Four-fermion heavy quark operators and light current amplitudes in heavy flavor hadrons

Dan Pirjol*

Department of Physics, Israel Institute of Technology, Technion, Haifa 32000, Israel

Nikolai Uraltsev

Department of Physics, University of Notre Dame du Lac, Notre Dame, Indiana 46556;
Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455;
and St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188350, Russia†

We introduce and study the properties of the ''color-straight'' four-quark operators containing heavy and light quark fields. They are of the form $(\bar{b}\Gamma_b b)(\bar{q}\Gamma_q q)$ where both brackets are color singlets. Their expectation values include the bulk of the nonfactorizable contributions to the nonleptonic decay widths of heavy hadrons. The expectation values of the color-straight operators in the heavy hadrons are related to momentum integrals of the elastic light-quark form factors of the respective heavy hadron. We calculate the asymptotic behavior of the light-current form factors of heavy hadrons and show that the actual decrease is $1/(q^2)^{3/2}$ rather than $1/q^4$. The two-loop hybrid anomalous dimensions of the four-quark operators and their mixing (absent in the first loop) are obtained. Using plausible models for the elastic form factors, we estimate the expectation values of the color-straight operators in the heavy mesons and baryons. Improved estimates will be possible in the future with new data on the radiative decays of heavy hadrons. We give the Wilson coefficients of the four-fermion operators in the $1/m_b$ expansion of the inclusive widths and discuss the numerical predictions for the lifetime ratios. Estimates of the nonfactorizable expectation values are given. [S0556-2821(99)07601-8]

PACS number(s): 13.20.He, 13.30.−a, 13.40.Gp

I. INTRODUCTION

Heavy quark expansion proved to be useful in describing the decay properties of b-flavored hadrons. At the level of nonperturbative effects a number of local heavy quark operators of increasing dimension appears whose expectation values in the heavy flavor hadrons determine the importance of preasymptotic effects. The first nontrivial operators, chromomagnetic $O_G = \bar{b}(i/2)\sigma_{\mu\nu}G_{\mu\nu}b$ and the kinetic operator $O_\pi = \bar{b}(i\vec{D})^2 b$ have $D=5$. The expectation value of O_G is known directly from the masses of b-flavored hadrons. The expectation value μ_π^2 of O_π is not yet known definitely, although a certain progress has been achieved over the last few years in evaluating it for B mesons.

More operators appear at $D=6$, in particular, four-fermion operators $\bar{b}\Gamma b \bar{q}\Gamma' q$ where q are light quarks and Γ, Γ' denote various Lorentz and color structures. In the inclusive widths of heavy hadrons such expectation values govern $1/m_b^3$ corrections. Their effect is still significant, especially due to specific accidental suppression of the impact of the leading $D=5$ operators.

Unfortunately, the expectation values of the four-fermion operators up to now remain rather uncertain. Since the mid-1980s [1], the vacuum factorization approximation has been used to estimate the mesonic matrix elements which then appear proportional to f_B^2. Such factorizable terms are absent in baryons, and a number of simple constituent quark model estimates have been employed. The validity of the assump-

tions implemented in such analyses is not clear, however. As a result, the expectation values where the factorizable contributions are absent or suppressed, remain uncertain.

On the other hand, the problem of a more reliable evaluation of the relevant four-fermion expectation values recently attracted a renewed attention since the lifetime ratios of the different b-flavored hadrons have been accurately measured. While data and predictions of the meson lifetimes are nontrivially consistent, the small experimental ratio $\tau_{\Lambda_b}/\tau_{B_d} = 0.78 \pm 0.07$ [2], if taken literally, seems to be in a conflict with the expectations based on the $1/m_b$ expansion.

In the framework of nonrelativistic quark description the four-fermion expectation values are all expressed via the wave function density at origin $|\Psi(0)|^2$ (for mesons) or the diquark density $\int d^3 y |\Psi(0,y)|^2$ (for baryons). All expectation values differ then by only simple color and spin factors [1]. For example, in B mesons one has

$$\frac{1}{2M_B}\langle B^-|(\bar{b}b)(\bar{u}u)|B^-\rangle = |\Psi(0)|^2,$$

$$\frac{1}{2M_B}\langle B^-|(\bar{b}i\gamma_5 u)(\bar{u}i\gamma_5 b)|B^-\rangle = N_c|\Psi(0)|^2 \qquad (1)$$

(color indices are contracted inside each bracket), and for baryons

$$\frac{1}{2M_{\Lambda_b}}\langle\Lambda_b|(\bar{b}b)(\bar{u}u)|\Lambda_b\rangle = \int d^3 y |\Psi(0,y)|^2,$$

$$\frac{1}{2M_B}\langle\Lambda_b|(\bar{b}u)(\bar{u}b)|\Lambda_b\rangle = \frac{1}{2}\int d^3 y |\Psi(0,y)|^2,$$

$$(2)$$

etc.

*Present address: Floyd R. Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853.

†Permanent address.

In actual QCD this simple picture does not hold, and the naive quantum mechanics (QM) relations between different expectation values are generally violated. Moreover, the notion itself of the nonrelativistic wave function used in the potential description, becomes ambiguous. Even in the perturbative domain the expectation values become scale-dependent, and the renormalization is in general different for different operators. This manifestly goes beyond the potential description, even extended for the price of introducing various light-quark spin wave functions in an attempt to account for the relativistic bispinor nature of the light quark fields.

In this paper we note that there exists nevertheless a natural generalization of the notion of the wave function density, in particular at origin (the origin is defined as the position of the heavy quark). It is associated with the expectation values of those four-fermion operators for which the $\bar{b}\Gamma b$ bracket is a color singlet. The color flow for such operators is not disturbed, and we call them ''color-straight'' operators. Their expectation values in the heavy quark limit $m_b \to \infty$ are related to the observable transition amplitudes. This fact suggests that they are better candidates for the operator basis used to parametrize hadronic expectation values in various applications. Moreover, they are more suitable also for applying general bounds of the type discussed in [3]. Such QM-type inequalities can be formulated more rigorously for these operators in full QCD.

Knowledge of the light-quark current elastic form factors of heavy hadrons would allow one to determine the color-straight expectation values. Unfortunately, they are practically unknown yet. Nevertheless, employing reasonable assumptions about their q^2 dependence allows more definite estimates of the expectation values. As the most conservative attitude, they can be viewed as educated dimensional analysis, with the added bonus of being free of ambiguities related to *ad hoc* powers of 2π inherent in various naive dimensional estimates. Such numerically significant uncertainties often cause controversy in the resulting expectations leading sometimes to rather surprising phenomenological conclusions. We also think that the derived relations can be used for an alternative, simple evaluation of the color-straight expectation values in the lattice heavy quark simulations.

II. COLOR-STRAIGHT OPERATORS AND LIGHT CURRENT AMPLITUDES

Our main object of interest is the expectation values of the color-straight operators of the generic type

$$\bar{b}_i \Gamma_b b^i \bar{q}_j \Gamma_q q^j \qquad (3)$$

where Γ_b, Γ_q are arbitrary matrices contracting Lorentz indices (Γ_q can be also a matrix in the light flavor space), and i,j are color indices. We will consider the heavy quark limit $m_b \to \infty$ assuming that the normalization point μ of the operators or currents is set much smaller than m_b. In this case there are two nonvanishing types of operators transforming under rotations of the heavy quark spin as spin-singlet and

spin-triplet, respectively. The corresponding Dirac structure on the heavy side is $\Gamma_b = 1$ and $\Gamma_b = \vec{\gamma}\gamma_5 = \vec{\sigma}$:

$$O_{s-s} = (\bar{b}b)(\bar{q}\Gamma_q q), \quad O_{s-tr} = (\bar{b}\sigma_k b)(\bar{q}\Gamma_q q) \qquad (4)$$

All possible Γ_b-structures are reduced to these operators. In our discussion we always assume that the heavy quark is at rest, $v_\mu = (1,\vec{0})$, and v_μ denotes the velocity of the b-hadron H_b. Since in the heavy quark limit the b quark spin decouples, we start for simplicity from considering the spin-singlet operators O_{s-s}. The straightforward generalization for O_{s-tr} will be formulated later.

If a heavy meson were a two-body QM system where, additionally, the light quark is nonrelativistic as well, the expectation values of O_i measure the meson wave function at origin, see the first of Eqs. (1), and likewise for other matrices Γ_q for which different spin wave functions $\Psi(x)$ can enter. In the momentum representation

$$\Psi(0) = \int \frac{d^3\vec{p}}{(2\pi)^3}\Psi(\vec{p}) \qquad (5)$$

[we use the normalization where $\int d^3 p/(2\pi)^3|\Psi(\vec{p})|^2 = 1$].

On the other hand, in such a nonrelativistic system the Fourier transform of the light quark density distribution measures the elastic transition amplitude (form factor) of the meson associated with the scattering on the light quark:

$$\mathcal{F}(\vec{q}) \equiv \frac{1}{2M_B}\langle B(\vec{q})|\bar{q}q(0)|B(0)\rangle = \int d^3\vec{x}\,\Psi(\vec{x})\Psi^*(\vec{x})e^{-i\vec{q}\vec{x}}. \qquad (6)$$

The following relation then obviously holds:

$$\int \frac{d^3\vec{q}}{(2\pi)^3}\mathcal{F}(\vec{q}) = |\Psi(0)|^2 = \frac{1}{2M_B}\langle B|(\bar{b}b)(\bar{q}q)(0)|B\rangle. \qquad (7)$$

Integrating the transition amplitude over all $\vec{q}$ yields the local four-fermion expectation value we are interested in. Since we study a transition induced by scattering on the light quark, the scale of the transferred momentum is the typical bound-state momentum and is much smaller than m_b.

In actual QCD the simple nonrelativistic picture does not apply. The light quark is certainly relativistic. Additionally, a two-body potential description (generally, any fixed-parton wave function) can only be approximately correct, with *a priori* unknown accuracy.

It appears, however, that in spite of the fact that neither Eqs. (1) nor (6) can be rigorously written in QCD, the final relation between the momentum integral of the (elastic) transition amplitudes and the color-straight expectation values holds exactly, up to corrections vanishing when $m_b \to \infty$. It is not difficult to see, for example, that proceeding from a two-body nonrelativistic meson to a three-body nonrelativistic baryon does not modify the relation. We do not illustrate it here, and instead give a general field-theoretic proof.

Let us start with the operator $O_{\text{s-s}} = \bar{b}b\bar{q}\Gamma q$ and consider the corresponding light quark current and its transition amplitude:

$$J_\Gamma(x) = \bar{q}\Gamma q(x); \quad \frac{1}{2M_{H_b}}\langle \tilde{H}_b(\vec{q})|J_\Gamma(0)|H_b(0)\rangle = \mathcal{A}_\Gamma(\vec{q}). \tag{8}$$

The current does not need to be scalar; any particular component can even be considered separately. Likewise, the transition amplitude may not be a true scalar. The initial and final states may differ. The following relation holds:

$$\frac{1}{2M_{H_b}}\langle \tilde{H}_b(0)|\bar{b}b\bar{q}\Gamma q(0)|H_b(0)\rangle = \int \frac{d^3\vec{q}}{(2\pi)^3}\mathcal{A}_\Gamma(\vec{q}). \tag{9}$$

To prove this relation, we write explicitly the state $\tilde{H}_b(\vec{q})$ with non-zero momentum as a result of the Lorentz boost from rest to the velocity $\vec{v} = \vec{q}/M_{\tilde{H}_b}$:

$$|\tilde{H}_b(\vec{q})\rangle = U\left[L\left(\frac{\vec{q}}{M_{\tilde{H}_b}}\right)\right]|\tilde{H}_b(0)\rangle, \tag{10}$$

where $U[L(\vec{q}/M_{\tilde{H}_b})]$ is the corresponding Lorentz boost unitary operator. This operator is given by [4]

$$U[L(\vec{v})] = e^{-i\vec{n}\cdot\vec{K}\theta}, \quad \sinh\theta = |\vec{v}|, \quad \vec{n} = \frac{\vec{v}}{|\vec{v}|}; \tag{11}$$

the boost generators $\vec{K}$ can be expressed in terms of the symmetric energy-momentum tensor $T_{\mu\nu}$:

$$K^i = \int d^3\vec{x}(x^i T^{00} - x^0 T^{0i}) \tag{12}$$

[x^0 is fixed in Eq. (10) and can be put to zero]. Since $\vec{q}$ does not scale with m_b, we actually need to retain only the linear in $\vec{v}$ terms, which leads to simplifications. For example, the polarization degrees of freedom of $\tilde{H}_b$ (if any) do not change under the boost.

The whole energy-momentum tensor consists of two parts:

$$T_{\mu\nu} = T_{\mu\nu}^{\text{light}} + T_{\mu\nu}^{\text{heavy}}$$

$$= T_{\mu\nu}^{\text{light}} + \frac{1}{4}\bar{b}[\gamma_\mu(i\vec{D})_\nu + \gamma_\nu(i\vec{D})_\mu - (i\overleftarrow{D})_\nu\gamma_\mu - (i\overleftarrow{D})_\mu\gamma_\nu]b, \tag{13}$$

where $T_{\mu\nu}^{\text{light}}$ is the usual QCD energy-momentum tensor including only light fields; it is free of the large parameter m_b. In the heavy quark limit we need to retain only the part of $T_{\mu\nu}$ which is proportional to m_b:

$$K^i = \int d^3x\, x^i T^{00}(x) = m_b \int d^3x\, x^i \bar{b}b(x) + \mathcal{O}(m_b^0). \tag{14}$$

Here we have used the equations of motion for the b field. The anomalous terms are included in the last term (see, e.g., [5], Sec. II). Therefore, we arrive at

$$\langle \tilde{H}_b(\vec{q})|J_\Gamma(0)|H_b(0)\rangle = \langle \tilde{H}_b(0)|e^{i\int d^3z(\vec{q}\vec{z})\bar{b}b(\vec{z})}J_\Gamma(0)|H_b(0)\rangle. \tag{15}$$

The heavy quark limit leads to further simplifications: the number of heavy quarks becomes fixed and b itself becomes static. Then in the single-b sector the following identity holds:

$$e^{i\int d^3z f(\vec{z})\bar{b}b(\vec{z})} = \int d^3z e^{if(\vec{z})}\bar{b}b(z). \tag{16}$$

Indeed, in the single-b sector any product of the static b quark bilinears is very simple:

$$(\bar{b}\sigma_1 b)(\vec{z}_1) \ldots (\bar{b}\sigma_n b)(\vec{z}_n)$$
$$= \delta^3(\vec{z}_1 - \vec{z}_n) \ldots \delta^3(\vec{z}_{n-1} - \vec{z}_n)$$
$$\times \bar{b}\sigma_1 \ldots \sigma_n b(z_n)|_{\text{single }b} \tag{17}$$

(σ_k are arbitrary spin matrices). Using this, we obtain

$$e^{i\int d^3z f(\vec{z})\bar{b}b(\vec{z})} = \sum_{n=0}^\infty \frac{i^n}{n!}\int d^3z_1 \ldots d^3z_n f(\vec{z}_1) \ldots f(\vec{z}_n)$$
$$\times \bar{b}b(\vec{z}_1) \ldots \bar{b}b(\vec{z}_n)$$
$$= \sum_{n=0}^\infty \frac{i^n}{n!}\int d^3z f^n(\vec{z})\bar{b}b(\vec{z})$$
$$= \int d^3z e^{if(\vec{z})}\bar{b}b(\vec{z}). \tag{18}$$

Taking $f(\vec{z}) = \vec{q}\vec{z}$ we rewrite Eq. (15) in the desired form:

$$\langle \tilde{H}_b(\vec{q})|J_\Gamma(0)|H_b(0)\rangle$$
$$= \langle \tilde{H}_b(0)|\int d^3z e^{i\vec{q}\vec{z}}\bar{b}b(\vec{z})J_\Gamma(0)|H_b(0)\rangle. \tag{19}$$

Equation (19) provides the discussed quantum field-theory generalization of the notion of the light-quark density $\bar{q}\Gamma q$ at arbitrary separation; one can define, for example,

$$|\Psi_\Gamma(x)|_{H_b}^2 = \int \frac{d^3\vec{q}}{(2\pi)^3}e^{i\vec{q}\vec{x}}\frac{1}{2M_{H_b}}\langle H_b(\vec{q})|J_\Gamma(0)|H_b(0)\rangle. \tag{20}$$

In what follows we are interested in local heavy quark operators, that is when the light field operators enter at the same point as the b quark field. It is these operators that appear in the heavy quark expansion. Integrating Eq. (19) over $\vec{q}$ we get

$$\langle \tilde{H}_b(0) | \bar{b}b(0) J_\Gamma(0) | H_b(0) \rangle$$

$$= \int \frac{d^3\vec{q}}{(2\pi)^3} \langle \tilde{H}_b(\vec{q}) | J_\Gamma(0) | H_b(0) \rangle. \qquad (21)$$

This is our master equation. We see that, in principle, it is even more general than was stated earlier: $J_\Gamma(0)$ can be an arbitrary gauge-invariant operator composed of the light fields, and not necessarily a light-quark bilinear. Besides, this relation holds not only for the truly forward transition matrix elements. The initial and final state hadrons can be different. Generally, they can even have different momenta; however, it must be assumed that these momenta are small compared to m_b – say, of the typical light hadron mass scale. Since this equation involves the integration over all transferred momenta, varying the relative momentum of the final and initial hadrons have no effect whatsoever, as it should be.

Informative relations emerge, on the other hand, if we vary the heavy flavor state $|\tilde{H}_b\rangle$ (or $|H_b\rangle$) within the corresponding heavy-spin multiplet. Since the b-quark spin decouples, this yields similar relations for the color-straight spin-triplet operators containing $\bar{b}\vec{\sigma}b$, that is, with the axial-vector b-quark current. In particular,

$$\langle \tilde{H}_b(0) | \bar{b}\sigma_k b(0) J_\Gamma(0) | H_b(0) \rangle$$

$$= \int \frac{d^3\vec{q}}{(2\pi)^3} \langle S_k \tilde{H}_b(\vec{q}) | J_\Gamma(0) | H_b(0) \rangle, \qquad (22)$$

where $\vec{S}/2$ is the b-quark spin operator. Formally one obtains this by using, for example, the representation

$$|S_k \tilde{H}_b\rangle = \int d^3\vec{x} \, \bar{b}\sigma_k b(x) | \tilde{H}_b \rangle$$

and applying relations (16),(17) generalized to include the b-quark spin matrices. Alternatively, it follows merely from the heavy-spin symmetry relation between matrix elements of the operators $\bar{b}b J_\Gamma(x)$ and $\bar{b}\sigma_k b J_\Gamma(x)$.

