## $B \rightarrow \tau \mu(X)$  decays in SUSY models without *R* parity

Dafne Guetta\*

*Dipartimento di Fisica—Universita´ degli Studi di Bologna, Via Irnerio 46, I-40126 Bologna, Italy*

Jesus M. Mira† and Enrico Nardi‡

*Departamento de Fı´sica—Universidad de Antioquia, A.A. 1226, Medellı´n, Colombia*

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Being strictly forbidden in the standard model, experimental detection of the lepton flavor violating decays  $B(\bar{B}) \to \tau^+ \mu^-$  and  $b(\bar{b}) \to X \tau^+ \mu^-$  would constitute an unmistakable indication of new physics. We study these decays in supersymmetric models without *R* parity and without lepton number. In order to derive order of magnitude predictions for the branching ratios, we assume a horizontal *U*(1) symmetry with horizontal charges chosen to explain the magnitude of fermion masses and quark mixing angles. We find that the branching ratios for decays with a  $\tau\mu$  pair in the final state are not particularly suppressed with respect to the lepton flavor conserving channels. In general in these models  $B[b \to \mu^+\mu^-(X)] \le B[b(\bar{b}) \to \tau^+\mu^-(X)]$  $\leq B[b \rightarrow \tau^+\tau^-(X)]$ . While in some cases the rates for final states  $\tau^+\tau^-$  can be up to one order of magnitude larger than the lepton flavor violating channel, due to better efficiencies for muon detection and to the absence of standard model contributions, decays into  $\tau\mu$  final states appear to be better suited to reveal this kind of new physics. [S0556-2821(99)00103-4]

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## **I. INTRODUCTION**

In the standard model (SM) the  $SU(2)\times U(1)$  gauge symmetry together with Lorentz invariance implies accidental baryon (*B*) and lepton (*L*) number conservation at the renormalizable level. Because of the larger Lorentz structure, supersymmetric (SUSY) versions of the SM allow for renormalizable *B* and *L* violating operators involving scalars with non-zero *B* and *L* charges that can induce fast proton decay as well as several other unobserved processes. Therefore, additional symmetries are required to enforce proton stability and to suppress *B* and *L* violating transitions. In most SUSY models, invariance under the additional parity quantum number  $R = (-1)^{3B+L+2S}$  (*S* being the spin) is assumed, and this enforces *B* and *L* conservation at the renormalizable level. However, today it is believed that *B* and *L* are not likely to be fundamental symmetries of nature, and in fact a much larger spectrum of models is known to be consistent with the data. To render phenomenologically viable SUSY extensions of the SM, the first priority is to ensure proton stability. In this respect other symmetries can be more effective than *R* parity, since *R* parity still allows for potentially dangerous dimension five *B* and *L* violating operators. Some interesting alternatives exist which forbid dimension four and five *B* violating terms, and hence are more effective to ensure proton stability  $[1]$ . Since in these models *L* number can be violated by renormalizable operators, they imply a quite different phenomenology from *R*-parity conserving SUSY models  $[2]$ .

Two new types of Lagrangian terms characterize this

class of models. First we have bilinear terms which couple the three lepton doublets superfields  $\hat{L}_e$ ,  $\hat{L}_\mu$  and  $\hat{L}_\tau$  with the up-type Higgs superfield  $\hat{\Phi}_{\mu}$ . The main effect of these terms is to induce neutrino masses via neutrino-neutralino mixing  $[3-6]$ . Requiring that the resulting masses do not exceed the experimental limits, implies constraints on the structure of these models  $[3-6]$ . In the models we will study these constraints are automatically satisfied, thanks to the presence of a horizontal symmetry that suppresses all the contributions to neutrino masses. Another effect of the bilinear terms is that of mixing fermions in different representations of *SU*(2). In turn, this can generate flavor changing couplings of the *Z* boson to the leptons. In our theoretical framework also these effects are safely suppressed below the experimental sensitivity. For these reasons the effects of the bilinear terms do not warrant further elaboration in the present context.

Secondly we have a set of renormalizable interactions in the superpotential which are responsible for *L* and lepton flavor  $(L_i, i = e, \mu, \tau)$  violating transitions. In the mass basis, these terms read

$$
\lambda_{ijk}\hat{L}_i\hat{L}_j\hat{l}_k^c + \lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{d}_k^c, \qquad (1.1)
$$

where  $\hat{Q}_i$  and  $\hat{d}_i^c$  denote the quark doublet and down-quark singlet superfields,  $\hat{l}_i^c$  are the lepton singlets, and  $\lambda_{ijk}$  $= -\lambda_{ijk}$  due to the antisymmetry in the *SU*(2) indices.

