

## Valence quark spin distribution functions

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The hyperfine interactions of the constituent quark model provide a natural explanation for many nucleon properties, including the  $\Delta$ - $N$  splitting, the charge radius of the neutron, and the observation that the proton's quark distribution function ratio  $d(x)/u(x) \rightarrow 0$  as  $x \rightarrow 1$ . The hyperfine-perturbed quark model also makes predictions for the nucleon spin-dependent distribution functions. Precision measurements of the resulting asymmetries  $A_1^p(x)$  and  $A_1^n(x)$  in the valence region can test this model and thereby the hypothesis that the valence quark spin distributions are "normal."

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### I. INTRODUCTION

The quark model has enjoyed so much success as a qualitative guide to hadronic structure that the discovery that only about 30% of the proton's spin could be attributed to quark spin came as a surprise. Since the quark model remains unjustified within QCD, it is a misnomer to call this "proton spin surprise" the "proton spin crisis." However, whatever we call it, this result has generated much very productive experimental and theoretical activity.

While in general the spin of the proton could reside on any mixture of its quark and gluon constituents or in their orbital angular momenta, a conservative interpretation [1] of the current situation is that the valence quarks carry the spin expected by the quark model but that the low  $x$  sea of  $q\bar{q}$  pairs is negatively polarized. In this case  $\Sigma$  (defined to be twice the expectation value of the quark plus antiquark spin along the spin direction of a polarized proton, so that  $\Sigma = 1$  would saturate the proton spin), when decomposed into its valence and sea components, would be

$$\Sigma = \Sigma_v + \sum_q \Delta(q + \bar{q})_{sea} \quad (1)$$

where  $\Sigma_v = \int dx \Sigma_v(x)$  is twice the spin on the valence quarks and  $\Delta q_{sea} = \int dx \Delta q_{sea}(x)$  and  $\Delta \bar{q}_{sea} = \int dx \Delta \bar{q}_{sea}(x)$  are, respectively, twice the spin on the sea quarks and antiquarks of flavor  $q$ . If the valence quarks were in nonrelativistic  $S$ -waves as in the naive quark model, then  $\Sigma_v$  would be unity. However, as has been appreciated for nearly 30 years [2], in realistic valence quark models lower components of quarks spinors convert about 25% of the quark spin into orbital angular momentum so that  $\Sigma_v \approx 0.75$ . If in addition each of the three light quark flavors carries  $\Delta(q + \bar{q})_{sea} \approx -0.15$ , a very modest per flavor effect,  $\Sigma \approx 0.30$  would follow. Sea quark polarizations of just this sign and magnitude have recently been obtained in a realistic model of  $q\bar{q}$  pair creation [1]. [In a more general context, such small  $\Delta(q + \bar{q})_{sea}$  values are perfectly consistent with a  $1/N_c$  expansion of QCD (where sea quarks appear at order  $1/N_c$  via quark-antiquark loops). Note that the condition  $\Delta(q + \bar{q})_{sea} \ll 1$ , not how accurately  $\Sigma_v$  approximates unity, determines the applicability of the  $1/N_c$  expansion: any nonzero  $\Delta(q$

$+\bar{q})_{sea}$  would lead to a "spin crisis" as  $N_f$  (the number of light flavors) tends to infinity.]

In the conservative scenario just described, both the 25% relativistic quenching of spin from  $\Sigma_v$  and the negative polarization of  $\Delta(q + \bar{q})_{sea}$  are compensated by orbital angular momentum. In general, however, we are only guaranteed that

$$\Sigma_v + \sum_q \Delta(q + \bar{q})_{sea} + 2L_q + \Sigma_g = 1 \quad (2)$$

(where  $L_q$  is the quark and antiquark orbital angular momentum and  $\frac{1}{2}\Sigma_g$  is the total angular momentum residing in the gluonic fields), so major experimental efforts are planned to measure the component parts of Eq. (2) in an effort to disentangle the "spin crisis." These efforts begin with planned extensions of deep inelastic lepton scattering measurements of the proton and neutron spin structure functions down to very small  $x$  to complete the integrals required to calculate  $\Sigma$  and studies of the  $Q^2$ -dependence of spin structure functions to make inferences about  $\Delta g(x)$ , the gluon helicity contribution to  $\Sigma_g(x)$ . Major efforts are also planned to directly measure  $\Delta g(x)$  based on helicity-dependent gluon-parton cross sections. In addition to these classical inclusive measurements, flavor-tagging semi-inclusive experiments are planned to measure separately  $\Delta s_{sea}(x)$ ,  $\Delta \bar{s}_{sea}(x)$ ,  $\Delta \bar{u}_{sea}(x)$ ,  $\Delta \bar{d}_{sea}(x)$ , and also the quark contributions  $\Delta u(x) \equiv \Delta u_v(x) + \Delta u_{sea}(x)$  and  $\Delta d(x) \equiv \Delta d_v(x) + \Delta d_{sea}(x)$ . [Note that it is not possible to experimentally separate the quark contributions  $\Delta u_{sea}(x)$  and  $\Delta d_{sea}(x)$  from  $\Delta u_v(x)$  and  $\Delta d_v(x)$ : this separation is conceptual only.] Additional complementary information on the  $s\bar{s}$  content of the proton is expected from planned measurements of the electric and magnetic form factors  $G_E^s$  and  $G_M^s$  of the  $\bar{s}\gamma^\mu s$  current using parity-violating electron-nucleon elastic scattering.

