Meson correlators at finite temperature

Varun Sheel,* Hiranmaya Mishra, and Jitendra C. Parikh
Theory Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India
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We evaluate equal time point to point spatial correlation functions of mesonic currents at finite temperature. For this purpose we consider the QCD vacuum structure in terms of quark-antiquark condensates and their fluctuations in terms of an irreducible four point structure of the vacuum. The temperature dependence of quark condensates is modeled using chiral perturbation theory for low temperatures and lattice QCD simulations near the critical temperature. For the four point function, we assume a simple T dependence so that it vanishes at T_C. We first consider the propagation of quarks in a condensate medium at finite temperature. We then determine the correlation functions in a hot medium. Parameters such as mass, coupling constant and threshold energy are deduced from the finite temperature correlators. We find that all of them decrease close to the critical temperature. [S0556-2821(99)05401-6]

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I. INTRODUCTION

The structure of a vacuum in quantum chromodynamics (QCD) is one of the most interesting questions in strong interaction physics [1]. The evidence for quark and gluon condensates in a vacuum is a reflection of its complex nature [2]. Determination of correlation functions [3,4] of hadronic currents in such a vacuum state provides rich information regarding interquark interaction as a function of their spatial separation as well as on hadron spectroscopy. These are some of the nonperturbative features of QCD and are of great value in understanding the ground state structure of the theory of strong interactions [3,4].

We have earlier studied mesonic and baryonic current correlators at zero temperature with a nontrivial structure for the ground state with quark-antiquark condensates [5,6]. It was shown that the square of the quark propagator does not reproduce the equal time or spatial correlation function for the pion deduced from observations. In order to match the data it was necessary to introduce an irreducible four point structure for the quarks in the vacuum. This may be looked upon as a combination of two effects—(i) an effective way of incorporating gluon condensate contributions to the correlator and (ii) the existence of explicit four point quark structure in the vacuum.

As is well known [7] the QCD vacuum state changes with temperature. Lattice Monte Carlo simulations suggest that chiral symmetry is restored around 150 MeV. In view of this the present note is aimed at looking at the behavior of the meson correlation functions at finite temperature. This is of great interest in the context of behavior of hadrons around the chiral phase transition associated with quark gluon plasma [8,9]. It may be noted that there is little phenomenological information in this regime but there are several theoretical studies [10–13] using operator product expansion (OPE) and sum rule methods as well as using instanton liquid model for QCD ground states [14,15]. The main objective here is to employ a different nonperturbative approach developed by us [16]. This has been successful at zero temperature, and its extension to finite temperature is therefore of interest. In particular, we will obtain the temperature dependence of masses, coupling constants, and threshold energies for the pion and ρ mesons.

It is worthwhile pointing out that OPE and our approach are based on intrinsically different assumptions. The former is an expansion which separates short distance (Wilson coefficients) and long distance (condensates) physics. In our method we assume an explicit vacuum structure in terms of quark condensates (two point function) plus an irreducible four point function. Having made an ansatz for the vacuum (as above), we do not make any further approximation in the evaluation of the correlators. The approach is phenomenological in the sense that the values of the parameters in the four point function [16] are chosen to reproduce the behavior of the correlators. As emphasized by Shuryak [3] and Schäfer and Shuryak [14] OPE is able to quantitatively describe the zero temperature pion correlation functions for small x (up to 0.25 fm) but underpredicts it for large x. In our work [6,16] the agreement is quantitative with experimentally deduced mesonic correlation function and lattice results [4] for the whole range. This covers the small x values, resonance region, and large x domain where the correlator vanishes. We have, however, more parameters. To the extent that the large x behavior of the pionic correlator depends on gluon condensates in OPE our parametrization would imply an effective way of including gluon condensate effects. For studying the correlators at finite temperature, we assume that the two point as well as the four point function vanish at $T=T_C$. The parameters in the four point function are assumed constant at their $T=0$ values—no additional T dependence is given to them.

We organize the paper as follows. In Sec. II we discuss the quark condensate at finite temperature to fix the parameter appearing in the ansatz of the ground state of QCD. We then discuss in Sec. III the quark propagation in the thermal vacuum. In Sec. IV we calculate meson correlation functions at finite temperature. Finally we discuss the results in Sec. V.

*Electronic address: varun@prl.ernet.in

II. QUARK CONDENSATE AT FINITE TEMPERATURE

To calculate the correlators at finite temperature we need the expression for the equal time propagator for the interacting quark field operators. We have earlier developed [5] a model of vacuum structure in terms of quark-antiquark condensates with a condensate function $h(k)$. The equal time propagator could then be calculated in terms of the condensate function [6]. One can generalize this to finite temperature using the method of thermofield dynamics. Here the thermal average is obtained as an expectation value of the operator over the thermal vacuum [17]. This leads to

$$\langle \psi^{i}_{\alpha}(\vec{x})\psi^{j\dagger}_{\beta}(\vec{0})\rangle_T = \frac{\delta^{ij}}{(2\pi)^3}\int e^{i\vec{k}\cdot\vec{x}}\Lambda_{+\alpha\beta}(\vec{k},T)d\vec{k},$$

$$\langle \psi^{i\dagger}_{\alpha}(\vec{x})\psi^{j}_{\beta}(\vec{0})\rangle_T = \frac{\delta^{ij}}{(2\pi)^3}\int e^{-i\vec{k}\cdot\vec{x}}\Lambda_{-\beta\alpha}(\vec{k},T)d\vec{k}. \tag{1}$$

