Determination of the scalar glueball mass in QCD sum rules

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(Received 19 June 1998; published 12 January 1999)

The 0^{++} glueball mass is analyzed in the QCD sum rules. We show that in order to determine the 0^{++} glueball mass by using the QCD sum rules method, it is necessary to clarify the following three ingredients: (1) to choose the appropriate moment with acceptable parameters which satisfy all of the criteria; (2) to take into account the radiative corrections; (3) to estimate an additional contribution to the glueball mass from the lowest lying $\overline{q}q$ resonance. We conclude that the key point is to choose suitable moments to determine the 0^{++} glueball mass; the radiative corrections do not affect it sensitively and the composite resonance has little effect on it. [S0556-2821(99)01803-2]

PACS number(s): 12.39.Mk, 11.55.Hx, 12.38.Lg

I. INTRODUCTION

The self-interaction among gluons is a distinctive feature in QCD theory. It may lead to bound gluon states, glueballs. Thus discovery of the glueball will be a direct test of QCD theory. Although there are several glueball candidates experimentally, there is no conclusive evidence of them. People recently paid particular attention to two scalar states: $f_0(1500)$(J=0) [1] and $f_J(1710)$ (J=0) [2], they seem like glueballs. However, the explicit analyses [3] on them reveal that neither of them appears to be a pure meson or a pure glueball. Most probably they are mixtures of a glueball and $\overline{q}q$ meson.

The properties of the glueball have been investigated in lattice gauge theory and in many models based on QCD theory. Even in the lattice gauge calculation, there are different predictions for the 0^{++} glueball [4,5,6]. Some years ago, the mass of the 0^{++} glueball was predicted around 700–900 MeV. Recently, the IBM group [4] predicted the lightest 0^{++} glueball mass to be (1710 ± 63) MeV, and the UK QCD group [5] gave the estimated mass (1625 ± 92) MeV, respectively. The improvement of the determination of the 0^{++} glueball mass originates from the more accurate lattice technique; however, at present uncertainty still exists .

Novikov *et al.* [7] first tried to estimate the scalar glueball mass by using QCD sum rules [8], but they only took the mass to be 700 MeV by hand because of uncontrolled instanton contributions. Since then, Pascual and Tarrach [9], Narison [10] and J. Bordes *et al.* [11] presented their calculation on the scalar glueball mass in the framework of QCD sum rules. They all got a lower mass prediction around 700–900 MeV when they used the moments R_{-1} or R_0 and neglected the radiative corrections in their calculation of the correlators. Bagan and Steele [12] first took account of the radiative corrections in the correlator calculation. Choosing appropriate moments (R_0 and R_1) for their calculation, they got a higher glueball mass prediction around 1.7 GeV. It seems that the radiative corrections make a big difference on the prediction of the scalar glueball mass. Obviously, there are some uncertainties in the determination of the scalar glueball mass; in order to give the reliable values in the QCD sum rules reasonably, an analysis of these uncertainties is necessary.

In this paper, we first give the criteria to choose the moments, which are obtained by the Borel transformation of the correlator weighted by different powers of q^2, according to application of QCD sum rules. It is important to choose suitable moments to determine the glueball mass [13]. From the criteria it follows that different moments have different results, but not all of them are reliable. By choosing the appropriate moment, we get the glueball mass without radiative corrections: $\sim$1.71 GeV. When the radiative corrections are included, glueball mass shifts a little: $\sim$1.66 GeV.

Secondly, we consider the effect of mixing between the lowest-lying 0^{++} glueball and $\overline{q}q$ meson, i.e., the gluonic currents and quark currents couple both to glueball states and $\overline{q}q$ states. Therefore, there are some exotic form factors to be determined. By using the low-energy theorem, we can construct a sum rule for the mixing correlation function (one gluonic current and one quark current). Through these relationships and based on the assumption of two states (lowest-lying states of glueball and $\overline{q}q$ meson) dominance, we find the mass for 0^{++} glueball is around 1.9 GeV, which is a little higher than the pure resonance prediction while the mass for 0^{++} meson is around 1.0 GeV, which is a little lower than the pure meson state prediction.