Given the substantial effort being devoted to this problem, it is surprising that we still do not know whether our original simple picture of the spin structure of the valence quarks is right. To some degree this is because this question is not well-defined: in contrast to other methods (e.g., QCD sum rules [3]), the quark model is not normally embedded in a field-theoretic framework. As a result, there are many diffi-

culties in making comparisons between the ‘‘predictions’’ of the quark model and the precisely defined quantities measured in deep inelastic scattering. As two illustrations of such difficulties, I note that (1) the separation of Eq. (2) is  $Q^2$ -dependent (e.g.,  $\Delta g$  might be small at low  $Q^2$  but very important at large  $Q^2$ ) and, as mentioned above, (2) the  $u$  and  $d$  contributions to  $\Delta u_v(x)$  and  $\Delta d_v(x)$  cannot be disentangled from those to  $\Delta u_{sea}(x)$  and  $\Delta d_{sea}(x)$ . However, beyond  $x \approx 0.3$ , sea quarks and antiquarks are scarce and, since gluons are too, such intrinsically field-theoretic issues as the factorization scheme dependence of  $\Sigma_v(x)$  associated with the gluon anomaly [5] may be neglected. Thus while the integral values  $\Delta u_v = \int dx \Delta u_v(x)$  and  $\Delta d_v = \int dx \Delta d_v(x)$  cannot be checked, those fractions of the distributions  $\Delta u(x)$  and  $\Delta d(x)$  extending beyond  $x \approx 0.3$  may be compared with valence quark model expectations with a residual ambiguity associated only with their  $Q^2$  evolution. Although in what follows I will imagine distribution functions devolved to the ‘‘quark model scale’’  $Q_0^2 \approx 1 \text{ GeV}^2$ , given this residual ambiguity I will avoid predictions of the  $x$ -dependence of distribution functions and focus instead on the polarization asymmetries  $A_1^p(x)$  and  $A_1^n(x)$  which depend only on ratios of distribution functions and which should therefore have minimal  $Q^2$ -dependence. It is unfortunate that the current experimental situation for  $A_1^p(x)$  and  $A_1^n(x)$  for  $x > 0.3$  leaves much to be desired (see Fig. 1): it is even consistent with naive  $SU(6)$  predictions.

What are the valence quark model predictions for the resulting polarization asymmetries  $A_1^p(x)$  and  $A_1^n(x)$  in the valence region? Ignoring  $Q^2$  evolution, they are

$$A_1^p(x) = \frac{4\Delta u_v(x) + \Delta d_v(x)}{4u_v(x) + d_v(x)} \quad (3)$$

$$A_1^n(x) = \frac{4\Delta d_v(x) + \Delta u_v(x)}{4d_v(x) + u_v(x)} \quad (4)$$

where  $u_v(x)$  and  $d_v(x)$  are the unpolarized valence distribution functions which integrate to 2 and 1, respectively. From these formulas it is clear that the predictions depend on knowing the interplay between the valence quark spin and momentum wave functions so that there can be no unique prediction of the valence quark model for these asymmetries. However, I will argue here that its predictions are sufficiently well-determined that they can be used to answer the simple question of whether the valence spin structure is ‘‘normal’’ or not.