Several of the  $\lambda$  and  $\lambda'$  couplings are strongly constrained by the existing phenomenology  $[2]$ . The best limits are for couplings involving fermions of the first two generations  $(i, j, k=1,2)$  while for couplings involving more than a single third generation field the existing limits are much weaker. From the theoretical point of view, the values of the  $\lambda$  and  $\lambda'$  couplings in Eq. (1.1) are not predicted by the model. However, general models that can explain the observed fermion mass hierarchy also predict that *R*-parity vio-

<sup>\*</sup>Email address: guetta@bo.infn.it

<sup>†</sup> Email address: jmira@pegasus.udea.edu.co

<sup>‡</sup>Email address: enardi@fisica.udea.edu.co

lating couplings involving more than a single third generation field are the largest ones. This suggests that this kind of new physics can be effectively searched for in rare decays involving *b*,  $\tau$  and  $\nu_{\tau}$  [7–9]. In the following, we will make this statement more precise, by imposing on the models additional theoretical constraints. Following Ref. [9] we embed into the *R*-parity violating model a particular horizontal symmetry that can account for the order of magnitude of the fermion masses and Cabibbo-Kobayashi-Maskawa (CKM) angles  $[10,11]$ . This framework allows us to estimate the size of the relevant  $L$  violating couplings in Eq.  $(1.1)$ .

A rather complete study of *B* decays into third generation leptons like  $B \to \tau \overline{\nu}_{\tau}$ ,  $B \to \tau^+ \tau^-$ ,  $b \to X \tau^+ \tau^-$ ,  $b \to X \nu_{\tau} \overline{\nu}_{\tau}$ has been recently presented in Ref. [9]. The sensitivity of these decay modes to new physics from SUSY models without *R*-parity was thoroughly investigated, and compared to the sensitivity of the corresponding decay modes into  $\mu$ ,  $\nu_{\mu}$ . It was found that the processes that are most sensitive to these effects are the leptonic decays  $B_{d,s} \to \tau^+\tau^-$  and the inclusive decay  $b \rightarrow X_s \tau^+ \tau^-$ . However, from the experimental point of view the efficiency for  $\tau$  identification is expected to be rather low at *B* factories. Similarly,  $\tau$ -tagging will be a very hard task at future high *B* statistics experiments at hadron colliders  $[12]$ . This constitutes a serious drawback for the search of new physics effects in decays with final state  $\tau$ 's, and it is unlikely that the theoretical enhancement of the decay rates could fully compensate for this.

In this paper we point out that the lepton flavor violating decays  $B_{d,s}(\overline{B}_{d,s}) \to \tau^+ \mu^-$  and  $b(\overline{b}) \to X_s \tau^+ \mu^-$  together with the corresponding *CP* conjugate decays, can provide the best compromise between the two requirements of large theoretical branching ratios and good efficiencies in searching for the experimental signatures. Indeed, for the two body decay  $B \rightarrow \tau^+ \mu^-$  the absence of a  $\mu^+$  with momentum opposite to the  $\mu^-$  in the *B* rest frame represents a clean signature, rather easy to search for. The first experimental limit on this decay  $B(B \to \tau^{\pm} \mu^{\mp})$  < 8.3×10<sup>-4</sup> has been recently established by the CLEO Collaboration [13]. The search for the three body decay  $b \rightarrow X_s \tau^+ \mu^-$  appears to be more difficult, because of the lack of knowledge of the momentum of the missing  $\mu^+$ . This is reflected in the present experimental situation. While a tight limit on the lepton flavor violating decay  $b \rightarrow X_s \mu^+ e^-$  has been recently established  $[B(b \rightarrow X_s \mu^{\pm} e^{\mp})$ <2.2×10<sup>-5</sup> [14]] to date no experimental limit exists on decays into  $X_s \tau^+ e^-$  or  $X_s \tau^+ \mu^-$  final states.

At hadron colliders, already the study of the two body decay  $B \rightarrow \tau^+ \mu^-$  will be a difficult task. This is because in this case the momentum of the decaying *B* is not known, hence the presence of large backgrounds, as for example from the semileptonic decays  $B \rightarrow D^{(*)} \mu \bar{\nu}$ , will render quite challenging the search for this rare decay.

From the theoretical point of view, the detection of lepton flavor violating decays would represent a striking evidence of physics beyond the SM. The absence of SM contributions, and in the case of the decay  $b \rightarrow X_s \tau^+ \mu^-$  the absence of long distance effects which are difficult to estimate in a reliable way  $[15]$ , render these decays well suited to reveal in a clean way new physics effects. In summary, because of the large theoretical enhancement of the branching ratios with respect to  $\mu^+\mu^-$  final states, and since in any case a muon is experimentally much easier to identify than a tau, we believe that these processes will allow to search for signals of SUSY models without *R* parity with a better sensitivity than lepton flavor conserving decays into  $\mu^+\mu^-$  or  $\tau^+\tau^-$ .

In the next section we will outline the main features of SUSY models without *R*-parity embedded in models with horizontal symmetries. In Sec. III we will present the relevant expressions for the effective new physics coefficients which appear in these models, and for the decay rates. Finally, in Sec. IV we will discuss our results and present our conclusions.