The thermal vacuum is obtained from the zero temperature vacuum by a thermal Bogoliubov transformation in an extended Hilbert space involving extra field operators (thermal doubling of operators) [17]. The functions $\Lambda_\pm$ [Eq. (1)] for the case of two flavor massless quarks are given as (with $k=|\vec{k}|$)

$$\Lambda_{\pm}(\vec{k},T)=\frac{1}{2}[1\pm\cos 2\theta(\gamma^0\sin 2h(k)+\vec{\alpha}\cdot\hat{k}\,\cos 2h(k))]. \tag{2}$$

In the above $h(k)$ is the condensate function [5,6,16] corresponding to the Bogoliubov transformation to include a condensate structure in the vacuum. The function θ is associated with the thermal Bogoliubov transformation and is related to the distribution function as [17]

$$\sin^2\theta(k)=\frac{1}{\exp[\beta\epsilon(k)]+1}, \tag{3}$$

β being the inverse temperature. Further $\epsilon(k)$ is the single particle energy given as $\epsilon(k)=\sqrt{k^2+m(k)^2}$. In the presence of condensate the dynamical mass is given as $m(k)=m+k\tan 2h(k)$, m being a possible current quark mass [5].

We had earlier taken a Gaussian ansatz for the condensate function $\sin 2h(k)=e^{-R^2k^2/2}$. In order to determine the parameter R, we had taken a value of R consistent with hadronic correlator phenomenology. We choose a similar structure for the condensate function at finite temperature, namely, $\sin 2h(k)=e^{-R(T)^2k^2/2}$ with $R(T)$ now being temperature dependent.

In order to determine $R(T)$ or equivalently the ratio $S(T)=R(T=0)/R(T)$, we first evaluate our expression of the order parameter (the condensate value) at finite temperature. In terms of the dimensionless variable $\eta=Rk$, this is given as

$$\frac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_{T=0}}=S(T)^3\left[1-2\sqrt{\frac{2}{\pi}}\int e^{-\eta^2/2}\sin^2(z,\eta)\,\eta^2 d\eta\right], \tag{4}$$

where $\sin^2\theta(z,\eta)=1/e^{z\epsilon(\eta)}+1$, with $z=\beta/R(T)$ and $\epsilon(\eta)=\eta/\cos 2h(\eta)$.

We can obtain $S(T)=R(T=0)/R(T)$ if we know the temperature dependence of the order parameter on the left hand side of Eq. (4). As there are no phenomenological inputs for this, we shall consider the results from chiral perturbation theory (CHPT) which is expected to be valid at least for small temperatures. For higher temperatures near the critical temperature, lattice simulations seem to yield the universal behavior [7] with a large correlation length associated with a second order phase transition for two flavor massless QCD. We shall use such a critical behavior to consider the temperature dependence of the order parameter near the critical temperature.

We quote here the results of CHPT obtained by Gerber and Leutwyler [18]. The condensate ratio at small temperatures compared to the pion mass is given as

$$\frac{\langle\bar{q}q\rangle_T}{\langle\bar{q}q\rangle_{T=0}}=1+\frac{c}{F^2}\Bigg[\frac{3}{2}T^4h_0'+4\pi T^4(a'h_1^2+2ah_1h_1') + \pi T^8(h'b_{\rm eff}+b'_{\rm eff}h)\Bigg], \tag{5}$$

where the functions h are defined as

$$\begin{aligned} h_0&=H^4(\mu)/(3\pi^2), \qquad h_0'=-H^2(\mu)/(2\pi^2T^2),\\ h_1&=H^2(\mu)/(2\pi^2), \qquad h_1'=-H^0(\mu)/(4\pi^2T^2),\\ h&=3h_0[h_0+\mu^2h_1], \quad h'=3h_0'[h_0+\mu^2h_1] + 3h_0[h_0'+h_1/T^2+\mu^2h_1'], \end{aligned} \tag{6}$$

with $\mu=M_\pi/T$. Also $b_{\rm eff}=b-(0.6T/\pi^3F^4M_\pi)$, $b'_{\rm eff}=-1/\pi^3F^4M_\pi^2[\frac{5}{16}-(0.3T/M_\pi)]$, and $a'=(2a/m_\pi^2)+(3/32\pi F^2)[1-(35m_\pi^2/32\pi^2F^2)]$. The constant $c\simeq 0.9$ and $F_\pi/F=1.057\pm 0.012$ with $F_\pi=93$ MeV [18]. The constants a and b are related to the S-wave and D-wave π-π scattering lengths, respectively [18]. Finally the functions $H^n(\mu)$ are given as [19]

$$H^n(\mu)=\int_0^\infty\frac{x^n dx}{\sqrt{x^2+\mu^2}}\frac{1}{e^{\sqrt{x^2+\mu^2}}-1}. \tag{7}$$

We have extracted the temperature dependence of the condensate as in Eq. (5) for low temperatures. For temperatures close to T_c, the critical behavior is that of O(4) spin model in three dimension [20] and has also been seen in lattice QCD simulations [21]. The order parameter here is given as $\langle\bar{q}q\rangle_T/\langle\bar{q}q\rangle_{T=0}=[1-(T/T_c)]^\beta$, where $\beta=0.39$ [7]. We have taken $T_c=150$ MeV [7]. The two regions are joined smoothly and the result is shown in Fig. 1(a). This result is fitted with Eq. (4) to determine $S(T)=R(0)/R(T)$, which is plotted in Fig. 1(b). We shall use it to calculate the quark propagator and the hadronic correlation functions. Note that

