Triple gauge boson couplings in rare *B* and *K* decays

Gustavo Burdman*

Department of Physics, University of Wisconsin, Madison, Wisconsin 53706 (Received 18 June 1998; published 16 December 1998)

We examine the sensitivity of flavor changing neutral current (FCNC) processes to anomalous triple gauge boson couplings. We show that in the non-linear realization of the electroweak symmetry breaking sector, these processes are very sensitive to two *CP* conserving anomalous couplings. A clean separation of their effects is possible in the next round of experiments probing $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ processes, as well as kaon decays such as $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. The obtained sensitivity is found to be competitive with that of direct measurements at high energy colliders. In particular, for one of the *WWZ* couplings the one-loop FCNC effects are enhanced by a logarithmic dependence on the scale of new physics. We also explore the potential signals of *CP* violating anomalous triple gauge boson couplings in rare *B* decays. [S0556-2821(99)03901-6]

PACS number(s): 12.60.Cn, 12.60.Fr, 13.20.Eb, 13.20.He

I. INTRODUCTION

The remarkable experimental success of the standard model (SM) suggests the possibility that at the weak scale there may be no other dynamics or particle content. On the other hand, several questions remain unanswered within the SM framework and may require new dynamics in order to be addressed. Among these questions are the origin of electroweak symmetry breaking and of fermion masses. In principle, it could be argued that the energy scales of the new dynamics related to these questions may be so large as to be irrelevant to observables at the weak scale. However, it is known that the physics behind the Higgs sector and responsible for the breaking of the electroweak symmetry cannot reside at scales much higher than a few TeV. Furthermore, it is possible that the origin of the top quark mass might be related to electroweak symmetry breaking. Thus, at least in some cases, the dynamics associated with new physics may not reside at arbitrarily high energies and there might be observable effects at lower energies. In cases where the underlying dynamics is not known or fully understood, the study of these non-decoupling effects is the realm of effective field theory. The non-decoupling effects of the Higgs mechanism in the electroweak symmetry breaking (EWSM) sector of the SM have been vastly studied in the literature [1,2,3]. In order to write down the effective theory at energies well below the new physics scale Λ , all states with masses above Λ must be integrated out. The result is an effective field theory for the gauge bosons of the electroweak gauge group and the Nambu-Goldstone bosons (NGB) associated with the spontaneous breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_{\rm EM}$. The effective theory at weak scale energies and below is in general non-renormalizable. However, it is possible to expand it in terms of the increasing dimension of the operators: the higher the dimension of the operator, the higher the inverse power of Λ suppressing its effects. Up to a given order (e.g., operators of dimension six, eight, etc.), it is possible to obtain a predictive effective theory. The effects of the physics above the scale Λ are encoded in the values of

the coefficients of the higher-dimension operators.

We will concentrate on a scenario without scalars with masses below Λ . This choice is motivated by the fact that the presence of a light scalar is usually either accompanied by other new particles with masses of the order of the weak scale (e.g., supersymmetry) or allows for the scale of new physics to be very large [4], thereby resulting in suppressed effects. This scenario is most appropriately described by a Higgs sector with non-linear transformation properties [3]. However, as we will see below, for the most part our results will be independent of this choice. We will stress the relevance of using the non-linear realization when necessary.

Important constraints on deviations from the SM through the coefficients of the effective Lagrangian of the EWSB sector come from electroweak observables at the weak scale. For instance, non-standard contributions to two-point functions are severely constrained by oblique parameters as measured at the Z-pole [5]. Contributions to anomalous triple gauge-boson couplings (TGC) are bound by measurements of gauge boson production at the CERN e^+e^- collider LEPII [6] and the Fermilab Tevatron [7] as well as by indirect measurements [8,9], whereas anomalous quartic couplings give one-loop contributions to oblique parameters. Finally, there is a set of operators in the EWSB sector that amounts to corrections to the NGB propagators and that result in four-fermion operators coupling to fermion masses. These are not bound at one-loop by oblique corrections, but by their contributions to vertices through top-quark loops [10,11].

We are interested in evaluating the sensitivity of flavor changing neutral current (FCNC) decay processes at low energies, such as loop-induced *B* and *K* decays, to new dynamics in the EWSB sector residing above the scale Λ . These processes, such as $b \rightarrow s \gamma$, $b \rightarrow s \ell^+ \ell^-$, $K \rightarrow \pi \nu \overline{\nu}$, etc., are affected in principle by all non-standard couplings of the gauge bosons and the NGB. In practice, since oblique corrections are directly probed with high precision at the *Z* pole, the corrections to two-point functions are already highly constrained and will give no effect when included in the oneloop processes named above. Moreover, non-standard quartic gauge boson couplings do not enter in these decays to leading order. Thus, we are left with two sources of devia-

^{*}Email address: burdman@pheno.physics.wisc.edu

tions from the SM expectations in these modes: corrections to the NGB propagators and anomalous TGC. In Ref. [11] the sensitivity of rare *B* and *K* decays to the corrections to NGB propagators was studied. It was there concluded that within the constraints imposed on the effective Lagrangian parameters by the measurements of R_b , and *B* and *K* mixing large deviations from the SM were still possible in most FCNC decay modes, with the exception of $b \rightarrow s\gamma$ and b $\rightarrow d\gamma$. In this paper, we want to evaluate the sensitivity of these decay modes to anomalous TGC originated, through the effective Lagrangian, at energies above Λ .

The effects of anomalous TGC in rare B decays have been previously studied in the literature. For instance, the effects of the dimension four anomalous $WW\gamma$ coupling $\Delta \kappa_{\gamma}$ in $b \rightarrow s \gamma$ transitions were first considered in [12], whereas this plus the dimension six coupling λ_{γ} where studied in [13,14]. These plus the corresponding CP violating couplings and their effects in the $b \rightarrow s \gamma$ branching fractions were also considered in [15]. Finally, the anomalous WWZcouplings and their effects in $b \rightarrow s \mu^+ \mu^-$ were studied in Ref. [17]. In this paper, we use the power counting of the non-linear realization of the EWSB sector to organize the anomalous TGC according to the dimension of the operator generating them in the effective theory. This will identify the relevant anomalous TGC in scenarios where the EWSB sector is strongly coupled. We will see that in these cases, FCNC transitions are very sensitive to one $WW\gamma$ and one WWZ anomalous couplings, thus offering very well defined constraints on the strongly coupled EWSB sector that are competitive, for these couplings, to those obtained at higher energies. We also add the constraints from present and future measurements of rare kaon decays such as $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Previous studies of the effects of anomalous TGC couplings in rare K decays were done for $K \rightarrow \ell^+ \ell^-$ decays [16], a mode largely affected by long distance contributions, and for K^+ $\rightarrow \pi^+ \nu \overline{\nu}$ by considering the effects of a parity violating anomalous TGC coupling [8]. In this paper we study the effects of the two relevant couplings and put the effects in the context of the a specific scenario for EWSB and with the effects in B decays. We complete the analysis by considering the effects of CP violating TGC in rare B decays, both in the rate as well as in CP asymmetries.

