass matrix. In this case, three mass eigenvalues become of $O(0)$, $O(0)$, and $O(\mu)$. In order to solve the solar neutrino problem, we must introduce small parameters in the coefficients of Eq. (9) as the mass perturbation [12].

Here we should comment on R-parity, which distinguishes $\overline{Q_i}$ from $\overline{H}$, and N_i from S. It is difficult to introduce R-parity at the preon level.[13] N_is can be easily distinguished from S by $U(1)_w$ gauge symmetry. On the other hand, for $\overline{\mathbf{5}}$ fields, we simply assume that operators $[\overline{Q_i}HS]$, $[\overline{H}HS\mathbf{1}_i]$, and $[\mathbf{10}_{i_1}\overline{Q_{i_2}}\ \overline{Q_{i_3}}]$ are forbidden. The absence of an operator which is consistent with all symmetries sometimes happens in string derived models. Then the conventional R-parity symmetry as well as $\mathrm{U}(1)_{B-L}$ global symmetry appear in the effective theory below the confinement scale.

We can also derive the same structures of mass matrices as Eqs. (4)–(7) by another mechanism. One example is represented in the Appendix, where the origin of mass hierarchy is produced not by the ''compositeness'' but by the discrete symmetry.

We would like to show another possibility before closing this section. We can obtain large (small) mixing in the lepton (quark) sector, even in the case that only $\mathbf{10}_i$ produces the mass hierarchy. In this case, contrary to our model, there is no mass hierarchy in the Dirac mass and Majorana mass of neutrinos, since $\mathbf{1}_i$ does not produce mass hierarchy. The model with three sets of vectorlike extra generations of $\mathbf{10}$ and $\overline{\mathbf{10}}$ which is built by Babu and Barr is just the case [15].

[9] The explicit values of V_{KM}^{quark} in Eq. (13) have no meaning. Here we would like to discuss the order of the values of V_{KM}^{quark} elements.

[10] Since m_ν is Hermite, it is diagonalized by $U^\nu m_\nu U^{\nu T}$.

[11] To be accurate, we need to know the coefficients in Eqs. (6) and (9) to check whether this $O(1)$ mixing is maximal or not. However, this model cannot predict the coefficients.

[12] We can also consider the situation that three mass eigenvalues of $O(\mu)$ are closely degenerated and have small mass squared differences of $O(10^{-5})$ eV^2 or $O(10^{-10})$ eV^2. This situation also needs small parameters in the coefficients of neutrino mass matrix.

[13] One of the simplest examples is the introduction of Z_4 symmetry. We assign Z_4 charge as $Q(i)$, $Q'(-i)$, $A(+)$, $N_3(-)$, $\overline{\chi_j}(+)$, $\overline{Q_i}(-)$, $H(+)$, $\overline{H}(+)$, and $S(+)$. However, N_1 and N_2 have wrong signs in this case.

Another example is the composite model built by Strassler [20], where the possibility of large (small) mixing in the lepton (quark) sector have been mentioned by Strassler and Yanagida [29].

We can easily build the composite model which induces the same results of Babu and Barr based on Ref. [20].[14] Here we show it briefly. We introduce the hypercolor of $\mathrm{Sp}(2N)$ with $N_f = N+2 (N>1)$ fundamental representations for each generation. Contrary to our composite model, there are three hypercolors in addition to the standard gauge group. The composite state of $\mathbf{10}_i$ induces the hierarchy parameter $\epsilon_i \equiv \Lambda_i/M_p$ $(i=1,2,3)$, and the mass hierarchy of quark and lepton is given by

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3,$$

$$m_u : m_c : m_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2. \tag{15}$$

Since the proton stability demands $10^{-2} < \epsilon_i < 1$, we consider $\epsilon_3 \sim 1$ and $\epsilon_i/\epsilon_{i+1} \simeq 10^{-1}$. It is the model of large $\tan\beta$. This model also induces large mixing in the lepton sector as $V^{\mathrm{lepton}}{}_{ij} \simeq 1$ and small mixing in the quark sector as $V_{KM}^{\mathrm{quark}}{}_{ij} \simeq \epsilon_i/\epsilon_j$.

III. IMPROVING THE MODEL

The previous model naturally induces large (small) mixing in the lepton (quark) sector. We can naturally solve the atmospheric neutrino problem. However, we must introduce small parameters in the coefficients of neutrino mass matrix in order to solve the solar neutrino problem. In this section we try to improve the model in order to obtain the natural solution of the solar neutrino problem.