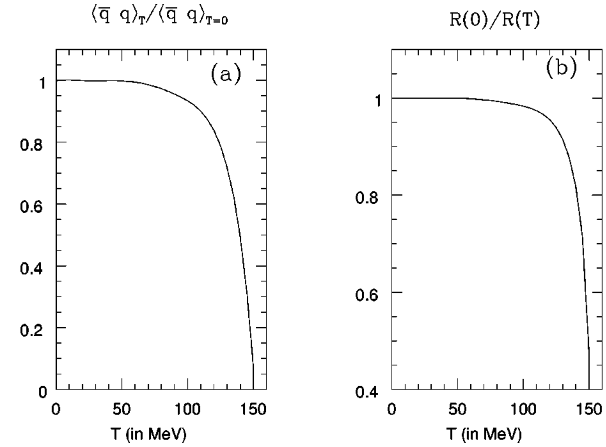

FIG. 1. (a) shows quark condensates at finite temperature normalized to that at zero temperature obtained from CHPT and Lattice. (b) shows $R(0)/R(T)$ as determined from (a).

in this framework we are unable to explicitly include gluon condensate effects at finite temperature. However, as discussed later we have included an irreducible four point structure for the quarks.

In the OPE studies, different models are considered for the behavior of a vacuum at finite temperature by Adami *et al.* [11]. These include lattice simulation, instanton liquid models, and string models. Further, the Wilson coefficients are temperature dependent. In a later work Hatsuda *et al.* [13] argue that the T dependence in the Wilson coefficients is due to an erroneous mixing of short and long distance physics. These authors [13] also stress the crucial importance of retaining Lorentz nonscalar operators in OPE because of a special choice of frame to define thermal equilibrium. A dilute pion gas model is used to define the thermal vacuum in this [13] work. Later we compare our findings with those of Hatusda *et al.* [13].

III. QUARK PROPAGATION IN THERMAL VACUUM

In the calculation of correlators, quark propagators enter in a direct manner and hence it is instructive to study aspects of the interacting propagator in some detail [6]. The equal time interacting quark Feynman propagator in the condensate vacuum is given as $S_{\alpha\beta}(\vec{x})=\langle\frac{1}{2}[\psi^i_\alpha(\vec{x}),\overline{\psi}^i_\beta(0)]\rangle$, which at finite temperature reduces to

$$S(\vec{x},T)=\frac{1}{2}\frac{\delta^{ij}}{(2\pi)^3}\int e^{i\vec{k}.\vec{x}}\cos 2\theta[\sin 2h-\vec{\gamma}\cdot\hat{k}\,\cos 2h]d\vec{k} \tag{8}$$

$$=\frac{i}{4\pi^2}\frac{\vec{\gamma}\cdot\vec{x}}{x^2}[I_1(x)-I_2(x)]+\frac{1}{4\pi^2}\frac{I_3(x)}{x}, \tag{9}$$

where

$$I_1(x)=\int_0^\infty k\left(\cos kx-\frac{\sin kx}{kx}\right)\cos 2\theta dk, \tag{10}$$

$$I_3(x)=\int_0^\infty k\,\sin kx\,\cos 2\theta e^{-R^2(T)k^2}dk, \tag{11}$$

$$I_2(x)=\int_0^\infty k\left(\cos kx-\frac{\sin kx}{kx}\right)\times\cos 2\theta\frac{e^{-R^2(T)k^2}}{1+(1-e^{-R^2(T)k^2})^{1/2}}dk, \tag{12}$$

with $x=|\vec{x}|$, $k=|\vec{k}|$.

The free massive propagator, which can be derived from $S(\vec{x},T)$ by the substitutions $\sin 2f(k)=m_q/\epsilon$ and $\cos 2f(k)=k/\epsilon$, is given as

$$S_0(m_q,\vec{x},T)=\frac{1}{(2\pi)^2}\frac{1}{x}\left[m_q(m_qK_1(m_qx)-2I_5(x))-i\frac{\vec{\gamma}\cdot\vec{x}}{x}(m_q^2K_2(m_qx)+2I_6(u))\right], \tag{13}$$

where

$$I_5(x)=\int_0^\infty\frac{\mathrm{k}}{\epsilon}\sin(kx)\sin^2\,\theta dk,\quad I_6(x)=\int_0^\infty\frac{k^2}{\epsilon}\left(\cos kx-\frac{\sin kx}{kx}\right)\sin^2\theta dk.$$

$K_1(m_qx)$ and $K_2(m_qx)$ are the first and second order modified Bessel functions of the second kind, respectively.

In Fig. 2 we plot the two components $\mathrm{Tr}\,S(\vec{x},T)$ and $\mathrm{Tr}(\gamma\cdot\hat{x})S(\vec{x},T)$ of the propagator for massless interacting quarks given by Eq. (9) at $T=0$ MeV, $T=100$ MeV, and $T=135$ MeV, corresponding to the chirality flip and nonflip components considered by Shuryak and Verbaarschot [22]. The normalization is discussed in our earlier work [6].

To compare with the constituent quark models with an effective constituent mass, we have also plotted the behavior of free massive quark propagators with masses 100 MeV, 200 MeV, and 300 MeV. In the chirality flip part, the propagator in the condensate medium starts from zero, consistent with zero quark mass at small distances, attains a maximum value of about 250 MeV at a distance of about 0.9 fm and then falls off gradually. Further the interacting propagator overshoots the massive propagators after about 0.6 fm. We