Aside from this observation, there is little in this paper that could not be extracted from earlier work on this subject to which I will refer below. However, the results of this earlier work vary widely since they are based on diverse methods of dealing with relativistic internal quark motion, various prescriptions for boosting to the infinite momentum frame, *ad hoc* versus dynamical origins for the assumed  $SU(6)$ -breaking, potential versus bag models, and choices of quark masses. Here I will *assume* that the hyperfine interaction is responsible for the  $d(x)/u(x)$  ratio as  $x \rightarrow 1$ , and then normalize predictions for the valence quark spin distribution

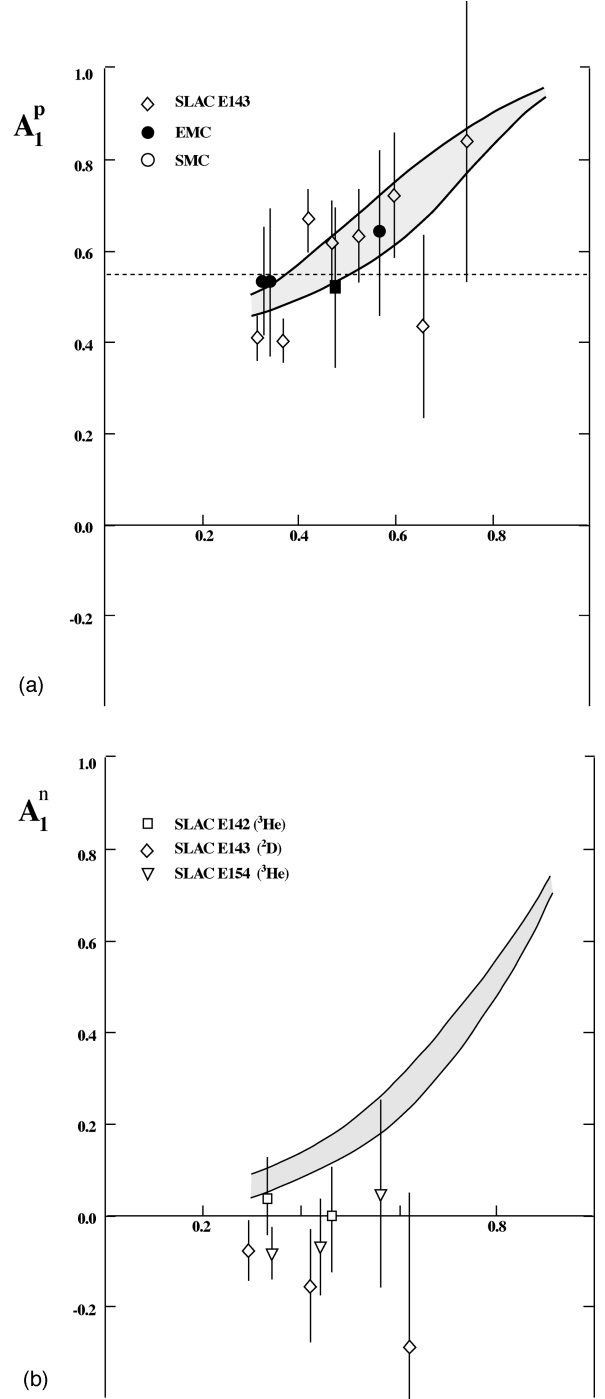


FIG. 1. (a). Data [4] on  $A_1^p$  and the prediction (shaded band) of the model described in the text; the  $SU(6)$  prediction is  $\frac{5}{9}$  (dotted line). (b). Data [4] on  $A_1^n$  and the prediction (shaded band) of the model described in the text; the  $SU(6)$  prediction is 0.

functions to the data on this ratio. In doing so, I will not only avoid much model dependence, but also most of the pitfalls discussed above associated with not knowing precisely how to embed the quark model in field theory.

The body of this paper builds up to these predictions in steps. In the next section, I will review the naive  $SU(6)$  predictions and then modify them within the context of

$SU(6)$  by allowing the quarks to have relativistic internal motions. I then describe the breaking of  $SU(6)$ -symmetric quark spin distributions in the hyperfine-perturbed quark model and close with a brief historical overview.

## II. $SU(6)$ AND “RELATIVIZED $SU(6)$ ” DISTRIBUTION FUNCTIONS

I begin by recalling that in  $SU(6)$  one may simply write

$$p\uparrow = uudC^A\psi^S\chi_+^\lambda \quad (5)$$

$$n\uparrow = dduC^A\psi^S\chi_+^\lambda \quad (6)$$

where since  $C^A$  and  $\psi^S$  are the antisymmetric color and symmetric  $L=0$  spatial wave functions,

$$\chi_+^\lambda = -\sqrt{\frac{1}{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \quad (7)$$

is the unique spin- $\frac{1}{2}$  wave function which is symmetric in the first two quarks as required by the Pauli principle [6]. In the nonrelativistic  $SU(6)$  quark model one therefore expects