For this purpose, we introduce the discrete symmetry Z_2' and one more gauge singlet field Φ. Under this discrete symmetry only $\overline{\mathbf{5}}_1$ and Φ have odd charges, while other fields have even charges. We assume that Φ takes VEV as $\langle\Phi\rangle/M_p \ll 1$.[15] Then the mass matrices of m_{ij}^d, m_{ij}^e, and m_{ij}^ν in Eqs. (5), (6), and (7) are modified as

$$m_{ij}^d \simeq \begin{pmatrix} \phi\epsilon^2 & \epsilon^2 & \epsilon^2 \\ \phi\epsilon & \epsilon & \epsilon \\ \phi & 1 & 1 \end{pmatrix} \overline{v}, \tag{16}$$

$$m_{ij}^e \simeq \begin{pmatrix} \phi\epsilon^2 & \phi\epsilon & \phi \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \overline{v}, \tag{17}$$

$$m_{ij}^\nu \simeq \begin{pmatrix} \phi\epsilon^2 & \phi\epsilon & \phi \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} g^2 v, \tag{18}$$

respectively. Where we define $\phi \equiv g\langle\Phi\rangle/M_p \ll 1$. m_{ij}^u and M_ν have the same form as Eq. (4) and Eq. (8), respectively. The mass matrix of three light neutrinos m_{ν_l} becomes

$$\begin{aligned} m_{\nu_l} &= -m_\nu M_\nu^{-1} m_\nu^T, \\ &\simeq -\begin{pmatrix} \phi\epsilon^2 & \phi\epsilon & \phi \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \begin{pmatrix} 1/\epsilon^4 & 1/\epsilon^3 & 1/\epsilon^2 \\ 1/\epsilon^3 & 1/\epsilon^2 & 1/\epsilon \\ 1/\epsilon^2 & 1/\epsilon & 1 \end{pmatrix} \\ &\quad \times \begin{pmatrix} \phi\epsilon^2 & \epsilon^2 & \epsilon^2 \\ \phi\epsilon & \epsilon & \epsilon \\ \phi & 1 & 1 \end{pmatrix} \frac{g^4 v^2}{M_p}, \\ &\simeq -\begin{pmatrix} \phi^2 & \phi & \phi \\ \phi & 1 & 1 \\ \phi & 1 & 1 \end{pmatrix} \frac{g^4 v^2}{M_p}. \end{aligned} \tag{19}$$

Let us estimate flavor mixing in the quark and the lepton sector. For the quark sector, the flavor mixing matrix Eq. (11) does not change. V_{cb} and V_{ub}/V_{cb} are the same as in Eq. (13), which are roughly consistent with experiments. On the other hand, $V_{us} \simeq \epsilon$ becomes larger than the value of ration m_d/m_s, because Eq. (10) is modified as

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \phi\epsilon^2 : \epsilon : 1,$$

$$m_u : m_c : m_t \sim \epsilon^4 : \epsilon^2 : 1. \tag{20}$$

If $\phi \simeq O(10^{-1})$, the value of V_{us} and the mass hierarchy of the first and the second generations of down-sector and charged lepton become realistic.

As for the lepton sector, unitary matrices U_L^e and U^ν in Eq. (14) are modified as

$$U_L^e \sim U^\nu \simeq \begin{pmatrix} 1 & \phi & \phi \\ \phi & \cos\theta & -\sin\theta \\ \phi & \sin\theta & \cos\theta \end{pmatrix}, \tag{21}$$

where θ is a mixing angle of $O(1)$. It suggests that $V^{\mathrm{lepton}} \equiv U_L^{e\dagger} U^\nu$ has the same form as Eq. (21). This form seems to be suitable for the small mixing solar neutrino MSW solution of ν_e and ν_μ, and the large mixing atmospheric neutrino solution of ν_μ and ν_τ. However, it is not true. Equation (21) is derived because we estimate masses of three light neutri-

[14] In the original model in Ref. [20], top Yukawa of $O(1)$ is generated dynamically. Here we consider the situation that top Yukawa is also induced from perturbative interaction by introducing elementary Higgs fields. The composite ''Higgs field'' in Ref. [20] corresponds to χ_js in our model.