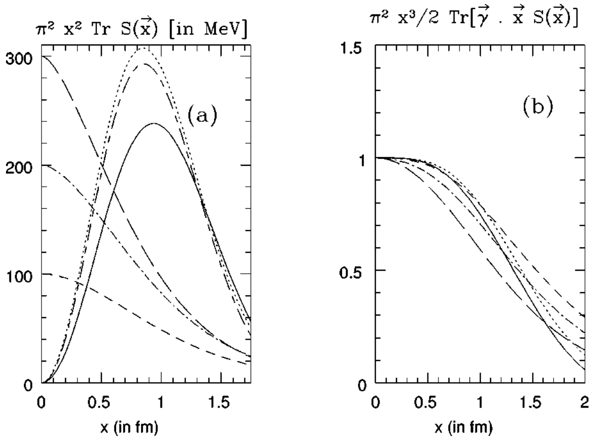

FIG. 2. The two components of the thermal quark propagator, (a) Tr(S) and (b) Tr[$(\vec{\gamma}\cdot\hat{x})$S] versus the distance x (in fm). The three lines, dot, short dash-long dash, and solid corresponds to massless quark interacting propagator $S(x,T)$ at temperatures of 0, 100, and 135 MeV, respectively. The three lines, short dashed, dot-short dashed, and long dashed correspond to a massive free propagator with a mass of 100, 200, and 300 MeV, respectively, at $T=135$ MeV.

also see that with increasing temperature, the chirality flip component has a lower peak and the position of the peak shifts towards higher distances indicating the decrease of the dynamical mass with temperature.

In the chirality nonflip part, the interacting propagator starts from 1, again consistent with the behavior expected from asymptotic freedom. But at a larger separation it falls rather fast indicative of an effective mass of the order of 150 MeV. These features are qualitatively similar to that of the quark propagator at zero temperature [6,22], though quantitatively there are differences. Also, the nonflip component falls faster with an increase of temperature.

Clearly therefore, the situation is similar to the one at zero temperature. We find that a constituent quark description is adequate to describe the behavior of the chirality nonflip part of the propagator, but it is not able to do so for the chirality flip part. We would like to mention that for $T=0$ the leading behavior of the propagator in our model [6] is identical to the OPE result.

IV. MESON CORRELATION FUNCTIONS

In our earlier work, we noted that phenomenology of correlation functions necessitated the introduction of an irreducible four point structure (or fluctuations of the condensate fields) in the vacuum [16]. In fact, the meson correlation functions were different from the square of the two point function (propagator) and the difference could be expressed in terms of the four point function. The expression of the meson correlation function at zero temperature defined in our earlier work [16] can be extended to finite temperatures as

$$R(\vec{x},T)=\mathrm{Tr}[S(\vec{x},T)\Gamma' S(-\vec{x},T)\Gamma] + \mathrm{Tr}\langle|[\Sigma(\vec{x})\Gamma'\Sigma(-\vec{x})\Gamma]|\rangle_T, \tag{14}$$

where $J(x)=\overline{\psi}^i_\alpha(x)\Gamma_{\alpha\beta}\psi^j_\beta(x)$ is a generic meson current with Γ being a 4×4 matrix $(1,\gamma_5,\gamma_\mu$ or $\gamma_\mu\gamma_5)$; x is a four vector; α and β are spinor indices; and i and j are flavor indices. The field $\Sigma(\vec{x})$ is the condensate fluctuation field introduced in Ref. [16] to include four point irreducible structures in QCD vacuum.

Thus, at finite temperature the correlator [Eq. (14)] is now the square of the interacting equal time *thermal* propagator plus the four point contribution at finite temperature. The thermal quark propagator was obtained in an earlier section. We keep the structure of the fields $\Sigma(x,T)$ the same as for zero temperature [16]:

$$\Sigma_{\alpha\beta}(\vec{x})=\Sigma^V_{\alpha\beta}(\vec{x})+\Sigma^S_{\alpha\beta}(\vec{x}), \tag{15}$$

$$=\mu_1^2(\gamma^i\gamma^j)_{\alpha\beta}\epsilon_{ijk}\phi^k(\vec{x})+\mu_2^2\delta_{\alpha\beta}\phi(\vec{x}), \tag{16}$$

where the first term corresponds to vector fluctuations and the second to scalar fluctuations. μ_1 and μ_2 in the above equations are dimensional parameters which give the strength of the fluctuations and $\phi^k(\vec{x})$ and $\phi(\vec{x})$ are vector and scalar fields such that, with $|\Omega\rangle$ as the ground state of QCD, we have

$$\begin{aligned}\langle\Omega|\phi^i(\vec{x})\phi^j(0)|\Omega\rangle&=\delta^{ij}g_V(\vec{x});\\ \langle\Omega|\phi(\vec{x})\phi(0)|\Omega\rangle&=g_S(\vec{x}).\end{aligned} \tag{17}$$

At finite temperature, the functions g_V and g_S will be temperature dependent. We do not know how to calculate it except for a general property that the effect of the four point structure should decrease with temperature. We take here a simple ansatz for the temperature dependence of g_V and g_S:

$$g_{S,V}(x,T)=\left(\frac{\langle\overline{q}q\rangle_T}{\langle\overline{q}q\rangle_{T=0}}\right)^2 g_{S,V}(x,T=0). \tag{18}$$

The parameters μ_i are chosen to have the values obtained earlier [16] while fitting the mesonic and baryonic correlators at $T=0$.

Similar to the calculations at zero temperature, we evaluate (for $T\neq 0$) the ratio of the physical correlation function to that of massless noninteracting quarks at zero temperature given as

$$R_0(x)=\mathrm{Tr}[S_0(x)\Gamma' S_0(-x)\Gamma]. \tag{19a}$$

The normalized correlation functions thus defined as

$$C(\vec{x},T)=\frac{R(\vec{x},T)}{R_0(\vec{x})} \tag{19b}$$

are plotted in Fig. 3 for the pseudoscalar and vector channels.