The purpose of this work is to evaluate the sensitivity of future B and K experiments to anomalous TGC in the context of a strongly coupled EWSB sector. Although modelindependent in nature, this context results in a hierarchy of anomalous TGC related to the power counting in the resulting effective theory. A complete treatment of the effects of this scenario in rare B and K decays is lacking in the literature. This forms part of a program started in Ref. [11], intended to explore the reach of sensitivity processes like the ones discussed here to a strongly coupled EWSB sector. It is possible that in a scenario like this one direct signals will not become available until the CERN Large Hadron Collider (LHC) begins taking data. We also evaluate the competitiveness of these measurements with the direct measurements at higher energies to take place at the CERN LEPII and the Fermilab Tevatron colliders. We find these two approaches complementary largely due to the fact that the rare decay modes are selectively sensitive to a handful of anomalous TGC allowing independent measurements of these couplings. In the next section we review the non-linear realization of the effective Lagrangian of the EWSB sector in relation to anomalous TGC. In Sec. III we compute the effects in rare B and K decays and we discuss the results and conclude in Sec. IV.

II. THE EFFECTIVE LAGRANGIAN AND ANOMALOUS TGC

In the absence of a light Higgs boson, the symmetry breaking sector is represented by a non-renormalizable effective Lagrangian corresponding to the non-linear realization of the σ model. The essential feature is the spontaneous breaking of the global symmetry $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$. To leading order, the interactions involving the NGB associated with this mechanism and the gauge fields are described by

$$\mathcal{L}_{LO} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \operatorname{Tr}[W_{\mu\nu} W^{\mu\nu}] + \frac{v^2}{4} \operatorname{Tr}[D_{\mu} U^{\dagger} D^{\mu} U],$$
(1)

where $B_{\mu\nu}$ and $W_{\mu\nu} = \partial_{\mu}W_{\nu} - \partial_{\nu}W_{\mu} + ig[W_{\mu}, W_{\nu}]$ are the the $U(1)_{Y}$ and $SU(2)_{L}$ field strengths, respectively, the electroweak scale is v = 246 GeV and the NGB enter through the matrices $U(x) = e^{i\pi(x)^{a}\tau_{a}/v}$. The covariant derivative acting on U(x) is given by $D_{\mu}U(x) = \partial_{\mu}U(x) + igW_{\mu}(x)U(x)$ $-(i/2)g'B_{\mu}(x)U(x)\tau_{3}$. To this order, there are no free parameters once the gauge bosons masses are fixed. The dependence on the dynamics underlying the strong symmetry breaking sector appears at next-to-leading order. A complete set of operators at next to leading order includes one operator of dimension two and operators of dimension four [1,2]. The effective Lagrangian to next to leading order is given by (see the Appendix for the expanded operator basis)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{LO} + \mathcal{L}_1' + \sum_{i=1}^{19} \alpha_i \mathcal{L}_i, \qquad (2)$$

where \mathcal{L}'_1 is a dimension two custodial symmetry violating term absent in the heavy Higgs limit of the SM. If we restrict ourselves to CP invariant structures, there remain fourteen operators of dimension four. As it was mentioned above, the coefficients of some of these operators are constrained by low energy observables. For instance, precision electroweak observables bound the coefficient of \mathcal{L}'_1 , which gives a contribution to $\Delta \rho$. The combinations ($\alpha_1 + \alpha_8$) and ($\alpha_1 + \alpha_{13}$) contribute to the oblique parameters *S* and *U*, defined in [5]. Corrections to the charged and neutral NGB propagators come from the operators \mathcal{L}_{11} and \mathcal{L}_{12} , respectively. Their effects in *B* and *K* FCNC processes were studied in Ref. [11]. The coefficients α_2 , α_3 , α_9 and α_{14} modify the TGC and are the object of our study. Imposing *CP* conservation,¹ the most general form of the *WWN* ($N = \gamma, Z$) couplings can be written as [18]

$$\mathcal{L}_{WWN} = g_{WWN} \Biggl\{ i \kappa_N W^{\dagger}_{\mu} W_{\nu} N^{\mu\nu} + i g_1^N \\ \times (W^{\dagger}_{\mu\nu} W^{\mu} N^{\nu} - W_{\mu\nu} W^{\dagger\mu} N^{\nu}) + g_5^V \epsilon^{\mu\nu\rho\sigma} \\ \times (W^{\dagger}_{\mu} \partial_{\rho} W_{\nu} - W_{\mu} \partial_{\rho} W^{\dagger}_{\nu}) N_{\sigma} + i \frac{\lambda_N}{M_W^2} W^{\dagger}_{\mu\nu} W^{\nu}_{\lambda} N^{\nu\lambda} \Biggr\},$$
(3)

with the conventional choices being $g_{WW\gamma} = -e$ and $g_{WWZ} = -g \cos \theta$. In principle, there are six free parameters, since gauge invariance implies $\Delta g_1^{\gamma} = g_5^{\gamma} = 0$. Making contact with the electroweak Lagrangian (2), these parameters can be expressed in terms of the next-to-leading order coefficients [19,2]

$$\Delta \kappa_{\gamma} \equiv \kappa_{\gamma} - 1 = g^{2}(\alpha_{2} - \alpha_{1} + \alpha_{3} - \alpha_{8} + \alpha_{9})$$

$$\Delta \kappa_{Z} \equiv \kappa_{Z} - 1 = g^{2}(\alpha_{3} - \alpha_{8} + \alpha_{9}) + g'^{2}(\alpha_{1} - \alpha_{2})$$

$$\Delta g_{1}^{Z} \equiv g_{1}^{Z} - 1 = \frac{g^{2}}{\cos^{2} \theta_{W}} \alpha_{3}$$

$$g_{5}^{Z} = \frac{g^{2}}{\cos^{2} \theta_{W}} \alpha_{14}$$

$$\lambda_{\gamma} = \lambda_{Z} = 0, \qquad (4)$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively, and the operator basis is the one defined in [1]. As we see from the last line in Eq. (4), to this order in the energy expansion (2), we obtain $\lambda_N = 0$. These TGC get contributions from operators of dimension six, suppressed by an extra factor of (v^2/Λ^2) . We are left with κ_γ , κ_Z , g_1^T and g_5^Z . Finally, when considering rare *B* and *K* decays, we can neglect the contributions from κ_Z , since they will be suppressed by powers of the squared of the external momenta over m_Z^2 . Thus, in this approach, there are only three parameters relevant at very low energies. The SM predictions for them are $\kappa_\gamma = g_1^T = 1$ and $g_5^Z = 0$.

III. THE EFFECTS IN FCNC DECAYS

The presence of the anomalous TGC $\Delta \kappa_{\gamma}$, Δg_1^Z and g_5^Z will result in deviations from the SM in various FCNC *B* and *K* decays.² We first concentrate on rare *B* decays, with focus on strategies to make use of large data samples for the vari-

ous modes. We then present the constraints from $K^+ \rightarrow \pi^+ \nu \overline{\nu}$ measurements, and finally study the possible effects of CP violating anomalous TGC.