## **II.** *R***-PARITY VIOLATION IN THE FRAMEWORK OF HORIZONTAL SYMMETRIES**

In order to evaluate the effects of the *R*-parity violating interactions in Eq.  $(1.1)$ , we need to estimate quantitatively the coefficients  $\lambda$  and  $\lambda'$ . We work in the framework of the supersymmetric models with horizontal symmetries that have been thoroughly investigated in Refs.  $[10,11]$ . We assign to each supermultiplet  $\hat{\psi}$  a charge  $H(\hat{\psi})$  of an Abelian horizontal group  $H = U(1)_H$  which is explicitly broken by a small parameter  $\varepsilon$  with charge  $H(\varepsilon) = -1$ . This gives rise to a set of selection rules for the effective couplings appearing in the low energy Lagrangian  $[10]$ . Assuming that each of the lepton, quark and Higgs superfields carries a positive or zero charge, the selection rule relevant for the present discussion is that the effective coupling *gabc* for a general trilinear superpotential term  $\hat{\psi}_a \hat{\psi}_b \hat{\psi}_c$  is of order  $g_{abc}$  $\sim \varepsilon^{H(\hat{\psi}_a) + H(\hat{\psi}_b) + H(\hat{\psi}_c)}$ . Therefore, the leptons and down-type quarks Yukawa couplings are respectively of order  $Y_{ij}^l$  $\sim \varepsilon^{H(\hat{\Phi}_d) + H(\hat{L}_i) + H(\hat{l}_j^c)}$  and  $Y_{ij}^d \sim \varepsilon^{H(\hat{\Phi}_d) + H(\hat{Q}_i) + H(\hat{d}_j^c)}$ . Most of the  $L$ -violating couplings in Eq.  $(1.1)$  are further suppressed with respect to the corresponding Yukawa couplings. They can be estimated as

$$
\lambda_{kij} \sim Y_{ij}^l \varepsilon^{H(L_k) - H(\Phi_d)} \sim \left(\frac{2\sqrt{2}G_F}{\cos^2 \beta}\right)^{1/2}
$$

$$
\times m_{l_i} \varepsilon^{H(l_j^c) - H(l_i^c) + H(L_k) - H(\Phi_d)}, \tag{2.1}
$$

and

$$
\lambda'_{kij} \sim Y_{ij}^d \varepsilon^{H(L_k) - H(\Phi_d)} \sim \left(\frac{2\sqrt{2}G_F}{\cos^2 \beta}\right)^{1/2}
$$

$$
\times m_{d_i} \varepsilon^{H(d_j^c) - H(d_i^c) + H(L_k) - H(\Phi_d)}, \tag{2.2}
$$

where  $G_F$  is the Fermi constant, and tan  $\beta = \langle \Phi_u \rangle / \langle \Phi_d \rangle$  with  $\Phi_u$  the up-type Higgs doublet. From Eqs. (2.1) and (2.2) it is apparent that in our framework the couplings  $\lambda$  and  $\lambda'$  involving fermions of the third generation are respectively enhanced by  $m_\tau$  and  $m_b$ .

In order to give a numerical estimate of the couplings, we need a set of *H* charges and a value for  $\varepsilon$ . The magnitude of the  $H$  breaking parameter is generally taken to be the value of the Cabibbo angle,  $\varepsilon \sim 0.22$ , while the quark, lepton and Higgs charges are chosen to reproduce the values of the fermion masses and CKM mixing angles. Besides reproducing the measured values, the model has some predictivity in the quark sector  $\vert 10 \vert$ , it yields estimates for ratios of neutrino masses  $[11,16]$  and, most important in the present context, it ensures that the *L*-violating couplings in Eqs.  $(1.1)$ ,  $(2.1)$  and  $(2.2)$  are safely suppressed below the present experimental limits  $[4]$ . The following  $H$ -charge assignments fit the order of magnitude of all the quark masses and CKM mixing angles  $[10]$ :

$$
\hat{Q}_1 \hat{Q}_2 \hat{Q}_3 \qquad \hat{d}_1^c \hat{d}_2^c \hat{d}_3^c \qquad \hat{u}_1^c \hat{u}_2^c \hat{u}_3^c \qquad \hat{\Phi}_d \hat{\Phi}_u
$$
\n(3) (2) (0) (3) (2) (2) (3) (1) (0) (0) (0) (0) (0) (2.3)

Following Ref. [9], we use for the leptons two different sets of charges which define two different models, and for each model we chose a different value of the squark masses  $m_{\tilde{q}}$ :

$\hat{L}_1 \hat{L}_2 \hat{L}_3$	$\hat{l}_1^c \hat{l}_2^c \hat{l}_3^c$	$m_1^c$ (GeV)	$m_{\tilde{q}}$ (GeV)	
Model I:	(4) (2) (0)	(4) (3) (3)	100	170
Model II:	(3) (0) (0)	(5) (5) (3)	100	350.

Model I tends to enhance new physics effects induced by operators arising from squark exchange, while in model II the effects of new scalar operators induced by slepton exchange tend to dominate. The choices  $(2.3)$  and  $(2.4)$  for the horizontal charges are not unique. Since the Yukawa interactions are invariant under a set of *U*(1) symmetries such as *B*, *L* and hypercharge, it is always possible to shift the *H*-charges of any amount proportional to one of the corresponding  $U(1)$  quantum numbers without affecting the predictions for the masses and mixing angles. In particular, the shift (proportional to *L*)  $H(\hat{L}_i) \rightarrow H(\hat{L}_i) + n$ ,  $H(\hat{l}_i^c) \rightarrow H(\hat{l}_i^c)$  $-n$  and  $H(\hat{\psi}) \rightarrow H(\hat{\psi})$  for all the other fields has the effect of suppressing (for  $n>0$ ) all the *L* violating couplings in Eqs.  $(1.1)$ ,  $(2.1)$  and  $(2.2)$  by a factor of  $\varepsilon^n$ . It turns out that already for  $n=1$  the suppression is large enough so that all the lepton flavor violating decays will be unobservable at most of the future *B*-physics experiments. We also notice that model II can be derived from model I by means of shifts proportional to lepton flavor numbers with  $n_e = -1, n_u$  $=$  - 2,*n*<sub>7</sub> = 0. This has the effect of enhancing some of the  $\lambda$ couplings without affecting the charged lepton masses. Of course, the predictions for the neutrino mixing angles will be different in the two models.