As expected (on physical grounds) the amplitude of the correlator decreases with increasing temperature. The peak of the vector correlator shifts towards the right after $T=0.9T_c$. We might remind ourselves that the position of

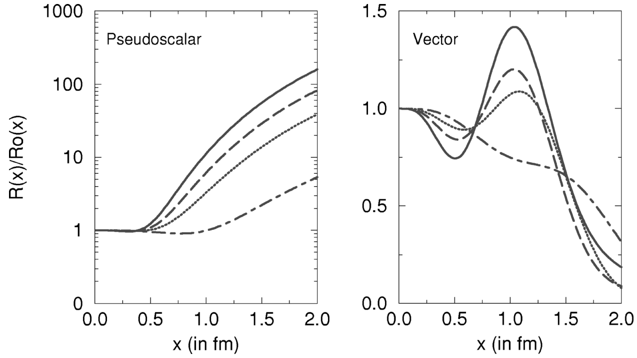

FIG. 3. The ratio of the meson correlation functions at finite temperature to the correlation functions for noninteracting massless quarks at zero temperature $R(x,T)/R_0(x,T=0)$ vs distance x (in fm). The solid, dashed, dotted, and dot-dashed lines correspond to temperatures $T=0$ MeV, $T=130$ MeV, $T=140$ MeV, and $T=148$ MeV, respectively.

the peak of the correlator is inversely proportional to the mass of the particle in the relevant channel [4].

The spatial hadronic correlators have been used to extract the hadronic screening masses and widths at finite temperature [9]. To extract the hadronic properties at finite temperature, we use a phenomenological parametrization as is usually done in sum rule calculations [11,13]. We may note here however, that the phenomenological inputs are not available

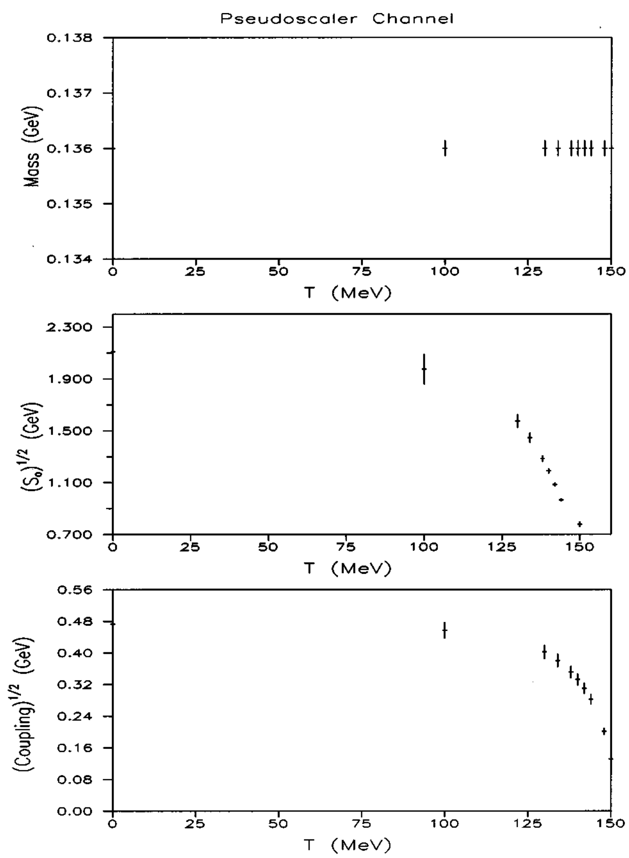

FIG. 4. The temperature dependence of mass, threshold (S_0), and coupling for the pseudoscalar channel. The vertical lines represent the errors obtained while fitting.

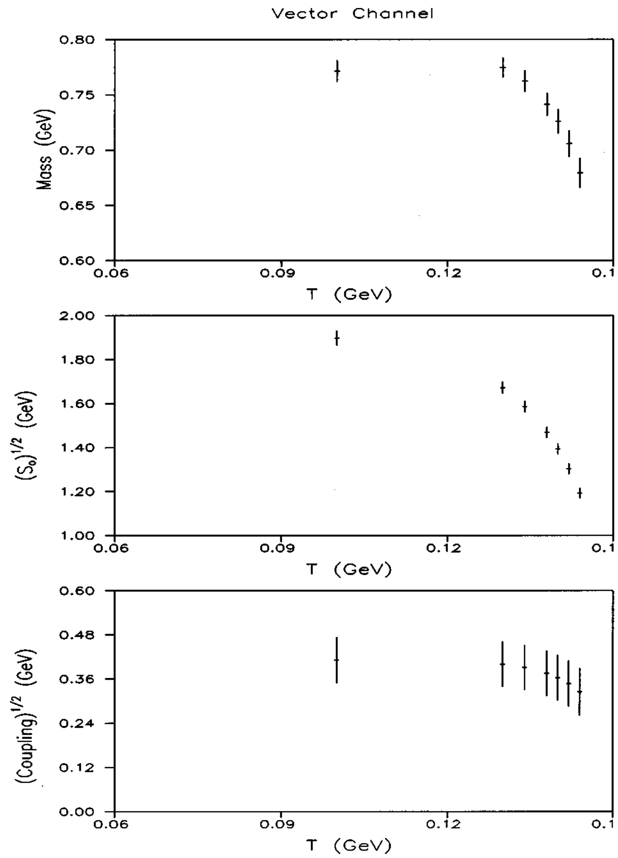

FIG. 5. The temperature dependence of mass, threshold (S_0), and coupling for the vector channel. The vertical lines represent the errors obtained while fitting.