The paper is organized as follows. In Sec. II, a brief review about the calculation of the mass of a physical state from QCD sum rules is given. In Sec. III, we discuss the criteria of choosing the moments and the effect of the radiative corrections. The mixing effect of the glueball with the meson state is studied in Sec. IV. Finally, the last section is reserved for a summary.

II. QCD SUM RULES AND MOMENTS

Let us consider the correlator

$$\Pi(q^2)=i\int e^{iqx}\langle 0|T\{j(x),j(0)\}|0\rangle dx, \tag{1}$$

where $j(x)$ is the current with definite quantum numbers.

In the deep Euclidean domain $(-q^2\to\infty)$, it is suitable to carry out an operator product expansion (OPE)

$$\Pi(q^2)=\sum_n C_n(q^2)O_n, \tag{2}$$

where the $C_n(q^2)$ are Wilson coefficients. Then, the correlator can be expressed in terms of vacuum expectation values of the local operators O_n.

On the other hand, the imaginary part of $\Pi(q^2)$ in the Minkowski domain (at positive values of q^2), which is called the spectral density, is relevant with the physical observables. Therefore, we can extract some information of the hadrons from QCD calculation by using the dispersion relation

$$\Pi(q^2)=\frac{(q^2)^n}{\pi}\int\frac{\mathrm{Im}\,\Pi(s)}{s^n(s-q^2)}ds+\sum_{k=0}^{n-1}a_k(q^2)^k, \tag{3}$$

where a_k are some subtraction constants originating from the facial divergence of $\Pi(q^2)$. In order to keep control of the convergence of the OPE series and enhance the contribution of the lowest lying resonance to the spectral density, the standard Borel transformation is used. However, in practice, it may be more convenient to use the moments R_k instead, which are defined by

$$\begin{aligned}R_k(\tau,s_0)&=\frac{1}{\tau}\hat{L}\,[(q^2)^k\{\Pi(Q^2)-\Pi(0)\}]\\&\quad-\frac{1}{\pi}\int_{s_0}^{+\infty}s^ke^{-s\tau}\,\mathrm{Im}\,\Pi^{\{pert\}}(s)ds\\&=\frac{1}{\pi}\int_0^{s_0}s^ke^{-s\tau}\,\mathrm{Im}\,\Pi(s)ds,\end{aligned} \tag{4}$$

where $\hat{L}$ is the Borel transformation and τ is the Borel transformation parameter; s_0 is the starting point of the continuum threshold. Using the higher rank moments, one can enchance the perturbative contribution and suppress resonance contribution. In the following, we will see the role of R_k in our analysis.

III. CRITERIA OF CHOOSING THE MOMENTS

In this paper, the 0^{++} gluonic current is defined as

$$j(x)=\alpha_sG^a_{\mu\nu}G^a_{\mu\nu}(x), \tag{5}$$

where $G^a_{\mu\nu}$ in Eq. (5) stands for the gluon field strength tensor and α_s is the quark-gluon coupling constant. The current $j(x)$ is the gauge-invariant and nonrenormalization (to two loops order) current in pure QCD.

Through the operator product expansion, the correlator without radiative corrections becomes

$$\begin{aligned}\Pi(q^2)&=a_0(Q^2)^2\ln(Q^2/\nu^2)+b_0\langle\alpha_sG^2\rangle\\&\quad+c_0\frac{\langle gG^3\rangle}{Q^2}+d_0\frac{\langle\alpha_s^2G^4\rangle}{(Q^2)^2},\end{aligned} \tag{6}$$

with $Q^2=-q^2>0$, and

$$a_0=-2\left(\frac{\alpha_s}{\pi}\right)^2,\quad b_0=4\alpha_s,$$

$$c_0=8\alpha_s^2,\quad d_0=8\pi\alpha_s.$$

For the nonperturbative condensates the following notations and estimates are used:

$$\langle\alpha_sG^2\rangle=\langle\alpha_sG^a_{\mu\nu}G^a_{\mu\nu}\rangle,$$

$$\langle gG^3\rangle=\langle gf_{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}\rangle,$$

$$\langle\alpha_s^2G^4\rangle=14\langle(\alpha_sf_{abc}G^a_{\mu\rho}G_{\rho\nu})^2\rangle-\langle(\alpha_sf_{abc}G^a_{\mu\nu}G^b_{\rho\lambda})^2\rangle.$$