It is worthwhile to give a less rigorous but a transparent QM derivation of the master equation Eq. (21). Let us represent the expectation value of the color-straight operator $\bar{b}sb J_\Gamma(0)$ (s is either the unit or a spin matrix) by the sum over possible intermediate states:

$$\langle \tilde{H}_b | \bar{b}sb(0) J_\Gamma(0) | H_b \rangle$$

$$= \sum_n \int \frac{d^3\vec{q}}{(2\pi)^3 2E_n} \langle \tilde{H}_b | \bar{b}sb(0) | n(\vec{q}) \rangle$$

$$\times \langle n(\vec{q}) | J_\Gamma(0) | H_b \rangle. \qquad (23)$$

The states $|n(\vec{q})\rangle$ are hadrons with a single b quark. In the effective theory the integral over momenta must converge at a hadronic scale which is much smaller than m_b. Then only the elastic transition (i.e., where $\tilde{H}_b$ and $|n\rangle$ belong to the
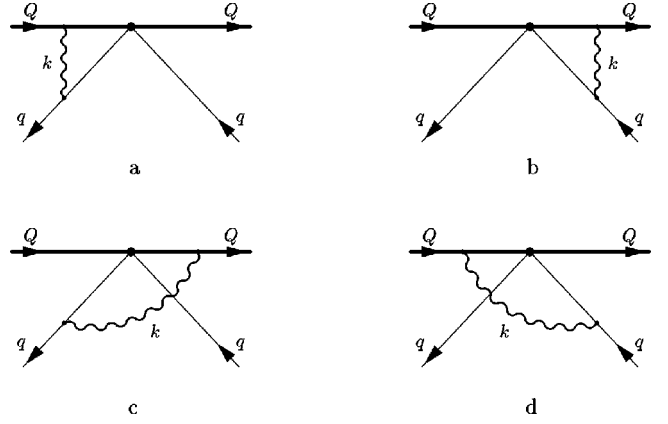

FIG. 1. Diagrams for the renormalization of the four-fermion operators.

same hyperfine multiplet differing, at most, by the heavy quark spin alignment if s is not a unit matrix) survive in the sum: all excited transition amplitudes generated by the heavy quark current $\bar{b}sb(0)$ are either proportional to $1/m_b$, or to velocity $\vec{v} \simeq \vec{q}/m_b$ of the heavy hadron state $|n(\vec{q})\rangle$ [6]. Moreover, since $\vec{v} \to 0$ the elastic amplitude is unity up to corrections $\sim q^2/m_b^2$ we neglect. Thus, Eq. (21) is reproduced.

Let us illustrate the validity of relation (21) diagrammatically, in respect to the perturbative corrections. Relevant order-α_s corrections to the expectation value of the four-fermion operator are drawn in Figs. 1 whereas Fig. 2 show the corrections to the form factor. The gluon exchanges involving only light quarks merely renormalize the current in question, and we do not consider them. The corrections dressing the heavy-quark part vanish due to conservation of the b-quark current (we consider gluon momenta much smaller than m_b).

In the nonrelativistic approximation for the light quark the ''crossed'' diagrams are suppressed, and the remaining diagrams Figs. 1(a) and 1(b) have obvious counterparts in the corresponding diagrams in Fig. 2. Going beyond a simple potential approximation (e.g., at $k^2 \gg m_q^2$), however brings in diagrams Figs. 1(c) and 1(d) as well. In fact, one should keep in mind that in Eq. (21) the integration of the formfactor is performed only over the spacelike components of $\vec{q}$. This fixes the spacelike separation of the $\bar{b}b$ and $\bar{q}\Gamma q$ currents to be zero, however *per se* does not specify the timelike separation of the vertices which is actually determined by the
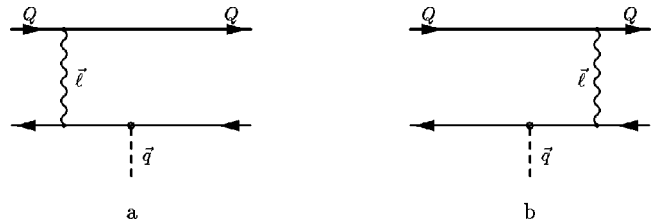

FIG. 2. Diagrams contributing to the light quark form factor of a heavy hadron.

heavy quark propagators. In reality, a single diagram Fig. 2(a) corresponds to the sum of diagrams (a) and (c) in Fig. 1, and likewise with diagrams (b). In the coordinate representation, the heavy quark propagator in Fig. 1(a) is $\vartheta(-x_0)$ and in Fig. 1(c) it is $\vartheta(x_0)$ thus yielding unity in the sum (unity means absence of any propagation, as in Fig. 2).

Let us illustrate it in the usual momentum representation. Denoting the gluon momentum in Fig. 1 by k, we keep the spacelike components of k fixed and consider the integral over $\omega \equiv k_0$. The diagrams (a) and (c) are given, respectively, by

$$\frac{1}{-\omega-i\epsilon} \cdot \mathcal{A}(\vec{k},\omega) \text{ and } \frac{1}{\omega-i\epsilon} \cdot \mathcal{A}(\vec{k},\omega), \quad (24)$$

where $\mathcal{A}(\vec{k},\omega)$ generically denotes the ''light'' part of the diagram (including the gluon propagator). Since

$$\frac{1}{-\omega-i\epsilon} + \frac{1}{\omega-i\epsilon} = 2\pi i \delta(\omega),$$

the integration $dk_0/2\pi i$ of the sum of Figs. 1(a) and 1(c) amounts merely to setting $k_0=0$ in the rest of the diagram. (This is a special case of the more general relations given in Appendix A.) Then it exactly coincides with Fig. 2(a) if $\vec{k}$ is identified with the gluon momentum $\vec{l}$ in the latter. Although the gluon momentum transfer $\vec{l}$ is not generally equal to $\vec{q}$ but can differ by a primordial momentum in the bound state, integration over all $\vec{q}$ is equivalent to integration over $d^3\vec{l}$. Similarly, the sum of the diagrams in Figs. 1(b) and 1(d) yields the integral of Fig. 2(b) over $\vec{q}$.

It is clear that this proof is generalized for an arbitrary number of gluon exchanges between the ''light'' and 'heavy' parts of the diagrams, or the case of the axial b-quark current (see Appendix A). It is imperative, however, that the b quark current is color-singlet.

III. APPLICATIONS

We now turn to some applications of the relations (21), (22).

A. Perturbative renormalization of the color-straight operators

In general, the composite heavy-quark operators depend on the renormalization point μ which is assumed to satisfy the ''hybrid'' hierarchy condition $\Lambda_{\text{QCD}} \ll \mu \ll m_b$. The most interesting is the logarithmic renormalization. This ''hybrid'' renormalization was first considered in [7–9] where the one-loop hybrid anomalous dimensions were calculated for the quark bilinears and four-fermion operators.

In the expressions of the matrix elements of the color-straight operators $\bar{b}(\sigma_k)b\bar{q}\Gamma q$ via the integral of the transition matrix element of the light quarks current, the normalization-point dependence can appear in two ways: first, as a μ-dependence of the light-quark current itself. This is a usual, ''ultraviolet'' renormalization since μ is an ultra-

violet cutoff in respect to the light degrees of freedom. The second way the dependence on the UV cutoff can enter is via the divergence of the integral over the momentum of the final state. Indeed, in the effective theory with the cutoff μ the perturbative states with momenta above μ are absent, while the form factors with $|\vec{q}| \ll \mu$ coincide with those in full QCD. Therefore, if in full QCD the integral of the amplitude in Eqs. (21),(22) does not converge at $\vec{q} \sim \Lambda_{\text{QCD}}$ but has a log behavior in the hybrid domain, this leads to the logarithmic dependence of the matrix element on μ.

In practice we are interested in vector or axial vector currents of light quarks. They are conserved and their anomalous dimensions vanish (for the flavor-singlet axial-vector current there is an anomalous dimension in higher orders in α_s related to the axial triangle anomaly). Therefore we will phrase our discussion neglecting this type of renormalization.

The asymptotics of the actual light quark current form factors of the heavy flavor hadrons is given by the perturbative diagrams where hard gluons transfer the high momentum from the light quark to the heavy one. The tree-level order-α_s diagrams are shown in Figs. 2(a) and 2(b). By virtue of the relations Eqs. (21),(22) they determine *one-loop* renormalization of the four-fermion operators. It is easy to see that these diagrams yield amplitudes fading out at least as $1/\vec{q}^4$ (the odd powers of $\vec{q}$ do not contribute to the integral).[1] In principle, depending on the particular form of Γ, the asymptotics of these diagrams may have the $1/|\vec{q}|^3$ term—it is given by

$$\frac{4\pi\alpha_s}{\vec{q}^4} \frac{1}{2M_{H_b}} \langle \widetilde{H}_b(0)|\bar{b}t^a b\bar{q}(\gamma_0 \slashed{q}\Gamma - \Gamma q \slashed{\gamma}_0)t^a q(0)|H_b(0)\rangle$$

(25)

$(\bar{b}t^a b \rightarrow \bar{b}\vec{\sigma}t^a b$ for the spin-flip transitions). The matrix element may not vanish for b-flavored hadrons with nonvanishing spin of light degrees of freedom (let us recall that $q_0=0$). However, in this matrix element both hadrons are at rest, therefore any such $1/|\vec{q}|^3$ term vanishes upon integrating over the direction of $\vec{q}$. The fact of vanishing of the leading-order hybrid anomalous dimension for the operators of the form $(\bar{b}b)(\bar{q}\gamma_\mu(\gamma_5)q)$ was noted in [8] already in the mid 1980s as a result of simple calculations of the one-loop diagrams. Our relation gives it an alternative interpretation.

A closer look reveals, however, that the cancellation of the leading $1/|\vec{q}|^3$ asymptotics does not hold already at the one-loop level. The asymptotics has actually the form $\sim \alpha_s^2(\vec{q})/|\vec{q}|^3$ which emerges from the diagrams shown in Figs. 3 (other diagrams decrease faster in the Feynman

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[1]Similar quark counting rules in heavy mesons for $|\vec{q}| \ll m_b$ have been applied, e.g., in [10].
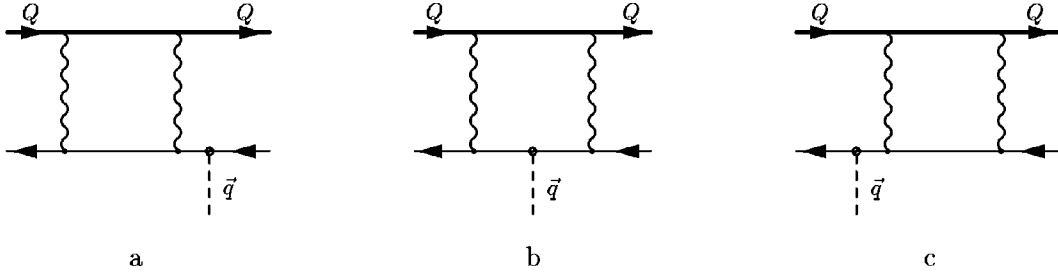
FIG. 3. Order-α_s^2 diagrams determining the asymptotics of the light quark form factor of a heavy hadron. Similar diagrams with the twisted gluon lines are not shown.

gauge). This leads to a nonzero anomalous dimension of the color-straight operators at order α_s^2 and their mixing with color-octet operators. In particular, the evaluation of the one-loop amplitudes leads to

$$\mathcal{A}_\Gamma(\vec{q}) = \frac{\frac{\pi^2}{2}\alpha_s^2}{|\vec{q}|^3}\left[\left(1-\frac{1}{N_c^2}\right)\frac{\langle\tilde{H}_b|\bar{b}b\bar{q}\Gamma q|H_b\rangle}{2M_{H_b}} + N_c\left(1-\frac{4}{N_c^2}\right)\frac{\langle\tilde{H}_b|\bar{b}t^a b\bar{q}\Gamma t^a q|H_b\rangle}{2M_{H_b}}\right]. \quad (26)$$

Equation (21) then yields for the UV part of the four-fermion operator

$$O_{s-s} = \frac{\alpha_s^2}{4}\left[\left(1-\frac{1}{N_c^2}\right)\bar{b}b\bar{q}\Gamma q + N_c\left(1-\frac{4}{N_c^2}\right)\bar{b}t^a b\bar{q}\Gamma t^a q\right]$$
$$\times\ln\frac{\Lambda_{\text{uv}}}{\mu} + \text{finite piece}, \quad (27)$$

and likewise for the spin-triplet operators.

A direct calculation of the two-loop anomalous dimensions confirms this. The computational details are described in Appendix B. Here we only quote the result. Let us denote[2]

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\binom{O^i}{T^i} = \hat{\gamma}\binom{O^i}{T^i}. \quad (28)$$

Then

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[2]The anomalous dimensions of the operators are often defined with the opposite sign. We prefer to use this convention where the meaning of the anomalous and canonical dimensions are the same. That is, the scaling properties of the operators are given by the sum (rather than difference) of their canonical and anomalous dimensions.

$$\gamma_{11} = 4\pi^2\left(1-\frac{1}{N_c^2}\right)\left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3),$$

$$\gamma_{12} = 4\pi^2 N_c\left(1-\frac{4}{N_c^2}\right)\left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3)$$

$$\gamma_{21} = \pi^2 N_c\left(1-\frac{1}{N_c^2}\right)\left(1-\frac{4}{N_c^2}\right)\left(\frac{\alpha_s}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3),$$

$$\gamma_{22} = 3N_c\frac{\alpha_s}{4\pi} + \mathcal{O}(\alpha_s^2). \quad (29)$$

We note that since γ_{11}, γ_{12}, and γ_{21} vanish to order α_s, these two-loop anomalous dimensions do not depend on the renormalization scheme. For γ_{22} the second-order terms depend on the scheme and we do not consider them.

Additional terms are present for the flavor-singlet operators: for the vector current only γ_{22} is modified, $\gamma_{22}\to\gamma_{22} -\frac{4}{3}n_f(\alpha_s/4\pi)$. If the operator has the flavor-singlet axial current then only the diagonal anomalous dimension for the color-straight operator changes, $\gamma_{11}\to\gamma_{11}-6n_f(N_c -1/N_c)(\alpha_s/4\pi)^2$.

It is interesting that, although γ_{11}, γ_{12}, and γ_{21} already appear in the second loop, they are universal. In particular, they are the same for both timelike and spacelike components of the light quark currents. *A priori* this does not need to hold. We expect that this universality will be violated in the next order in α_s.

The two-loop anomalous dimensions are enhanced, they contain a large factor π^2. Neglecting them introduces a numerical uncertainty in the running of operators. We can estimate it by simply setting $\ln(\mu'/\mu)$ to unity. The corresponding corrections at $\alpha_s=1$ constitute about 15 to 30%. This provides additional justification for the standard choice of $\alpha_s(\mu)=1$ as the low (hadronic) normalization scale.

We point out that the naive estimate of the power of the asymptotics $1/\vec{q}^4$ of the light current form factors existing in the literature [10] is not correct: the actual fall off is only $1/|\vec{q}|^3$ as shown in Eq. (26), which, however, is generated only by the exchange of two gluons with momenta $\sim\vec{q}$. (The modification for the spin-triplet operators is obvious.) This asymptotics can be easily RG improved using relations Eqs. (21),(22). To the NLO it amounts to adding the factor

$[\alpha_s(\vec{q})/\alpha_s(\mu)]^{-3N_c/2\beta_0}$ (or $[\alpha_s(\vec{q})/\alpha_s(\mu)]^{(-3N_c+4/3n_f)/2\beta_0}$ for the flavor-singlet vector current) in front of the color-octet operator, with μ the normalization point of the operators. Since the gap between the typical hadronic mass μ_{had} and m_b is not too large in the logarithmic scale, this observation has, probably, rather theoretical than practical significance.

It turns out that in the relativistic system the Coulomb interaction is still strong enough to make the wave function density not approaching a literal constant at zero separation but having the small logarithmic dependence on distance which appears only at the level of loop corrections, at order $\sim \alpha_s^2$.

Based on the application to the lifetimes of heavy hadrons (first of all, in B mesons) and routine application of factorization, a standard choice for the basis for four-fermion operators ascending to the original papers on the subject was [11,12]

$$O_{\mathrm{singl}}=(\bar{b}\Gamma^{(1)}q)(\bar{q}\Gamma^{(2)}b), \quad O_{\mathrm{oct}}=(\bar{b}t^a\Gamma^{(1)}q)(\bar{q}t^a\Gamma^{(2)}b)$$
(30)

$(t^a=\lambda^a/2, \lambda^a$ are the usual Gell-Mann color matrices), that is the s-channel color-singlet and color-octet operators. It appears, however, that a better choice is to classify the operators according to the color structure in the t-channel:

$$O=(\bar{b}_i\Gamma^{(1)}q^j)(\bar{q}_k\Gamma^{(2)}b^l)\cdot \delta_{il}\delta_{kj},$$

$$T=(\bar{b}_i\Gamma^{(1)}q^j)(\bar{q}_k\Gamma^{(2)}b^l)\cdot t^a_{il}t^a_{kj}.$$
(31)

In the large-N_c limit these two bases coincide (up to permutation):

$$O_{\mathrm{singl}}=2T+\frac{1}{N_c}O, \quad O_{\mathrm{oct}}=\frac{1}{2}\left(1-\frac{1}{N_c^2}\right)O-\frac{1}{N_c}T \quad (32)$$

$$O=2O_{\mathrm{oct}}+\frac{1}{N_c}O_{\mathrm{singl}}, \quad T=\frac{1}{2}\left(1-\frac{1}{N_c^2}\right)O_{\mathrm{singl}}-\frac{1}{N_c}O_{\mathrm{oct}}.$$
(33)

The t-channel octet operators T also diagonalize the one-loop anomalous dimension matrix; its value depends on the type of the current, flavor-singlet or octet [8].

We parametrize these generic expectation values encountered in actual weak decays as

$$\frac{1}{2M_B}\langle B|O_V|B\rangle=\omega_V, \quad \frac{1}{2M_B}\langle B|T_V|B\rangle=\tau_V \quad (34)$$

$$\frac{1}{2M_B}\langle B|O_A|B\rangle=\omega_A, \quad \frac{1}{2M_B}\langle B|T_A|B\rangle=\tau_A.$$
(35)

The parameters ω, τ have dimension m^3 and are constants in the heavy quark limit. They can be valence or nonvalence; the flavor of the light quark in the operator will be indicated as a superscript.

For Λ_b-baryons we denote

$$\frac{1}{2M_{\Lambda_b}}\langle \Lambda_b|O_V|\Lambda_b\rangle=\lambda, \quad \frac{1}{2M_{\Lambda_b}}\langle \Lambda_b|T_V|\Lambda_b\rangle=-\frac{2}{3}\lambda';$$
(36)

in the valence approximation $\lambda'=\lambda$. The values of λ for u and d quarks are equal, likewise for λ'. These valence expectation values will be normally used without flavor index.

A remark is appropriate to conclude the discussion of the perturbative renormalization. Strictly speaking, the flavor-singlet operators can be renormalized in somewhat different ways depending on the prescription to treat the tadpole-type closed loops. The free quark loop by dimensional counting scales with the UV cutoff μ like μ^3, and describes a possible power mixing with the $D=3$ "unit" heavy-quark operator $\bar{b}b$ already at order α_s^0. Although for practically relevant operators such a "bare" mixing vanishes for the usual way to regulate the light quark loop, one can raise the question where this freedom is reflected in relations (21),(22) for a generic Γ. The resolution is rather straightforward: the flavor-singlet current $\bar{q}\Gamma q$ also requires regularization of the closed fermion loop and, a priori admits mixing with the unit operator (the tadpole graph). This operator does not lead to any physical transition at $q\neq 0$ but to the forward amplitude with $\vec{q}=0$. A formally defined current $\bar{q}\Gamma q$ may thus lead to an additional term proportional to $\delta^3(\vec{q})$ in the transition amplitude $\mathcal{A}(\vec{q})$, which would reproduce the tadpole term in the expectation value.