[15] Z_2 symmetry is broken at the scale of $\langle\Phi\rangle$. However, mass matrices of quark and lepton do not change since we consider the case of $g\langle\Phi\rangle/M_p \ll 1$.

nos as $O(\phi^2\mu)$, $O(\mu)$, and $O(\mu)$ in Eq. (19). Since two mass squared differences are both of $O(\mu^2)$ in this case, we cannot solve the solar neutrino problem without introducing small parameters in the coefficients of neutrino mass matrix Eq. (19).

Here we assume that the determinant of 2×2 submatrix of the second and the third generations is zero at order μ^2 in Eq. (19).[16] In this case, three mass eigenvalues are of $O(\phi\mu)$, $O(\phi\mu)$, and $O(\mu)$. The unitary matrix U^ν is modified as

$$U^\nu \sim \begin{pmatrix} 1/\sqrt{2} & 1/2 & 1/2 \\ -1/\sqrt{2} & 1/2 & 1/2 \\ \phi & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{22}$$

This form is justified for the sufficient small ϕ. Then lepton flavor mixing $V^{\rm lepton} \equiv U_L^{e\dagger} U^\nu$ can induce large mixings of $(\nu_e-\nu_\mu)$ and $(\nu_\mu-\nu_\tau)$.[17] This case is so-called ''bimaximal mixing.'' If we input $\phi\simeq10^{-1.5}$, we obtain $\delta m^2_{12} \simeq 10^{-5}$ eV^2, which is nothing but the large angle MSW solution. Besides, this case suggests the realistic mass hierarchy of quark and lepton, and realistic value of V_{us} as we have seen before. Thus, we can solve not only the atmospheric neutrino problem but also the solar neutrino problem by the large angle MSW solution in this model. On the other hand, if we input $\phi\simeq10^{-4}$, we obtain $\delta m^2_{12}\simeq10^{-10}$ eV^2, which is suitable for the vacuum oscillation solutions.[18] Unfortunately, masses of electron and down quark in Eq. (20) become too small when $\phi\simeq10^{-4}$. This case might also be a realistic solution by extending the model.

IV. SUMMARY AND DISCUSSION

In this paper we suggest a composite model which can naturally induce large flavor mixing in the lepton sector and small mixing in the quark sector. This model has only one hypercolor in addition to the standard gauge group, which makes composite states of preons. In this model, **10** and **1** representations in SU(5) are composite states, and they produce the mass hierarchy. This can explain why large mixing is realized in the lepton sector, while small mixing is realized in the quark sector. This model can naturally solve the atmospheric neutrino problem. In the improved model, we can derive the mass scale which is suitable for the solution of the solar neutrino problem. We can solve not only the atmospheric neutrino problem but also the solar neutrino problem by the large angle MSW solution in this model. The vacuum oscillation solution might be possible by extending the model.

TABLE III. Quantum numbers of chiral superfields.

Field	Sp(8)	SU(5)	$Z_5\times Z_2''$
P	$\square$	1	$(\omega,-)$
$\mathbf{10}_3$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$(1,+)$
$\mathbf{10}_2$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$(\omega^3,+)$
$\mathbf{10}_1$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$(\omega,+)$
$\overline{\mathbf{5}}_3$	1	$\overline{\square}$	$(\omega^3,+)$
$\overline{\mathbf{5}}_2$	1	$\overline{\square}$	$(\omega^3,+)$
$\overline{\mathbf{5}}_1$	1	$\overline{\square}$	$(\omega^3,+)$
$\mathbf{1}_3$	1	1	$(\omega^2,+)$
$\mathbf{1}_2$	1	1	$(\omega,+)$
$\mathbf{1}_1$	1	1	$(\omega^4,+)$
H	1	$\square$	$(1,+)$
$\overline{H}$	1	$\overline{\square}$	$(1,+)$

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APPENDIX: A MODEL WITH DISCRETE SYMMETRY

Here we suggest the model where the discrete symmetry plays a crucial role of producing mass hierarchy, which induces large (small) mixing of the lepton (quark) sector. We introduce a gauge group Sp(8) and its ten fundamental representations Ps. We also introduce the discrete symmetry $Z_5\times Z_2''$. The representations of fields are shown in Table III.