## **III. COEFFICIENTS AND RATES FOR THE DECAYS**

In the models under investigation, the lepton flavor violating decay  $\overline{B}_q \rightarrow \tau^+ \mu^-$  (with  $q = d, s$ ) and  $b \rightarrow X_s \tau^+ \mu^-$  are induced by the effective Lagrangian

$$
-\mathcal{L}_{\text{eff}}^- = C_{1S}^- (\bar{q}_L b_R) (\bar{\mu}_R \tau_L) + C_{2S}^- (\bar{q}_R b_L) (\bar{\mu}_L \tau_R)
$$
  
+ 
$$
C_V^- (\bar{q}_R \gamma^\mu b_R) (\bar{\mu}_L \gamma_\mu \tau_L) + \text{H.c.}
$$
 (3.1)

The first two operators arise from sneutrino exchange dia-

grams, while the last one corresponds to squark exchange diagrams after Fierz rearrangement. The coefficients appearing in Eq.  $(3.1)$  read

$$
C_{1S}^{-} = \sum_{i \neq 3} \frac{\lambda'_{iq}^{*} \lambda_{i32}}{m_{\tilde{\nu}_i}^2}; \quad C_{2S}^{-} = \sum_{i \neq 2} \frac{\lambda'_{i3q} \lambda_{i23}^{*}}{m_{\tilde{\nu}_i}^2};
$$

$$
C_V^{-} = \sum_{i} \frac{\lambda'_{2i3}^{*} \lambda'_{3iq}}{2m_{\tilde{q}_i}^2},
$$
(3.2)

where the index values  $i=3$  in  $C_{1S}^-$  and  $i=2$  in  $C_{2S}^-$  are excluded because of the antisymmetry in the first two indices of the  $\lambda$  couplings.

For the decays  $B_q \rightarrow \tau^+ \mu^-$  and  $\bar{b} \rightarrow X_s \tau^+ \mu^-$  the effective Lagrangian reads

$$
- \mathcal{L}_{eff}^{+} = C_{1S}^{+} (\bar{b}_{L} q_{R}) (\bar{\mu}_{R} \tau_{L}) + C_{2S}^{+} (\bar{b}_{R} q_{L}) (\bar{\mu}_{L} \tau_{R}) + C_{V}^{+} (\bar{b}_{R} \gamma^{\mu} q_{R}) (\bar{\mu}_{L} \gamma_{\mu} \tau_{L}) + \text{H.c.},
$$
(3.3)

where

$$
C_{1S}^{+} = \sum_{i \neq 3} \frac{\lambda_{i3q}^{*} \lambda_{i32}}{m_{\tilde{\nu}_{i}}^{2}}; \quad C_{2S}^{+} = \sum_{i \neq 2} \frac{\lambda_{iq3}' \lambda_{i23}^{*}}{m_{\tilde{\nu}_{i}}^{2}}; C_{V}^{+} = \sum_{i} \frac{\lambda_{2iq}^{*} \lambda_{3i3}^{'}}{2m_{\tilde{q}_{i}}^{2}}.
$$
 (3.4)

The expressions for the various branching ratios are presented in the next two subsections. In order to simplify the formulas we have neglected the muon mass (however,  $m<sub>\mu</sub>$  $\neq 0$  has been kept in the numerical analysis).

## **A.** The decays  $\overline{B} \rightarrow \tau^+ \mu^-$  and  $B \rightarrow \tau^+ \mu^-$

The amplitude for the decay  $\overline{B}_q \rightarrow \tau^+ \mu^-$  can be written as

$$
\mathcal{A}_{q}^{-} = i f_{B_{q}} m_{B} \frac{1}{4} \left\{ \left[ \frac{m_{B}}{m_{b}} (C_{2S}^{-} - C_{1S}^{-}) - \frac{m_{\tau}}{m_{B}} C_{V}^{-} \right] (\bar{\mu} \tau) + \left[ \frac{m_{B}}{m_{b}} (C_{2S}^{-} + C_{1S}^{-}) - \frac{m_{\tau}}{m_{B}} C_{V}^{-} \right] (\bar{\mu} \gamma_{5} \tau) \right\}, \qquad (3.5)
$$

where we have used the PCAC (partial conserved axial vector current) relations  $\langle 0|\bar{u}\gamma^{\mu}\gamma_5 b|\bar{B}\rangle = i f_B p_B^{\mu}$ and  $\langle 0|\bar{u}\gamma_5 b|\bar{B}\rangle \approx -i f_B m_B^2/m_b$ . This yields the branching ratio

$$
B(\overline{B}_{q} \to \tau^{+}\mu^{-}) = f_{B_{q}}^{2} \tau_{B} \frac{m_{B}^{3}}{64\pi} \left(1 - \frac{m_{\tau}^{2}}{m_{B}^{2}}\right)^{3}
$$

$$
\times \left[ \left| \frac{m_{B}}{m_{b}} C_{2S}^{-} - \frac{m_{\tau}}{m_{B}} C_{V}^{-} \right|^{2} + \frac{m_{B}^{2}}{m_{b}^{2}} |C_{1S}^{-}|^{2} \right]. \quad (3.6)
$$