Similarly, strictly speaking one could have chosen an arbitrary convention for the phases of the states $|\tilde{H}_b(\vec{q})\rangle$ with different momenta $\vec{q}$. This would redefine the phase of the transition amplitude $\mathcal{A}(\vec{q})$. In our relations such a freedom was eliminated by adopting Eqs. (11),(12),(14) which ensures, for example, the proper analytic properties of the transition amplitudes.

In the purely perturbative calculations one can, in principle, consider not only the actual physical amplitudes, but also similar transition amplitude induced by the light quark currents carrying color. Applying to them relations similar to Eq. (21) and (22) one would need to consider the color-nonsinglet quark *in* or *out* states. This case requires certain care since such amplitudes may have additional (gauge-dependent) infrared singularities.

It is worth reiterating that in our analysis it is assumed that all heavy quark operators are renormalized at a scale well below m_b, which implies a nontrivial—even if finite—renormalization when passing from the full QCD fields. In particular, the vector $\bar{b}b$ and axial $\bar{b}\vec{\sigma}b$ currents both do not renormalize in this domain; however, they run differently when evolved down from the scale $\sim m_b$. While $\bar{b}\gamma_0 b \to \bar{b}b$ is not renormalized, the short-distance renormalization of $\bar{b}\gamma_k\gamma_5 b \to \bar{b}\sigma_k b$ slightly suppresses it:

$$\zeta \simeq 1-\frac{2\alpha_s}{3\pi}+\mathcal{O}(\alpha_s^2)$$

(the second-order correction has been also calculated [13]). This is not, however, the only short-distance contribution differentiating the renormalization which, in general, depends on the exact form of the operators. We will not further dwell on these corrections in our numerical analysis.

B. Estimates of the color-straight expectation values in B mesons

Relations (21),(22) open up a possibility for an alternative evaluation of the expectation values of the color-straight operators. This requires knowledge of the light-quark-current formfactors of heavy hadrons. The direct experimental information about them is scarce. Therefore, we have to assume a reasonable model. Our general strategy for all expectation values of interest is the same: decompose the transition amplitude into the invariant formfactors, and adopt a model for the formfactors satisfying known constraints.

For the family of Λ_b baryons, the number of possible amplitudes is limited due to the fact that the light degrees of freedom are spinless—one can construct only scalar, vector and tensor currents while pseudoscalar and axial amplitudes vanish. There are no axial analogues of the expectation values in Eq. (36). For mesons all amplitudes are possible. Our main attention will be devoted to the vector and axial currents, due to the chiral invariance of phenomenologically relevant four-fermion operators. We do not consider the tensor current, and only briefly comment on the scalar one.

There is only one formfactor for the vector current (for each flavor content) for both B and Λ_b describing the only nonvanishing timelike component:

$$\langle B(\vec{q})|J_\mu|B(0)\rangle = -v_\mu F_B(q^2)$$

$$\langle \Lambda_b(\vec{q})|J_\mu|\Lambda_b(0)\rangle = v_\mu F_{\Lambda_b}(q^2)\bar{u}(v,s')u(v,s).$$
(37)

One important constraint on the formfactors is their value at $q^2=0$. The values of $F_{B,\Lambda_b}(0)$ are fixed by the corresponding charge of the hadron: it is 1 for the current of a valence quark, and zero for a ''sea'' light flavor.[3] For the amplitude in Eqs. (37) the integration over $\vec{q}$ yields

$$v_\mu \int \frac{d^3\vec{q}}{(2\pi)^3} F(q^2) = \frac{v_\mu}{4\pi^2}\int_0^\infty dt\sqrt{t}F(-t).$$
(38)

The valence form factors are expected to decrease for nonzero $\vec{q}^2$. For the isovector formfactor the slope at $q^2=0$ (related to the corresponding charge radius) can be estimated in terms of experimentally observable quantities by an analogue of the Cabibbo-Radicati sum rule for heavy hadrons [14]:

———

[3]We adopt the convention where B mesons have the quark content $b\bar{q}$.

$$\frac{dF(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{1}{8\alpha Q^2}\sum_{\text{exc}}(2J+1)\frac{\Gamma(B_{\text{exc}}\rightarrow B\gamma)}{|\vec{k}|^3}$$
(39)

where the sum runs over excitations of B with spin J and Q is the light quark charge in units of e (a similar relation holds for baryons as well), and the nonresonant contributions are neglected. This slope is not yet known experimentally well enough.

Relations following from Eq. (38) can be used in the lattice simulations to evaluate the expectation values, by measuring the transition formfactors in a few kinematic points and interpolating between them. This type of lattice measurements can be simpler than for the heavy-quark current transitions, since the heavy quarks remain at rest and the momenta involved in the process do not scale with m_Q. This makes the static approximation rather straightforward.

If we represent the form factor as a sum over singularities in the t-channel

$$F(q^2)=\sum_n \frac{c_n M_n^2}{M_n^2-q^2},$$
(40)

the integral Eq. (38) takes the form

$$\int \frac{d^3\vec{q}}{(2\pi)^3} F(q^2) = -\frac{1}{4\pi}\sum_n c_n M_n^3;$$
(41)

we have an additional constraint

$$\sum_n c_n M_n^2 = 0$$
(42)

following from the fact that the transition amplitudes decrease faster than $1/\vec{q}^2$.

It is natural to consider the simplest model of saturation containing only two lowest-lying 1^- states with appropriate isospin quantum numbers. For example, for $I=1$ we use $\rho(770)$ and $\rho(1450)$. With fixed normalization at $q^2=0$ and the constraint (42) this model predicts the value of the integral in terms of the two masses. It is worth noting that such a model would obviously lead to equal expectation values of the operators in B and Λ_b. Imposing an additional constraint from the slope of the form factors would allow one to fix all residues in a three-pole model as well, which can be hoped to yield a more accurate estimate.

A word of reservation is in order at this point. Such a saturation of the nucleon form factors by two lowest t-channel resonances is known to provide a good approximation for moderate q^2 where the experimental form factors are described by the double-pole expressions. There is no general theoretical justification for such a coincidence, and more resonances are expected to play a role for larger q^2. In particular, at $-q^2>1$ GeV^2 the form factor can decrease faster. Because of the phase space factor the role of the domain of large q^2 is enhanced. The contribution of higher states, while affecting a little the formfactors near $q^2=0$, still can significantly change the integral (41). We will return to this point later.

It is clear that since the asymptotics of the amplitudes has an odd power of $1/|\vec{q}|$, their representation by a finite number of the t-channel resonances is not possible. The true spectrum of the t-channel states must extend to arbitrary high masses. It applies even if there were no typical for QCD log-like dependence of the asymptotics $1/|\vec{q}|^3$. It does not affect our estimates since we evaluate the operators in the effective theory where the high-momentum component of the hadrons is peeled off.

Addressing the color-straight operators containing the axial light-quark current (which does not vanish in B mesons) we note that its matrix elements are generally described by two form factors, just as in the well-known case of spin-$\frac{1}{2}$ fermions. These are analogues of the axial-charge and weak magnetism terms. Spontaneous breaking of the chiral symmetry modifies the value of the axial-charge form factor from its symmetric limit of 1 at $q^2=0$. Nevertheless, for the isovector current, its conservation $\partial_\mu J_{\mu 5}(x)=0$ in the chiral limit leads to a relation between the form factors, so that only one, the axial-charge form factor is independent, as in the case of the vector current. At $q^2=0$ this relation equates the axial-charge form factor to the $B^*B\pi$ coupling g (the heavy-quark analogue of the Goldberger-Treiman relation). Given the value of g, therefore, one can evaluate the expectation value exactly as outlined for the vector currents.

For the isosinglet axial current, one has to take into consideration the anomalous term, the topological charge density Q:

$$\partial_\mu J_{\mu 5}^{(0)}(x)=2i\bar{q}\hat{m}_q\gamma_5 q(x)+n_f Q(x),$$

$$Q(x)=\frac{\alpha_s}{4\pi}\mathrm{Tr}G_{\alpha\beta}\widetilde{G}^{\alpha\beta}(x) \tag{43}$$

with $\hat{m}_q$ the light quark mass matrix. The matrix elements of Q over the B meson states are not known, and the above relation appears to be less constraining. In the large-N_c limit the difference between singlet and nonsinglet formfactors is expected to disappear; however, the practical validity of this approximation for the anomalous term is questionable. These problems are addressed in the next section.

Let us briefly mention the case of the scalar current. Although the corresponding formfactor is not fixed at $q^2=0$, its value for the valence quarks can be obtained from the $SU(3)$ mass splittings:

$$\frac{1}{2M_B}\langle B^+|\bar{u}u(0)|B^+\rangle\simeq\frac{M_{B_s}-M_B}{m_s}\simeq 0.7,$$

$$\frac{1}{2M_{\Lambda_b}}\langle\Lambda_b|\bar{u}u(0)|\Lambda_b\rangle\simeq\frac{M_{\Xi_b}-M_{\Lambda_b}}{m_s}\simeq 1.4. \tag{44}$$

This estimate is obtained with the help of the Zweig rule in a similar way as done in [15] to extract the value of the nucleon σ-term from the $SU(3)$ splittings in the baryon octet. We neglected here the light quark masses $m_{u,d}$ and took

$m_s(1\ \mathrm{GeV})\simeq 130\ \mathrm{MeV}$. The mass of the baryon Ξ_b has not been yet measured; the above estimate used the prediction $M_{\Xi_b}=5805.7\pm 8.1\ \mathrm{MeV}$ [16]. The normalization point dependence of m_s in these relations reproduces the dependence of the scalar current.

In what follows we will apply the described strategy to the evaluation of a few expectation values of operators with the vector and axial light quark currents. First, however, we make a few qualitative observations.

In the case of the (valence) vector current we have $F(0)=1$. It is usually believed that $|F(q^2)|<1$ at spacelike q. Let us further assume that $F(q^2)$ is small enough above a certain scale μ, so that we can neglect it there:

$$F(q^2)\simeq 0\quad\text{at}\quad -q^2>\mu^2. \tag{45}$$

Then we get an upper bound

$$\left|\frac{1}{2M_B}\langle H_b|\bar{b}\gamma_\mu b\bar{q}\gamma_\mu q|H_b\rangle\right|<\frac{\mu^3}{6\pi^2}=0.017\ \mathrm{GeV}^3 \tag{46}$$

for $\mu=1\ \mathrm{GeV}$. This bound is of the type discussed in [3]; the numerical coefficient coincides with the one given there.

A possible justification for the assumption, Eq. (45), can be given as follows. In the effective theory with the normalization point μ the momenta of fields exceeding μ are absent, whether or not the full theory yields a logarithmic "tail" at large momenta. For example, it is not possible to exchange a gluon with momentum $|\vec{q}|>\mu$ in such a theory. The exact shape of the formfactor would depend on the concrete realization of the effective theory. The amplitude may not vanish exactly due to multiple gluon exchanges with $|\vec{q}|<\mu$, however would then decrease exponentially.

A literal steplike formfactor saturating the bound (46) is clearly unrealistic. Therefore, we can assume instead that

$$|F(q^2)|<\mathrm{e}^{-\vec{q}^2/\mu^2}, \tag{47}$$

which results in

$$\left|\frac{1}{2M_B}\langle H_b|\bar{b}\gamma_\mu b\bar{q}\gamma_\mu q|H_b\rangle\right|<\frac{\mu^3}{8\pi^{3/2}}=0.022\ \mathrm{GeV}^3 \tag{48}$$

with the same value for μ as in Eq. (46). Moreover, the exponential ansatz $\mathrm{e}^{-\vec{q}^2/\mu^2}$ for the formfactor with μ^2 adjusted to reproduce the "charge" radius, seems a reasonable model for the possible behavior of the valence form factor of purely soft degrees of freedom. As expected, in this model the momentum integrals of the formfactors are noticeably smaller than in the two-pole ansatz with the same slope at $q^2=0$:

$$\frac{1}{2M_B}\langle H_b|\bar{b}\gamma_\mu b\bar{q}\gamma_\mu q|H_b\rangle=F(0)\frac{1}{8\pi^{3/2}}\left(\frac{M_1^2 M_2^2}{M_1^2+M_2^2}\right)^{3/2}, \tag{49}$$

$$\mu^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}.$$

The above discussed bounds rely on the assumption that $|F(q^2)| < 1$. It always holds in the nonrelativistic QM and usually is taken for granted. However, we do not know a general rigorous proof in QCD. Nevertheless, there are some indirect arguments that it must hold, which we outline here. For the isovector current one can employ the equal-time commutation relation (J_μ^a can be both the vector $V_\mu^a = \bar{q}\gamma_\mu \frac{1}{2}\tau^a q$ or the axial current $A_\mu^a = \bar{q}\gamma_\mu \gamma_5 \frac{1}{2}\tau^a q$)

$$[J_0^+(\vec{x}), J_0^-(\vec{y})] = \delta^3(\vec{x} - \vec{y}) 2 V_0^3(\vec{x}) \tag{50}$$

to represent $1 - |F(q^2)|^2$ at $q^2 < 0$ as a difference of two sums of the distinct transition probabilities:

$$|F(\vec{q}^2)|^2 = 1 - \left(\sum_n |F_n^+|^2 - \sum_m |F_m^-|^2 \right). \tag{51}$$

Here $|F_n^+|^2, |F_m^-|^2$ schematically denote the transition probabilities in, say, B^- meson induced by the currents $\bar{u}\gamma_0 d$ and $\bar{d}\gamma_0 u$ with the momentum transfer $\vec{q}$, respectively (and similarly for Λ_b). In the second sum only the states with $I = \frac{3}{2}$ contribute. Since there are no valence $\bar{d}$ quarks in B^-, in the large-N_c limit the last term with the wrong sign would vanish. Additionally, in this limit the isoscalar meson form factor is expected to coincide with the isovector one. Therefore, the large-N_c arguments allow to establish such a QM bound for all form factors of interest.

The situation seems different for the axial (pseudoscalar) formfactors. They do not have a natural normalization at small momentum. Moreover, the amplitudes generally have an enhancement due to the pion pole (the effect absent in nonrelativistic QM). However, the domain $\vec{q}^2 \sim m_\pi^2$ yields a very small contribution to the integral [see, e.g., Eq. (46)]. The significant contribution can originate only from momenta $\gtrsim 1$ GeV where one expects the effects of chiral symmetry breaking to become insignificant. The equal-time commutation relation (50) can still be used to derive a sum rule of the type (51) for the matrix elements of the axial isovector current. Its explicit form is similar to Eq. (51) and reads

$$|G_1(\vec{q}^2)|^2 = 1 - \left(\sum_n |G_n^+|^2 - \sum_m |G_m^-|^2 \right) \tag{52}$$

with $G_1(q^2)$ defined below in Eq. (68) and $|G_n^+|^2, |G_m^-|^2$ are the analogues of the F_n amplitudes for transitions induced by the axial vector current acting on a B meson. At $q^2 = 0$ this sum rule is just the familiar Adler-Weisberger sum rule and the amplitudes G_n are related to pion couplings between the ground and excited states. The explicit form of these sum rules for heavy mesons and baryons can be found in [14,17]. Thus, we expect the type of bounds (46), (48) to hold for the axial vector current expectation values as well.

IV. NUMERICAL ESTIMATES

In this section we estimate the expectation values of the color-straight four-fermion operators relevant for the lifetimes of b-flavored hadrons. The light quark fields are left-handed; the Penguin diagrams bring in the right-handed fields as well. Nevertheless, the chiral structure of the currents admits only the vector or axial vector light quark currents. Since the coefficient functions can include the momentum of the decaying b hadron (its velocity), the timelike and spacelike components enter, in general, with different weights; the three-dimensional rotation invariance is still preserved. Finally, since the forward matrix elements are considered, only the parity-conserving (three-dimensional) scalar expectation values survive. Therefore, we need to consider the operators

$$O_V = (\bar{b}b)(\bar{q}\gamma_0 q) = (\bar{b}\gamma_\mu b)(\bar{q}\gamma_\mu q),$$

$$O_A = -(\bar{b}\vec{\sigma}b)(\bar{q}\vec{\gamma}\gamma_5 q) = (\bar{b}\gamma_\mu \gamma_5 b)(\bar{q}\gamma_\mu \gamma_5 q). \tag{53}$$

As was mentioned, the expectation value of the operator O_A in Λ_b vanishes.

A. Vector current

We first consider the case of B mesons. Assuming only isospin symmetry, we define the isovector and isoscalar four-quark matrix elements by

$$\frac{1}{2M_B}\langle B_i | \bar{b}\gamma_\mu b \bar{q}\tau^a \gamma_\mu q | B_j \rangle = V_3 \tau_{ij}^a$$

$$\frac{1}{2M_B}\langle B_i | \bar{b}\gamma_\mu b \sum_{q=u,d} \bar{q}\gamma_\mu q | B_j \rangle = V_1 \delta_{ij}. \tag{54}$$

The indices i, j label the respective state in the isospin doublet $i = (\bar{d}, -\bar{u})$. Accordingly, we introduce the isospin-triplet and singlet vector form factors

$$\frac{1}{2M_B}\langle B_i(\vec{q})| \bar{q}\tau^a \gamma_\mu q | B_j(0) \rangle = v_\mu \mathcal{F}_3(q^2)\tau_{ij}^a$$

$$\frac{1}{2M_B}\langle B_i(\vec{q})| \sum_{q=u,d} \bar{q}\gamma_\mu q | B_j(0) \rangle = -v_\mu \mathcal{F}_1(q^2)\delta_{ij}, \tag{55}$$

with the normalization conditions $\mathcal{F}_1(0) = \mathcal{F}_3(0) = 1$. Using the two-pole ansatz saturated by $\rho(770)$ and $\rho(1450)$ for the nonsinglet current $\mathcal{F}_3$, we get from Eqs. (40)–(42)

$$V_3 \simeq \frac{1}{4\pi}\frac{M_1^2 M_2^2}{M_1 + M_2} \simeq 0.045 \text{ GeV}^3. \tag{56}$$

It is natural to saturate the $I = 0$ form factor $\mathcal{F}_1$ by the states $\omega(782)$ and $\omega(1420)$. It then leads to almost the same numerical estimate for V_1 as for V_3. The reason is obviously an almost exact degeneracy of the vector states in the isovector and isosinglet channels. Although it perfectly fits the large-

N_c picture, we cannot be sure what is the actual accuracy of such a conclusion. Nevertheless, in view of such a suppression of the difference, we will take the two different combinations of the expectation values which actually parametrize the valence and nonvalence contributions:

$$\frac{V_1 - V_3}{2} = \frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{u} \gamma_\mu u | B^- \rangle \simeq -0.044 \ \text{GeV}^3 \tag{57}$$

$$\frac{V_1 + V_3}{2} = \frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{d} \gamma_\mu d | B^- \rangle \simeq \mathcal{O}(10^{-4} \ \text{GeV}^3). $$

The last number, clearly, is at best an order of magnitude estimate.

We can try to estimate the violation of the $SU(3)$ flavor symmetry considering the expectation value of $\bar{b} \gamma_\mu b \bar{s} \gamma_\mu s$ in B_s mesons. For this we saturate the form factor with the vector $\bar{s}s$ states $\phi(1020)$ and $\phi(1680)$, which corresponds to the ''ideal'' mixing in the $\omega - \phi$ system [18]. In this case we would get

$$\frac{1}{2M_B} \langle B_s | \bar{b} \gamma_\mu b \bar{s} \gamma_\mu s | B_s \rangle \simeq -0.085 \ \text{GeV}^3, \tag{58}$$

i.e., almost twice larger than the first estimate (57).