$$u_v\uparrow(x) = \frac{5}{3}v_{SU(6)}(x) \quad (8)$$

$$u_v\downarrow(x) = \frac{1}{3}v_{SU(6)}(x) \quad (9)$$

$$d_v\uparrow(x) = \frac{1}{3}v_{SU(6)}(x) \quad (10)$$

$$d_v\downarrow(x) = \frac{2}{3}v_{SU(6)}(x) \quad (11)$$

where  $v_{SU(6)}(x)$  is the universal  $SU(6)$  distribution function associated with  $\psi^S$ . These distributions lead to the standard  $SU(6)$  predictions  $d(x)/u(x)=1/2$ ,  $A_1^p(x)=5/9$  and  $A_1^n(x)=0$ , and  $G_A=5/3$ . When the relativistic quenching mentioned above [2] is turned on, it creates an  $x$ -dependent probability which we denote by  $\frac{1}{2}c_A(x)$  for a spin up (down) quark to be flipped to down (up). This reshuffling of probability leads to the “relativistic  $SU(6)$ ” spin distributions

$$u_v\uparrow(x) = \left[ \frac{5}{3} - \frac{2}{3}c_A(x) \right] v_{SU(6)}(x) \quad (12)$$

$$u_v\downarrow(x) = \left[ \frac{1}{3} + \frac{2}{3}c_A(x) \right] v_{SU(6)}(x) \quad (13)$$

$$d_v\uparrow(x) = \left[ \frac{1}{3} + \frac{1}{6}c_A(x) \right] v_{SU(6)}(x) \quad (14)$$

$$d_v\downarrow(x) = \left[ \frac{2}{3} - \frac{1}{6}c_A(x) \right] v_{SU(6)}(x) \quad (15)$$

where with the nucleon expectation value  $\langle 1 - c_A(x) \rangle_N \approx \frac{3}{5}G_A \approx 0.75$ , the integrated valence spins become

$$\Delta u_v \approx +\frac{4}{5}G_A \quad (16)$$

$$\Delta d_v \approx -\frac{1}{5}G_A, \quad (17)$$

so that the “relativistic  $SU(6)$ ” spin distributions satisfy the Bjorken sum rule. However, among other problems, the “relativistic  $SU(6)$ ” model still makes the incorrect prediction  $d(x)/u(x)=1/2$ . Note that the model also predicts that  $A_1^p(x)=[1-c_A(x)]5/9$  and  $A_1^n(x)=0$  as  $x \rightarrow 1$ , which, since  $c_A(x) \rightarrow 0$  as  $x \rightarrow 1$ , is not obviously wrong (see Fig. 1).

## III. PREDICTIONS OF THE HYPERFINE-PERTURBED QUARK MODEL

Since the zeroth-order nucleons are pure  $S$ -waves, in the hyperfine-perturbed quark model [7], only the Fermi contact part of the hyperfine interaction [the  $\vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij})$  force responsible for the  $\Delta$ - $N$  mass splitting] is operative in perturbing the nucleon’s energy in first order. What does this perturbation do? In the nucleon rest frame, quark pairs with spin 1 have their energies raised (as in the  $\Delta$ ) while pairs with spin zero have their energies lowered. Since  $\chi^\lambda$  has the two  $u$  quarks in a pure spin 1 state, while each  $ud$  pair is in a mixture of spin 1 and spin 0 (with spin 0 dominant so that the net perturbation in a nucleon decreases its energy), up quarks acquire higher average energy than down quarks. This physics then immediately suggests that the neutron will have a negative charge radius and that  $d(x)/u(x)$  will vanish as  $x \rightarrow 1$  [8–10]. Since the individual spin components of  $\chi^\lambda$  are *not* in an eigenstate of the hyperfine interaction, it is less obvious what the effects are on the spin-dependent distribution functions.

These effects are encoded in the  $L=0$  component of the hyperfine-perturbed wave function

$$uudC^A \left[ \cos \theta_m \psi^S \chi_+^\lambda + \sin \theta_m \sqrt{\frac{1}{2}} (\psi^\rho \chi_+^\rho - \psi^\lambda \chi_+^\lambda) \right] \quad (18)$$

where  $(\psi^\rho, \psi^\lambda)$  are mixed symmetry wave functions of the permutation group  $S_3$  which are antisymmetric ( $\rho$ ) and symmetric ( $\lambda$ ) under  $1 \leftrightarrow 2$  interchange, where

$$\chi_+^\rho = \sqrt{\frac{1}{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow, \quad (19)$$