A. Rare B decays

For the $b \rightarrow s \gamma$ and $b \rightarrow s \ell^+ \ell^-$ transitions, it is useful to cast the contributions of the anomalous couplings as shifts in the matching conditions at M_W for the Wilson coefficient functions in the weak effective Hamiltonian

$$H_{\rm eff} = -\frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \qquad (5)$$

with the operator basis defined in Ref. [21]. Of interest in our analysis are the electromagnetic penguin operator

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \qquad (6)$$

and the four-fermion operators corresponding to the vector and axial-vector couplings to leptons,

$$\mathcal{O}_{9} = \frac{e^{2}}{16\pi^{2}} (\bar{s}_{L} \gamma_{\mu} b_{L}) (\bar{\ell} \gamma^{\mu} \ell)$$
(7)

and

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell). \tag{8}$$

We first turn to the effects of $\Delta \kappa_{\gamma}$ in $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ transitions. This modification of the $W^+W^-\gamma$ coupling gives a shift in the one-loop $b \rightarrow q\gamma$ vertex, with (q=d,s). For q=s this is given by

$$\delta\Gamma^{b\to s\gamma}_{\mu} = i \frac{e}{4\pi^2} \frac{G_F}{\sqrt{2}} V^*_{ls} V_{lb} [\delta C_7(M_W) m_b \overline{s}_L \sigma_{\mu\nu} b_R q^{\nu} + \delta C_9(M_W) \overline{s}_L (\not q q_\mu - q^2 \gamma_\mu) b_L], \qquad (9)$$

where q_{μ} is the photon four-momentum, only the top quark contributions are kept and terms suppressed by m_s/m_b have been neglected. These shifts in the Wilson coefficients at M_W are given by

$$\delta C_7(M_W) = \frac{1}{2} \Delta \kappa_\gamma A_1(x_t) \tag{10}$$

$$\delta C_9(M_W) = \Delta \kappa_\gamma A_2(x_t), \tag{11}$$

with $x_t = m_t^2 / M_W^2$. The functions $A_1(x)$ and $A_2(x)$ are given by [12]

$$A_1(x) = \frac{x}{2} \left(\frac{2x}{(1-x)^2} + \frac{(3-x)}{(1-x)^3} \ln x \right),$$
 (12)

and

¹We discuss possible effects from CP violating anomalous TGC later in the paper.

²Charm FCNC decays are generally affected by large longdistance contributions that tend to obscure the extraction of short distance physics. Although there are some exceptions to this statement, the effects of anomalous TGC are not among them [20].



FIG. 1. The $BR(b \rightarrow s\gamma)$ vs $\Delta \kappa_{\gamma}$. The dashed horizontal lines correspond to the 1σ CLEO measurement [22], whereas the dotted lines are the 1σ measurement from ALEPH [23].

$$A_2(x) = -\frac{x}{4} \left(\frac{(1-5x)}{(1-x)^2} + \frac{(7-15x+4x^2)}{(1-x)^3} \ln x \right).$$
(13)

In the language of the effective Hamiltonian formalism, these contributions translate into modifications of the matching conditions for the Wilson coefficient functions C_7 and C_9 at the scale M_W . The first term in Eq. (9) modifies $C_7(M_W)$ and therefore contributes to both $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$, whereas the second term only enters in the offshell photon amplitude and gives a contribution to $C_9(M_W)$. The anomalous TGC diagrams containing the $\Delta \kappa_{\gamma}$ have the same divergent structure as the SM TGC contributions, and therefore the GIM mechanism renders them finite by decreasing their degree of divergence by one, thus eliminating an initially logarithmic divergence.

In order to compute the effects in *B* decays, we evolve the Wilson coefficients down to the scale $\mu \simeq m_b$ using standard procedures [21]. In Fig. 1 we plot the $b \rightarrow s \gamma$ branching fraction as a function of $\Delta \kappa_{\gamma}$. Also shown for reference are the 1σ intervals from the latest measurements of the CLEO Collaboration [22]: $BR(b \rightarrow s \gamma) = (2.50 \pm 0.47 \pm 0.39) \times 10^{-4}$, as well as from the ALEPH Collaboration [23]: $BR(b \rightarrow s \gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$. Combining these two results gives an approximate 1σ interval for $\Delta \kappa_{\gamma}$ (-0.20, 0.20). Future measurements of the $b \rightarrow s \gamma$ branching ratio will greatly improve these constraints. For instance, a 20% measurement of the $b \rightarrow s \gamma$ branching ratio would translate into the more stringent 1σ bound $-0.15 < \Delta \kappa_{\gamma} < 0.15$, if centered at the SM prediction.

The dilepton modes $b \rightarrow s \ell^+ \ell^-$ receive contributions from $\Delta \kappa_{\gamma}$ through both δC_7 and δC_9 . In Fig. 2 the branching ratio $BR(b \rightarrow s \ell^+ \ell^-)$, normalized by the SM value, is plotted versus $\Delta \kappa_{\gamma}$. Although the sensitivity of these decay channels is similar to the one obtained in $b \rightarrow s \gamma$, the bounds are somewhat less stringent. This is more so when we consider that, unlike in $b \rightarrow s \gamma$, other anomalous TGC may significantly affect this amplitude. However, we will later come back to this point to show that it is possible to cleanly sepa-



FIG. 2. The $b \rightarrow s \ell^+ \ell^+$ branching ratio, normalized to its SM value, plotted vs $\Delta \kappa_{\gamma}$.

rate the contributions from the various relevant couplings even if only $b \rightarrow s \ell^+ \ell^-$ decays are considered.

The sensitivity of these rare B decays to $\Delta \kappa_{\gamma}$ is certainly comparable to that of higher energy experiments such as LEPII and the Tevatron. For instance, the 95% C.L. limits from LEPII [6] combining the data taken at 162 GeV and at 172 GeV ($10pb^{-1}$ at each energy) are (-1.10, 1.80). On the other hand, the most recent measurements at the Fermi-Tevatron [7] put this coupling in the range lab (-0.36, 0.45). The Tevatron bounds depend on the scale of suppression introduced with the momentum dependence of the couplings, necessary to respect unitarity constraints. Both the Tevatron and the LEP bounds are obtained within a certain set of assumptions. Future LEPII measurements at higher energies, as well as Tevatron measurements with higher luminosity, will result in bounds similar to the ones that will be obtained from FCNC processes named above.

We now turn to the effects of Δg_1^Z , an anomalous W^+W^-Z coupling. Its presence affects the $b \rightarrow q \ell^+ \ell^-$ amplitude as well as the one of the neutrino modes $b \rightarrow q \nu \overline{\nu}$ and $K \rightarrow \pi \nu \overline{\nu}$. The modes governed by $b \rightarrow s \ell^+ \ell^+$ are the most accessible experimentally among the B processes. Unlike the $\Delta \kappa_{\gamma}$ contribution, the diagrams including Δg_1^Z are still divergent, even after summing over all the intermediate up-quark states. This divergence originates in the contributions from the longitudinal pieces in the W propagator and reflects the non-decoupling behavior of the Higgs sector. This logarithmic dependence of the loop effect on the high energy scale Λ is a manifestation of the dynamics above this scale, and is presumably related to electroweak symmetry breaking. Thus, this logarithmic enhancement of the one-loop effect of Δg_1^Z is of a rather fundamental origin [24] and makes FCNC particularly sensitive to this anomalous coupling.