Now, we can apply the standard dispersion representation for the correlator

$$\Pi(Q^2)=\Pi(0)-\Pi'(0)+\frac{(Q^2)^2}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}\,\Pi(s)}{s^2(s+Q^2)}ds \tag{7}$$

to connect the QCD calculation with the resonance physics. From the low energy theorem [7] it follows that

$$\Pi(0)=\frac{32\pi}{11}\langle\alpha_sG^2\rangle. \tag{8}$$

For the physical spectral density $\mathrm{Im}\Pi(s)$, one can divide it into two parts: low energy part and high energy part. Its high-energy behavior is known as trivial,

$$\mathrm{Im}\,\Pi(s)\to\frac{2}{\pi}s^2\alpha_s^2(s), \tag{9}$$

while at low energy region, $\mathrm{Im}\Pi(s)$ can be expressed in the single narrow width approximation. The single resonance model for $\mathrm{Im}\,\Pi(s)$ leads to

$$\mathrm{Im}\,\Pi(s)=\pi f^2M^4\delta(s-M^2), \tag{10}$$

where M, f are the glueball mass and coupling of the gluon current to the glueball. Thus we can proceed with the following calculation.

To construct the sum rules, we use the moments R_k defined above, then the standard dispersion relation is transformed into

$$R_k(\tau,s_0)=\frac{1}{\pi}\int_0^{s_0}s^ke^{-s\tau}\,\mathrm{Im}\,\Pi(s)ds, \tag{11}$$

and from Eq. (4) we have (for $k \geqslant -1$)

$$R_k(\tau,s_0)=\left(-\frac{\partial}{\partial\tau}\right)^{k+1} R_{-1}(\tau,s_0). \tag{12}$$

Renormalization-group improvement of the sum rules amounts to the substitution

$$\nu^2 \rightarrow \frac{1}{\tau},$$

$$\langle gG^3\rangle \rightarrow \left[\frac{\alpha_s}{\alpha_s(\nu^2)}\right]^{7/11} \langle gG^3\rangle.$$

$R_{-1}(\tau,s_0)$ without radiative corrections can be obtained from Eq. (6).

If we had a complete knowledge of resonances and QCD, we would be able to fix the glueball mass, then different moments R_k would give the same result definitely, but we are far from this goal. In practice, we cannot calculate the infinite terms in OPE. Therefore, the result will depend on the choice of the moments. So there should be certain criteria to choose some suitable moments at appropriate s_0. As shown in Ref. [12], the R_{-1} sum rule leads to a much smaller mass scale due to the anomalously large contribution of the low-energy part $[\Pi(0)]$ to the sum rule and it violates asymptotic freedom at the large energy region. They claimed that R_{-1} was not reliable to predict the 0^{++} glueball mass and employed the R_0 and R_1 moments to predict the 0^{++} glueball mass by fitting the stability criteria with the radiative corrections considered. Their approach showed that the R_0 and R_1 sum rules with the radiative corrections result in a higher mass scale compared to previous mass determination. They did not analyze how reliable these moments R_k are for determining the glueball mass. After analyzing the different moments with the criteria of QCD sum rules, one can find that R_0 is not reliable also for the calculation of 0^{++} glueball in the single narrow width resonance approximation. In order to determine which moment is more suitable and give a reliable mass prediction, we reexamine the R_k sum rules.

To improve the convergence of the asymptotic series, we study the ratio R_{k+1}/R_k, such as R_0/R_{-1} and R_1/R_0. In the narrow width approximation, we have

$$M^{2k+4} f^2 \exp(-\tau M^2)=R_k(\tau,s_0),$$

and (with $k \geqslant -1$)

$$M^2(\tau,s_0)=\frac{R_{k+1}(\tau,s_0)}{R_k}. \tag{13}$$

To proceed with the calculation, we choose the following parameters:

$$\langle\alpha_s G^2\rangle = 0.06 \ \text{GeV}^4,$$

$$\langle gG^3\rangle = (0.27 \ \text{GeV}^2)\langle\alpha_s G^2\rangle,$$

$$\langle\alpha_s^2 G^4\rangle = \frac{9}{16}\langle\alpha_s G^2\rangle^2,$$

$$\Lambda_{\overline{MS}} = 200 \ \text{MeV},$$

$$\alpha_s = \frac{-4\pi}{11\ln(\tau\Lambda^2_{\overline{MS}})}.$$

There are some uncertainties arising from the uncertainties in evaluating various gluon condensates, but they affect the mass prediction little. The value of $\Lambda_{\overline{MS}}$ has little effect on the mass determination also.