and where  $\theta_m$  is a small mixing angle induced by  $SU(6)$ -breaking interactions [9]. (Since, as explained above, the  $L=0$  ground state energies are perturbed in first order only by the  $\vec{S}_i \cdot \vec{S}_j$  interaction, one can ignore  $L=2$  and totally antisymmetric  $L=0$  admixtures.) It follows that the rest frame probabilities of spin up and spin down  $d$  quarks are, to first order in  $\theta_m$ ,

$$P(d\uparrow) = \frac{1}{3} \left| \psi^S - \sqrt{\frac{1}{2}} \theta_m \psi^\lambda \right|^2 \quad (20)$$

$$P(d\downarrow) = \frac{2}{3} \left| \psi^S - \sqrt{\frac{1}{2}} \theta_m \psi^\lambda \right|^2, \quad (21)$$

since the  $\psi^p \chi_+^p$  piece of the wave function does not interfere with the other terms in the probability distribution. In these formulas I have suppressed coordinate labels which indicate that the probability  $P(d\uparrow)$  [ $P(d\downarrow)$ ] is that for finding a spin up [spin down]  $d$  quark at a point  $\vec{r}_d$  while the two up quarks are at positions  $\vec{a}$  and  $\vec{b}$ .

Similarly one finds

$$P(u\uparrow) = \frac{5}{3} \left| \psi^S - \sqrt{\frac{1}{2}} \theta_m \psi^\lambda \right|^2 - \sqrt{\frac{2}{3}} \theta_m \psi^S \psi^p \quad (22)$$

$$P(u\downarrow) = \frac{1}{3} \left| \psi^S - \sqrt{\frac{1}{2}} \theta_m \psi^\lambda \right|^2 + \sqrt{\frac{2}{3}} \theta_m \psi^S \psi^p \quad (23)$$

where now the wave function  $\psi^p$  does play a role. I have now suppressed coordinate labels which indicate that the probability  $P(u\uparrow)$  [ $P(u\downarrow)$ ] is that for finding a spin up [spin down]  $u$  quark at a point  $\vec{r}_u$  while the other up quark is at position  $\vec{a}$  and the  $d$  quark is at position  $\vec{b}$ .

Note that, as advertised, the net leading-order effect of the  $SU(6)$ -breaking in the spin-averaged probabilities is to create distributions of mixed symmetry that allow the  $d$  quark to have a different probability distribution from the two  $u$  quarks. With the calculated quark model value [9]  $\sin \theta_m \simeq -0.23$ , the distortion of the  $SU(6)$ -symmetric probabilities is substantial. I now make the natural assumption that this distortion translates into the observation that  $d(x)/u(x) \rightarrow 0$  as  $x \rightarrow 1$ , and associate the measured  $u(x)$  and  $d(x)$  with functions  $u_v(x)$  and  $d_v(x)$  associated with the spin-averaged probability  $|\psi^S - \sqrt{\frac{1}{2}} \theta_m \psi^\lambda|^2$ . This remarkably simple picture then leads to the ‘‘standard’’ prediction  $F_2^n/F_2^p \rightarrow \frac{1}{4}$  as  $x \rightarrow 1$ .

The predictions of Eqs. (20)–(23) for  $A_1^p(x)$  and  $A_1^n(x)$  may easily be deduced using the properties of the mixed symmetry pair of wave functions ( $\psi^p$ ,  $\psi^\lambda$ ) under the permutation group  $S_3$ . Such an analysis reveals that the hyperfine interactions have distorted the distributions of  $u\downarrow$ ,  $d\uparrow$ , and  $d\downarrow$  identically, and that the entire dominance of  $u$  quarks as  $x \rightarrow 1$  is due to  $u\uparrow(x)$ . This means that

$$u_v\uparrow(x) = \left[ 1 - \frac{1}{2} c_A(x) \right] u_v(x) - \frac{1}{3} [1 - c_A(x)] d_v(x) \quad (24)$$

$$u_v\downarrow(x) = \frac{1}{3} [1 - c_A(x)] d_v(x) + \frac{1}{2} c_A(x) u_v(x) \quad (25)$$

$$d_v\uparrow(x) = \frac{1}{3} \left[ 1 + \frac{1}{2} c_A(x) \right] d_v(x) \quad (26)$$

$$d_v\downarrow(x) = \frac{2}{3} \left[ 1 - \frac{1}{4} c_A(x) \right] d_v(x) \quad (27)$$

The resulting predictions for  $A_1^p(x)$  and  $A_1^n(x)$  in the valence region, shown in Fig. 1, can be obtained without engaging in an elaborate parametrization of structure functions. Using the rough parametrizations  $d(x)/u(x) \simeq \kappa(1-x)$  as  $x \rightarrow 1$  (with  $0.5 < \kappa < 0.6$ ) and  $c_A(x) = nx(1-x)^n$  [which builds in  $c_A(x) \rightarrow 0$  as  $x \rightarrow 1$  and  $x \rightarrow 0$  and for  $2 < n < 4$  gives the required quenching of  $G_A$ ] produces the narrow bands shown in the figure. As  $x \rightarrow 1$  both  $A_1^p$  and  $A_1^n$  tend to 1, but I show the predictions only in the region where the valence quark wave function is large since very small effects might become important at the end point [11].