Equation  $(3.6)$  also accounts for the *CP* conjugate decay  $B_q \rightarrow \tau^- \mu^+$ , therefore experimental searches for both the decay channels  $\overline{B} \rightarrow \tau^+ \mu^-$  and  $B \rightarrow \tau^- \mu^+$  will yield informations on the same set of operators. The decay mode *B*  $\rightarrow \tau^+\mu^-$  is controlled by the coefficients  $C_{1S}^+$ ,  $C_{2S}^+$  and  $C_V^+$ in Eq.  $(3.4)$ . The amplitude is given by  $A_q^+ = -A_q^- (C_{1S}^+, C_{2S}^+, C_V^+)$  with  $A_q^-$  defined as in Eq. (3.5). Therefore the branching ratios  $B(B \to \tau^+ \mu^-)$  and  $B(\overline{B} \to \tau^- \mu^+)$  are again given by Eq. (3.6) with the substitution  ${C}_{1s}^-, C_{2s}^-, C_V^-\} \rightarrow {C}_{1s}^+, C_{2s}^+, C_V^+$ .

# **B.** The decays  $b \rightarrow X \tau^+ \mu^-$  and  $\overline{b} \rightarrow X \tau^+ \mu^-$

For the double differential distribution for the inclusive decay  $b(p) \rightarrow s(p') \tau^+(k') \mu^-(k)$  with respect to the invariants  $x = p' \cdot k/m_b^2$  and  $y = p \cdot k/m_b^2$ , we find

$$
\frac{d^2\Gamma}{dx dy} = \frac{m_b^5}{16\pi^3} \left[ 4|C_V^-|^2 \left( \frac{1 - \hat{m}_\tau^2 - \hat{m}_s^2}{2} - y \right) y \right. \\
\left. - (|C_{1S}^-|^2 + |C_{2S}^-|^2) \left( x - y + \frac{1 - \hat{m}_\tau^2 + \hat{m}_s^2}{2} \right) \right. \\
\left. \times (x - y) - 2 \operatorname{Re}(C_{2S}^- C_V^{-*}) \hat{m}_\tau x \right],\n\tag{3.7}
$$

where  $\hat{m}_f = m_f/m_b$ , and *x* and *y* range between the following limits:

$$
0 \le y \le \frac{1 - (\hat{m}_s + \hat{m}_\tau)^2}{2}, \quad x_- \le x \le x_+, \tag{3.8}
$$

with

$$
x_{\pm} = \frac{1}{2\hat{s}} [\hat{m}_s^2 + (1-\hat{s})(\hat{s} - \hat{m}_\tau^2 \pm \lambda^{1/2}(\hat{s}, \hat{m}_s^2, \hat{m}_\tau^2))], (3.9)
$$

where  $\hat{s} = (p - p')^2 / m_b^2$  and  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy$  $-2xz-2yz$ .

The corresponding expressions for the distribution  $d^2\Gamma^+$ /*dxdy* describing the decay  $\overline{b} \rightarrow \overline{s} \tau^+ \mu^-$  can be obtained from Eq. (3.7) by interchanging  $p \leftrightarrow -p'$  which yields  $x \leftrightarrow -y$  and  $\hat{m}_s^2 \leftrightarrow 1$  in the terms inside square brackets, and by substituting  ${C}_{1S}^-$ ,  ${C}_{2S}^-$ ,  ${C}_V^$ with  $\{C_{1S}^+, C_{2S}^+, C_{V}^+\}.$ 

We now introduce the forward-backward asymmetries  $A_{FB}^{\pm}$  of the two distributions  $\Gamma^{\pm}$ . The asymmetries are defined with respect to the angular variable  $c_{\theta} = \cos \theta$ , where  $\theta$ is the angle between the  $\mu^-$  momentum  $\vec{k}$  and the *s* momentum  $\vec{p}'$  in the *B* rest frame:

$$
A_{\text{FB}}^{\pm}(y) = \frac{1}{d\Gamma^{\pm}(y)/dy} \left[ \int_0^1 dc_{\theta} \frac{d^2\Gamma^{\pm}(y,c_{\theta})}{dydc_{\theta}} - \int_{-1}^0 dc_{\theta} \frac{d^2\Gamma^{\pm}(y,c_{\theta})}{dydc_{\theta}} \right].
$$
 (3.10)

As we will see, the dependence of the asymmetries with respect to the normalized muon energy  $y = E<sub>u</sub>/m<sub>b</sub>$  can provide important informations on the underlying new physics, which are complementary to the measurements of the branching ratios. The average values of the asymmetries are computed as

$$
\langle A_{\text{FB}}^{\pm} \rangle = \frac{1}{\Gamma^{\pm}} \int dy \frac{d\Gamma^{\pm}(y)}{dy} A_{\text{FB}}^{\pm}(y), \quad (3.11)
$$

where  $\Gamma^{\pm} = \int dy [d\Gamma^{\pm}(y)/dy]$ . Finally, we also study the total  $\mu^-$  asymmetry  $A_{FB}$  of the distribution  $d^2[\Gamma^-(y,c_\theta)]$  $+\Gamma^+(y,c_\theta)/dydc_\theta$  which turns out to be a useful quantity when untagged *B* samples are used for the measurements.