A closer look reveals, however, that the above expectation values are saturated at rather high momenta. Half of the ''valence'' value comes from $|\vec{q}| > 1.5$ GeV, and from even higher momenta in B_s. For this reason these estimates exceed the bounds (46),(48) discussed in the previous section, for a reasonable scale $\mu \simeq 1$ GeV. Adopting the exponential ansatz for the form factor we get

$$\frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{u} \gamma_\mu u | B^- \rangle \simeq -0.007 \ \text{GeV}^3$$

$$\frac{1}{2M_B} \langle B_s | \bar{b} \gamma_\mu b \bar{s} \gamma_\mu s | B_s \rangle \simeq -0.015 \ \text{GeV}^3. \tag{59}$$

A somewhat unexpected result of these simple estimates is the apparently large amount of $SU(3)$ breaking in Eqs. (57),(58). While it is not clear to what extent this is an artifact of our use of the simple two-pole ansatz for the form factors over a wide domain of q^2, it is worth noting that a simple mechanism exists which could account for it. It is well-known that the isovector charge radius of a hadron diverges in the chiral limit [19]. This indicates that the contribution of the low-momentum region in the integral over the form factor (38) is more suppressed in nonstrange B mesons compared to the B_s case. Since the two-pole model does not capture the origin of this phenomenon (the contribution of the two-body $\pi\pi$ intermediate state in the t-channel), it is conceivable that the magnitude of $SU(3)$ violation in the matrix elements does exceed a few percent.

It is interesting to compare the above estimates with the evaluation based on vacuum factorization. Both types of estimates have the same $\mathcal{O}(N_c^0)$ scaling in N_c. However, the vacuum contribution for the color-straight operators is $\mathcal{O}(N_c^0)$, similar to other meson states. This is in contrast to the case of the t-channel octet operators where the vacuum state is N_c-enhanced. Therefore, the factorization estimate is not expected to give an accurate result. For the valence expectation value one has

$$\frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{u} \gamma_\mu u | B^- \rangle_{\text{factor}} = -\frac{1}{4N_c} \widetilde{f}_B^2(\mu) M_B \tag{60}$$

(the nonvalence value vanishes). Here $\widetilde{f}_B$ denotes the annihilation constant of B for the $\bar{b} \gamma_\alpha \gamma_5 u$ current normalized at a low point μ where factorization must be applied [7,8], in contrast to the physical f_B defined for the current normalized at $\mu \gg m_b$:

$$\widetilde{f}_B(\mu) \simeq f_B \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right]^{-2/\beta_0}. \tag{61}$$

The physical value of f_B lies, probably, around 160 MeV. However, to the leading order in $1/m_b$ we work in, it is more consistent to use the asymptotic value which differs from the physical one by $1/m_b$ and nonlogarithmic perturbative corrections. These decrease the physical value of f_B by about 20% [20], so that we use in the right-hand side (RHS) of Eq. (61) $f_B^{\text{static}} = 200$ MeV. Therefore, we adopt $\widetilde{f}_B = 160$ MeV for $\alpha_s(\mu) = 1$, yielding

$$\frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{u} \gamma_\mu u | B^- \rangle_{\text{factor}} \simeq -0.011 \ \text{GeV}^3, \tag{62}$$

which is significantly lower than Eq. (57).

The fact that the corrections to factorization can be significant, is expected. Unfortunately, there are good reasons to question the accuracy of the alternative estimate (57) either, and a too large $SU(3)$ breaking is another indication. We think that it is justified to consider the estimate (57) for the valence expectation value rather as an upper bound, while the number obtained in the exponential ansatz a reasonable lower bound. A conservative estimate then is

$$\frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{u} \gamma_\mu u | B^- \rangle = -(0.025 \pm 0.015) \ \text{GeV}^3$$

$$\frac{1}{2M_B} \langle B^- | \bar{b} \gamma_\mu b \bar{d} \gamma_\mu d | B^- \rangle \approx \mathcal{O}(5 \times 10^{-4} \ \text{GeV}^3). \tag{63}$$

Similar estimates can be adopted for strange quarks in B_s.

Next we turn to baryons. Under the light flavor $SU(3)$ group the Λ_b and Ξ_b states transform as an antitriplet $T_i = (\Xi_b^d, -\Xi_b^u, \Lambda_b)$. In the limit of $SU(3)$ symmetry there are only two independent form factors, which can be defined as

$$\frac{1}{2M_{\Lambda_b}}\langle T_i(\vec{q})|\bar{q}\lambda^a\gamma_\mu q|T_j(0)\rangle$$

$$=v_\mu \mathcal{F}_8^\Lambda(q^2)\lambda_{ji}^a\bar{u}(v,s')u(v,s)$$

$$\frac{1}{2M_{\Lambda_b}}\langle T_i(\vec{q})|\sum_{q=u,d,s}\bar{q}\gamma_\mu q|T_j(0)\rangle$$

$$=v_\mu \mathcal{F}_1^\Lambda(q^2)\delta_{ij}\bar{u}(v,s')u(v,s). \qquad (64)$$

The normalization at $q^2=0$ is $\mathcal{F}_1^\Lambda(0)=2$, $\mathcal{F}_8^\Lambda(0)=-1$. Using a similar model for the formfactors as in the meson case we get the same expectation values (up to the sign) for the valence matrix elements, and strongly suppressed nonvalence contributions:

$$\frac{1}{2M_{\Lambda_b}}\langle \Lambda_b|\bar{b}\gamma_\mu b\bar{u}\gamma_\mu u|\Lambda_b\rangle$$

$$=\frac{1}{2M_{\Lambda_b}}\langle \Lambda_b|\bar{b}\gamma_\mu b\bar{d}\gamma_\mu d|\Lambda_b\rangle$$

$$\simeq \begin{cases} 0.007 \ \text{GeV}^3 & \text{(exponential)} \\ 0.045 \ \text{GeV}^3 & \text{(two-pole)} \end{cases}. \qquad (65)$$

For the same reasons as before it is natural to consider the two-pole value as an upper bound. The expectation values of the nonstrange operators in the Ξ_b states emerge the same as in (65), whereas $\langle\Xi_b|\bar{b}\gamma_\mu b\bar{s}\gamma_\mu s|\Xi_b\rangle$ again literally appears twice larger than in the $SU(3)$ limit. As discussed in the B meson case, such a large symmetry violation can be suspected to be, at least partially, an artifact of the two-pole model.

It is worth noting that in the case of heavy baryons the light quark form factors have anomalous Landau thresholds associated with the $N\bar{N}B$ triangle diagrams. It is well known that it is such singularities that determine the low-momentum behavior of the form factors and, in particular, the large charge radius of weakly-bound states like deuteron [21]. For the Λ_b form factor the anomalous singularity starts at

$$t_{\text{thr}}=4M_N^2\left(1-\frac{(M_{\Lambda_b}^2-M_N^2-M_B^2)^2}{4M_B^2M_N^2}\right)=3.2 \ \text{GeV}^2. \qquad (66)$$

In this system, however, the corresponding mass lies higher than the states we use to saturate the form factors. Moreover, there is no reason to expect the residues to be significant (for example, they are $1/N_c$ suppressed). Therefore, we believe that these singularities do not play a role in the expectation values we study. In any case, a refined estimate will be possible with a better knowledge of the form factors, say using the determination of the slope based on an application of the Cabibbo-Radicati sum rule to the radiative decays of excited baryons.

B. Operators with axial current

The expectation value of the operator $\bar{b}\gamma_\mu\gamma_5 b\bar{q}\gamma_\mu\gamma_5 q$ vanishes in the Λ_b baryon family, and we consider it only for B mesons employing the relation Eq. (22). In this case

$$(\vec{S}_b\vec{\epsilon})|B(\vec{q})\rangle=|B^*(\vec{q},\vec{\epsilon})\rangle, \quad \vec{S}_b=\int \text{d}^3x\,\bar{b}\vec{\gamma}\gamma_5 b(x). \qquad (67)$$

Since the light degrees of freedom carry spin $\frac{1}{2}$, the axial current is parametrized by two form factors; the third possible structure has wrong T parity and vanishes. This is an exact analogue of the absence of the second-class currents in β-decays of light baryons. Thus one has

$$\frac{1}{2M_B}\langle B_i^*(\vec{q},\vec{\epsilon})|\sum_{q=u,d,s}\bar{q}\gamma_\mu\gamma_5 q(0)|B_j(0)\rangle$$

$$=\{\epsilon_\mu^* G_1^{(0)}(q^2)-(\epsilon^*q)q_\mu G_0^{(0)}(q^2)\}\delta_{ij},$$

$$\frac{1}{2M_B}\langle B_i^*(\vec{q},\vec{\epsilon})|\bar{q}\lambda^a\gamma_\mu\gamma_5 q(0)|B_j(0)\rangle$$

$$=\{\epsilon_\mu^* G_1(q^2)-(\epsilon^*q)q_\mu G_0(q^2)\}\lambda_{ji}^a. \qquad (68)$$

Absence of the structure $(\epsilon^*q)v_\mu$ is easy to show explicitly (note that in any case the timelike component of the axial current does not enter the four-fermion operators). Using Eq. (67) and the fact that $\vec{S}_b$ commutes with all light-quark field operators, we get an equality

$$\langle B^*(\vec{q},\vec{\epsilon})|J_{\mu5}(0)|B(0)\rangle^*=\langle B(\vec{0})|J_{\mu5}(0)|B^*(\vec{q},\vec{\epsilon})\rangle$$

$$=\langle B^*(\vec{0},\vec{\epsilon^*})|J_{\mu5}(0)|B(\vec{q})\rangle.$$

Inserting here the form factor decompositions (68) for these matrix elements and taking into account the fact that G_i are real from T invariance, one finds that the structure $(\epsilon q)v_\mu$ appears with opposite signs on the two sides of the equality. Hence its coefficient must vanish.

We thus get

$$\frac{1}{2M_B}\langle B^-|\bar{b}\gamma_\mu\gamma_5 b(\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d)|B^-\rangle$$

$$=-\frac{1}{4\pi^2}\int_0^\infty \text{d}t\,\sqrt{t}(3G_1(-t)+tG_0(-t)) \qquad (69)$$

and a similar expression for the singlet matrix elements.

In the chiral limit, which will be assumed in what follows, the isovector form factor G_1 at $q^2=0$ is related to the $BB^*\pi$ coupling:

$$G_1(0)=g. \qquad (70)$$

The nonrelativistic quark model predicts $g=-0.75$. However, QCD sum rules estimates yield lower values [22–25]. The most recent analyses predict $g=-0.3$ [23,24] which is consistent with the existing experimental bounds $g^2=0.09$ -0.5 [26]. Moreover, the equation of motion $\partial_\mu J_{\mu5}=0$

leads to $q^2 G_0(q^2) = G_1(q^2)$ at all q^2, therefore for the non-singlet expectation value Eq. (69) takes the form

$$\frac{1}{2M_B}\langle B^-|\bar{b}\gamma_\mu\gamma_5 b(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)|B^-\rangle$$

$$= -\frac{1}{2\pi^2}\int_0^\infty dt\, \sqrt{t} G_1(-t). \tag{71}$$

The only nonvanishing contribution to G_0 from the pseudoscalar states in the isovector channel comes from the massless pion. The $J^{PC}=1^{++}$ states contribute to both G_1 and G_0:

$$G_1(t) = \sum_n \frac{g_n M_n^2}{M_n^2 - t}$$

$$G_0(t) = -\frac{g}{m_\pi^2 - t} + \sum_n \frac{g_n}{M_n^2 - t} \tag{72}$$

with the condition $\Sigma_n g_n = g$ replacing the zero momentum-transfer normalization of the vector formfactor. A faster than $1/q^2$ fall-off of the transition amplitude requires additionally

$$\sum_n g_n M_n^2 = 0, \tag{73}$$

which is analogous to the second constraint in Eq. (42) for the vector current.

In the numerical estimates for the isotriplet current we will consider both a two-pole ansatz for $G_1(q^2)$ and the exponential ansatz

$$G_1(q^2) = g e^{-\vec{q}^2/\mu^2},$$

$$G_0(q^2) = g \frac{e^{-\vec{q}^2/\mu^2}}{q^2} \quad \text{with} \quad \mu^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \tag{74}$$

Such a choice of μ ensures that the two ansätze have the same behavior at small q^2.

In the $I,J^{PC}=1,1^{++}$ channel only the lowest-lying state $a_1(1260)$ has been observed. For the numerical estimates we will need also the mass of its first radial excitation a_1'. This has been extracted in [27] from an analysis of the Weinberg sum rules. The value obtained in [27] for the mass of the a_1' resonance is 1869 MeV, which is what we will use in our estimates.

First, with the two-pole ansatz we obtain

$$\frac{1}{2M_B}\langle B^-|\bar{b}\gamma_\mu\gamma_5 b(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)|B^-\rangle$$

$$= -\frac{g}{2\pi}\frac{M_1^2 M_2^2}{M_1 + M_2} \approx 0.084 \text{ GeV}^3. \tag{75}$$

As explained above, we have adopted in this estimate the value $g = -0.3$ [23,24]. For the exponential form factor one obtains a smaller value

$$\frac{1}{2M_B}\langle B^-|\bar{b}\gamma_\mu\gamma_5 b(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)|B^-\rangle$$

$$= -\frac{g}{4\pi^{3/2}}\left(\frac{M_1^2 M_2^2}{M_1^2 + M_2^2}\right)^{3/2}$$

$$\approx 0.015 \text{ GeV}^3. \tag{76}$$

In a completely analogous way one can estimate the matrix element of the $I=0$ octet axial current with the flavor content of η. With the mass of the state $f_1(1285)$ close to mass of a_1 and assuming a similar degeneracy for the second excitation we do not get appreciable $SU(3)$ violation and, therefore, obtain for

$$\frac{1}{2M_B}\langle B^-|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)|B^-\rangle$$

the same value as in Eqs. (76) and (75).

In the case of the singlet axial vector current we need to account for the presence of the anomalous term in its divergence,

$$\partial_\mu J_{\mu 5}^{(0)}(x) = n_f Q(x), \quad Q(x) = \frac{\alpha_s}{4\pi}G\tilde{G}(x).$$

For simplicity, we will assume the exact $SU(3)$ chiral limit. The value of the isosinglet axial form factor at small q^2 is given by the matrix element of the anomalous divergence $Q(x)$:

$$\frac{1}{2M_B}\langle B^*(\epsilon,\vec{q})|Q(0)|B(0)\rangle = \frac{i}{n_f}G_1^{(0)}(0)(\epsilon^*q) + \mathcal{O}(q^2). \tag{77}$$

We used here the fact that there are no massless particles in the singlet channel and consequently $G_0^{(0)}(0)$ is finite. The contribution of the pseudoscalar states to the form factor $G_0^{(0)}$ does not vanish and is determined by their coupling to $Q(x), \langle n|Q(0)|0\rangle$. Similarly, the $G_1^{(0)}$ and $G_0^{(0)}$ form factors at arbitrary q^2 are not directly related to each other, but the difference $i(G_1^{(0)} - q^2 G_0^{(0)})$ equals to the matrix element of $n_f Q(x)$. Very little is known directly about these flavor-singlet expectation values or $B^*B\eta^{(\prime)}$ coupling.

Nevertheless, for estimates one usually employs an approximation in which the matrix elements of $Q(x)$ are saturated by the η' pole. Moreover, the couplings of the whole nonet of the pseudoscalar mesons π, K, η, η' are assumed $SU(3)$-symmetric. This assumption is incorporated in the simple σ-models proved to be successful in describing the properties of light hadrons. This model [28,29] naively has an $U(3) \times U(3)$ chiral symmetry; the $U(1)$ problem is solved by adding the anomalous term with $Q(x)$ and assuming the nonvanishing (in the quenched approximation, that is,

in QCD without light flavors) value of the zero-momentum correlator of the topological charge densities $Q(x)$

$$\lambda^4 = \int d^4x \langle 0|iT\{Q(x)Q(0)\}|0\rangle_{n_f=0} \qquad (78)$$

which leads, basically, to the nonzero anomalous mass of η' meson $m_{\eta'}^2 = \lambda^4/f_\pi^2$. Adopting such a model, we also have $G_1^{(0)}(0) \simeq G_1(0)$.

A possible justification for such a picture lies in the large-N_c approximation. However, in this limit $m_{\eta'}^2 \propto 1/N_c$ and the anomalous $U(1)$ symmetry effectively restores, which seems not be close to actual world where the anomalous mass of η' is numerically large and the octet-singlet mixing in pseudoscalars is small. It is probable that there exists a deeper dynamic reason explaining the practical validity of such an approximation.

The model with a single η' state in the pseudoscalar channel which merely shifts the pole in the nonsinglet amplitudes from $q^2=0$ to $m_{\eta'}^2$, while describing reasonably well the low-q^2 matrix elements of the topological charge density, leads to their too mild suppression at large q^2. In reality they are expected to decrease very fast above a typical momentum scale of the nonperturbative vacuum configurations. In order to mimic this behavior, we have to employ at least two pseudoscalar states saturating the correlators of Q, and we take the state η' with a mass of $M_{\eta'} = 1295$ MeV as the second pole. One expects an $J^{PC} = 0^{-+} SU(3)$ singlet in this mass region, accompanying the observed octet of pseudoscalars containing $\pi(1300), \eta(1295)$. In reality, the wide ''gluonium'' states can give a significant contribution. Probably, an exponential ansatz is a better approximation here.

In principle, the spectrum of the axial-vector singlet states has no direct relation to the anomaly and the $U(1)$ problem. Hence we take for the corresponding masses the experimental values, namely $f_1(1285)$ and its first radial excitation f_1', neglecting their mixing with the octet states. $f_1(1285)$ lies close to the isotriplet state $a_1(1260)$, indicating smallness of the annihilation effects. Therefore we will take for the mass of the first radial excitation $M_{f_1'} = M_{a_1'} \simeq 1870$ MeV in the numerical estimates below. In the two-pole model we have

$$G_1^{(0)}(q^2) = G_1^{(0)}(0) \frac{M_{f_1}^2 M_{f_1'}^2}{(M_{f_1}^2 - q^2)(M_{f_1'}^2 - q^2)}$$

$$G_1^{(0)}(q^2) - q^2 G_0^{(0)}(q^2)$$

$$= G_1^{(0)}(0) \frac{M_{\eta'}^2 M_{\eta'(1295)}^2}{(M_{\eta'}^2 - q^2)(M_{\eta'(1295)}^2 - q^2)}. \qquad (79)$$

The last equation replaces the second of Eqs. (72).

As a result, the difference in the estimates compared to the isotriplet current lies basically in the anomalous term, and is not too significant. Numerically, we get for the two-pole ansatz

$$\frac{1}{2M_B} \langle B^-|\bar{b}\gamma_\mu\gamma_5 b \sum_{q=u,d,s} \bar{q}\gamma_\mu\gamma_5 q|B^-\rangle$$

$$= -\frac{G_1^{(0)}(0)}{2\pi} \left(\frac{M_{f_1}^2 M_{f_1'}^2}{M_{f_1} + M_{f_1'}} + \frac{1}{2} \frac{M_{\eta'}^2 M_{\eta'(1295)}^2}{M_{\eta'} + M_{\eta'(1295)}} \right)$$

$$\simeq -(0.29 + 0.054)G_1^{(0)}(0) \text{ GeV}^3$$

$$= 0.1 \text{ GeV}^3 \quad \text{at } G_1^{(0)}(0) = -0.3. \qquad (80)$$

In the numerical estimate we used the equality $G_1^{(0)}(0) = G_1(0) = g$ which holds in the large-N_c limit, as discussed above.