The matching conditions for the Wilson coefficients $C_9(M_W)$ and $C_{10}(M_W)$ are shifted by

$$\delta C_9(M_W) = \Delta g_1^Z \left(\frac{1 - s^2 \theta_w}{s^2 \theta_w} \right) \left(s^2 \theta_w - \frac{1}{4} \right) B_1(x_t) \quad (14)$$

$$\delta C_{10}(M_W) = \Delta g_1^Z \left(\frac{1 - s^2 \theta_W}{s^2 \theta_W} \right) \frac{B_1(x_t)}{4}.$$
 (15)



FIG. 3. The $b \rightarrow s \ell^+ \ell^+$ branching ratio, normalized to its SM value vs Δg_1^Z (solid line) and g_5^Z (dashed line).

The function $B_1(x)$ is given simply by the leading logarithmic dependence,

$$B_1(x) = \frac{3}{2} x \ln \frac{\Lambda^2}{M_W^2} + \dots$$
 (16)

In Eq. (16), the dots stand for terms that are finite in the Λ $\rightarrow \infty$ limit. These terms are regularization scheme dependent and, although formally subleading, could be numerically important. However, it is expected that the overall size of the effect is correctly estimated by the leading logarithmic behavior, barring precise cancellations with the finite terms. Thus, the results we present for Δg_1^Z are meant to be indicative of the sensitivity to this coupling, but not a precise prediction,³ something that cannot be achieved without knowledge of the full theory above the energy scale Λ . The solid line in Fig. 3, shows the branching ratio for b $\rightarrow s\ell^+\ell^-$, normalized to the SM model prediction, as a function of Δg_1^Z , where the high energy scale scale in Eq. (16) is taken to be $\Lambda = 2$ TeV. Although at present, only upper limits on $b \rightarrow s \ell^+ \ell^-$ processes exist [25], sensitivity to the SM predictions is expected to be achieved in the next round of *B* physics experiments. For instance, measurements of $b \rightarrow s \ell^+ \ell^-$ branching ratios with 30% accuracy, can explore the region $|\Delta g_1^Z| < 0.10$, a very competitive performance even when compared with the high energy machines. For instance, LEPII is expected to just explore this region [26], whereas the Tevatron experiments, assuming an integrated luminosity of $1 f b^{-1}$, will bound Δg_1^Z to be in the interval (-0.18, 0.48) [27]. The main difference between these measurements and the FCNC decay modes is that the latter have an additional dependence on Λ from the logarithmic divergence.

Next, we study the effects of the *C* and *P* violating, but *CP* conserving coupling g_5^Z . These are simply obtained by



FIG. 4. The forward-backward asymmetry for leptons in $B \rightarrow K^* \ell^+ \ell^-$, for $\Delta g_1^Z = 0$, 0.1 and 0.20 (solid, dashed, dot-dashed, respectively). Although these give large effects in the branching ratio, the position of the asymmetry zero is almost unaffected.

the replacement $\Delta g_1^Z B_1(x_t) \rightarrow g_5^Z B_2(x_t)$ in Eqs. (14) and (15), where $B_2(x)$ is given by

$$B_2(x) = -\frac{3x}{1-x} \left(1 + \frac{x \ln x}{1-x} \right).$$
(17)

Unlike the contribution from Δg_1^Z , the resulting loop amplitude is finite, due to the fact that the $\epsilon_{\mu\nu\rho\sigma}$ tensor accompanying g_5^Z does not couple to the longitudinal portion of the *W* propagators. As a result, the contributions from this parameter to one-loop FCNC processes are not sensitive to the scale Λ . This, in turn, implies that in this case there is no logarithmic enhancement as in the case of the Δg_1^Z contribution and that these processes are not very sensitive to this coefficient, as can be seen from the dashed line in Fig. 3. This is in agreement with the conclusions of Ref. [8].

From the above results, we conclude that B decay processes involving one-loop FCNC are most sensitive to two CP conserving anomalous TGC, namely $\Delta \kappa_{\gamma}$ and Δg_1^2 . As we will see below, the analogous K decay modes have a similar sensitivity to Δg_1^Z . This is an important difference with the high energy searches for these effects, where the experiments are sensitive to several parameters giving room to possible cancellations with the consequent weakening of the bounds. The limited sensitivity of the low energy FCNC transitions permits the clean identification of the anomalous TGC. The obvious example is the fact that $b \rightarrow s \gamma$ is sensitive only to $\Delta \kappa_{\gamma}$, among the CP conserving couplings. However, even when only considering $b \rightarrow s \ell^+ \ell^-$ processes, where both $\Delta \kappa_{\gamma}$ and Δg_1^Z contribute, it is possible to separate their effects. This results from a very distinct pattern of shifts of the short distance Wilson coefficients. As it can be seen in Eq. (14), the shift in the coefficient $C_{9}(M_{W})$ will be negligible due to the suppression factor $(\sin^2 \theta_w - 1/4)$, whereas this is not the case for $C_{10}(M_W)$. This is reflected in Fig. 4, where we plot the forward-backward asymmetry for leptons in $B \rightarrow K^* \ell^+ \ell^-$ as a function of the dilepton mass. The asymmetry has a zero, the position of which depends on the values of C_7 and C_9 , but not on C_{10} [28]. Thus, values of

³In Ref. [17] this contribution was computed in the unitary gauge, and the scheme dependent terms were kept. Here we argue that only the logarithmic divergence can be trusted. The dependence on the scale Λ is common to both results.



FIG. 5. The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio, normalized to the SM prediction, plotted vs the anomalous *WWZ* couplings Δg_1^Z (solid line), and g_5^Z (dashed line).

 Δg_1^Z resulting in large deviations of the branching fractions in $b \rightarrow s \ell^+ \ell^-$ decays, do not change the position of the asymmetry zero. On the other hand, non-zero values of $\Delta \kappa_{\gamma}$ affect both C_7 and C_9 shifting the position where the asymmetry vanishes. In this way the angular information makes possible the separation between $\Delta \kappa_{\gamma}$ and Δg_1^Z effects that otherwise could be unresolvable in the branching ratio or even in the dilepton mass distribution.