The Sp(8) hypercolor makes the composite state $M \equiv PP$. The nonperturbative effects of $Sp(8)$ gauge symmetry induce the superpotential [33]

$$W_{dyn} = X({\rm Pf}M - \Lambda^{10}), \tag{A1}$$

which makes the vacuum $\langle M\rangle \simeq \Lambda^2$. Λ is the strong coupling scale of Sp(8). $Z_5\times Z_2''$ symmetry reduces to Z_5 symmetry below the confinement scale Λ.

Let us show the mass matrices of quark and lepton. Mass terms which are not singlet under the discrete symmetry Z_5 are produced from the irrelevant operators suppressed by the Planck scale M_p. The mass hierarchy is expressed by the small dimensionless parameter $\eta \equiv \langle M\rangle/M_p^2 \ll 1$. The mass matrix of the up-sector m^u_{ij}, down-sector m^d_{ij}, charged lepton m^e_{ij}, and Dirac neutrino m^ν_{ij} are given by

[16]This situation is also the fine-tuning at this stage. We expect this situation might be realized by the string derived model.

[17]To be accurate, $\theta\simeq O(1)$ of U_L^e in Eq. (21) should be sufficiently small in order to realize ''bimaximal mixing.'' This situation can be easily realized by suitable coefficients of charged lepton mass matrix.

[18]The ''bi-maximal mixing,'' which explains the atmospheric neutrino oscillation and the vacuum oscillation for the solar neutrino, have been studied in Refs. [30–32].

$$m^u_{ij} \simeq \begin{pmatrix} \eta^4 & \eta^3 & \eta^2 \\ \eta^3 & \eta^2 & \eta \\ \eta^2 & \eta & 1 \end{pmatrix} v, \tag{A2}$$

$$m^d_{ij} \simeq \begin{pmatrix} \eta^3 & \eta^3 & \eta^3 \\ \eta^2 & \eta^2 & \eta^2 \\ \eta & \eta & \eta \end{pmatrix} \overline{v}, \tag{A3}$$

$$m^e_{ij} \simeq \begin{pmatrix} \eta^3 & \eta^2 & \eta \\ \eta^3 & \eta^2 & \eta \\ \eta^3 & \eta^2 & \eta \end{pmatrix} \overline{v}, \tag{A4}$$

$$m^\nu_{ij} \simeq \begin{pmatrix} \eta^4 & \eta^3 & 1 \\ \eta^4 & \eta^3 & 1 \\ \eta^4 & \eta^3 & 1 \end{pmatrix} v, \tag{A5}$$

respectively. The Majorana mass of right-handed neutrinos M_ν is given by

$$M_\nu \simeq \begin{pmatrix} \eta & 1 & \eta^2 \\ 1 & \eta^4 & \eta \\ \eta^2 & \eta & \eta^3 \end{pmatrix} M_p. \tag{A6}$$

The mass matrix of three light neutrinos m_{ν_l} is given by

$$\begin{aligned} m_{\nu_l} &= -m_\nu M_\nu^{-1} m_\nu^T, \\ &\simeq -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (\eta^4 \;\; \eta^3 \;\; 1) \\ &\quad \times \begin{pmatrix} 1/\eta & 1 & 1/\eta^2 \\ 1 & \eta & 1/\eta \\ 1/\eta^2 & 1/\eta & 1/\eta^3 \end{pmatrix} \begin{pmatrix} \eta^4 \\ \eta^3 \\ 1 \end{pmatrix} (1\;1\;1) \frac{v^2}{M_p}, \\ &\simeq -\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{v^2}{M_p \eta^3}. \end{aligned} \tag{A7}$$

The following mass hierarchy is derived in this model:

$$\begin{aligned} m_d : m_s : m_b &\sim m_e : m_\mu : m_\tau \sim \eta^3 : \eta^2 : \eta \\ m_u : m_c : m_t &\sim \eta^4 : \eta^2 : 1. \end{aligned} \tag{A8}$$

This model suggests small tan β. This model naturally induces large mixing in the lepton sector and small mixing in the quark sector. It is because $\mathbf{10}_i$ and $\mathbf{1}_i$ produce the mass hierarchy[19] as the composite model presented in this paper. Here the mass hierarchy is produced not by the ''compositeness'' but by the discrete symmetry.

[19]Three $\overline{\mathbf{5}}_i$s have the same discrete charge while $\mathbf{10}_i$s and $\mathbf{1}_i$s have different charges corresponding to the generation.

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