We present also a calculation of the matrix element (80) employing the exponential ansatz. This is constructed in the same way as for the axial charge form factor. For the topological charge form factor we use an exponential normalized at $q^2 = 0$ by the same value $G_1^{(0)}(0)$ and vary the slope parameter μ_Q^2 from $\mu_Q^2 = m_{\eta'}^2 = 0.92$ GeV^2 [corresponding to the pole dominance by η' alone) to 0.59 GeV^2 [corresponding to the two-pole model, see Eq. (49)]. This yields the following numerical estimate:

$$\frac{1}{2M_B} \langle B^-|\bar{b}\gamma_\mu\gamma_5 b \sum_{q=u,d,s} \bar{q}\gamma_\mu\gamma_5 q|B^-\rangle$$

$$\simeq -\frac{G_1^{(0)}(0)}{4\pi^{3/2}} \left(\left[\frac{M_{f_1}^2 M_{f_1'}^2}{M_{f_1}^2 + M_{f_1'}^2} \right]^{3/2} + \frac{1}{2}\mu_Q^3 \right)$$

$$\simeq -(0.053 + (0.010 \text{ to } 0.020))G_1^{(0)}(0) \text{ GeV}^3$$

$$\simeq (0.02 \text{ to } 0.023) \text{ GeV}^3 \qquad (81)$$

with the same value for $G_1^{(0)}(0)$ as before. One could try to estimate the effects of $SU(3)$ breaking by accounting for the known shifts in masses and mixing. We think, however, that such models are too crude to capture correctly details of $SU(3)$ violation, and we do not attempt it here.

Combining the above results for the octet and singlet expectation values, we get the following estimates for the valence and nonvalence axial expectation values:

$$\frac{1}{2M_B} \langle B^-|\bar{b}\gamma_\mu\gamma_5 b \bar{u}\gamma_\mu\gamma_5 u|B^-\rangle \simeq 0.018 \text{ GeV}^3 \qquad (82)$$

$$0.09 \text{ GeV}^3$$

$$\frac{1}{2M_B} \langle B^-|\bar{b}\gamma_\mu\gamma_5 b \bar{d}\gamma_\mu\gamma_5 d|B^-\rangle \simeq 0.002 \text{ GeV}^3 \qquad (83)$$

$$0.007 \text{ GeV}^3$$

where the upper (lower) value corresponds to the exponential (two-pole) form factor model.

We note that the effect of the axial anomaly can be numerically important, although it is formally of order $1/N_c$. In the approximations considered here, it makes a contribution

of about 15% of the total singlet expectation values (80) and (81), respectively, and it can dominate the nonvalence matrix elements.

Finally, we quote also the factorization approximation estimate for the valence expectation value of the axial vector current. We obtain

$$\frac{1}{2M_B}\langle B^-|\bar{b}\gamma_\mu\gamma_5 b\bar{u}\gamma_\mu\gamma_5 u|B^-\rangle_{\text{factor}}$$

$$=\frac{3}{4N_c}\tilde{f}_B^2(\mu)M_B\simeq 0.034 \text{ GeV}^3. \quad (84)$$

Just as for the color-straight operators containing the vector current, we do not expect the factorization approximation to be accurate. However, it is interesting that the factorization value for the axial operators is less suppressed compared to the case of the vector current, and appears to be closer to the estimates given above.

C. Estimates from the fourth sum rule

One of the color-octet operators, the flavor-singlet vector four-fermion operator can be estimated in an alternative way. This operator $\tilde{O}_D$

$$\tilde{O}_D=\sum_{q=u,d,s}(\bar{b}\gamma_\mu t^a b)(\bar{q}\gamma_\mu t^a q) \quad (85)$$

is the QCD generalization of the Darwin term in atomic physics [31,5,32]:

$$\frac{1}{2M_{H_b}}\langle H_b|2\pi\alpha_s\tilde{O}_D|H_b\rangle=-\frac{1}{2M_{H_b}}\langle H_b|O_D|H_b\rangle$$

$$=-(\rho_D^3)_{H_b}. \quad (86)$$

On the other hand, it determines the third moment of the small velocity (SV) structure function, the so-called fourth sum rule, and is related to quantities measurable in the semileptonic decays. For example, in B mesons this sum rule in the resonant approximation takes the form [32]

$$\frac{1}{3}\rho_D^3=E_{1/2}^3|\tau_{1/2}|^2+2E_{3/2}^3|\tau_{3/2}|^2+\cdots, \quad (87)$$

where τ_j are so-called "oscillator strengths" which determine the small velocity transition amplitudes into the excited p-wave states with spin of light degrees of freedom j. The excitation energies of these states with respect to the ground state s-wave mesons are denoted with E_j (for a recent discussion see review [33]).

It should be noted that the literal application of the fourth sum rule requires a specific regularization scheme for the operators. In view of the tentative nature of our estimates we neglect these subtleties here. Some of them were discussed in [5] and more recently in [34] and [35]. We only mention that the large negative logarithmic anomalous dimension of the Darwin operator [1]

$$(\alpha_s\tilde{O}_D)_{\mu'}\simeq\left(\frac{\alpha_s(\mu')}{\alpha_s(\mu)}\right)^{13/(2\beta_0)}(\alpha_s\tilde{O}_D)_\mu, \quad \beta_0=\frac{11}{3}N_c-\frac{2}{3}n_f \quad (88)$$

to a large extent offsets the apparent scale dependence in the evaluation of $\langle\tilde{O}_D\rangle$ via ρ_D^3 due to the μ-dependence of α_s (or, similarly, the scale dependence of the factorization estimate of ρ_D^3 in B mesons). According to the standard practice we use for our estimates the scale corresponding to $\alpha_s(\mu)=1$.

A recent discussion of the status of the sum rule evaluation of ρ_D^3 in B mesons can be found in the review [33], Sec. IV. The corresponding value is in a reasonable agreement with the factorization estimates [see Appendix C, Eq. (C7)].

Since the straightforward factorization cannot be used for baryons, we will apply the fourth sum rule to evaluate the operator $\tilde{O}_D$ in Λ_b. The fourth sum rule for it takes the form

$$\frac{1}{2M_{\Lambda_b}}\langle\Lambda_b|2\pi\alpha_s\tilde{O}_D|\Lambda_b\rangle=-3\sum_n E_n^3|\sigma^{(n)}|^2. \quad (89)$$

The states appearing in the RHS are orbital excitations of the Λ_b baryon with quantum numbers of the light degrees of freedom $s_l^{\pi_l}=1^-$. Their excitation energies are E_n and $\sigma^{(n)}$ are the corresponding oscillator strengths describing semileptonic decays of Λ_b to the analogous excitations of the Λ_c baryon; they are defined as in [36].

The first excited states appearing in this sum rule have been identified as the doublet of negative-parity baryons $\Lambda_{c_1}^+(2593)$ and $\Lambda_{c_1}^+(2625)$. Unfortunately no experimental information is available to date on the transition amplitude $\sigma^{(1)}$ governing the decays of Λ_b into both of them, although it will be ultimately measured.

The important piece of information would be the slope $\rho_{\Lambda_b}^2$ of the elastic IW function in Λ_b. This quantity is more accessible than $\sigma^{(1)}$ and will be measured in the near future at LEP. In the absence of the data we can use the second sum rule (Voloshin's "optical" sum rule) [37] for $\bar{\Lambda}_{\Lambda_b}=M_{\Lambda_b}-m_b\simeq M_{\Lambda_c}-m_c$. As discussed in [33] (for earlier application see also [32,38]), we can estimate ρ_D^3 using just the excitation energy of the first states. We simply take $M_{\Lambda_c^*}\simeq 2.615$ GeV and the first excitation energy $\Delta_1\simeq 330$ MeV. Assuming $\bar{\Lambda}_B\simeq 600$ MeV and, therefore, $\bar{\Lambda}_{\Lambda_b}\simeq 900$ MeV, we then have

$$(\rho_D^3)_{\Lambda_b}\approx\frac{3}{2}\Delta_1^2\bar{\Lambda}_{\Lambda_b}\approx 0.15 \text{ GeV}^3. \quad (90)$$

[A similar estimate $(\mu_\pi^2)_{\Lambda_b}\simeq\frac{3}{2}\Delta_1\bar{\Lambda}_{\Lambda_b}\simeq 0.45$ GeV^2 agrees well with the mass relations [39,12] for charm and beauty in the meson and baryon sectors.] We note in passing that, most probably, the large mass of the heavy baryon implying larger

TABLE I. Estimated valence expectation values in B and Λ_b, in GeV^3; factorized contributions assume $\tilde{f}_B = 160$ MeV.

	ω_V	ω_A	τ_V	τ_A	λ_u	λ'_u	$(\rho^3_D)_B$	$(\rho^3_D)_{\Lambda_b}$
Secs. IV A–IV B	-0.020	0.045			0.020			
Factorization	-0.011	0.034	-0.015	0.045			0.10	
4th sum rule			-0.028			0.018	0.18	0.15

$\bar{\Lambda}$ compared to B meson is due to larger slope $\rho^2_{\Lambda_b}$; the higher-dimension operators, therefore, can be even smaller than in mesons.

The sum of the expectation values for all three light flavors $2\lambda' + \lambda'_s$ is related to the expectation value of the Darwin operator [31]:

$$2\lambda' + \lambda'_s = \frac{3}{4\pi\alpha_s}(\rho^3_D)_{\Lambda_b}.$$

Hence, we estimate the $SU(3)$-singlet color-octet expectation value in Λ_b as

$$\lambda'_{u+d+s} \approx \frac{3}{4\pi}(\rho^3_D)_{\Lambda_b} \approx 0.036 \text{ GeV}^3 \qquad (91)$$

with uncertainty about 30–40%. The estimate of λ'_u can be obtained if we neglect the small contribution of the nonvalence strange quarks:

$$\lambda'_u = \lambda'_d \simeq \frac{3}{8\pi}(\rho^3_D)_{\Lambda_b} \simeq 0.018 \text{ GeV}^3. \qquad (92)$$

We note that we get a reasonable agreement with the quark model relation $\lambda' \approx \lambda$ between the straight and octet expectation values in Λ_b for our central estimates, Eq. (65). It is interesting that the corresponding value of the diquark density at origin appears close to our central estimate for mesons $-\omega_V$ (but larger than the alternative analogue of $|\Psi(0)|^2$ in mesons $\tilde{f}^2_B M_B/12$). It also exceeds the estimates obtained in the QCD sum rules [40] or quoted from bag models [41]; these analyses determined the combination $\frac{4}{3}\lambda' - \frac{1}{3}\lambda$ which generally emerged in the ball park of 0.004 GeV^3.

V. NONFACTORIZABLE PIECES IN THE MATRIX ELEMENTS OF THE FOUR-QUARK OPERATORS

As was mentioned earlier, there are four operators (for a given light quark flavor) determining the corrections to the mesons widths. These are color-straight O and color-octet T. Each of these can contain either timelike (vector O_V, T_V) or spacelike (axial O_A, T_A) components of light and heavy quark currents.

The color-octet expectation values τ in general can be estimated using vacuum factorization, since the operators T coincide with the operators colorless in the s-channel up to $1/N_c$ terms [see Eq. (33)]. For such operators vacuum factorization is expected to work up to $1/N_c$ corrections. This gives

$$\tau_V = -\frac{\tilde{f}^2_B(\mu)M_B}{8}\left(1 - \frac{1}{N^2_c}\right) \simeq -0.015 \text{ GeV}^3 \qquad (93)$$

$$\tau_A = \frac{3\tilde{f}^2_B(\mu)M_B}{8}\left(1 - \frac{1}{N^2_c}\right) \simeq 0.045 \text{ GeV}^3. \qquad (94)$$

It is interesting to note that the leading $1/N_c$ corrections to the factorization approximation can be estimated in a phenomenological approach. For this the expectation value of the color singlet O_{singl} in (33) is written as

$$\langle H_b|(b\bar{\Gamma}q)(q\bar{\Gamma}b)|H_b\rangle = \langle H_b|b\bar{\Gamma}q|0\rangle\langle0|q\bar{\Gamma}b|H_b\rangle$$
$$+ \sum_n \langle H_b|b\bar{\Gamma}q|n\rangle\langle n|q\bar{\Gamma}b|H_b\rangle. \qquad (95)$$

The leading corrections to the vacuum factorization approximation are of order 1 and come from one-particle intermediate states like $\pi(\eta,\eta'),\rho(\omega),a_1$ for B mesons, or light baryons for Λ_b. The corresponding transition amplitudes have been evaluated in the QCD sum rules [44] and lattice simulations with an accuracy sufficient for determining the scale of the effects. Alternatively, they can be approximately obtained from the corresponding decays of charmed particles.

Large-N_c arguments *per se* do not ensure that the vacuum factorization approximation works in the case of the color-straight operators, for the nonfactorizable contribution appears at the same order in N_c as the factorizable one. Their Wilson coefficients are not suppressed compared to those of the color-octet operators T (see Table I), and they can be important even if their expectation values are formally subleading in $1/N_c$. Moreover, the factorizable part of the expectation values has only a specific Lorentz structure which is subject to the strong chirality suppression $\sim m^2_c/m^2_b$ in the effects of weak annihilation (WA) in mesons. Nonfactorizable contributions there can be dominant [11,12].

Nonfactorizable effects also appear as expectation values of the nonconstituent quark operators. Although they do not split the widths of $B^\pm$ and B^0, they can differentiate the meson vs baryon lifetimes. Numerically they seem to be strongly suppressed, with a possible exception of the Darwin operator which will be discussed below.

The nonfactorizable effects in B mesons were first discussed in the framework of the $1/m$ expansion in [11] where the parametrization

$$\frac{1}{2M_B}\langle B|\bar{b}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma_\nu(1-\gamma_5)b|B\rangle$$

$$=\frac{\widetilde{f}_B^2 M_B}{2}(v_s v_\mu v_\nu - g_s g_{\mu\nu}) \qquad (96)$$

$$\frac{1}{2M_B}\langle B|\bar{b}\gamma_\mu(1-\gamma_5)t^a q\bar{q}\gamma_\nu(1-\gamma_5)t^a b|B\rangle$$

$$=\frac{\widetilde{f}_B^2 M_B}{2}(v_o v_\mu v_\nu - g_o g_{\mu\nu}) \qquad (97)$$

was suggested motivated by the analysis of the WA effects: neglecting the c quark mass WA is governed by g_o and g_s. (In the factorization approximation $v_s=1$ and $v_o=g_o=g_s=0$.) These parameters are related to ω,τ in the following way:

$$\widetilde{f}_B^2 M_B v_s = -\frac{2}{N_c}\omega_V + \frac{2}{3N_c}\omega_A - 4\tau_V + \frac{4}{3}\tau_A \qquad (98)$$

$$\widetilde{f}_B^2 M_B g_s = -\frac{1}{N_c}\omega_V - \frac{1}{3N_c}\omega_A - 2\tau_V - \frac{2}{3}\tau_A \qquad (99)$$

$$\widetilde{f}_B^2 M_B v_o = -\left(1-\frac{1}{N_c^2}\right)\omega_V + \frac{1}{3}\left(1-\frac{1}{N_c^2}\right)\omega_A$$
$$+\frac{2}{N_c}\tau_V - \frac{2}{3N_c}\tau_A \qquad (100)$$

$$\widetilde{f}_B^2 M_B g_o = -\frac{1}{2}\left(1-\frac{1}{N_c^2}\right)\omega_V - \frac{1}{6}\left(1-\frac{1}{N_c^2}\right)\omega_A$$
$$+\frac{1}{N_c}\tau_V + \frac{1}{3N_c}\tau_A . \qquad (101)$$

The inverse relations expressing ω and τ via v and g are given in Appendix C.

The color counting rules suggest that $\tau_{V,A}\sim N_c$ while $\omega_{V,A}\sim N_c^0$. The factorization estimates for $\tau_{V,A}$ in the large-N_c limit are expected to hold with the $1/N_c$ accuracy. Therefore, knowledge of the color-straight operators allows to estimate the leading, $1/N_c$ terms in v_o and g_o:

$$v_o \simeq -\frac{3\omega_V - \omega_A}{3\widetilde{f}_B^2 M_B} - \frac{1}{2N_c} \quad \text{(valence)} \qquad (102)$$

$$g_o \simeq -\frac{3\omega_V + \omega_A}{6\widetilde{f}_B^2 M_B} . \qquad (103)$$

[The term $-1/(2N_c)$ is absent for non-valence expectation values.]

Since the nonvalence expectation values appear to be suppressed, we only consider the valence matrix elements generically denoted by the superscript (v). Let us consider for

definiteness the charged B meson; the corresponding parameters are then defined by Eqs. (96),(97) with $q=u$. Although ω_V and ω_A are not precisely evaluated, we still observe a clear tendency to cancellations in g_o and, therefore, suppression of the effects of WA. Say, for the exponential ansatz we get

$$v_o^{(v)}\approx -0.07, \quad g_o^{(v)}\approx 0.004. \qquad (104)$$

For the two-pole ansatz representing the upper limit in our estimates, we get

$$v_o^{(v)}\approx 0.4, \quad g_o^{(v)}\approx 0.05. \qquad (105)$$

The nonfactorizable octet parameters seem to emerge suppressed. In particular, the expectation value of the operator responsible for WA is very small. The color-singlet expectation value g_s was not estimated in the literature. It is natural to think [12] that the scale of g_s does not exceed that of g_o. The above estimates then illustrate the degree of suppression of the effects of WA when the c quark mass is neglected.

It is appropriate to note at this point that there is convincing experimental evidence that WA is indeed strongly suppressed in heavy mesons. The width difference between D_s and D^0 is very sensitive to WA. Even though the literal $1/m_c$ expansion in charmed particles is hardly applicable at the quantitative level, the significance of such effects would have led to a large $\tau_{D_s}-\tau_{D^0}$ difference. Barring accidental cancellations one gets a typical estimate [12]

$$|g_o,g_s|\lesssim 10^{-2}.$$

We note, therefore, that our estimates, whatever tentative they are, indicate a strong enough suppression. It is interesting that the QCD sum rule estimates of the parameters g made in 1992 by V. Braun[4] yielded close values, a few units $\times 10^{-2}$. Later evaluations gave $v_o\simeq 0.05$, $g_o\simeq 0.05$ [30], and $v_o\simeq 0.1$, $g_o\simeq 0.03$ [42]; they were simplified in many aspects, though. While the expectation value of v_o generally emerges of the order of 0.05, our estimates for g_o seem to predict typically somewhat smaller values $\sim 10^{-2}$, in a better agreement with the experimental indications.

The nonvalence expectation values are probably even further suppressed. Our estimates yielded $v_o^{(nv)}\approx(0.5\ \text{to}\ 2)\times 10^{-2}$ and $g_o^{(nv)}\approx -(0.25\ \text{to}\ 1)\times 10^{-2}$, with the dominant part coming from the axial current via the anomalous terms.

For baryonic expectation values there is no vacuum factorization approximation. This does not mean, of course, that they are suppressed. The color-straight expectation values λ were estimated in the preceding sections to vary from 0.007 to 0.045 GeV^3; the interval above 0.03 GeV^3 seems improbable, though. The estimate of the color-octet $\lambda'_{u,d}$ based on the evaluation of the Darwin operator yielded about 0.018 GeV^3.