B. Rare K decays

Effects similar to those discussed above for *B* decays are present in the analogous *K* processes, due to the one loop contributions to the $s \rightarrow d\gamma$ and $s \rightarrow dZ$ vertices. The photon mediated transitions, such as $K \rightarrow \pi \ell^+ \ell^-$ and hyperon radiative decays, are largely affected by long distance contributions which are theoretically uncertain and make difficult the extraction of interesting short distance information. On the other hand, $s \rightarrow d\nu\bar{\nu}$ transitions such as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ and $K_L \rightarrow \pi^0 \nu\bar{\nu}$ are theoretically cleaner. There are two diagrams contributing to these processes, the box and the $s \rightarrow dZ$ penguin. The latter is sensitive to Δg_1^Z and g_5^Z . The anomalous contribution to the $s \rightarrow d\nu\bar{\nu}$ amplitude can be written as

$$\delta \mathcal{A}(s \to d \nu \bar{\nu}) = \frac{4G_F}{\sqrt{2}} \frac{\alpha \cot^2 \theta_w}{8\pi} V_{td}^* V_{ts} (\Delta g_1^Z B_1(x_t) + g_5^Z B_2(x_t)) (\bar{d}_L \gamma_\mu s_L) (\bar{\nu}_L \gamma^\mu \nu_L), \quad (18)$$

with the functions $B_1(x)$ and $B_2(x)$ defined in Eqs. (16) and (17). As discussed in the previous section, only the effect of Δg_1^Z is sensitive to the logarithmic dependence on the high energy scale Λ , due to its coupling to the longitudinal gauge bosons. In Fig. 5 we plot the branching fraction for K^+ $\rightarrow \pi^+ \nu \bar{\nu}$, normalized to the SM expectation, as a function of Δg_1^Z . We observe that this decay mode has a sensitivity to Δg_1^Z comparable to that of the $b \rightarrow s \ell^+ \ell^-$ decays. However, the effect here is anti-correlated with the analogous one in *B* processes. Currently, this branching ratio is measured to be [29] $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (4.2^{+9.7}_{-3.5}) \times 10^{-10}$, whereas the SM



FIG. 6. The $BR(b \rightarrow s\gamma)$ vs the CP violating $WW\gamma$ coupling $\tilde{\kappa}_{\gamma}$. The dashed horizontal lines correspond to the 1σ CLEO measurement [22], whereas the dotted lines are the 1σ measurement from ALEPH [23].

prediction is in the range $(0.60-1.00) \times 10^{-10}$ [30]. Thus, as it can be seen in Fig. 5, there is room for relatively large values of Δg_1^Z in both *B* and *K* FCNC decays.

C. CP Violating anomalous TGC

In this section we discuss the possible effects of CP violating TGC. The most general form of the CP violating couplings of a neutral gauge boson $N = \gamma$, Z to a W pair is

$$\mathcal{L}_{CPV} = g_{WWN} \Biggl\{ i \tilde{\kappa}_N W^{\dagger}_{\mu} W_{\nu} \tilde{N}^{\mu\nu} - g_4^N W^{\dagger}_{\mu} W_{\nu} (\partial^{\mu} N^{\nu} + \partial^{\nu} N^{\mu}) + i \frac{\tilde{\lambda}_N}{M_W^2} W^{\dagger}_{\lambda\nu} W^{\mu}_{\nu} \tilde{N}^{\nu\lambda} \Biggr\},$$
(19)

with $\tilde{N}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} N_{\alpha\beta}$. The effects of the ZW^+W^- CP violating couplings in rare *B* and *K* decays are suppressed by powers of the typical external momentum divided by m_Z^2 , since all terms in Eq. (19) involve derivatives of the *Z* field. On the other hand, the only γW^+W^- coupling corresponding to a dimension four operator and satisfying gauge invariance is $\tilde{\kappa}_{\gamma}$, since $\tilde{\lambda}_{\gamma}$ corresponds to a dimension six operator in the non-linear realization. In the effective Lagrangian (2), there are eight dimension four operators contributing to the various CP violating terms in Eq. (19). The complete set of CP violating operators is given in the Appendix. In that basis, the contributions to $\tilde{\kappa}_{\gamma}$ are

$$\tilde{\kappa}_{\gamma} = 2g^2(-\alpha_{16} - 4\alpha_{17}). \tag{20}$$

The $\tilde{\kappa}_{\gamma}$ contributions to $b \rightarrow s\gamma$ and $b \rightarrow s\ell^+\ell^-$ take the form of complex shifts of the Wilson coefficients C_7 and C_9 . The CP violating contribution to the coefficient of the magnetic moment operator $\bar{s}_L \sigma_{\mu\nu} b_R$ takes the form

$$C_7(M_W) = C_7^{\text{SM}}(M_W) - \frac{i}{2} \tilde{\kappa}_{\gamma} A_1(x_t),$$
 (21)

where the function $A_1(x)$ is given in Eq. (12). On the other hand, the leading order contributions from $\tilde{\kappa}_{\gamma}$ to the second term in Eq. (9) corresponding to the shift in the coefficient $C_9(M_W)$, are of order $\mathcal{O}(m_b^2/M_W^2)$ and therefore negligible.

The $\tilde{\kappa}_{\gamma}$ contribution to $C_7(M_W)$ results always in a constructive effect in the $b \rightarrow s \gamma$ branching ratio, since there is no interference with the SM. This translates into a rather tight bound on $\tilde{\kappa}_{\gamma}$, as it can be seen from Fig. 6. Taking the 95% C.L. upper bound from the CLEO result, for instance, constrains this coupling to be in the range

$$-0.60 \leqslant \tilde{\kappa}_{\gamma} \leqslant 0.60. \tag{22}$$

The $b \rightarrow s \ell^+ \ell^-$ modes give looser bounds. More stringent bounds than these come from the upper limits on the electric dipole moment (EDM) of the neutron [31], giving the constraint $|\tilde{\kappa}_{\gamma}| < (2-3) \times 10^{-4}$. The neutron EDM bound is sensitive to the cutoff Λ in the same way the Δg_1^Z contributions to rare *B* and *K* decays are. On the other hand, the present bounds are cutoff independent by virtue of the Glaskow-Iliopoulos-Marcene (GIM) cancellation. Direct limits at hadron colliders are similar to the ones to be obtained in $b \rightarrow s \gamma$. For instance, in Ref. [32] is is estimated that the Tevatron with an integrated luminosity of $1 f b^{-1}$, will result in the bound $|\tilde{\kappa}_{\gamma}| < 0.33$.