at finite temperature. Consequently, the correlators are parametrized using the spectral density function with the mass, decay width, and the coupling of the particle to the vacuum, all three parameters being temperature dependent. We first express the correlator in terms of spectral density function:

$$R_{ph}(\vec{x}) = \int_0^\infty ds \frac{\sqrt{s}}{4\pi^2 x} K_1(\sqrt{s}x)\rho(s). \tag{20}$$

Then we use the following phenomenological parametrization for the spectral density function [11,13]:

$$\rho^V(s) = 3\lambda_\rho^2 \delta(s - M_\rho^2) + \frac{3s}{4\pi^2} \tanh\left[\frac{\sqrt{s}}{4T}\right] \theta(s - s_o) + T^2 S_\rho \delta(s), \tag{21}$$

$$\rho^P(s) = \lambda_\pi^2 \delta(s - M_\pi^2) + \frac{3s}{8\pi^2} \tanh\left[\frac{\sqrt{s}}{4T}\right] \theta(s - s_o), \tag{22}$$

where λ is the coupling of the bound state to the current, M is mass of the bound state, and s_0 is the threshold for continuum contributions. The last term in Eq. (21) is the scattering term for soft thermal dissociations (mainly through pions), which exists only at finite temperature [11]. This term is given as

TABLE I. Fitted parameters.

Channel	Temp. (MeV)	M (GeV)	λ	$\sqrt{s_0}$(GeV)
Vector	0	0.780±0.005	$(0.420\pm0.041\text{ GeV})^2$	2.070±0.035
	100	0.771±0.001	$(0.411\pm0.062\text{ GeV})^2$	1.897±0.033
	130	0.774±0.001	$(0.402\pm0.061\text{ GeV})^2$	1.672±0.027
	134	0.762±0.001	$(0.391\pm0.061\text{ GeV})^2$	1.586±0.026
	138	0.741±0.001	$(0.375\pm0.061\text{ GeV})^2$	1.468±0.024
	140	0.726±0.001	$(0.363\pm0.062\text{ GeV})^2$	1.393±0.024
	142	0.706±0.001	$(0.347\pm0.063\text{ GeV})^2$	1.303±0.024
	144	0.679±0.001	$(0.325\pm0.064\text{ GeV})^2$	1.192±0.024
Pseudoscalar	0	0.136±0.00014	$(0.473\pm0.021\text{ GeV})^2$	2.110±0.152
	100	0.136±0.00014	$(0.457\pm0.021\text{ GeV})^2$	1.974±0.118
	130	0.136±0.00014	$(0.402\pm0.018\text{ GeV})^2$	1.574±0.052
	134	0.136±0.00014	$(0.380\pm0.017\text{ GeV})^2$	1.445±0.038
	138	0.136±0.00014	$(0.351\pm0.016\text{ GeV})^2$	1.284±0.025
	140	0.136±0.00014	$(0.332\pm0.015\text{ GeV})^2$	1.190±0.020
	142	0.136±0.00014	$(0.310\pm0.014\text{ GeV})^2$	1.084±0.014
	144	0.136±0.00014	$(0.282\pm0.013\text{ GeV})^2$	0.965±0.011
	148	0.136±0.00014	$(0.201\pm0.009\text{ GeV})^2$	0.970±0.020
	150	0.136±0.00014	$(0.131\pm0.006\text{ GeV})^2$	0.777±0.020

$$S_\rho = \lim_{|\vec{p}|\to 0} \frac{1}{2\pi} \int_0^{|\vec{p}|^2} d\omega^2 \int_v^\infty x^2 \left(n\left(\frac{|\vec{p}|x-\omega}{2T} \right) - n\left(\frac{|\vec{p}|x+\omega}{2T} \right) \right). \tag{23}$$

The derivation of the above expression is slightly tricky and we have given it in the Appendix. Following Ref. [11] we take $S_\rho \approx T^2/9$.

The mass, threshold and coupling are then extracted such that the correlators as obtained from Eq. (20) agree with the normalized correlation functions as calculated by us (Fig. 3) [16]. This is done for each temperature. The results are plotted in Fig. 4 for the pseudoscalar channel and in Fig. 5 for the vector channel. The results are also shown in Table I.

V. SUMMARY AND CONCLUSIONS

As can be seen from Fig. 3, with increase in temperature, the correlation functions have a lower peak indicating lack of correlations with temperature. In the vector channel the mass of the ρ meson appears to decrease beyond 120 MeV. The threshold for the continuum also decreases around the same temperature. The behavior with the temperature of these quantities is qualitatively similar to that found by Hatsuda *et al.* [13]. We have also plotted the temperature dependence of the coupling of the bound state to the current which decreases with temperature but rather slowly as compared to mass or the threshold for the continuum. The temperature dependence of these parameters can be used to calculate the lepton pair production rate from ρ in the context of ultrarelativistic heavy ion collision experiments to estimate vector meson mass shift in the medium.

We see that our results are broadly in agreement with those of Hatsuda *et al.* [13]. All the same, it is not possible to carry out a term by term comparison of our results with OPE results. This is because the assumptions are different in the two approaches. We recall that Hatsuda *et al.* [13] attribute the decrease in the ρ mass to contributions coming from four point function. They also find variation in the gluon condensates to be less than 5% over the temperature region that considered. In our work we do not include the gluon condensates and the four point structure vanishes as $T\to T_C$. This is not the case for gluon condensates ($\langle GG\rangle$) in lattice calculations or in the dilute pion gas model of Ref. [13]. Consequently, we may be tempted to infer that the decrease in ρ mass in our model is due to ''genuine'' four point function effects (as in Hatsuda *et al.* [13]) and not from the ''effective'' gluon condensate contribution. One, however, has to be very cautious. This is because in the present work the parameters in the four point function were kept constant at their $T=0$ values. It should be recalled that these values were determined so as to correctly describe the behavior of the correlators (in particular pion) at $T=0$. Hence the parameters do reflect some effective gluon condensate effects. The results at finite temperature would certainly be modified if these parameters are given significant T dependence. Our parametrization is such that if the parameters decrease with T then the contribution to the correlator will decrease. The crucial question, however, still remains as to the behavior of $\langle GG\rangle$ for $T<T_C$. If it varies very little in this range then our assumption that the parameters remain constant would be reasonable.