M^2 and f^2 are the functions of s_0, $s_0 > M^2$. Since the glueball mass M in Eq. (13) depends on τ and s_0, we take the stationary point of M^2 versus τ at an appropriate s_0 as the square of the glueball mass.

To determine the suitable moment and the appropriate s_0, the following criteria are employed. (1) The moments should be chosen to have a balance between the perturbative and the lowest lying resonance contribution to the sum rule, which means that both the perturbative contribution and the lowest resonance contribution to the sum rule are dominant in the sum rules. Besides, the contribution of the highest dimension operator in the sum rule should be suppressed less than 15 percent. (2) s_0 should be a little higher than the physical mass and approach it as near as possible due to the continuum threshold hypothesis and the narrow width approximation. (3) The choice of moments and a suitable s_0 should lead to not only the widest flat portions of the plots of M^2 versus τ but also an appropriate parameter region of τ with the parameter region compatible to the value of the glueball mass. Here we should give some comments on these three points. On one hand, a good sum rule needs a large perturbative contribution (which hints at good convergence of OPE); on the other hand, a large perturbative contribution means a large uncertainty from the excited states (continuum state) and it is dangerous. So a balance between the perturbative contribution and the lowest resonance contribution to the sum rule is necessary. Since the perturbative part of the correlator is not equal to the continuum part in the sum rules and especially since they have different percentages in different moments, it is possible to obtain the balance when we choose suitable moments. Although s_0 varies in a certain region according to the criteria and the uncertainty resulting from the varying of s_0 is obvious because of our little knowledge of the continuum states, we expect the glueball mass is not sensitive to it. In the case of glueball, when the s_0 is set as a free parameter (larger than the mass square), there is an error for the glueball mass with the varying of s_0, but the upper and lower bound of the glueball mass are limited and the error is about 10–20%.

Let us begin our analysis through the R_k sum rules without radiative corrections. It is known that different moments have different suppressions to the nonperturbative contribu-

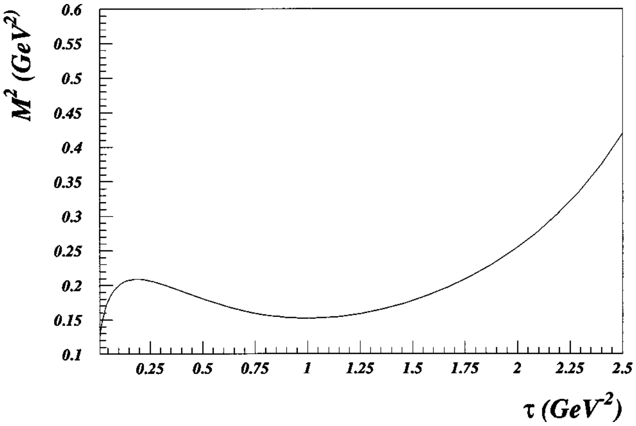

FIG. 1. R_0/R_{-1} versus τ at $s_0=3.6$ GeV^2 without radiative corrections.

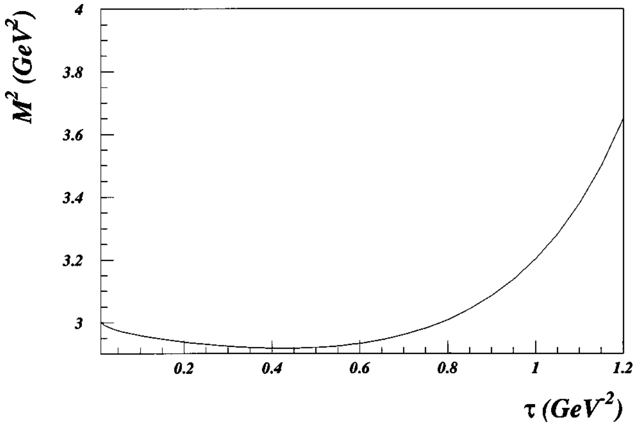

FIG. 3. R_2/R_1 versus τ at $s_0=3.6$ GeV^2 without radiative corrections.

tion and the lowest resonance contribution, moments with higher rank enhance the perturbative contribution and suppress the lowest resonance contribution to the sum rules.