#### IV. SOME HISTORY

The history of the prediction of the effects of  $SU(6)$ -breaking on the quark distribution functions in the valence region is somewhat convoluted. It perhaps begins with the parton model discussion by Feynman [12] who argues that as a  $u$  or  $d$  quark approaches  $x=1$ , it must leave behind ‘‘wee’’ partons with either  $I=0$  or  $I=1$ , and that these two configurations are unlikely to have the same  $x$ -dependence. He then notes that if the  $I=0$  configuration dominates as  $x \rightarrow 1$ , the observed ratio  $F_2^n/F_2^p = 1/4$  would follow. If we take the modern view that this high  $x$  behavior will be controlled by the valence quarks, and note the quark model correlation between isospin and spin in the valence quark sector, this argument would also naively lead to the conclusion that  $u_v\uparrow(x)$  will dominate as  $x \rightarrow 1$ . While correct, since Feynman’s argument relies on the ‘‘wee’’ partons being uncorrelated with the leading quark, and so does not take into account the required antisymmetrization between the leading  $u$  quark and the ‘‘wee’’  $u$  quark, its predictions for the full valence region are unclear.

A more complete quark model argument is given in the papers of Close [13] and Carlitz and Kaur [14]. They argued that  $SU(6)$ -breaking changes Eq. (5) into

$$p\uparrow = uud C_A \left[ \frac{\sqrt{3}}{2} \uparrow \chi_{ud}^0 \psi^0 + \frac{1}{2} \left( \sqrt{\frac{1}{3}} \uparrow \chi_{ud}^{10} - \sqrt{\frac{2}{3}} \downarrow \chi_{ud}^{11} \right) \psi^1 \right] \quad (28)$$

where  $\chi_{ud}^0 = \sqrt{\frac{1}{2}}(\uparrow\downarrow - \downarrow\uparrow)$ ,  $\chi_{ud}^{11} = \uparrow\uparrow$ , and  $\chi_{ud}^{10} = \sqrt{\frac{1}{2}}(\uparrow\downarrow + \downarrow\uparrow)$  are the  $S=0$  and two  $S=1$   $ud$  spin wave functions. For  $\psi^0 = \psi^1 = \psi^S$ , this wave function collapses to Eq. (5). Referring to hyperfine forces as driving the physics (which is equivalent to Feynman’s assumption in this case), these papers posit that  $SU(6)$ -breaking leads to  $\psi^0 \neq \psi^1$ , which would in turn lead to the relations  $u_v\uparrow(x) = \frac{1}{6}v_1(x) + \frac{3}{2}v_0(x)$ ,  $u_v\downarrow(x) = \frac{1}{3}v_1(x)$ ,  $d_v\uparrow(x) = \frac{1}{3}v_1(x)$ , and  $d_v\downarrow(x) = \frac{2}{3}v_1(x)$  in terms of the distribution functions  $v_0$  and  $v_1$  associated with  $\psi^0$  and  $\psi^1$ , respectively, with  $v_1/v_0 \rightarrow 0$  as  $x \rightarrow 1$ . This model thus also leads to  $F_2^n/F_2^p \rightarrow \frac{1}{4}$  as  $x \rightarrow 1$  and it predicts that both  $A_1^p(x)$  and  $A_1^n(x) \rightarrow 1$  as  $x \rightarrow 1$ . While assuming  $\psi^0 \neq \psi^1$  is very natural, since these diquark spin states are eigenstates of the hyperfine interaction, this as-

sumption is not consistent with the Pauli principle unless  $\psi^0$  and  $\psi^1$  have very special properties under the permutation group  $S_3$  or the wave function (28) is antisymmetrized. Thus, as with Feynman's argument, it is unclear what these models predict for the full valence region. The closely related model of Close and Thomas [15] is based on examining the energy of the spectator diquark *after* the deep inelastic scattering in a rest frame calculation of deep inelastic structure functions. Although, as pointed out by the authors, their calculation suffers from the fact that the diquark is a colored object which cannot have a well-defined energy, this calculation emphasizes the same physics and reaches the same conclusions as Refs. [13,14]. Given that the impact of the hyperfine interaction is implemented somewhat intuitively in this work, it is once again unclear whether the results presented are reliable for anything other than the  $x \rightarrow 1$  behavior.