Taking into account the bound from Eq. (22), we now consider possible CP violating observables. In the SM, CP violating asymmetries in $b \rightarrow d\gamma$ and $b \rightarrow d\ell'^+ \ell'^-$ are expected to be in the few percent range [33]. On the other hand, they are negligibly small in the corresponding $b \rightarrow s$ transitions, due to an extra factor of the Cabibbo angle. Thus, processes with strange mesons, such as $B \rightarrow K\ell'^+\ell'^-$, are free of SM sources of CP violation. For a partial rate asymmetry to arise, it is necessary that a CP-invariant phase be present in the amplitude. In the case of $b \rightarrow s$ transitions this is provided, for instance, by the imaginary part of the oneloop insertion of the four-fermion operators such as $(\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$ in the $b \rightarrow s \gamma^{(*)}$ vertex. The mixing of this operator with \mathcal{O}_9 results in [21]

$$C_{9}^{\text{eff}} = C_{9}(m_{b}) + g(s)(3C_{1} + C_{2} + 3C_{3} + C_{4} + 3C_{5} + C_{6}),$$
(23)

where the coefficients of the four-quark operators can be found in Ref. [21] and the function g(s) is given by

$$g(s) = -\frac{4}{9}\ln z^{2} + \frac{8}{27} + \frac{16}{9}\frac{z^{2}}{s} - \frac{2}{9}\left(2 + \frac{4z^{2}}{s^{2}}\right) \begin{cases} 2\sqrt{4z^{2}/s^{2} - 1} \arctan\left(\frac{1}{\sqrt{z - 1}}\right), & \text{for } s < 4m_{c}^{2}, \\ \sqrt{1 - 4z^{2}/s^{2}}\left[\ln\left(\frac{1 + \sqrt{1 - 4z^{2}/s^{2}}}{1 - \sqrt{1 - 4z^{2}/s^{2}}}\right) + i\pi\right] & \text{for } s > 4m_{c}^{2}, \end{cases}$$

$$(24)$$

where $z = m_c/m_b$. The imaginary part present in Eq. (24), in combination with the CP violating phase coming from $\tilde{\kappa}_{\gamma}$, results in a small CP asymmetry. For instance, when the constraint from Eq. (22) is considered, the partial rate CP asymmetries in $B \rightarrow K \ell^+ \ell^-$ are bound to be

$$A_{\rm CP}(B \to K \ell^+ \ell^-) = \frac{\Gamma(B^+ \to K^+ \ell^+ \ell^-) - \Gamma(B^- \to K^- \ell^+ \ell^-)}{\Gamma(B^+ \to K^+ \ell^+ \ell^-) + \Gamma(B^- \to K^- \ell^+ \ell^-)} \lesssim 1\%.$$

$$\tag{25}$$

Similar asymmetries are obtained in other strange meson modes, such as K^0 , K^* , etc. Given the suppression of the SM asymmetries observation of CP violation at this level would indicate the presence of new physics. On the other hand, several thousand reconstructed events would be needed for a significant measurement. We therefore conclude that the experimental observation of the effects of CP violating anomalous TGC is beyond the capabilities of first generation *B* factories, where only a few hundred events are expected in these decay channels.

IV. CONCLUSIONS

We have carried out a comprehensive study of the effects of anomalous TGC in FCNC *B* and *K* decays. We have seen that these processes are sensitive to two *CP* conserving couplings, $\Delta \kappa_{\gamma}$ and Δg_1^Z , as well as to the CP violating coupling $\tilde{\kappa}_{\gamma}$. The reach of the next round of measurements at *B* physics experiments such as Babar, Belle, CDF and D0 will put bounds on the CP conserving couplings that are comparable to the limits to be obtained from direct gauge boson production at LEPII and an upgraded Tevatron. For comparison, in Table I we quote the 95% C.L. bounds on $\Delta \kappa_{\gamma}$ and Δg_1^Z projected for LEPII [26] at 190 GeV and with 500 fb^{-1} of integrated luminosity, as well as the limits for an upgraded Tevatron [27] with $1 fb^{-1}$.

For the future bounds from FCNC *B* decays, we use very conservative estimates of 1σ bounds that include *current* the-

TABLE I. Comparison of bounds on anomalous TGC.

	LEPII 190 GeV	Tevatron RunII 1 fb ⁻¹	FCNC Decays
$\Delta \kappa_{\gamma}$	(-0.25, 0.40)	(-0.38, 0.38)	(-0.20, 0.20)
$\Delta g_1^{\dot{Z}}$	(-0.08, 0.08)	(-0.18, 0.48)	(-0.10, 0.10)
$\tilde{\kappa}_{\gamma}$	-	(-0.33, 0.33)	(-0.50, 0.50)

oretical uncertainties present in the calculation of these modes. For instance, as mentioned earlier and can be seen from Fig. 1, a 20% measurement of the $b \rightarrow s \gamma$ branching ratio would bound $\Delta \kappa_{\gamma}$ to be in the range (-0.15, 0.15). For the bounds on $\Delta g_1^{Z'}$, we rely on the projections for various $b \rightarrow s \ell^+ \ell^-$ decay modes to be observed at B experiments at the SM level. For instance, several hundred events in the $B \rightarrow K^* \ell^+ \ell^+$ channel will be available at the Tevatron experiments in the incoming run. This will allow not only tight bounds from the effect in the rate (Fig. 3), but also the clean separation of the Δg_1^Z coupling from possible effects from anomalous $WW\gamma$ couplings by analyzing dilepton angular information. The forward-backward asymmetry for leptons shown in Fig. 4 is an example of this separation: the position of the asymmetry zero is immune to Δg_1^Z , whereas it is very sensitive to changes in the $WW\gamma$ couplings. On the other hand, it is possible to extract the short distance information from these exclusive modes by using a variety of techniques mostly related to heavy and light quark symmetry arguments, and with relatively small hadronic uncertainties [28,34].

The limits on Δg_1^Z can be further improved by future measurements of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching fraction. This mode is as sensitive to the *WWZ* anomalous coupling as the $b \rightarrow s \ell^+ \ell^-$ modes, with the advantage that it is not polluted by the *WW* γ couplings.

We have also studied the effects of CP violating anomalous TGC, among which only $\tilde{\kappa}_{\gamma}$ is of relevance in FCNC decays. As seen in Fig. 6, the current 1σ bound from the $b \rightarrow s\gamma$ branching ratio measurement is $-0.60 < \tilde{\kappa}_{\gamma} < 0.60$. Thus, the range quoted in Table I is a rather conservative estimate of what can be achieved by the next generation measurements of this decay channel. It compares well with what can be obtained by direct measurements, for instance, through $W\gamma$ production at the Tevatron [32].

On the other hand, we have seen that the identification of an effect in the radiative channels as coming from a CP violating coupling would require measurements of CP asymmetries below 1%. This can only be obtained with several thousand reconstructed events in channels such as $B \rightarrow K \ell^+ \ell^-$, a goal that is beyond the first generation *B* factories and perhaps to be attained by future dedicated *B* experiments at hadron colliders, such as the LHC-B at CERN or BTeV at the Tevatron.

We now briefly discuss the potential impact of these bounds on our understanding of the EWSB sector of the SM. As mentioned earlier, we focused on the non-linear realization of the EWSB sector, which is the appropriate description in the absence of scalars with masses below the cutoff Λ . Within this framework, the anomalous TGC λ_{γ} and λ_{Z} vanish at next-to-leading order in the effective theory (2), since they correspond to operators that are suppressed by v^2/Λ^2 relative to the dimension four set { \mathcal{L}_i }. The only consequence this power counting has in the analysis of low energy signals such as FCNC *B* and *K* decays, is the vanishing of the λ_{γ} contributions, since the λ_{Z} effects are suppressed by the factor q^2/M_Z^2 and are therefore negligible in any description of the EWSB sector. Thus, as far as the anomalous *WWZ* couplings are concerned, the present analysis is valid in both the linear and non-linear realizations.