In the pseudoscalar channel the mass remains almost constant until the critical temperature, whereas the threshold and the coupling decrease with the temperature. We have found that in the pseudoscalar channel, the contribution to the correlation function mostly comes from the fluctuating fields. Further, the temperature behavior as taken in Eq. (18) essentially does not shift the position of the peak, whereas the magnitude of the correlator decreases. That is reflected in the above behavior of the parameters in the pseudoscalar channel. We may note here that a similar behavior of the pion mass becoming almost insensitive to temperature below the critical temperature was also observed in Ref. [23], where correlation functions were calculated in a QCD motivated effective theory, namely, the Nambu–Jona-Lasinio model.

We would like to add here that the present analysis will be valid for temperatures below the critical temperature. Above the critical temperature there have been calculations essentially using finite temperature perturbative QCD in random phase approximations (RPA) [24]. However, in the region above T_C, nonperturbative features have been known to exist from studies in lattice QCD simulations [7]. In view of this, one may have to carry out a hard thermal loop calculation where a partial resummation is done [25]. Alternatively, one may use other nonperturbative approaches such as QCD sum rules at finite temperature [26] or RPA approach in an instanton liquid model for the QCD vacuum [15].

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APPENDIX

Here we shall derive the scattering term S_ρ. This may be calculated by considering the imaginary part of the longitudinal correlator for spacelike four momenta and can be written as

$$\rho_l^s(\omega,\vec{p})=\frac{Im\Pi_{00}}{|\vec{p}^2|}, \tag{A1}$$

which is explicitly written as [27]

$$\rho_l^s(\omega,\vec{p})=2\times\frac{(2\pi)^4}{|\vec{p}^2|}\int\frac{d^3k_1}{2E_1(2\pi)^3}\frac{d^3k_2}{2E_2(2\pi)^3}\times|\langle\pi(\vec{k}_1)|J_0|\pi(\vec{k}_2)\rangle|^2 \tag{A2}$$

$$\times\delta(\omega-E_1+E_2)\,\delta^{(3)}(\vec{p}-k_1+k_2)(n_2-n_1). \tag{A3}$$

Here $E_1=\sqrt{\vec{k}_1^2+m_\pi^2}$; $E_2=\sqrt{\vec{k}_2^2+m_\pi^2}$ and $n_i\equiv n(E_i)$ is the Bose distribution function for pions.

In general the expectation of a vector current with respect to a pion state is given as [28]

$$\langle\pi(k_1)|J_\mu|\pi(k_2)\rangle=(k_1+k_2)_\mu G_\pi(p), \tag{A4}$$

where $p=k_1-k_2$ and $G_\pi(p)$ is the pion form factor with $G_\pi(0)=1$. Substituting this in Eq. (A3) and integrating over k_2 we obtain

$$S_\rho(\omega,\vec{p})=2\times\frac{2}{(2\pi)^2}4|\vec{p}^2|\int\frac{d^3k_1}{E_1}(G_\pi(p))^2\times\delta(\omega-E_1+E-2)(2E_1-\omega)^2(n_2-n_1), \tag{A5}$$

with $\vec{k}_2=\vec{p}-\vec{k}_1$. Next, since the δ function above contributes to the spacelike $(p^2<0)$ region we write it as

$$\delta(\omega-E_1+E_2)=2E_2\delta((\omega-E_1)^2-E_2^2)\theta(-p^2).$$

To simplify further, we may change the integration over three momentum $\vec{k}_1$ to the integration over energy E_1 and the angle $\cos\theta_{\vec{p},\vec{k}}$. Performing the integration over angles restricts the lower limit of the energy integral E_1 as $E_{1\,\min}=\frac{1}{2}(\omega+|\vec{p}|v)$, where $v=[1-(4m_\pi^2/p^2)]^{1/2}$. Thus we have

$$\rho_l^s(\omega,\vec{p})=\frac{1}{4\pi}|\vec{p}|^3\int_{E_{\min}}^\infty G_\pi^2(p)(2E_1-\omega)^2(n_2-n_1), \tag{A6}$$

with $E_2-E_1-\omega$. Next, defining the variable x through $E_1=\frac{1}{2}(\omega+|\vec{p}|x)^{1/2}$ leads to

$$\rho_l^s(\omega,\vec{p})=\frac{1}{8\pi}\int_v^\infty dxx^2 n\left(\frac{|\vec{p}|x-\omega}{2T}\right)\times G_\pi^2(p)(2E_1-\omega)^2(n_2-n_1). \tag{A7}$$

We shall consider the longitudinal form factor $S_\rho(\omega,\vec{p})$ in a frame which is at rest with respect to the medium which implies that $\vec{p}\to 0$. In this limit the constraint $0<\omega<\vec{p}^2$ also forces ω to approach zero. However the above integral becomes increasingly large as $\vec{p}\to 0$ such that the integrated quantity of $S_\rho(\omega,\vec{p})$ within the phase space for ω remains finite. Thus we first integrate over this region with $\vec{p}$ finite and then take the limit $\vec{p}\to 0$. Thus let

$$I=\lim_{|\vec{p}|\to 0}\int_0^{|\vec{p}|^2}d\omega^2\rho_l^s(\omega,\vec{p})=\frac{S_\rho}{2\pi}, \tag{A8}$$

so that $\rho_l^s(\omega,\vec{p})$ effectively becomes a δ function. Thus the spectral density reduces to

$$\lim_{|\vec{p}|\to 0}\rho_l^s(\omega,\vec{p})=\delta(\omega^2)\frac{S_\rho}{2\pi}. \tag{A9}$$