In the sum rule of the moments R_{-1}, although there is a platform for mass prediction (see Fig. 1), the perturbative contribution in R_{-1} is less than 30%, which does not fit the criteria (1), so it is not acceptable.

Using the moment R_0, one can obtain a balance between the perturbative and the lowest resonance contribution to the sum rules; however there is no platform for mass prediction (see Fig. 2). It does not satisfy the criteria (3), so this moment is not suitable for the mass prediction either. All the previous calculations without radiative corrections were based on either moment R_{-1} or moment R_0, so the results are not very reliable .

The moment R_1 gives an excellent platform in the region 3.0 GeV$^2<s_0<4.3$ GeV^2; the result for the best s_0 at 3.6 GeV^2 is shown in Fig. 3. In the meanwhile, we can find a balance between the perturbative and the lowest resonance contribution to it, i.e., the ratio of the nonperturbative part to the perturbative part is less than 30–40 % and the ratio of the continuum part to the moment R_1 is less than 30–40 % too. Besides, the highest order term (four gluon condensate) contribution to the R_1 is less than 10% in the parameter region

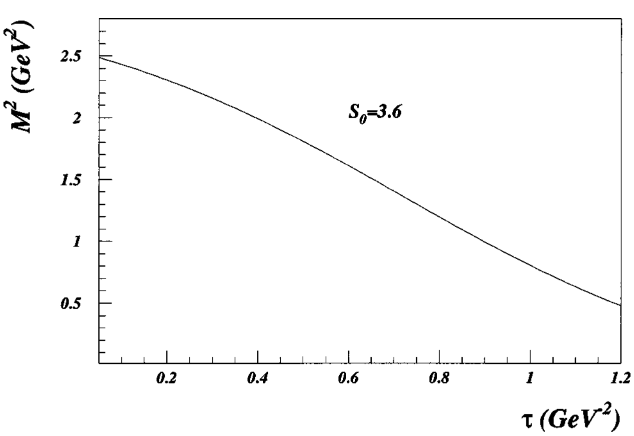

FIG. 2. R_1/R_0 versus τ at $s_0=3.6$ GeV^2 without radiative corrections.

of τ where there is a broad mass platform, so OPE is well convergent. Therefore, the moment R_1 satisfies all of the criteria and is reliable for the glueball mass determination. The curve shows that the 0^{++} glueball mass is $\sim$1710 MeV. In the acceptable region of s_0, the 0^{++} glueball mass is 1710 ± 110 MeV.

The moments with higher rank cannot stress the lowest resonance contribution in the sum rule since the higher dimension condensates will not be negligible (we have little knowledge about higher dimension condensates at present). Therefore, we have no way to proceed with our prediction from R_k with $k>2$.

After taking into account radiative corrections, the correlator is [12]

$$\Pi(q^2)=(a_0+a_1\ln(Q^2/\nu^2))(Q^2)^2\ln(Q^2/\nu^2)+(b_0+b_1\ln(Q^2/\nu^2))\langle\alpha_s G^2\rangle+(c_0+c_1\ln(Q^2/\nu^2))\frac{\langle gG^3\rangle}{Q^2}+d_0\frac{\alpha_s^2G^4}{(Q^2)^2}, \quad (14)$$

where

$$a_0=-2\left(\frac{\alpha_s}{\pi}\right)^2\left(1+\frac{51}{4}\frac{\alpha_s}{\pi}\right),$$

$$b_0=4\alpha_s\left(1+\frac{49}{12}\frac{\alpha_s}{\pi}\right),$$

$$c_0=8\alpha_s^2, \quad d_0=8\pi\alpha_s,$$

$$a_1=\frac{11}{2}\left(\frac{\alpha_s}{\pi}\right)^3, \quad b_1=-11\frac{\alpha_s^2}{\pi}, \quad c_1=-58\alpha_s^3.$$

The predicted mass from ratio R_2/R_1 is $\sim$1.66 GeV (see Fig. 4). The value is a little lower than the one without radiative corrections.