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[4]Private communication to N. Uraltsev.

A different parametrization of the valence expectation values was used in [43]:

$$\frac{1}{2M_B}\langle B^-|(\bar{b}\gamma_\mu(1-\gamma_5)u)(\bar{u}\gamma_\mu(1-\gamma_5)b)|B\rangle$$

$$=\frac{\tilde{f}_B^2(\mu)M_B}{2}B_1(\mu) \tag{106}$$

$$\frac{1}{2M_B}\langle B^-|(\bar{b}(1-\gamma_5)u)(\bar{u}(1+\gamma_5)b)|B^-\rangle$$

$$=\frac{\tilde{f}_B^2(\mu)M_B}{2}B_2(\mu) \tag{107}$$

$$\frac{1}{2M_B}\langle B^-|(\bar{b}\gamma_\mu(1-\gamma_5)t^a u)(\bar{u}\gamma_\mu(1-\gamma_5)t^a b)|B^-\rangle$$

$$=\frac{\tilde{f}_B^2(\mu)M_B}{2}\varepsilon_1(\mu) \tag{108}$$

$$\frac{1}{2M_B}\langle B^-|(\bar{b}(1-\gamma_5)t^a u)(\bar{u}(1+\gamma_5)t^a b)|B^-\rangle$$

$$=\frac{\tilde{f}_B^2(\mu)M_B}{2}\varepsilon_2(\mu). \tag{109}$$

These parameters are related as follows:

$$\tilde{f}_B^2 M_B B_1=\tilde{f}_B^2 M_B(v_s-4g_s)=4(\tau_V+\tau_A)+\frac{2}{N_c}(\omega_V+\omega_A) \tag{110}$$

$$\tilde{f}_B^2 M_B B_2=\tilde{f}_B^2 M_B(v_s-g_s)=-2(\tau_V-\tau_A)-\frac{1}{N_c}(\omega_V-\omega_A) \tag{111}$$

$$\tilde{f}_B^2 M_B \varepsilon_1=\tilde{f}_B^2 M_B(v_o-4g_o)$$

$$=-\frac{2}{N_c}(\tau_V+\tau_A)+\left(1-\frac{1}{N_c^2}\right)(\omega_V+\omega_A) \tag{112}$$

$$\tilde{f}_B^2 M_B \varepsilon_2=\tilde{f}_B^2 M_B(v_o-g_o)$$

$$=\frac{1}{N_c}(\tau_V-\tau_A)-\frac{1}{2}\left(1-\frac{1}{N_c^2}\right)(\omega_V-\omega_A). \tag{113}$$

The above estimates for the octet expectation values neglecting $1/N_c^2$ terms read for $\varepsilon_{1,2}$ as

$$\varepsilon_1\approx-0.085 \text{ to } 0.17, \quad \varepsilon_2\approx-0.07 \text{ to } 0.33, \tag{114}$$

while the QCD sum rule calculations give $\varepsilon_1\simeq-0.15$, $\varepsilon_2\simeq0$ [30] and $\varepsilon_1\simeq-0.04\pm0.02$, $\varepsilon_2\simeq0.06\pm0.03$ [42].

For convenience, we give in Table I the central estimates of the four-fermion expectation values discussed above in B mesons ($\omega_{V,A}$, $\tau_{V,A}$) and in Λ_b(λ,λ' for a fixed flavor, u or d). Since the nonvalence expectation values are strongly suppressed, we do not quote them here.

Concluding this section, we note that there are two exact positivity constraints on the expectation values of the s-channel colorless operators: $v_s-g_s>1$ ($v_s-g_s>0$ for non-valence) and $g_s>0$.

The first inequality follows from inserting a complete set of intermediate states $|n\rangle$ in the matrix element $\langle B|(\bar{b}\gamma^0(1-\gamma_5)q)(\bar{q}\gamma^0(1-\gamma_5)b)|B\rangle$. We obtain

$$v_s-g_s=1+\frac{1}{\tilde{f}_B^2 M_B^2}\sum_n\int d\mu(n)|\langle n|\bar{q}\gamma^0(1-\gamma_5)b|B\rangle|^2>1, \tag{115}$$

where $d\mu(n)$ stands for the phase space. The $1/N_c$ contributions in the RHS come when the intermediate states n are π,ρ,a_1, etc. For nonvalence quarks the vacuum factorization contribution 1 in the RHS explicitly showing $|n\rangle=|0\rangle$ vanishes.[5] In terms of the parameters B_i,ε_i, the constraint (115) reads $B_2>1$. Estimates of [42] give values for B_i compatible with 1.

The second inequality is obtained by taking spacelike $\mu=\nu=i$ in Eq. (96). Summing over i yields

$$g_s=\frac{1}{3\tilde{f}_B^2 M_B^2}\sum_n\int d\mu(n)\sum_i|\langle n|\bar{q}\gamma^i(1-\gamma_5)b|B\rangle|^2>0. \tag{116}$$

(For the B parameters this is $B_2>B_1$.)

A similar inequality can be obtained for the baryonic matrix elements:

$$\lambda'-\frac{1}{4}\lambda=\frac{3}{4M_{\Lambda_b}}\sum_n\int d\mu(n)|\langle n|\bar{q}(1+\gamma_5)b|\Lambda_b\rangle|^2>0, \tag{117}$$

where we used the identity

$$\langle\Lambda_b|(\bar{b}(1-\gamma_5)q)(\bar{q}(1+\gamma_5)b)|\Lambda_b\rangle$$

$$=-\frac{1}{2N_c}\langle\Lambda_b|O_V|\Lambda_b\rangle-\langle\Lambda_b|T_V|\Lambda_b\rangle. \tag{118}$$

In the constituent quark model the bounds for B mesons become equalities; the relation Eq. (117) merely expresses the fact that the diquark wave function at origin is positive. It does not seem to be very restrictive for our estimates. There is an additional constraint on the expectation values of the operators with chirality flip for light quarks, however we are

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[5]These inequalities assume a physical regularization scheme for composite operators in which, for example, there is a power mixing of the four-fermion operators with the unit one, $\bar{b}b(0)$. For a recent discussion, see e.g. [33,35].

not interested in them. The above bounds can be refined and even actual approximate estimates can be obtained evaluating the contributions of a few lowest intermediate states in the hadronic saturation Eqs. (115)–(117).

VI. CORRECTIONS TO THE DECAY WIDTHS

In this section we give the expressions for the corrections to the widths in terms of the effective four-fermion operators normalized at a low scale. These expressions were originally derived in [1]. We present them here for completeness and book-keeping purposes, in a more convenient form.

Let us introduce the notation $\Delta\hat{\Gamma}$ for the operator describ-

ing the corrections to the inclusive width $\Delta\Gamma_{H_b}$ of the beauty hadron H_b:

$$\Delta\Gamma = \frac{1}{2M_{H_b}}\langle H_b|\Delta\hat{\Gamma}|H_b\rangle. \qquad (119)$$

In what follows we neglect the effects generated in the KM suppressed decays which have a factor $|V_{ub}/V_{cb}|^2$, and by penguin operators in $\mathcal{H}_{\mathrm{weak}}(\Delta B=1)$ at the scale m_b.

Without accounting for the perturbative QCD effects in the domain $k\ll m_b$ one has

$$\Delta\hat{\Gamma} = \frac{G_F^2 m_b^2}{2\pi}|V_{cb}|^2(1-y)^2\left\{\left(c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\right)[O_V^u+O_A^u]+4c_1c_2[T_V^u+T_A^u]\right\} - \frac{G_F^2 m_b^2}{4\pi}|V_{cb}|^2(1-y)^2\left\{\left(c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\right)\right.$$
$$\times\left[(1+y)O_V^{d'}+\frac{1}{3}(1-y)O_A^{d'}\right]+2(2c_1c_2+N_cc_2^2)\left[(1+y)T_V^{d'}+\frac{1}{3}(1-y)T_A^{d'}\right]\right\}$$
$$-\frac{G_F^2 m_b^2}{4\pi}|V_{cb}|^2\sqrt{1-4y}\left\{\left(c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\right)\left[O_V^{s'}+\frac{1}{3}(1-4y)O_A^{s'}\right]+2(2c_1c_2+N_cc_2^2)\left[T_V^{s'}+\frac{1}{3}(1-4y)T_A^{s'}\right]\right\}$$
$$-\frac{G_F^2 m_b^2}{2\pi}|V_{cb}|^2\left\{\left(c_1^2+c_2^2+\frac{2c_1c_2}{N_c}+\frac{n_l}{2N_c}\right)\left[O_V^c+\frac{O_A^c}{3}\right]+2\left(2c_1c_2+N_cc_1^2+\frac{n_\ell}{2}\right)\left[T_V^c+\frac{T_A^c}{3}\right]\right\}. \qquad (120)$$

We denoted here $y=m_c^2/m_b^2$ and $d'=d\cos\theta_c+s\sin\theta_c$, $s'=s\cos\theta_c-d\sin\theta_c$. The m_c-dependence for the operators with external c quark legs is completely neglected. (Eventually they will lead only to penguin-type operators which are estimated, basically, in the leading-log approximation.) We included the contribution from the semileptonic decays with $n_l=2$ species of light leptons (the τ contribution is suppressed by phase space). Since numerically $m_c^2/m_b^2\approx\mu_{\mathrm{had}}/m_b$, keeping m_c^2/m_b^2 corrections apparently is not legitimate in practice at all. We retain these terms only for getting an idea of the scale of the finite-m_c corrections in the coefficient functions.

The perturbative evolution below m_b in the LLA is particularly simple in this basis: the color-straight operators O do not renormalize. The color-octet operators T renormalize in a universal way with $\gamma_T=3N_c$, except for the flavor singlet vectorlike operator similar to $\tilde{O}_D$ which has anomalous dimension $\gamma_{\tilde{D}}=3N_c-\frac{4}{3}n_f$ where n_f is the number of open flavors ($\gamma_D=-\frac{13}{3}N_c$). At the scale below the charm mass the operators with the c-quark fields merely vanish. As a result, at the low normalization point μ we have

$$\Delta\hat{\Gamma} = \frac{G_F^2 m_b^2}{2\pi}|V_{cb}|^2\left\{(1-y)^2\left(c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\right)\left[O_V^u+O_A^u-\frac{1+y}{2}O_V^{d'}-\frac{1-y}{6}O_A^{d'}-\frac{\sqrt{1-4y}}{2(1-y)^2}O_V^{s'}-\frac{(1-4y)^{3/2}}{6(1-y)^2}O_A^{s'}\right]\right.$$
$$\left.+\zeta(1-y)^2\left[4c_1c_2(T_V^u+T_A^u)-(2c_1c_2+N_cc_2^2)\left((1+y)T_V^{d'}+\frac{1-y}{3}T_A^{d'}+\frac{\sqrt{1-4y}}{(1-y)^2}T_V^{s'}+\frac{(1-4y)^{3/2}}{3(1-y)^2}T_A^{s'}\right)\right]+2\pi c_D\frac{O_D}{2\pi}\right\},$$
$$(121)$$

where

$$\zeta=\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{3N_c/(2\beta_0-4/3)}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{3N_c/2\beta_0}, \qquad \beta_0=\frac{11}{3}N_c-2=9. \qquad (122)$$

We wrote the contribution of the Darwin operator separately although it is related to the sum $T_V^u+T_V^d+T_V^s$. In this form c_D emerges from the penguin-type diagrams while the other terms do not include the annihilation diagrams.

The coefficient c_D of the Darwin operator Eq. (86) takes the following LLA form:

TABLE II. Values of Wilson coefficients for the total width.

$\alpha_s(m_b)$	$A_{V,A}^u$	A_V^d	A_V^s	A_A^d	A_A^s	$B_{V,A}^u$	B_V^d	B_V^s	B_A^d	B_A^s
0.3	0.98	-0.53	-0.48	-0.15	-0.11	-2.1	0.70	0.64	0.20	0.15
0.2	0.98	-0.53	-0.48	-0.15	-0.11	-2.6	0.84	0.77	0.24	0.18

$\alpha_s(m_b)$	$2\pi c_D$	$\widetilde{B}_V^u$	$\widetilde{B}_V^d$	$\widetilde{B}_V^s$
0.3	-0.80	-1.3	1.5	1.4
0.2	-0.56	-2.0	1.4	1.3

$$c_D = -\frac{1}{2\pi\alpha_s(\mu)}\zeta\left\{(1-y)^2\left(\frac{\eta-1}{n_f}+\frac{\eta(\xi-1)}{n_f+1}\right)\left[4c_1c_2-(2c_1c_2+N_cc_2^2)\left(1+y+\frac{\sqrt{1-4y}}{(1-y)^2}\right)\right]\right.$$
$$\left. -2\frac{\eta(\xi-1)}{n_f+1}\left[2c_1c_2+N_cc_1^2+\frac{n_l}{2}\right]\right\},$$
(123)

where

$$\xi=\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{-2(n_f+1)/(3\beta_0-2)},\qquad \eta=\left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{(-2n_f/3\beta_0)};$$

$$n_f=3. \tag{124}$$

In principle, there is another source of the Darwin term in the width which comes from the $1/m_b$ expansion of the expectation value of $\bar{b}b$ and from the nonlogarithmic terms in the expansion of the transition operator. They were calculated for the case of the semileptonic width in [45] and [46]. We can estimate this correction to the LLA by neglecting the deviation of the Wilson coefficients c_1,c_2 from their bare values 1 and 0, respectively, and neglecting the mass of the quarks (leptons) produced by the virtual W boson. In this approximation the possible effect from the $\bar{u}d(\bar{c}s)$ loop cancels and we can use the calculations for the semileptonic widths. This yields to the leading order in α_s

$$\delta c_D \simeq \frac{2N_c+n_l}{576\pi^2}(77-88y+24y^2-8y^3-5y^4+36y^2\ln y). \tag{125}$$

The overall nonlog term in c_D appears to be of the opposite sign to the LLA result and is roughly a half of it in magnitude. We conclude that the LLA estimate is accurate within a factor of 2 being, probably, on the upper side. We use the LLA expressions for numerical estimates below.

At the next-to-leading order the $\Delta B=1$ weak decay Wilson coefficients $c_1(m_b),c_2(m_b)$ are[6]

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[6]Note that these NLO values of $c_{1,2}$ are immediately reproduced

$$c_1=1.13,\qquad c_2=-0.29, \tag{126}$$

corresponding to $\alpha_s(M_Z)=0.118$. This gives

$$c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\simeq1.15,\qquad 2c_1c_2+N_cc_2^2\simeq-0.40,$$

$$4c_1c_2\simeq-1.3,\qquad 2c_1c_2+N_cc_1^2\simeq3.2. \tag{127}$$

With this input the resulting values for the coefficients in $\Delta\hat{\Gamma}$ are given in Table II. To illustrate the uncertainty associated with the LLA we quote two sets of values corresponding to using $\alpha_s(m_b)=0.3$ and to $\alpha_s(m_b)=0.2$ (the former option can represent the choice of the V-scheme strong coupling in the LLA expressions, which seems more appropriate to us). The coefficients A and B are defined as

$$\Delta\hat{\Gamma} = \frac{G_F^2m_b^2}{2\pi}|V_{cb}|^2\left(A_V^uO_V^u+A_V^dO_V^{d'}+A_V^sO_V^{s'}+A_A^uO_A^u\right.$$
$$+A_A^dO_A^{d'}+A_A^sO_A^{s'}+B_V^uT_V^u+B_V^dT_V^{d'}+B_V^sT_V^{s'}+B_A^uT_A^u$$
$$\left.+B_A^dT_A^{d'}+B_A^sT_A^{s'}+2\pi c_D\frac{O_D}{2\pi}\right) \tag{128}$$

(i.e., using the "redundant" basis including the Darwin operator to show explicitly the loop contributions). We also quote the values of $\widetilde{B}_V^u$, $\widetilde{B}_V^d$ and $\widetilde{B}_V^s$ given by

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in the simple LLA if one uses the more physical V-scheme α_s coupling [47].

$$\tilde{B}_V^u = B_V^u - 2\pi\alpha_s c_D, \quad \tilde{B}_V^d = B_V^d - 2\pi\alpha_s c_D,$$

$$\tilde{B}_V^s = B_V^s - 2\pi\alpha_s c_D. \tag{129}$$

The Cabibbo mixing is neglected here.

For numerical estimates we use the values of the running low-scale masses $m_b \simeq 4.6$ GeV, $m_c \simeq 1.25$ GeV and normalize the width correction $\Delta\Gamma_{H_b}$ to the semileptonic width which is reliably evaluated in the OPE (for a review, see [33]). The final estimates for these corrections then read as

$$\frac{\Delta\Gamma_{H_b}}{\Gamma_{\rm sl}} \simeq \frac{1}{{\rm BR}_{\rm sl}}\frac{\Delta\Gamma_{H_b}}{\Gamma_{H_b}} \simeq 0.36\left[A_V^u \frac{\langle O_V^u \rangle}{0.02\ {\rm GeV}^3} + A_V^d \frac{\langle O_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + A_V^s \frac{\langle O_V^{s'} \rangle}{0.02\ {\rm GeV}^3} + A_A^u \frac{\langle O_A^u \rangle}{0.02\ {\rm GeV}^3} + A_A^d \frac{\langle O_A^{d'} \rangle}{0.02\ {\rm GeV}^3} \right.$$

$$+ A_A^s \frac{\langle O_A^{s'} \rangle}{0.02\ {\rm GeV}^3} + B_V^u \frac{\langle T_V^u \rangle}{0.02\ {\rm GeV}^3} + B_V^d \frac{\langle T_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + B_V^s \frac{\langle T_V^{s'} \rangle}{0.02\ {\rm GeV}^3} + B_A^u \frac{\langle T_A^u \rangle}{0.02\ {\rm GeV}^3} + B_A^d \frac{\langle T_A^{d'} \rangle}{0.02\ {\rm GeV}^3}$$

$$\left. + B_A^s \frac{\langle T_A^{s'} \rangle}{0.02\ {\rm GeV}^3} + 0.8\ (2\pi c_D) \frac{\rho_D^3}{0.1\ {\rm GeV}^3} \right] = 0.36\left[A_V^u \frac{\langle O_V^u \rangle}{0.02\ {\rm GeV}^3} + A_V^d \frac{\langle O_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + A_V^s \frac{\langle O_V^{s'} \rangle}{0.02\ {\rm GeV}^3} \right.$$

$$+ A_A^u \frac{\langle O_V^u \rangle}{0.02\ {\rm GeV}^3} + A_A^d \frac{\langle O_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + A_A^s \frac{\langle O_V^{s'} \rangle}{0.02\ {\rm GeV}^3} + \tilde{B}_V^u \frac{\langle T_V^u \rangle}{0.02\ {\rm GeV}^3} + \tilde{B}_V^d \frac{\langle T_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + \tilde{B}_V^s \frac{\langle T_V^{s'} \rangle}{0.02\ {\rm GeV}^3}$$

$$\left. + B_A^u \frac{\langle O_V^u \rangle}{0.02\ {\rm GeV}^3} + B_A^d \frac{\langle O_V^{d'} \rangle}{0.02\ {\rm GeV}^3} + B_A^s \frac{\langle O_V^{s'} \rangle}{0.02\ {\rm GeV}^3} \right], \tag{130}$$

where $\langle O_V^u \rangle = (1/2M_{H_b})\langle H_b | O_V^u | H_b \rangle$, etc. We recall that the expectation values in B are denoted by ω for color-straight operators O and by τ for the octet ones T, Eq. (35); for Λ_b these are λ and $-\frac{2}{3}\lambda'$, Eq. (36).