Although they do not use the hyperfine-perturbed quark model, the formalism required to deal explicitly with the fully antisymmetrized nucleon wave function seems to have first been applied to the valence quark spin distribution functions by Le Yaouanc *et al.* [16]. They introduce an  $SU(6)$  intraband mixing between the ground state  $[56,0^+]$  and the mixed symmetry  $[70,0^+]$  in an attempt to account for the observed behavior  $F_2^n/F_2^p \rightarrow \frac{1}{4}$  as  $x \rightarrow 1$ , i.e.,  $d(x)/u(x) \rightarrow 0$  as  $x \rightarrow 1$ . This is precisely the kind of mixing introduced in Eq. (18) as required by color hyperfine interactions. (In fact, using this formalism makes calculations much simpler than in the  $uds$  basis, though perhaps less physically transparent.) They then make a prescription to boost this mixed wave function into the infinite momentum frame, fit the mixing angle to the data, and discuss the implications of such mixing to a wide range of phenomena.

More recently, a number of authors [17–20] have addressed the connection between the hyperfine-perturbed quark model (either potential-based or bag-like) and the quark distribution functions. Most of these papers find the same two key effects I have emphasized here: axial current quenching by internal quark motion and  $u$  quark dominance as  $x \rightarrow 1$ .

Despite this extensive body of work [12–20], it does not seem to be widely appreciated that the hyperfine-perturbed

valence quark model makes quite clear predictions for the asymmetries  $A_1^n(x)$  and  $A_1^p(x)$  in the valence region. I attribute this state of affairs to the fact that this work has been very ambitious: most authors have attempted “absolute” calculations of structure functions. In doing so they encountered many obstacles, which forced them to a variety of assumptions, approximations, and “procedures.” The result is a wide range of predictions for the structure functions with apparent agreement *only* on their qualitative features.

## V. CONCLUSIONS

In this paper I have shown that once it is assumed that the hyperfine perturbations of the quark model are responsible for the  $SU(6)$ -breaking observed in the spin-independent structure functions, a very narrow band of predictions for the spin-dependent structure functions follows. In a broader context, I have argued that the extensive measurements and theoretical studies engendered by the “spin crisis” should be anchored in knowledge of whether the valence quark spin distributions are in fact anomalous. Thus whether the distributions described here prove to be correct when confronted with the data will be interesting, but not as important as the fact that such data will indicate whether the valence spin structure functions are in fact anomalous, and thus guide the search for where the resolution of the “spin crisis” is to be found.

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- [1] P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997); N. Isgur, in *The Spin Structure of the Nucleon*, Proceedings of the 1995 Erice School, edited by B. Frois, V. W. Hughes, and N. de Groot (World Scientific, Singapore, 1997), p. 260.
- [2] N. N. Bogoliubov, Ann. Inst. Henri Poincaré, Sect. A **8**, 163 (1968); F. E. Close, Nucl. Phys. **B80**, 269 (1974); A. LeYaouanc *et al.*, Phys. Rev. D **9**, 2636 (1974); **15**, 844 (1977) and references therein; A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, *ibid.* **10**, 2599 (1974); Michael J. Ruiz, *ibid.* **12**, 2922 (1975); C. Hayne and N. Isgur, *ibid.* **25**, 1944 (1982).
- [3] See, for example, the pedagogical review by B. L. Ioffe, in *The Spin Structure of the Nucleon* [1], p. 215.
- [4] E142 Collaboration, P. L. Anthony *et al.*, Phys. Rev. Lett. **71**,

- 959 (1993); Phys. Rev. D **54**, 6620 (1996); E143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995); **75**, 25 (1995); E154 Collaboration, K. Abe *et al.*, *ibid.* **79**, 26 (1997); EMC Collaboration, J. Ashman *et al.*, Nucl. Phys. **B328**, 1 (1989); SMC Collaboration, D. Adams *et al.*, Phys. Lett. B **329**, 199 (1994); **339**, 332(E) (1994); **357**, 248 (1995).
- [5] For a pedagogical discussion see A. H. Mueller, in *Internal Spin Structure of the Nucleon*, edited by V. Hughes and C. Cavata (World Scientific, Singapore, 1995), p. 91; for recent developments see R. D. Ball, S. Forte, and G. Ridolfi, Phys. Lett. B **378**, 255 (1996) and references therein.
- [6] The  $uds$  basis used here sacrifices explicit flavor symmetry in favor of a more compact and physically transparent wave func-