In this section, we show how the predicted glueball mass depends on the choice of the moment. We give the criteria on choosing suitable moments and s_0 to calculate the glue-

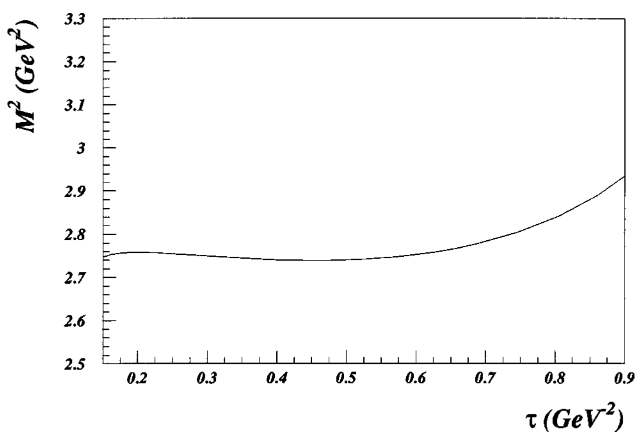

FIG. 4. R_2/R_1 versus τ at $s_0=3.6$ GeV^2 with radiative corrections.

ball mass in QCD sum rules. From the criteria, only R_1 are reliable for determination of the 0^{++} glueball mass and the result is ~ 1.71 GeV. The radiative corrections do not affect the mass determination sensitively; they shift the glueball mass a little lower: ~ 1.66 GeV.

IV. LOW ENERGY THEOREM TO THE MIXING PICTURE

Now we proceed to discuss the mixing effect in the determination of the 0^{++} glueball mass. Let us consider the 0^{++} quark current with isospin $I=0$:

$$j_2(x)=\frac{1}{\sqrt{2}}(\bar{u}u(x)+\bar{d}d(x)). \tag{15}$$

Through operator product expansion, the correlator of the $j_2(x)$ is given by [14]

$$\Pi_2(q^2)=a_0'(Q^2)^2\ln(Q^2/\nu^2)+\frac{3}{Q^2}\langle m\bar{q}q\rangle + \frac{1}{8\pi Q^2}\langle\alpha_s G^2\rangle+\frac{b_0'}{(Q^2)^2}\langle\bar{q}q\rangle^2, \tag{16}$$

where $Q^2=-q^2>0$, and

$$a_0'=\frac{3}{8\pi^2}\left(1+\frac{13\alpha_s}{3\pi}\right), \quad b_0'=-\frac{176}{27}\pi\alpha_s .$$

The correlator of the $j_1(x)$ without radiative corrections is not changed.

In order to estimate the vacuum expectation values of higher dimension operators, the vacuum intermediate states dominance approximation [8] has been employed

$$\langle\bar{q}\sigma_{\mu\nu}\lambda^a q\bar{q}\sigma_{\mu\nu}\lambda^a q\rangle=-\frac{16}{3}\langle qq\rangle^2,$$

$$\langle\bar{q}\gamma_\mu\lambda^a q\bar{q}\gamma_\mu\lambda^a q\rangle=-\frac{16}{9}\langle qq\rangle^2.$$

To proceed with the numerical calculation, in addition to the parameters we have chosen above, the following parameters are taken:

$$\langle\bar{q}q\rangle=-(0.25\ \text{GeV})^3,$$

$$\langle m\bar{q}q\rangle=-(0.1\ \text{GeV})^4,$$

$$\alpha_s=0.28,$$

where the scale of the running coupling is set at the glueball mass.