The effects of the coupling Δg_1^Z in FCNC processes are enhanced by a logarithmic dependence on the high energy scale Λ . In the effective field theory language, this leading logarithm coexists with finite counterterms which are naturally of comparable size. As discussed in Sec. III in relation to Eq. (16), the finite counterterms are model-dependent, whereas the coefficient of the leading logarithm is determined at low energies independently of the specific theory above the scale Λ . Thus, although not the full answer, the logarithmic dependence provides us with the correct size of the effect, implying that the limits on Δg_1^Z should be considered rough estimates of the effects, designed to evaluate the sensitivity of a given experiment to this physics. Furthermore, this logarithmically divergent behavior with the scale Λ arises as a consequence of the contributions of longitudinal components of W^{\pm} in the loops, and is a manifestation of the non-standard behavior of the NGB of the electroweak symmetry breaking. All other anomalous TGC give finite one-loop contributions to FCNC processes due to the GIM mechanism and the fact that they only couple to the transverse piece of the gauge boson propagators. The GIM cancellation ensures that the bounds obtained on $\Delta \kappa_{\gamma}$ and g_5^Z are more precise. Therefore the bounds from rare B and K decays on Δg_1^Z will be less precise (perhaps good up to factors of two or so), but is the coupling to which FCNC B and Kdecays are most sensitive and potentially the most interesting one.

With respect to the expected size of the effects, we emphasize that the present study is model-independent and that to compute the coefficients $\{\alpha_i\}$ of the effective Lagrangian (2) knowledge of the full theory above the matching scale Λ is needed. However, it is possible to apply dimensional arguments to these couplings. For instance, naive dimensional analysis (NDA) [35] suggests that

$$\alpha_i \simeq \mathcal{O}(1) \times \frac{v^2}{\Lambda^2},$$
 (26)

with the scale of new physics obeying $\Lambda \leq 4\pi v$. However, in practice this power counting can only be applied to those coefficients that respect the custodial SU(2) symmetry that ensures that $\Delta \rho_* = \alpha T$ is small compared to one. As discussed in Ref. [8], this constraint implies that custodial breaking terms in \mathcal{L}_{eff} should naturally be further suppressed by an extra factor of $\mathcal{O}(10^{-2})$ or so. In terms of the anomalous TGC, this means that it is natural to expect that g_5^2 is no larger than $\mathcal{O}(10^{-4}-10^{-3})$. On the other hand, $\Delta \kappa_{\gamma}$ and Δg_1^Z receive contributions from custodial conserving terms and then are expected to be in the $\mathcal{O}(10^{-3}-10^{-1})$ range in these scenarios. A sizeable fraction of this range can be reached by FCNC processes, which are sensitive to anomalous TGC as small as a few percent. For the coefficient Δg_{\perp}^{Z} , this is true even in the first generation of B factory experiments and for $\simeq 30\%$ measurements of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ branching ratio. For both the CP conserving $\Delta \kappa_{\gamma}$ and CP violating $\tilde{\kappa}_{\gamma} WW\gamma$ anomalous couplings, a few percent precision will only be achieved with at least one order of magnitude more reconstructed events, to be available at the proposed LHC-B and BTeV experiments.

ACKNOWLEDGMENTS

The author thanks Kaoru Hagiwara for suggesting to look at the *CP* violating TGC, and to Guo-Hong Wu for clarifying comments on the operator basis of Ref. [2]. Useful conversations with Dieter Zeppenfeld are also acknowledged. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-95ER40896 and the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

APPENDIX

Here we specify the operator basis used for the effective Lagrangian of the EWSB sector of the SM. Defining

$$T \equiv U\tau^3 U^{\dagger}, \quad V \equiv (D_{\mu}U)U^{\dagger}, \tag{A1}$$

with U and the covariant derivative defined in Sec. I, all operators up to dimension four that are invariant under $SU(2)_L \times U(1)_Y$ can be written in terms of the gauge fields, T, V_{μ} and

$$\mathcal{D}_{\mu}\mathcal{O} \equiv \partial \mathcal{O} + ig[W_{\mu}, \mathcal{O}]. \tag{A2}$$

The dimension two operator $\mathcal{L}'_1 = (v^2/4)[\operatorname{Tr}(TV_{\mu})]^2$, gives a contribution to the *T* parameter and thus its coefficient is greatly constrained [2]. The *CP*-invariant dimension-four operators of Eq. (2) are given by

$$\mathcal{L}_{1} = \frac{1}{2} g g' B_{\mu\nu} \operatorname{Tr}(TW^{\mu\nu})$$
(A3)
$$\mathcal{L}_{2} = \frac{1}{2} i g' B_{\mu\nu} \operatorname{Tr}(T[V^{\mu}, V^{\nu}])$$

$$\mathcal{L}_{3} = i g \operatorname{Tr}(W_{\mu\nu}[V^{\mu}, V^{\nu}])$$

$$\mathcal{L}_{4} = [\operatorname{Tr}(V_{\mu}V_{\nu})]^{2}$$

$$\mathcal{L}_{5} = [\operatorname{Tr}(V_{\mu}V^{\mu})]^{2}$$

$$\mathcal{L}_{6} = \operatorname{Tr}(V_{\mu}V_{\nu})\operatorname{Tr}(TV^{\mu})\operatorname{Tr}(TV^{\nu})$$

$$\mathcal{L}_{7} = \operatorname{Tr}(V_{\mu}V^{\mu})\operatorname{Tr}(TV_{\nu})\operatorname{Tr}(TV^{\nu})$$

$$\mathcal{L}_{8} = \frac{1}{4}g^{2}[\operatorname{Tr}(TW_{\mu\nu})]^{2}$$

$$\mathcal{L}_{9} = \frac{1}{2}ig \operatorname{Tr}(TW_{\mu\nu})\operatorname{Tr}(T[V^{\mu}, V^{\nu}])$$

$$\mathcal{L}_{10} = \frac{1}{2}[\operatorname{Tr}(TV_{\mu})\operatorname{Tr}(TV_{\nu})]^{2}$$

$$\mathcal{L}_{11} = \operatorname{Tr}[(\mathcal{D}_{\mu}V^{\mu})^{2}]$$

$$\mathcal{L}_{12} = \operatorname{Tr}(T\mathcal{D}_{\mu}\mathcal{D}_{\nu}V^{\nu})\operatorname{Tr}(TV^{\mu})$$

$$\mathcal{L}_{13} = \frac{1}{2}[\operatorname{Tr}(T\mathcal{D}_{\mu}V_{\nu})]^{2}$$

$$\mathcal{L}_{14} = g \epsilon^{\mu\nu\rho\sigma} \operatorname{Tr}(TV_{\mu})\operatorname{Tr}(V_{\nu}W_{\rho\sigma}).$$