## **IV. DISCUSSION**

In this section we discuss the numerical predictions for the branching ratios for the decays  $B_{d,s}(\overline{B}_{d,s}) \rightarrow \tau^+ \mu^-$ ,  $b(\bar{b}) \rightarrow X_s \tau^+ \mu^-$  and for the  $\mu^-$  forward-backward asymmetries measurable in the decays into three body final states. In our estimates, we have used the following set of values for the relevant SM parameters:  $m_b$ =4.8 GeV,  $m_s$ =200 MeV,  $m_{\mu}$  = 106 MeV,  $m_{\tau}$  = 1.777 GeV,  $f_{B_d}$  = 200 MeV,  $f_{B_s}$  $=$  230 MeV,  $m_B$ = 5.3 GeV,  $\tau_B$ = 1.6 ps and  $B(B\rightarrow X_c l\bar{\nu})$  $=10.4\%$ , while the magnitude of the various new physics coefficients is determined by the sets of *H*-charges and SUSY masses listed in Eqs.  $(2.3)$  and  $(2.4)$ .

Our results are collected in Table I and in Figs. 1–3. Table I lists the numerical results for *B* decays involving the channels  $b \rightarrow \mu^-$  and  $\overline{b} \rightarrow \mu^-$ , which are respectively controlled by the two different sets of coefficients  ${C_{1S,2S,V}^-}$  and  ${C_{15,25,V}^{\dagger}}$ . These coefficients are evaluated in the two different models defined by the charges in Eqs.  $(2.3)$  and  $(2.4)$ . The entries in the first column in Table I refer to model I. We recall that in this case the choice of the leptonic horizontal charges tends to enhance the effects of squark exchange. The entries in the second column refer to model II. Here squark exchange diagrams are suppressed by a different choice of

TABLE I. Predictions for the branching ratios for the lepton flavor violating *B* and  $\overline{B}$  decays into  $\tau^+\mu^-$  and for the forwardbackward  $\mu^-$  asymmetries, in the *R*-parity violating models discussed in the text. Model I is defined by the lepton horizontal charges  $H(\hat{L}) = (4,2,0), H(\hat{l}^c) = (4,3,3)$  and by the SUSY masses  $m_l$ <sup> $\approx$ </sup> 100 GeV, $m_{\tilde{q}}$  = 170 GeV. Model II corresponds to the horizontal charges  $H(\hat{L}) = (3,0,0), H(\hat{l}^c) = (5,5,3)$  and to  $m\tilde{l} = 100 \text{ GeV}$ and  $m_{\tilde{q}} = 350$  GeV. In both models the value of the horizontal symmetry breaking parameter is  $\varepsilon = 0.22$ . The existing experimental limits are given in the third column.



the horizontal charges and by the relatively large value of the squark masses, so the effects of slepton exchange tend to dominate. We stress that the aim of the numerical predictions given in the first two columns in Table I is that of suggesting the level of precision that future *B*-physics experiments will have to reach in order to detect, or to effectively constrain, new physics from SUSY without *R*-parity. In the last column we list the existing experimental limits. It is apparent that most of the decay modes we have studied are presently still unconstrained.



FIG. 1. Predictions for the  $\mu^-$  forward-backward asymmetry  $A_{FB}^- (y)$  in the decay  $b \rightarrow X_s \tau^+ \mu^-$  as a function of the normalized *B* rest frame muon energy  $y = E_\mu / m_b$ . The solid (dashed) line correspond to model  $I$  (II) discussed in the text. Model I is defined by the lepton horizontal charges  $H(\hat{L}) = (4,2,0), H(\hat{l}^c) = (4,3,3)$  and by the SUSY masses  $m_l^{\sim} = 100 \text{ GeV}, m_{\tilde{q}} = 170 \text{ GeV}$ . Model II corresponds to the horizontal charges  $H(\hat{L}) = (3,0,0), H(\hat{l}^c) = (5,5,3)$ and to  $m_l^-$  = 100 GeV and  $m_{\tilde{q}}$  = 350 GeV. In both models the value of the horizontal symmetry breaking parameter is  $\varepsilon = 0.22$ .



FIG. 2. Predictions for the  $\mu^-$  forward-backward asymmetry  $A_{FB}^+(y)$  in the decay  $\overline{b} \rightarrow X_s \tau^+ \mu^-$  as a function of the normalized *B* rest frame muon energy  $y = E<sub>\mu</sub> / m<sub>b</sub>$ . The solid (dashed) line correspond to model I  $(II)$  discussed in the text. The new physics model parameters are as in Fig. 1.

The first four lines in Table I collect the results for the two body leptonic decays, while the results for the three body final states are given in the following two lines. Next we present the results for the average values of the  $\mu^$ forward-backward asymmetries  $A_{FB}^{\pm}$  corresponding respectively to  $b \rightarrow \mu^-$  and  $\overline{b} \rightarrow \mu^-$  decays. The average value of the untagged  $\mu^-$  asymmetry  $A_{FB}$  is given in the last line.