We also note that there arises no ambiguity from the pion form factor as $G_\pi(p=0)=1$. Now the integral I can be written as

$$I=\frac{1}{8\pi}\lim_{|\vec{p}|\to 0}\int_0^{|\vec{p}|^2}d\omega^2\int_v^\infty dxx^2\times[n((|\vec{p}|x-\omega)/2T)-n((|\vec{p}|x+\omega)/2T)]. \tag{A10}$$

We change the integration variables [29] by putting $\omega = |\vec{p}|\lambda$ and $x=\sqrt{1+[y^2/|\vec{p}|^2(1-\lambda^2)]}$. Hence the spectral density function can be written as

$$I=\frac{1}{4\pi}\lim_{|\vec{p}|\to 0}\int_0^1 d\lambda\lambda\int_{2m_\pi}^{\infty}\frac{xydy}{(1-\lambda^2)^2}\times\left[n\left(\frac{|\vec{p}|x-\omega}{2T}\right)-n\left(\frac{|\vec{p}|x+\omega}{2T}\right)\right] \tag{A11}$$

In the limit of $|\vec{p}|\to 0$, we may Taylor expand the difference of the distribution functions in the square bracket of Eq. (A11) and have

$$\left[n\left(\frac{|\vec{p}|x-\omega}{2T}\right)-n\left(\frac{|\vec{p}x+\omega}{2T}\right)\right]\approx-\frac{2x|\vec{p}|^2(1-\lambda^2)^2}{y^2}\frac{dn}{d\lambda}.$$

Substituting back in Eq. (A11) and performing an integration by parts for $d\lambda$ integration we have

$$I=\frac{1}{2\pi}\int_0^1 d\lambda\int_{2m_\pi}^{\infty}dyn\left(\frac{y}{2T\sqrt{1-\lambda^2}}\right)y. \tag{A12}$$

In the limit of vanishing pion mass we have $I=2\pi T^2/9$ so that $S_\rho=T^2/9$.

[1] E. V. Shuryak, *The QCD Vacuum, Hadrons and the Superdense Matter* (World Scientific, Singapore, 1988).
[2] M. A. Shifman, A. I. vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); **B147**, 519 (1979).
[3] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
[4] M.-C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Phys. Rev. D **48**, 3340 (1993); **49**, 6039 (1994).
[5] A. Mishra, H. Mishra, S. P. Misra, P. K. Panda, and Varun Sheel, Int. J. Mod. Phys. E **5**, 93 (1996).
[6] Varun Sheel, Hiranmaya Mishra, and Jitendra C. Parikh, Int. J. Mod. Phys. E **6**, 275 (1997).
[7] E. Laermann, Nucl. Phys. **A610**, 1c (1996).
[8] E. V. Shuryak, Nucl. Phys. **A544**, 65c (1992).
[9] T. Schäfer and E. V. Shuryak, Phys. Rev. D **54**, 1099 (1996).
[10] A. I. Bocharev and M. E. Shaposhnikov, Nucl. Phys. **B268**, 220 (1986).
[11] C. Adami, T. Hatsuda, and I. Zahed, Phys. Rev. D **43**, 921 (1991).
[12] C. Adami and G. E. Brown, Phys. Rev. D **46**, 478 (1992).
[13] Tetsuo Hatsuda, Yuji Koike, and Su Houng Lee, Nucl. Phys. **B394**, 221 (1993).
[14] T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70**, 223 (1998).
[15] M. Velkovsky and E. V. Shuryak, Phys. Rev. D **56**, 2766 (1996).
[16] Varun Sheel, Hiranmaya Mishra, and Jitendra C. Parikh, Phys. Lett. B **382**, 173 (1996).
[17] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermofield Dynamics and Condensed States* (North-Holland, Amsterdam, 1982); P. A. Henning, Phys. Rep. **253**, 235 (1995).
[18] P. Gerber and H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).
[19] H.-S. Roh and T. Matsui, Eur. Phys. J. A **A1**, 205 (1998).
[20] K. Rajgopal and F. Wilczek, Nucl. Phys. **B399**, 395 (1993).
[21] F. Karsch, Phys. Rev. D **49**, 3791 (1994); F. Karsch and E. Laermann, *ibid.* **50**, 6954 (1994).
[22] E. V. Shuryak and J. J. M. Verbaarschot, Nucl. Phys. **B410**, 37 (1993).
[23] T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. Suppl. **91**, 284 (1987).
[24] Jitendra C. Parikh and Philip J. Siemens, Phys. Rev. D **37**, 3246 (1988); R. B. Thayyullathil and J. C. Parikh, *ibid.* **44**, 3964 (1991).
[25] Eric Braaten and Robert D. Pisarski, Phys. Rev. Lett. **64**, 1338 (1990); Nucl. Phys. **B339**, 310 (1990).
[26] J. Hansson and I. Zahed, ''QCD sumrules at high temperature,'' SUNY Report No. NTG 90-338 (1990).
[27] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Engelwood Cliffs, NJ, 1963).
[28] J. F. Donoghue, E. Golowich, and B. R. Holostein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, England, 1992).
[29] S. Mallik and K. Mukherjee, Phys. Rev. D **58**, 096011 (1998).