It is interesting to note that, regarding the N_c counting rules one can view the Wilson coefficients of the color-straight operators to be N_c^0 while the coefficients of the octet operators as $1/N_c$. This is true if we recall that formally $c_1(m_b) = \mathcal{O}(1)$ while $c_2(m_b) = \mathcal{O}(1/N_c)$. These are not mandatory assumptions for the large-N_c analysis: smallness of a particular perturbative renormalization can always be compensated by large logarithms of M_W/m_b; in any case the nonleptonic weak decay coefficients $c_{1,2}$ are external to QCD itself and can be taken completely arbitrary. Nevertheless, their numerical values fit well such a naive assignment.

Our procedure of evaluating the $1/m_b^3$ corrections to the widths then gets justification in the N_c counting rules: we take at face value the N_c^0 color-straight expectation values appearing with the coefficients $\sim N_c^0$, and take only the leading factorizable values $\sim N_c$ for the color-octet operators which come with the subleading coefficient $1/N_c$. This formally sums up all leading corrections $\sim N_c^0$ in the decay widths. We can expect therefore that the matrix elements of the color-straight operators include the dominant nonfactorizable contributions to the nonleptonic decay widths of heavy hadrons.

VII. DISCUSSION

We have considered the expectation values of the four-fermion operators which are encountered in the $1/m_Q$ expansion of the inclusive widths of beauty hadrons. The size of the color-straight operators used to be most uncertain in B mesons, since the factorization approximation *a priori* is not expected to be accurate for them. On the other hand, just these operators have the most direct meaning being analogues of the usual wave function density $|\Psi(0)|^2$. Using the exact relation of their expectation values to the momentum integral of the elastic transition amplitudes, we estimated these expectation values employing reasonable assumptions about the behavior of the form factors. We showed that the actual large-q^2 asymptotics of the light quark amplitudes in heavy hadrons is $1/(q^2)^{3/2}$ rather than $1/q^4$ as has been believed based on simple-minded quark counting rules. We also calculated the anomalous dimension of the color-straight operators and their mixing with the octet operators, the effects absent at order α_s. The order-α_s^2 corrections appear to be numerically enhanced.

In our estimates of the valence expectation values their size obtained from the two-pole ansatz can be considered as an upper bound. A more reasonable exponential approximation which suppresses the contributions of momenta above 1 GeV, yields smaller results. We accept it as a typical lower bound for the color-straight expectation values. Although the accuracy of the central estimates cannot be too good, they probably hold better than within a factor of two.

Our estimates, in principle, include a source for nonvalence expectation values. It is related to a different q^2-behavior of form factors describing different isospin amplitudes at $q^2 < 0$. We have it mainly as the different masses of the isosinglet resonances saturating the form factors in the t-channel, compared to the corresponding flavor nonsinglet

particles (i.e., annihilation shift of masses). Except for η', experimentally these splittings are rather small, and our literal estimates thus yield a strong suppression. We are not sure if this really applies to the color-straight expectation values; the actual suppression can be softer.

We observe a weaker suppression of the nonvalence color-straight matrix elements for the operators with the axial current. It is related to the nonperturbative ''annihilation'' effect, in particular, the axial anomaly in QCD and its solution of the $U(1)$ problem. We conjecture that the dominant effect is the mass shift of the lowest pseudoscalar state η' while the splitting of the massive resonances (in particular, axial) or the effect of the possible difference in the singlet and triplet couplings $G_A(0)$ and $G_A^{(0)}(0)$ is smaller. Then we get a tentative relation

$$\frac{1}{2M_B}\langle B^+|\bar{b}\gamma_\mu\gamma_5 b\bar{d}\gamma_\mu\gamma_5 d+\bar{b}\gamma_\mu\gamma_5 b\bar{s}\gamma_\mu\gamma_5 s|B^+\rangle$$

$$\approx -\frac{G_A^{(0)}(0)}{8\pi^{3/2}}\frac{M_{\eta'}^2 M_{\eta'(1295)}^2}{M_{\eta'}+M_{\eta'(1295)}}. \tag{131}$$

This estimate has the correct scaling with $1/N_c$. Numerically, the axial nonvalence expectation values appear to be suppressed by a factor about 0.1. We note that the numerical suppressions of various nonvalence effects typically is stronger than the naive factor 1/3 which would be expected if their justification was merely the large N_c approximation.

An interesting indication from our estimates is that the possible nonperturbative vitiation of the chirality suppression of WA in B mesons emerges at a rather low level [it is governed by the combinations $(\omega_V+\frac{1}{3}\omega_A)$, $(\tau_V+\frac{1}{3}\tau_A)]$. For the color-straight operators (where the effect *a priori* can be significant), the literal suppression is by more than an order of magnitude, in accord with the evidences from charmed mesons. In our approach the origin of the suppression roots to the fact that $-G_A(0)\lesssim 1/3$. The WA effect of the octet operators can be probed in the difference of the semileptonic $b\to u$ distributions in B^+ and B^0 [11].

The chirality suppression of WA can be eliminated already in the perturbative evolution of the effective operators. This does not happen in the LLA [48]. Our NLO calculations show that it does not happen at this level as well. It is interesting to check this property for the two-loop diagonal renormalization of the color-octet operators. In any case, we expect it to be lifted in three loops; also, the nonlogarithmic gluon corrections at $k\sim m_b$ defining the initial values of the Wilson coefficients must generate the chirality nonsuppressed effect at some level.

Let us now turn to the phenomenological consequences of our analysis. The estimated expectation values are typically of the order of, or somewhat larger than the factorization values (when the latter are possible) at $\tilde{f}_B=160$ MeV (the factorization value of ω_V is additionally suppressed, and our estimates only partially reproduce this). The actual expectation values of the color-straight operators can be smaller if, for example, the form factors change sign at $-q^2 \lesssim 1$ GeV^2. Such subtleties are not properly captured by the simple models we relied upon. On the other hand, larger values than quoted in Table I are improbable, at least if the nonperturbative dynamics we account for are dominated by the momenta not exceeding 1 GeV.

The relevance of the latter assumption for the analysis of the inclusive widths is easy to see, say, on the example of the effect of interference (dominant in B mesons). The decay rate of the process $b\to\bar{u}_k+(cd)_q$ is proportional to $q^2=(p_b-k)^2$. At $k^2=0$ one has $q^2=m_b^2-2p_b k$, and this constitutes only about 12 GeV^2 vs $m_b^2\simeq 21$ GeV^2 already for $|\vec{k}|=1$ GeV. At the same time the usual relation of the $1/m_b^3$ effects via the expectation values of the corresponding four-fermion operators assumes that $q^2=m_b^2$. Therefore, if $|\vec{k}|$ becomes as large as taken above, the validity of the leading-order expressions breaks down. In any case, accounting for effects like interference in the usual way is legitimate only if their impact is much smaller than the partonic width of a particular quark channel. It is worth noting, on the other hand, that the assumption that the nonperturbative contributions to the expectation values come from momenta not exceeding 1 GeV is built in the approach of the QCD sum rules.

At first sight, WA in mesons and ''weak scattering'' (WS) in baryons can get enhanced, in contrast to interference, if the quark momenta saturating the expectation values of the operators are large. Such a conclusion, even though eventually may prove to be correct, cannot be justified *a priori*, and even the sign of the corresponding corrections to the standard expressions is not known. All such effects manifestly go beyond the $1/m$ expansion truncated after $1/m_b^3$ terms. For this reason, simply assuming large expectation values in B particles does not allow one to boost significantly the lifetime differences respecting the self-consistency of the simplest $1/m_b$ expansion.

Bearing in mind all reservations made above, we still quote the central values for the corrections to the inclusive widths stemming from our analysis:

$$\frac{\delta\Gamma_{B^-}}{\Gamma_{sl}}\simeq 0.36(-1.1_{PI}-1.2_D),$$

$$\frac{\delta\Gamma_{B^0}}{\Gamma_{sl}}\simeq 0.36(-0.15_{WA}-1.2_D), \tag{132}$$

$$\frac{\delta\Gamma_{\Lambda_b}}{\Gamma_{sl}}\simeq 0.36(2.2_{WS}-1_{PI}-1_D).$$

Here we showed separately the effects of different light flavors: of the operators $(\bar{b}b)(\bar{u}u)$ responsible for PI in B and WS in Λ_b, and of $(\bar{b}b)(\bar{d}d)$ generating WA in B and PI in Λ_b. We singled out the contribution of the Darwin term. Even though it may seem to be a computational separation, it is legitimate, for it can be formally carried through the dependence on the number of light flavors. Being a flavor singlet, the Darwin operator does not differentiate the lifetimes of charged and neutral B [also of B_s to the extent that $SU(3)_{fl}$ is a good symmetry].

The above estimates generally support the original theoretically predicted pattern of the lifetimes. The nonvalence effects seem to be strongly suppressed. The main effect is destructive PI in B^-, about -4%, while WA is small, at a half percent level. Moreover, literally we get the effect of WA decreasing the width, the possibility originally discussed in [48,11,12] and which may seem to contradict the naive interpretation of WA. The overall difference of $\Gamma(B^-)$ and $\Gamma(B^0)$ appears about -4%. The major effect is WS in Λ_b, 8.5%, but it is partially offset by interference, -3.5%. The difference between $\Gamma(\Lambda_b)$ and $\Gamma(B^0)$ is literally 6%. These estimates fall close to the expectations quoted in the review [49]. We note that the often discarded Darwin term (e.g., in [43]) typically decreases the width by about 4%, although literally we get its effect in B and Λ_b close to each other. Including it, the overall decrease in the Λ_b lifetime from the four-fermion operators at the order $1/m_b^3$ comes out only at a percent level while τ_{B^0} increases by 5% and τ_{B^-} by 9%. The overall absolute shift is not too interesting by itself though, since it depends on the exact definition of the parton width.

It is worth noting that the corrections we addressed do not formally exhaust the $1/m_b^3$ terms in the asymptotic expansion of Γ_{H_b}—they come implicitly as well from the expectation values of the kinetic and chromomagnetic operators which appear at the level of $1/m_b^2$ corrections. These expectation values in the actual b hadrons differ from their asymptotic values at $m_b \to \infty$ by terms $\sim 1/m_b$ [5]. In particular, these deviations contain the expectation value ρ_{LS}^3 of one new local heavy quark operator, the convection current (or spin-orbital) one. (This operator cannot appear independently in the expansion of the transition operator describing the inclusive width since it is not Lorentz-invariant.) These corrections do not affect $\Gamma_{B^-} - \Gamma_{B^0}$ but, in principle, are present in $\Gamma_B - \Gamma_{\Lambda_b}$. Their practical neglection nevertheless is legitimate: such effects are included in the existing uncertainty of the differences of the expectation values μ_π^2 and μ_G^2 of the $D = 5$ operators between B and Λ_b. So far these expectation values are estimated without considering corrections to the heavy quark limit; for example, the value of μ_G^2 in Λ_b is nonzero but generally of the order of $\Lambda_{\text{QCD}}^3/m_b$. All such effects are also expected to be numerically insignificant. Let us recall that in B mesons the ρ_{LS}^3 expectation value is suppressed to the extent that their two-particle description is applicable [5].

Our analysis does not indicate a crucial impact of the nonfactorizable contributions in the low-scale expectation values on the B lifetimes conjectured in [43] or later speculations that $\Gamma(B^+)$ can even exceed $\Gamma(B^0)$ by a significant amount.

The small experimental lifetime of Λ_b thus remains a challenge for the straightforward $1/m_b$ expansion. An accurate measurement of the semileptonic width of Λ_b [or $\text{BR}_{\text{sl}}(\Lambda_b)$] would help to shed light on the origin of the problem. The gap between the experimental value of τ_{Λ_b} and the theoretical expectations could have been reduced by a significant enhancement of WS and suppression of PI in Λ_b,

according to the natural guess about the role of the spectator momentum we mentioned above. Since these effects originate from the quark decay mode $b \to c\bar{u}d$ constituting about 60% of the total width, a 25% effect in the lifetime would signal a more than 50% enhancement of this channel. Clearly, such an effect is not possible for the spectator quark occupying only a small fraction of the total phase space in the decay, and would require a nonconventional composition of the heavy hadron. The standard calculation of the $1/m_b^3$ terms neglecting the effect of finite spectator momenta is not applicable for quantitative description of such large corrections. For example, the expectation values of the Darwin operator would be in general much larger, likewise the mass scale governing the size of higher-dimension operators for $1/m_b^4$ and higher-order corrections must be higher in this situation.

Note added. When this paper was prepared for publication, a new improved QCD sum rule calculation of the four-fermion expectation values appeared [50]; the quoted results correspond to $v_o \simeq -0.03$, $g_o \simeq 0.003$, and $B_1 = 0.60 \pm 0.01$, $B_2 = 0.61 \pm 0.01$. It can be suspected, however, that the stated small errors did not adequately reflect the uncertainties inherent in the determination from the sum rules *per se*.

APPENDIX A: COMBINATORIAL RELATIONS

Here we quote two general algebraic relations which are useful in calculating the renormalization of amplitudes containing static heavy quarks.

For any set of N numbers $x_1, \ldots, x_N$

$$\sum_{k=0}^{N} \left(\Pi_{j=1}^{k} \frac{1}{\sum_{l=1}^{j} -x_l} \right) \left(\Pi_{j=k+1}^{N} \frac{1}{\sum_{l=j}^{N} x_l} \right) = 0 \quad \text{(A1)}$$

(it is assumed that $\Pi_{n_1}^{n_2} = 1$ if $n_1 > n_2$), and

$$\sum_{k=0}^{N} \left(\Pi_{j=k+1}^{N} \frac{1}{\sum_{l=k+1}^{j} -x_l} \right) \left(\Pi_{j=1}^{k} \frac{1}{\sum_{l=j}^{k} x_l} \right) = 0. \quad \text{(A2)}$$

The proof will be given below.

The sums of the type (A1) are reminiscent to those appearing in calculating the renormalization of any color-
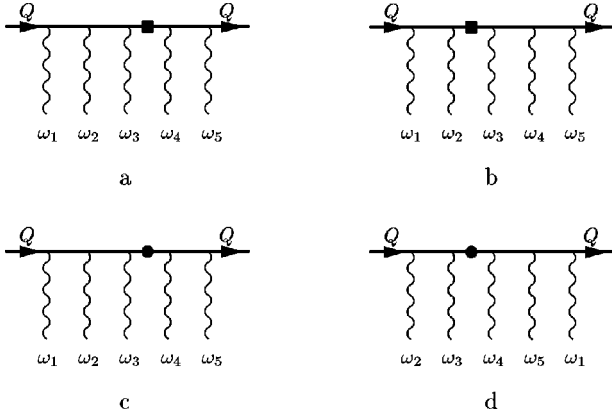
FIG. 4. The diagrams combined for the color-straight operators [(a),(b)] and for the mixing into the color-straight operators [(c),(d)]. The solid box denotes a color-straight operator, the blob in the diagrams (c) and (d) stands for an arbitrary heavy quark operator. Only two of six ($N=5$) diagrams to be combined are shown in both cases.

straight operator of the type $\bar{b}b \cdot O_{\text{light}}$. The sums similar to Eq. (A2) emerge in calculations of mixing of an arbitrary heavy quark operator into the color-straight operators, $\bar{b}Tb \cdot \tilde{O}_{\text{light}} \to \bar{b}b \cdot O_{\text{light}}$ where T is any color matrix.

For the color-straight weak vertex $\bar{b}b$ the product of color matrices on the heavy quark line does not depend on the location of the weak vertex in respect to the gluon vertices. The kth term in the sum Eq. (A1) corresponds to the diagram where the first k gluons attach to the initial b quark while the last $N-k$ gluons attach to the final-state quark, Figs. 4(a) and 4(b). We thus do not sum over permutations of gluons (their time ordering is fixed) but combine $N+1$ possibilities to place the weak vertex. The analogues of x_k are ω_k, the energies of gluons flowing into the quark line. With this identification the structure of the product of the heavy quark propagators is reproduced.

In dressing a nonstraight operator the gluon and weak vertices do not commute and moving the weak vertex would change the product of color matrices. However, calculating mixing *into* the color-straight operators amounts to taking trace over color indices of the initial and final state quarks. Then one, instead, can perform a cyclic move of the leftmost gluon in the initial state to the latest position in the final state, and vice versa, Figs. 4(c) and 4(d). In considering Eq. (A2) we thus imply combining all graphs obtained by the cyclic permutations of a particular diagram. Both considerations apply for any color representation of the quarks and gluons.

Taken naively, the relation (A1) would suggest that the renormalization of the color-straight operators vanishes to all orders (already in the sum of the above groups of diagrams, before actual integration over all gluon momenta). Likewise the identity (A2) would look like the property that the octet operators never mix with the straight operators. This is not so, however. The reason is that the identities Eqs. (A1) and (A2) apply only if the external b quarks are exactly on shell so that their nonrelativistic energy vanishes, $E=0$. In this

case the quark propagators generally become IR singular when integrated over the gluon momenta, and must be regularized by a small imaginary part $-i\epsilon$ in each heavy quark denominator. Alternatively, this is done by a shift of the external heavy quark energies by an infinitesimal imaginary amount. This regularization translates into the shift of all x_l which, however, is of the opposite sign for the initial-state and final-state gluons. For example, for the sum in Eq. (A1)

$$x_l \to x_l + i\epsilon \quad \text{for } l \le k \quad \text{and} \quad x_l \to x_l - i\epsilon \quad \text{for } l > k$$

[and the opposite shift in the sum in Eq. (A2)]. This infinitesimal shift of denominators leads to the fact that the sum of all diagrams does not vanish exactly but contains certain δ-functions of combination of energies corresponding to a certain on-shell heavy quark inside the diagrams. Nevertheless this kills some of the integrations over ω and simplifies the remaining integrals.

Let us prove the identities (A1) and (A2). This can be done most simply by using the following trick. We can consider the sum as a rational function of the variable x_N (for example), at $x_1, \ldots, x_{N-1}$ arbitrary but fixed. If we show that the residue of this function at any potential pole vanishes, this would mean that the whole function vanishes identically.
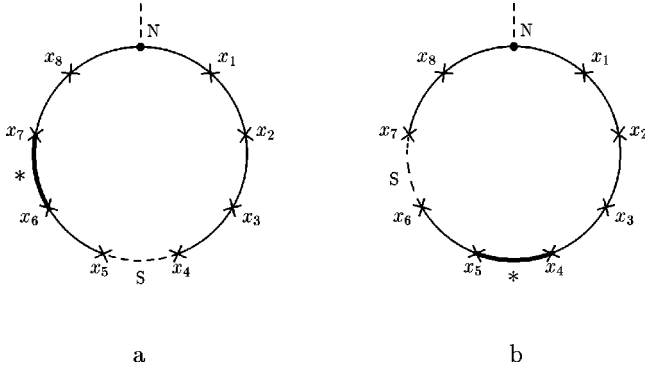
For the sum in Eq. (A1) this is particularly simple. Presence of a pole means that at certain k some of the denominators with $j=j_0$ vanish, with either $j_0 \le k$ (to the left of the weak vertex) or $j_0 > k$ (to the right of it). Let $j_0 < k$, for example, and therefore $\Sigma_{l=1}^{j_0} -x_l = 0$. Then the same vanishing denominator will be present for all diagrams corresponding to $k > j_0$, and it will change only for $k \le j_0$. Moreover, all terms with $k > j_0$ will have the common factor

$$\Pi_{j=1}^{j_0-1} \frac{1}{\displaystyle\sum_{l=1}^{j} -x_l}$$

which is the product of the propagators to the left of the one which vanishes.