- tion. See, e.g., N. Isgur and G. Karl, Phys. Rev. D **19**, 2653 (1979).
- [7] See, e.g., the review in [1] and the references therein.
- [8] R. Carlitz, S. D. Ellis, and R. Savit, Phys. Lett. **64B**, 85 (1976).
- [9] N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. **41**, 1269 (1978); **45**, 1738(E) (1980).
- [10] In lectures at the 1977 Zakopane Summer School [N. Isgur, Acta Phys. Pol. B **8**, 1081 (1977)], I argued that the  $\Delta$ - $N$  splitting suggests that  $SU(6)$  is broken in such a way that the neutron charge radius and the observation that  $d/u \rightarrow 0$  as  $x \rightarrow 1$  in deep inelastic scattering might be explained as related effects. This discussion was based on the observation that the spin wave function (7) may be written in the form  $\chi_+^\lambda = \sqrt{\frac{1}{6}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) + \sqrt{\frac{1}{6}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)$  and interpreted as an expansion of  $\chi_+^\lambda$  in the nonorthogonal basis  $[\sqrt{\frac{1}{2}}(\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow), \sqrt{\frac{1}{2}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow)]$  in which the  $ud$  pairs (13) and (23), respectively, form a tightly bound spin zero ‘‘core.’’ While qualitatively correct, as with Refs. [12–15] it is unclear whether the conclusions drawn in this framework about the full valence region survive antisymmetrization.
- [11] In addition to valence quark models based on the nucleon’s confinement-dominated wave function, there is a proposal by Farrar and Jackson that as  $x \rightarrow 1$  the distribution functions will be dominated by hard perturbative precesses [G. Farrar and D. Jackson, Phys. Rev. Lett. **35**, 1416 (1975); for a recent exposition see S. J. Brodsky, M. Burkardt, and I. Schmidt, Nucl. Phys. **B441**, 197 (1995)]. In that case near  $x=1$ ,  $u_v\uparrow(x) = \frac{5}{3}v_p(x)$ ,  $u_v\downarrow(x) = \frac{1}{3}v_a(x)$ ,  $d_v\uparrow(x) = \frac{1}{3}v_p(x)$ , and  $d_v\downarrow(x) = \frac{2}{3}v_a(x)$ , where now the parallel distribution function  $v_p$  dominates over the antiparallel distribution function  $v_a$  as  $x \rightarrow 1$ . This model thus leads to  $F_2^n/F_2^p \rightarrow \frac{3}{7}$  as  $x \rightarrow 1$ , but still predicts that both  $A_1^n(x)$  and  $A_1^p(x) \rightarrow 1$  as  $x \rightarrow 1$ . While  $F_2^n/F_2^p$  seems to have fallen well below 3/7, it could be argued that this discrepancy is due to errors in extracting  $F_2^n$  from deuterium data. This issue would be worth examining in the light of new high precision data and modern theories of the deuteron’s high momentum structure. See, e.g., the analysis of S. Liuti and F. Gross, Phys. Lett. B **356**, 157 (1995) which supports  $F_2^n/F_2^p = 1/4$  and that of W. Melnitchouk and A. W. Thomas, *ibid.* **377**, 11 (1996), which argues that  $F_2^n/F_2^p = 3/7$  is allowed.
- [12] R. P. Feynman, *Photon Hadron Interactions* (Benjamin, New York, 1972), p. 150.
- [13] F. E. Close, Phys. Lett. **43B**, 422 (1973); Nucl. Phys. **B80**, 269 (1974); see also F. E. Close, *Introduction to Quarks and Partons* (Academic, New York, 1979), p. 197.
- [14] J. Kaur, Nucl. Phys. **B128**, 219 (1977); see also R. Carlitz and J. Kaur, Phys. Rev. Lett. **38**, 673 (1977); R. Carlitz, Phys. Lett. **58B**, 345 (1975).
- [15] F. E. Close and A. W. Thomas, Phys. Lett. B **212**, 227 (1988).
- [16] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D **11**, 680 (1975); **12**, 2137 (1975); **13**, 1519(E) (1976); **15**, 844 (1976); **18**, 1733 (1978).
- [17] Z. Dziembowski, C. J. Martoff, and P. Zyla, Phys. Rev. D **50**, 5613 (1994) and references therein.
- [18] X. Song and J. S. McCarthy, Phys. Rev. D **49**, 3169 (1994).
- [19] C. J. Benesh, T. Goldman, and G. J. Stephenson, Phys. Rev. C **48**, 1379 (1993).
- [20] H. J. Weber, Phys. Rev. D **49**, 3160 (1994).