To put in evidence the advantage of studying the lepton flavor violating decay modes with respect to the lepton flavor conserving decays  $b \rightarrow \tau^+ \tau^-(X)$  and  $b \rightarrow \mu^+ \mu^-(X)$ , we list in Table II (taken from Ref.  $[9]$ ) the numerical predictions for these decays as derived in the SM, in model I and in model II.

Our results for the two body decays are as follows: the rates for *B* decays are more than two orders of magnitude larger than the corresponding *B* decays. This is mainly due to the fact that the dominant contribution to the two body decays of  $\bar{B}$  mesons is given by the coefficient  $C_{2S}$ , while for the *B*,  $C_{2S}^+$  is suppressed by the *H*-charge difference  $H(\hat{d}_3^c)$  $-H(\hat{Q}_3)=2$ , so that it gives negligible contributions to the decay rates. This is interesting, because—modulo generation



FIG. 3. Predictions for the  $\mu^-$  forward-backward asymmetry  $A_{FB}(y)$  in the decay of an untagged *b* sample into  $X_s \tau^+ \mu^-$  as a function of the normalized *B* rest frame muon energy  $y = E_{\mu}/m_b$ . The solid (dashed) line correspond to model I  $(II)$  discussed in the text. The new physics model parameters are as in Fig. 1.

TABLE II. Predictions taken from Ref. [9] for the lepton flavor conserving *B* decays into  $\tau^+\tau^-$  and  $\mu^+\mu^-$ , in the standard model and in the presence of new physics. Model I and model II coincide with the two models of Table I, and are discussed in the text. The existing experimental limits are listed in the last column.

Process	Standard model	Model 1	Model 2	Limit [Ref.]
B $(B_s \rightarrow \tau^+ \tau^-)$	$9.1 \times 10^{-7}$	$5.7 \times 10^{-6}$	$1.8 \times 10^{-4}$	$5.0 \times 10^{-2}$ a [7]
B $(B_d \rightarrow \tau^+ \tau^-)$	$4.3 \times 10^{-8}$	$1.9 \times 10^{-7}$	$6.3 \times 10^{-6}$	$1.5 \times 10^{-2}$ a [7]
B $(B_s \rightarrow \mu^+ \mu^-)$	$4.3 \times 10^{-9}$	$7.9 \times 10^{-7}$	$7.2 \times 10^{-7}$	$2.6 \times 10^{-6}$ [17]
B $(B_d \rightarrow \mu^+ \mu^-)$	$2.1 \times 10^{-10}$	$2.9 \times 10^{-8}$	$2.7\times10^{-8}$	$8.6 \times 10^{-7}$ b [17]
B $(b \rightarrow X_s \tau^+ \tau^-)_{\text{no cut}}$	$4.9\times 10^{-6}$	$7.3 \times 10^{-6}$	$7.9 \times 10^{-6}$	
B $(b \rightarrow X_s \tau^+ \tau^-)_{s>0.6}$	$1.5 \times 10^{-7}$	$2.2 \times 10^{-6}$	$2.7 \times 10^{-6}$	$5.0 \times 10^{-2}$ a [7]
B $(b \rightarrow X_s \mu^+ \mu^-)_{\text{no cut}}$	$3.1 \times 10^{-4}$	$3.1 \times 10^{-4}$	$3.4 \times 10^{-4}$	
B $(b \rightarrow X_s \mu^+ \mu^-)_{s<0.4}$	$4.3 \times 10^{-6}$	$4.4 \times 10^{-6}$	$8.4 \times 10^{-6}$	$5.8 \times 10^{-5}$ c [14]

<sup>a</sup>Limit estimated from the non-observation of large missing energy events at LEP.

<sup>b</sup>95% C.L.

<sup>c</sup>90% C.L.

independent shifts proportional to baryon number or hypercharge—the set of charges in Eq.  $(2.3)$  is unique for fitting the quark masses and mixing angles. Therefore, the hierarchy in the pattern of the decay rates shown in Table I can be taken as a general qualitative prediction of SUSY models without *R*-parity embedded in models with an *U*(1) horizontal symmetry. Confronting the figures of the leading  $\overline{B}_q \rightarrow \tau^+ \mu^-$  decay rates with the corresponding figures in Table II, we see that while in model I the rates are much smaller than the rates for  $B_q \rightarrow \tau^+\tau^-$ , in model II there is no similar suppression. In this case the larger phase space available for the lepton flavor violating decays yields a slight enhancement of the rates. Confronting with the experimental limit on this decay mode given in the third column in Table I, we see that an improvement of two orders of magnitude is needed in order to test the model. Such an improvement can be within the reach of forthcoming *B*-factory experiments.

In contrast to the two body decays, the decay rates for *b* and  $\overline{b}$  into three body final states predicted in model I are comparable in size. This is because the leading contribution to both decays now comes from vector operators, and confronting Eq.  $(3.2)$  with Eq.  $(3.4)$  it is easy to check that  $C_V^ =C_V^+$ . Model II tends to enhance the effect of scalar operators, and this again results in a relative enhancement of the *b* with respect to the  $\overline{b}$  decay rates. A confrontation with the figures given in Table II shows that in model II the lepton flavor violating *b* decay can be up to one order of magnitude larger than the rate for  $b \rightarrow X_s \tau^- \tau^+$ . Again, this is mainly due to phase space effects. Table II also shows that without kinematic cuts, the SM rates for three body final states are comparable with the rates predicted by the new physics models. Then, in order to single out the new physics short distance effects in the lepton flavor conserving channels, it is necessary to impose suitable cuts. Table II shows the effects of two different cuts on the  $\tau^+\tau^-$  and  $\mu^+\mu^-$  invariant masses. Clearly, one of the advantages of studying lepton flavor violating channels is the complete absence of this background to the new physics effects.