This CP-conserving basis contains three additional operators with respect to Ref. [2]. The operators \mathcal{L}_{11} , \mathcal{L}_{12} and \mathcal{L}_{13} either vanish or can be written as linear combinations of the others in the limit of massless fermions, in which $\mathcal{D}_{\mu}V^{\mu}$ $\simeq 0$. They are generally neglected when considering on-shell amplitudes. However, here we will insert these operators in one loop processes. Finally, there are three independent CP violating operators, as found in Ref. [2]. They are

$$\mathcal{L}_{15} = g \operatorname{Tr}(TV_{\mu})\operatorname{Tr}(V_{\nu}W^{\mu\nu})$$

$$\mathcal{L}_{16} = gg' \epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}\operatorname{Tr}(TW^{\rho\sigma})$$

$$\mathcal{L}_{17} = g^{2} \epsilon^{\mu\nu\rho\sigma}\operatorname{Tr}(TW_{\mu\nu})\operatorname{Tr}(TW^{\rho\sigma}) \qquad (A4)$$

$$\mathcal{L}_{18} = \operatorname{Tr}(V_{\mu}\mathcal{D}_{\nu}V^{\nu})\operatorname{Tr}(TV^{\mu})$$

$$\mathcal{L}_{19} = \operatorname{Tr}([V_{\mu}, T]\mathcal{D}^{\mu}\mathcal{D}^{\nu}V_{\nu}).$$

Only \mathcal{L}_{16} and \mathcal{L}_{17} contain $\tilde{F}_{\mu\nu}$ terms which then will contribute to $\tilde{\kappa}_{\gamma}$, as it can be seen in Eq. (20). The last two operators vanish in the limit of massless fermions, in which case the CP violating basis coincides with the one in Ref. [2].

- [1] A. Longhitano, Phys. Rev. D 22, 1166 (1980); Nucl. Phys. B188, 118 (1981).
- [2] T. Appelquist and G.-H. Wu, Phys. Rev. D 48, 3235 (1993).
- [3] For reviews see for example, F. Feruglio, Int. J. Mod. Phys. A 8, 4937 (1993); A. Dobado, A. Gómez-Nicola, A. L. Maroto, and J. R. Peláez, *Effective Lagrangians for the Standard Model* (Springer-Verlag, Berlin, 1997).
- [4] M. Quirós, in *Perspectives in Higgs Physics II*, edited by G. L. Kane (World Scientific, Singapore, 1997), and references therein.
- [5] J. Erler and P. Langacker, in Particle Data Group, R. M. Bar-

nett et al., Phys. Rev. D 54 103 (1996); 52, 441 (1995).

- [6] A Combination of Preliminary Measurements of Triple Gauge Boson Coupling Parameters at the LEP Experiments, the LEP electroweak working group, LEPEWWG/TGC/97-01.
- [7] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **78**, 4536 (1997); D0 Collaboration, S. Abachi *et al.*, *ibid.* **78**, 3634 (1997); D0 Collaboration, B. Abott *et al.*, contributed to the XVIII International Symposium on Lepton-Photon Interactions, Hamburg, Germany, 1997.
- [8] S. Dawson and G. Valencia, Phys. Rev. D 49, 2188 (1994).
- [9] O. J. P. Eboli, S. M. Lietti, M. C. González-Garcia, and S. F.

Novaes, Phys. Lett. B **339**, 119 (1994); S. Dawson and G. Valencia, *ibid.* **333**, 207 (1994); **341**, 452(E) (1994); S. Dawson and G. Valencia, Nucl. Phys. **B439**, 2 (1995).

- [10] J. Bernabéu, D. Comelli, A. Pich, and A. Santamaria, Phys. Rev. Lett. 78, 2902 (1997).
- [11] G. Burdman, Phys. Lett. B 409, 443 (1997).
- [12] S. Chia, Phys. Lett. B 240, 465 (1990).
- [13] K. Numata, Z. Phys. C 52, 691 (1991).
- [14] T. Rizzo, Phys. Lett. B 315, 471 (1993).
- [15] X.-G. He and B. McKellar, Phys. Lett. B 320, 168 (1994).
- [16] X.-G. He, Phys. Lett. B 319, 327 (1993).
- [17] G. Baillie, Z. Phys. C **61**, 667 (1994).
- [18] K. Hagiwara, K. Hikasa, R. D. Peccei, and D. Zeppenfeld, Nucl. Phys. B282, 253 (1987).
- [19] B. Holdom, Phys. Lett. B 258, 156 (1991).
- [20] G. Burdman, E. Golowich, J. Hewett, and S. Pakvasa (in preparation).
- [21] B. Grinstein, R. Springer, and M. B. Wise, Nucl. Phys. B339, 269 (1990); M. Ciuchini *et al.*, Phys. Lett. B 316, 127 (1993);
 M. Ciuchini, E. Franco, L. Reina, and L. Silverstrini, Nucl. Phys. B421, 41 (1994); A. Buras, M. Misiak, M. Münz, and S. Pokorski, *ibid.* B424, 374 (1994). For a review see G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
- [22] CLEO Collaboration, M. S. Alam *et al.*, Phys. Rev. Lett. 74, 2885 (1995); CLEO Collaboration, S. Glenn, presented at the Joint APS/AAPT meeting, Columbus, Ohio, 1998.
- [23] ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. B **429**, 169 (1998).

- [24] C. P. Burgess and D. London, Phys. Rev. Lett. 69, 3428 (1992); Phys. Rev. D 48, 4337 (1993).
- [25] UA1 Collaboration, C. Albajar *et al.*, Phys. Lett. B 262, 163 (1991); CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. 76, 4675 (1996); CLEO Collaboration, R. Balest *et al.*, presented at the International Conference on High Energy Physics, Glasgow, Scotland, 1994 (unpublished), Report No. CLEO-CONF-94-4.
- [26] Triple Gauge Boson Couplings Working Group, G. Gounaris et al., presented at "Physics at LEPII" Workshop, hep-ph/ 9601233.
- [27] Future Electroweak Physics at the Tevatron, edited by D. Amidei and R. Brock (Report No. FERMILAB-PUB-96/082, Batavia, Illinois, 1996).
- [28] G. Burdman, Phys. Rev. D 57, 4254 (1998).
- [29] BNL 787 Collaboration, S. Adler *et al.*, Phys. Rev. Lett. **76**, 1421 (1996).
- [30] See last reference in Ref. [21].
- [31] W. Marciano and A. Queijeiro, Phys. Rev. D 33, 3449 (1983);
 T. Appelquist and G.-H. Wu, *ibid.* 51, 240 (1995).
- [32] S. Dawson, X.-G. He, and G. Valencia, Phys. Lett. B 390, 431 (1997).
- [33] F. Krüger and L. M. Sehgal, Phys. Rev. D 56, 5452 (1997).
- [34] G. Burdman, Phys. Rev. D 52, 6400 (1995); G. Buchalla and
 G. Isidori, Nucl. Phys. B525, 333 (1998); D. Melikhov, N. Nikitin, and S. Simula, Phys. Lett. B 430, 332 (1998).
- [35] A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984); H. Georgi, Weak Interactions and Modern Particle Theory (Benjamin/Cummings, Menlo Park, California, 1984).