Through the R_k defined above, we can get the corresponding moments R_k and R_k' for $\Pi(q^2)$ and $\Pi_2(q^2)$

$$R_0(\tau,s_0)=-\frac{2a_0}{\tau^3}[1-\rho_2(s_0\tau)]+c_0\langle gG^3\rangle+d_0\langle\alpha_s^2G^4\rangle\tau, \tag{17}$$

$$R_1(\tau,s_0)=-\frac{6a_0}{\tau^4}[1-\rho_3(s_0\tau)]-d_0\langle\alpha_s^2G^4\rangle, \tag{18}$$

$$R_2(\tau,s_0)=-\frac{24a_0}{\tau^5}[1-\rho_4(s_0\tau)], \tag{19}$$

$$R_0'(\tau,s_0)=\frac{a_0'}{\tau^2}[1-\rho_1(s_0\tau)]+3\langle m\bar{q}q\rangle+\frac{1}{8\pi}\langle\alpha_s G^2\rangle + b_0'\tau\langle\bar{q}q\rangle^2, \tag{20}$$

$$R_1'(\tau,s_0)=\frac{2a_0'}{\tau^3}[1-\rho_2(s_0\tau)]-b_0'\langle\bar{q}q\rangle^2, \tag{21}$$

where

$$\rho_k(x)\equiv e^{-x}\sum_{j=0}^{k}\frac{x^j}{j!}. \tag{22}$$

By using the low-energy theorem [15], we can construct another correlator for the quark current with the gluonic current

$$\lim_{q\to 0} i\int dx e^{iqx}\langle 0|T\left[\frac{1}{\sqrt{2}}(\bar{u}u+\bar{d}d),\alpha_s G^2\right]|0\rangle=\frac{72\sqrt{2}\pi}{29}\langle\bar{u}u\rangle. \tag{23}$$

In order to factorize the spectral density, we define the couplings of the currents to the physical states in the following way:

$$\langle 0|j_1|Q\rangle = f_{12}m_2, \quad \langle 0|j_1|G\rangle = f_{11}m_1,$$
$$\langle 0|j_2|Q\rangle = f_{22}m_2, \quad \langle 0|j_2|G\rangle = f_{21}m_1, \tag{24}$$

where m_1 and m_2 refer to the glueball (including a few parts of the quark component) mass and the $\bar{q}q$ meson (including a few parts of the gluon component) mass, and $|Q\rangle$ and $|G\rangle$ refer to the $\bar{q}q$ meson state and the glueball state, respectively.

We indicate that the gluon current couples to both the glueball and quark states, as does the quark current. In the real physical world, the physical state is not pure glueball state or quark state; the mixing effect should not be omitted without any reasonable argument. After choosing the two resonances plus the continuum state approximation, the spectral density of the currents of $j_1(x)$ and $j_2(x)$ read in following, respectively,

$$\mathrm{Im}\,\Pi_1(s) = m_2^2 f_{12}^2 \delta(s-m_2^2) + m_1^2 f_{11}^2 \delta(s-m_1^2) + \frac{2}{\pi} s^2 \alpha_s^2 \theta(s-s_0), \tag{25}$$

$$\mathrm{Im}\,\Pi_2(s) = m_2^2 f_{22}^2 \delta(s-m_2^2) + m_1^2 f_{21}^2 \delta(s-m_1^2) + \pi a_0' s\, \theta(s-s_0). \tag{26}$$

Then it is straightforward to get the moments

$$R_0 = \frac{1}{\pi}\{m_2^2 e^{-m_2^2\tau} f_{12}^2 + m_1^2 e^{-m_1^2\tau} f_{11}^2\}, \tag{27}$$

$$R_1 = \frac{1}{\pi}\{m_2^4 e^{-m_2^2\tau} f_{12}^2 + m_1^4 e^{-m_1^2\tau} f_{11}^2\}, \tag{28}$$

$$R_2 = \frac{1}{\pi}\{m_2^6 e^{-m_2^2\tau} f_{12}^2 + m_1^6 e^{-m_1^2\tau} f_{11}^2\}, \tag{29}$$

$$R_0' = \frac{1}{\pi}\{m_2^2 e^{-m_2^2\tau} f_{22}^2 + m_1^2 e^{-m_1^2\tau} f_{21}^2\}, \tag{30}$$

$$R_1' = \frac{1}{\pi}\{m_2^4 e^{-m_2^2\tau} f_{22}^2 + m_1^4 e^{-m_1^2\tau} f_{21}^2\}. \tag{31}$$