In spite of some large enhancements from the new phys-

ics, the figures in Table I make apparent that even in the most favorable cases, the predicted branching ratios remain rather small. Therefore, in order to measure the corresponding rates, or to put significant constraints on the models, a large statistics and a good experimental efficiency are required. It is worth noticing that while a separate *measurement* of the relevant combinations of the new physics coefficients  ${C_{1S,2S,V}^-}$  and  ${C_{1S,2S,V}^+}$  requires the identification of the flavor of the decaying *B*, with a corresponding loss in the experimental efficiency, in order to establish *limits* on the new physics coefficients it is sufficient to search for  $\tau\mu$  pairs production in the decays of untagged *B* samples. This procedure can ensure a large gain in statistics and will yield the strongest bounds. If  $\tau\mu$  pairs *are* detected in the decay of a sample of untagged  $B$ 's, then a measurement of the  $\mu$ forward-backward asymmetry could provide the additional information needed to identify the flavor of the initial state.

Figures 1 and 2 depict respectively the forward-backward  $\mu^{-}$  asymmetries for *b* and  $\overline{b}$  as a function of the normalized muon energy in the *B* rest frame  $y = E<sub>\mu</sub>/m<sub>b</sub>$ . The solid lines correspond to model I, while the dashed lines refer to model II. We see that while the shape of the asymmetries is quite similar for the two models, there are large differences in the  $\mu$ <sup>-</sup> angular distributions for *b* or  $\overline{b}$  initial states. Namely, the asymmetry for decaying  $\overline{b}$  is negative in the whole energy range, yielding the large negative averages listed in Table I. In contrast, for decaying *b*'s the asymmetry changes sign. This induces large cancellations in the energy averages as is apparent from the figures in the next-to-last line in Table I. Figure 3 depicts the asymmetry for an untagged sample of an equal number of *b* and  $\overline{b}$  initial states. Since in model II (dashed line) the decay rate for  $b \rightarrow X_s \tau^+ \mu^-$  dominates over the rate for  $\overline{b}$ , the asymmetry for the untagged sample reproduces quite closely the energy dependence of the asymmetry in Fig. 1 (namely there is a change in the sign). In model I the two decay modes have comparable branching ratios, and accordingly it is not possible to identify in a reliable way the flavor of the initial state just from an inspection of the untagged asymmetry (solid line in Fig. 3). From these results it is clear that a measurement of a change of sign in the energy dependence of the  $\mu^-$  asymmetry in an untagged *B* sample would signal that most of the  $\tau^+\mu^-$  events originates from decays of *b*'s, while the measurement of a negative asymmetry over the whole energy range would suggest that the contribution from decaying  $\bar{b}$ 's is at least comparable in size.

To check to what extent this remarkable feature of the asymmetries depends on our particular models, we have arbitrarily varied the values of the scalars and vector coefficients in the two models. Our results indicate that in the limit of very heavy squarks  $m_{\tilde{q}} \ge 350$  GeV, which implies  $C_V^{\pm}$  $\ll C_S^{\pm}$ , the difference in the energy dependence of the two asymmetries tends to be washed out: namely also for initial  $\overline{b}$ we find a sign inversion. In the opposite limit  $C_S^{\pm} \ll C_V^{\pm}$ , corresponding to very heavy sleptons, both the asymmetries become negative over the whole energy range, even if there are large differences in the shapes. However, if the difference between the squarks and sleptons masses is kept within a few hundred GeV, the qualitative features of the energy dependence of the asymmetries are maintained, rendering possible in principle the identification of the flavor of the initial state.

In conclusion, in this paper we have investigated *b* and  $\bar{b}$ lepton flavor violating decays involving a  $\tau\mu$  pair in the final state. These decays are expected to occur in SUSY models

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without *R*-parity. Assuming a *U*(1) horizontal symmetry with fermion charges chosen to explain a known set of parameters (the quark masses, the CKM mixing angles, and the lepton masses) allowed us to derive order of magnitude predictions for the various branching ratios. A straightforward prediction of this theoretical framework is that new physics effects are stronger in decays like  $b \rightarrow \tau^+ \tau^-(X)$  when several third generation fermions are involved. However, our results indicate that in general decays involving a  $\tau\mu$  pair in the final state are not particularly suppressed with respect to the decays involving a pair of  $\tau$ 's. On the other hand, because of the presence of a single muon in the decay products and to the absence of any SM contribution (and in particular of the backgrounds from long distance effects) the lepton flavor violating channels are experimentally much easier to be searched for. Therefore, the decays  $\overline{B}(B) \rightarrow \tau^+ \mu^-$  and  $b(\bar{b}) \rightarrow X_s \tau^+ \mu^-$ , together with the *CP* conjugated decays, appear to be better suited than the lepton flavor conserving decays to search for signals of *R*-parity violation.

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