In the meanwhile, assuming the states $|G\rangle$ and $|Q\rangle$ saturate the left-hand side of Eq. (23), we can obtain

$$\lim_{q\to 0} i \int dx e^{iqx} \langle 0| T\left[\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \alpha_s G^2\right]|0\rangle = f_{22}f_{12} + f_{21}f_{11}. \tag{32}$$

The next step is to equate the QCD side with the hadron side one by one, and we get a set of equations about masses and couplings. Starting from a series of reasonable parameters s_0 and τ and after solving this series of equations, we can get a set of the two states' masses. We illustrate our result in Fig. 5. In this figure, the solid line corresponds to the glueball and the dotted line corresponds to the meson, the points of the plateau compatible to the parameters are regarded as the mass prediction points. The masses vary slightly with the s_0 and $s_0 = 3.7\ \mathrm{GeV}^2$ is found to be the best favorable value for the mass determination. There is no platform for τ above 0.6 GeV^{-2}. The mass predictions from the figure are glueball with mass around 1.9 GeV and meson with mass around 1.0 GeV. The results obtained are reasonable; they are the two lowest-lying states and dominate the spectral density, other excited states are suppressed by a factor $\exp(-m^2\tau)$. The glueball mass is a little higher than the pure glueball state while the quark state mass is a little lower than the pure quark state.

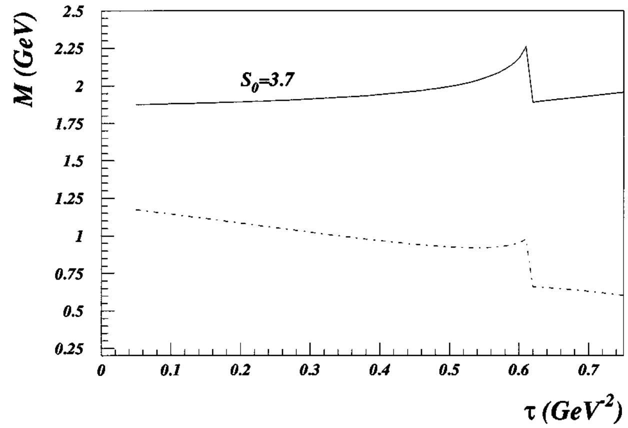

FIG. 5. M versus τ at $s_0 = 3.7\ \mathrm{GeV}^2$.

V. SUMMARY

In this paper, we analyze the determination of the scalar glueball mass based on the duality among resonance physics and QCD. The modified Borel transformation has been employed; it makes the calculation more convenient and reasonable.

We first conclude that it is important to choose suitable moments for the determination of the 0^{++} glueball mass. To stress the contribution of the lowest resonance and make the perturbative contribution dominant in sum rules, the criteria on the choice of the moment and continuum threshold are given. These criteria make it reliable to choose a suitable moment for the calculation of the glueball mass. We find moments R_{-1}, R_0 and R_k with higher rank $k>2$ are not suitable for the mass determination in the single narrow width resonance approximation. The moment R_1 is most preferable for the determination of the 0^{++} glueball mass. The numerical calculation shows that the mass is around 1.7 GeV without radiative corrections.

When the radiative correction is taken into account, it shifts to 1.66 GeV.

Secondly, we consider the physical states as composite resonances, which include both gluon component and quark component, so we saturate the spectral density with two physical resonances; in this way we consider not only the couplings of gluonic current to both glueball state and quark state, but also the couplings of quark current to quark state and glueball state. Employing the low-energy theorem and different moments, we predict the masses of glueball and normal meson from a set of coupled equations: glueball mass is around 1.9 GeV, which is a little higher than the one without mixing ($\sim$1.7 GeV), while mass of the quark state is around 1.0 GeV, a little lower than the pure quark state ($\sim$1.1 GeV). We conclude that the mixing between the glueball and the quark state does not affect their masses largely.

When we finished our paper, we found excellent papers in which the author used Monte Carlo to discuss uncertainties of determination of the vector mesons and nucleon spectral properties in the QCD sum rule [16]. Our many opinions already appeared in it.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of P. R. China.

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