The remaining factors will be different, but for $k=j_0+1,\ldots,N$ their sum exactly reproduces the LHS of Eq. (A1) for the set of $x_{j_0+1}, \ldots, x_N$ (that is, the case of $N-j_0$ gluons) owing to the on-shellness of the j_0-th propagator (the condition $\Sigma_{l=1}^{j_0} -x_l=0$). The induction from the obvious case $N=1$ immediately proves Eq. (A1) for arbitrary N.

The proof of the identity Eq. (A2) is a little more complicated. To phrase it, it is convenient to close the heavy quark line and map it onto the circle, Figs. 5. The weak vertex can be referred to as North Pole whereas the infinity can be called (with some reservations) South Pole. Every arc on the circle can be attributed the corresponding energy denominator. Proceeding from the kth arc to the $k+1$-th arc clockwise decreases the denominator by x_k. The values of all denominators are fixed by the condition that the arc containing the South Pole (the Infinity arc) has vanishing denominator [correspondingly, it is excluded from the product of propagators in Eq. (A2)].

FIG. 5. Graphic illustration for the case $N=8$.

consider, the overall one-loop renormalization vanishes. For the octet operator an additional contribution to the diagonal renormalization comes from the gluon exchange between the heavy and light lines.

In order α_s^2 both the renormalization of the color-straight operators and the straight–octet mixing occur. We do not consider the $\mathcal{O}(\alpha_s^2)$ diagonal anomalous dimension of the octet operators. Since the $\mathcal{O}(\alpha_s)$ contribution does not vanish, the $\mathcal{O}(\alpha_s^2)$ anomalous dimension is scheme-dependent. For the light-quark currents we are interested in, only non-factorizable diagrams must be considered where at least one gluon connects the heavy quark line with the light part of the diagram.

The hybrid anomalous dimensions are given by a (single) logarithmic UV divergence of the diagrams in the limit $m_Q \to \infty$, $|k| \ll m_Q$. In the Feynman gauge we adopt for computations, only 18 "double exchange" diagrams where two gluons connect light quark line with the heavy quark line each, yield the log. All other diagrams where there is only one gluon vertex either on the heavy quark or on the light quark lines, are finite for symmetry reasons similar to the one-loop case, or (in the case of dressing the octet operator) yield only the octet structure we are not interested in.

Combining the diagrams into groups of three according to the rules described in Appendix A (all locations of the weak vertex on the heavy quark line for the color-straight operators, or cyclic permutations of the $\bar{Q}Qg$ vertices for the octet operators) we get, at fixed values of the gluon momenta k_1, k_2 the sum of the heavy quark propagators in the form

With this image it is easy to establish the vanishing of the residues in the sum Eq. (A2) as well. A pole would appear due to the vanishing of the denominator of some other arc with $j=j_0$; it is indicated by the star in Fig. 5(a) (the Zero arc). This figure shows the case of $j_0=6$ and $k=4$. It is easy to see that the residue is exactly canceled by the configuration with $j_0 \leftrightarrow k$, that is, when the Infinity arc and the Zero arc are interchanged, Fig. 5(b).

Indeed, due to vanishing of the denominators at the both arcs all other denominators in Fig. 5(b) are equal to the corresponding denominators in Fig. 5(a). To get the residue one must merely remove the two vanishing propagators from the product and take it with the factor -1 when the Zero arc is clockwise from the Infinity arc and with the factor $+1$ otherwise. This cancellation in Eq. (A2) reads as

$$\left(\sum_{l=k+1}^{j_0} -x_l\right)\Pi_{j=k+1}^{N}\frac{1}{\displaystyle\sum_{l=k+1}^{j} -x_l}\cdot\Pi_{j=1}^{k}\frac{1}{\displaystyle\sum_{l=j}^{} x_l}$$

$$\to\left(\sum_{l=k+1}^{j_0} -x_l\right)\Pi_{j=j_0+1}^{N}\frac{1}{\displaystyle\sum_{l=j_0+1}^{} -x_l}$$

$$\times\Pi_{j=1}^{j_0}\frac{1}{\displaystyle\sum_{l=j}^{} x_l}\quad\text{at }\sum_{l=k+1}^{j_0} -x_l\to 0.$$

Thus, both identities (A1) and (A2) are proved.

$$-2\pi i\,\delta(\omega_1+\omega_2)\frac{1}{\omega_1+i\epsilon}\quad\text{or}\quad -2\pi i\,\delta(\omega_1+\omega_2)\frac{1}{\omega_2+i\epsilon}\tag{B1}$$

for the color-straight operators, or

$$-2\pi i\,\delta(\omega_1+\omega_2)\frac{1}{\omega_1+i\epsilon}+2\pi i\left(\mathcal{P}\frac{1}{\omega_1}\delta(\omega_2)-\mathcal{P}\frac{1}{\omega_2}\delta(\omega_1)\right)\tag{B2}$$

(and $\omega_1\leftrightarrow\omega_2$) for the octet operators. In view of the $\omega\to-\omega$ symmetry of the integration only the structure $-2\pi^2\delta(\omega_1)\delta(\omega_2)$ survives, and the resulting integrals contain simple purely three-dimensional expressions given below . By dimensional counting they all are logarithmic; they do not vanish since integrations runs over two spacelike vectors.

APPENDIX B: TWO-LOOP ANOMALOUS DIMENSIONS

For the order-α_s hybrid renormalization of the heavy quark operators $\bar{Q}Q\bar{q}q$ the identities discussed in Appendix A say that summing over all attachments of the gluon to the heavy quark line results in $\delta(\omega)$. Therefore, the integration over d^3k cannot produce an UV logarithm since it would require an odd power of $\vec{k}$ in the integrand. This is not possible in the simple one-loop diagram. The one loop renormalization of the straight operators coincides, therefore, with that of the light quark bilinear, while the octet-to-straight mixing is absent. For the vector or axial vector currents we

1. Dressing of color-straight operators $\bar{Q}Q\bar{q}\Gamma q$

The six groups of three diagrams in turn fall into three types which differ by the location of the gluon vertices on the light quark line, Figs. 6(a)–6(c). Each diagram can have gluon lines twisted or not. Their expressions are
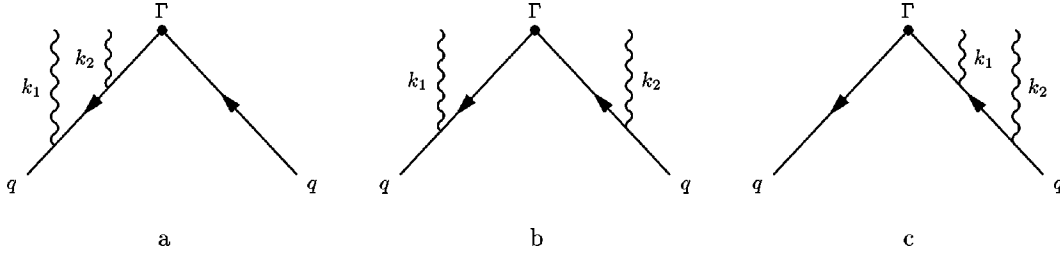
FIG. 6. Diagrams showing the different attachments of gluons to the light quark line.

$$I_a = \frac{g_s^4}{2} C[\Gamma\gamma_\mu\gamma_0\gamma_\nu\gamma_0] \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{(k_1+k_2)_\mu(k_1)_\nu}{\vec{k}_1^4\vec{k}_2^2(\vec{k}_1+\vec{k}_2)^2}$$

$$I_b = \frac{g_s^4}{2} C[\gamma_0\gamma_\mu\Gamma\gamma_\nu\gamma_0] \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{-(k_2)_\mu(k_1)_\nu}{\vec{k}_1^4\vec{k}_2^4}$$

$$(B3)$$

$$I_c = \frac{g_s^4}{2} C[\gamma_0\gamma_\mu\gamma_0\gamma_\nu\Gamma] \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{(k_2)_\mu(k_1+k_2)_\nu}{\vec{k}_1^2\vec{k}_2^4(\vec{k}_1+\vec{k}_2)^2}.$$

The color factors C are

$$C_1 = [t^a t^b]_l [t^a t^b]_h = \frac{1}{4}\left(1-\frac{1}{N_c^2}\right)[1]_l[1]_h - \frac{1}{N_c}[t^a]_l[t^a]_h$$

$$(B4)$$

$$C_2 = [t^a t^b]_l [t^b t^a]_h$$

$$= \frac{1}{4}\left(1-\frac{1}{N_c^2}\right)[1]_l[1]_h + \frac{N_c}{2}\left(1-\frac{2}{N_c^2}\right)[t^a]_l[t^a]_h$$

$$(B5)$$

for "twisted" and "nontwisted" diagrams, respectively.

For $\Gamma = \gamma_0$ or $\gamma_0\gamma_5$, twisted or non-twisted separately, we have

$$I_a + I_b + I_c = -g_s^4 C\Gamma \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{(\vec{k}_1\cdot\vec{k}_2)^2-\vec{k}_1^2\vec{k}_2^2}{\vec{k}_1^4\vec{k}_2^4(\vec{k}_1+\vec{k}_2)^2}$$

$$= g_s^4 C\Gamma \frac{1}{32\pi^2}\int^\Lambda \frac{dk}{k}.$$

$$(B6)$$

For $\Gamma = \gamma_i$ or $\gamma_i\gamma_5$

$$I_a + I_b + I_c = \frac{1}{3}g_s^4 C\Gamma \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3}$$

$$\times \frac{(\vec{k}_1\cdot\vec{k}_2)^2+2(\vec{k}_1\cdot\vec{k}_2)(\vec{k}_1^2+\vec{k}_2^2)+3\vec{k}_1^2\vec{k}_2^2}{\vec{k}_1^4\vec{k}_2^4(\vec{k}_1+\vec{k}_2)^2}$$

$$= g_s^4 C\Gamma \frac{1}{32\pi^2}\int^\Lambda \frac{dk}{k}.$$

$$(B7)$$

The sum of all diagrams for arbitrary Γ takes the form

$$\bar{Q}Q\bar{q}\Gamma q \rightarrow \left(1+\frac{\alpha_s^2}{4}\left(1-\frac{1}{N_c^2}\right)\ln\Lambda\right)\bar{Q}Q\bar{q}\Gamma q$$

$$+\frac{\alpha_s^2}{4}N_c\left(1-\frac{4}{N_c^2}\right)\ln\Lambda\,\bar{Q}t^a Q\bar{q}t^a\Gamma q. \quad (B8)$$

2. Mixing of octet operators $\bar{Q}t^a Q\bar{q}t^a\Gamma q$ into color-straight operators

Taking the trace over the heavy quark color indices we likewise can combine the 18 diagrams into 6 groups belonging again to the pairs, where each pair has the same location of the $\bar{q}qg$ vertices but different trace of color matrices along the heavy line. For example, for $\Gamma = \gamma_0$ the projection onto the straight operator yields

$$(I_a + I_b + I_c)|_{straight} = -g_s^4(C_3+C_4)\Gamma$$

$$\times \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{(\vec{k}_1\cdot\vec{k}_2)^2-\vec{k}_1^2\vec{k}_2^2}{\vec{k}_1^4\vec{k}_2^4(\vec{k}_1+\vec{k}_2)^2}$$

$$= g_s^4(C_3+C_4)\Gamma \frac{1}{32\pi^2}\int^\Lambda \frac{dk}{k}, \quad (B9)$$

where the color factors are given by

$$C_3 = -\frac{1}{4N_c}\left(1-\frac{1}{N_c^2}\right), \quad C_4 = \frac{N_c}{8}\left(1-\frac{1}{N_c^2}\right)\left(1-\frac{2}{N_c^2}\right).$$

$$(B10)$$

The same renormalization emerges for other Lorentz structures Γ as well.

For the flavor-singlet operators additional, annihilation diagrams are possible where the $q\bar{q}$ line forms a closed loop. It is easy to see that for the vector current it does not contribute. If the operator is color-straight, only two gluons can come out of the quark loop. The analogue of the Furry theorem leads to the cancellation of the two possible diagrams.

For any color-octet operator the sum of the diagrams where one of the gluons connects the external light and heavy quark lines yields only the octet operator in analogy with the one-loop diagrams. All other diagrams can obviously produce only the octet operators as well.

For the color-straight operator with the axial light quark current, both gluons must come out of the quark loop. The expression for the triangle subgraph does not differ from the

Abelian case [51]. The sum of the diagrams where one of the gluons is attached to the external light quark and another ends on the heavy quark line yields only nonlogarithmic contribution. The diagrams when both gluons are absorbed by the light quark legs describe the two-loop renormalization of the singlet axial current and differ from the classic Abelian result [51] only by the color factor $C_F/2$.

Using Eqs. (B8)–(B10) and the definition Eq. (28), we arrive at the $\mathcal{O}(\alpha_s^2)$ matrix of the anomalous dimensions given in Eq. (29) and in the text following it.

APPENDIX C: RELATIONS BETWEEN PARAMETRIZATIONS

Here we collect the relations between different parametrizations of the expectation values of the four-fermion operators in B mesons.

The hadronic parameters suggested in Ref. [11] are given by

$$\widetilde{f}_B^2 M_B v_s = -\frac{2}{N_c}\omega_V + \frac{2}{3N_c}\omega_A - 4\tau_V + \frac{4}{3}\tau_A \qquad (C1)$$

$$\widetilde{f}_B^2 M_B g_s = -\frac{1}{N_c}\omega_V - \frac{1}{3N_c}\omega_A - 2\tau_V - \frac{2}{3}\tau_A \qquad (C2)$$

$$\widetilde{f}_B^2 M_B v_o = -\left(1 - \frac{1}{N_c^2}\right)\omega_V + \frac{1}{3}\left(1 - \frac{1}{N_c^2}\right)\omega_A$$
$$+ \frac{2}{N_c}\tau_V - \frac{2}{3N_c}\tau_A \qquad (C3)$$

$$\widetilde{f}_B^2 M_B g_o = -\frac{1}{2}\left(1 - \frac{1}{N_c^2}\right)\omega_V - \frac{1}{6}\left(1 - \frac{1}{N_c^2}\right)\omega_A$$
$$+ \frac{1}{N_c}\tau_V + \frac{1}{3N_c}\tau_A . \qquad (C4)$$

The inverse relations read as

$$\omega_V = \widetilde{f}_B^2 M_B\left[-\frac{1}{2}v_o - g_o - \frac{1}{4N_c}v_s - \frac{1}{2N_c}g_s\right] \qquad (C5)$$

$$\omega_A = \widetilde{f}_B^2 M_B\left[\frac{3}{2}v_o - 3g_o + \frac{3}{4N_c}v_s - \frac{3}{2N_c}g_s\right] \qquad (C6)$$

$$\tau_V = \widetilde{f}_B^2 M_B\left[\frac{1}{4N_c}v_o + \frac{1}{2N_c}g_o - \frac{1}{8}\left(1 - \frac{1}{N_c^2}\right)v_s\right.$$
$$\left. -\frac{1}{4}\left(1 - \frac{1}{N_c^2}\right)g_s\right] \qquad (C7)$$

$$\tau_A = \widetilde{f}_B^2 M_B\left[-\frac{3}{4N_c}v_o + \frac{3}{2N_c}g_o + \frac{3}{8}\left(1 - \frac{1}{N_c^2}\right)v_s\right.$$
$$\left. -\frac{3}{4}\left(1 - \frac{1}{N_c^2}\right)g_s\right]. \qquad (C8)$$

We recall that for valence quarks $v_s^{\text{fact}} = 1$ while $v_o^{\text{fact}} = g_s^{\text{fact}} = g_o^{\text{fact}} = 0$.

For the parametrization of [43]

$$\widetilde{f}_B^2 M_B B_1 = \widetilde{f}_B^2 M_B(v_s - 4g_s) = 4(\tau_V + \tau_A) + \frac{2}{N_c}(\omega_V + \omega_A) \qquad (C9)$$

$$\widetilde{f}_B^2 M_B B_2 = \widetilde{f}_B^2 M_B(v_s - g_s) = -2(\tau_V - \tau_A) - \frac{1}{N_c}(\omega_V - \omega_A) \qquad (C10)$$

$$\widetilde{f}_B^2 M_B \varepsilon_1 = \widetilde{f}_B^2 M_B(v_o - 4g_o)$$
$$= -\frac{2}{N_c}(\tau_V + \tau_A) + \left(1 - \frac{1}{N_c^2}\right)(\omega_V + \omega_A) \qquad (C11)$$

$$\widetilde{f}_B^2 M_B \varepsilon_2 = \widetilde{f}_B^2 M_B(v_o - g_o)$$
$$= \frac{1}{N_c}(\tau_V - \tau_A) - \frac{1}{2}\left(1 - \frac{1}{N_c^2}\right)(\omega_V - \omega_A), \qquad (C12)$$

with the inverse relations

$$\omega_V = \widetilde{f}_B^2 M_B\left[\frac{1}{4N_c}B_1 - \frac{1}{2N_c}B_2 + \frac{1}{2}\epsilon_1 - \epsilon_2\right] \qquad (C13)$$

$$\omega_A = \widetilde{f}_B^2 M_B\left[\frac{1}{4N_c}B_1 + \frac{1}{2N_c}B_2 + \frac{1}{2}\epsilon_1 + \epsilon_2\right] \qquad (C14)$$

$$\tau_V = \widetilde{f}_B^2 M_B\left[\frac{1}{8}\left(1 - \frac{1}{N_c^2}\right)B_1 - \frac{1}{4}\left(1 - \frac{1}{N_c^2}\right)B_2\right.$$
$$\left. -\frac{1}{4N_c}\epsilon_1 + \frac{1}{2N_c}\epsilon_2\right] \qquad (C15)$$

$$\tau_A = \widetilde{f}_B^2 M_B\left[\frac{1}{8}\left(1 - \frac{1}{N_c^2}\right)B_1 + \frac{1}{4}\left(1 - \frac{1}{N_c^2}\right)B_2\right.$$
$$\left. -\frac{1}{4N_c}\epsilon_1 - \frac{1}{2N_c}\epsilon_2\right]. \qquad (C16)$$

All these relations hold for each light quark flavor separately.

In the $\Delta B = 2$ transitions $B_{(s)}^0 \to \bar{B}_{(s)}^0$ determining the width splitting in the B-$\bar{B}$ systems one encounters two four-

fermion operators [9], both color-nonsinglet in the *t*-channel. They are naturally parametrized as

$$\langle B_i | \bar{b}_i \gamma_\mu (1 - \gamma_5) q^i \bar{b}_j \gamma_\nu (1 - \gamma_5) q^j | \bar{B}_q \rangle$$
$$= -2 \tilde{f}_B^2 (\tilde{v} P_\mu P_\nu - \tilde{g} g_{\mu\nu} M_B^2). \qquad (C17)$$

The nonvalence matrix elements vanish. There is a standard notation $\tilde{B}_B$ for $[1/(1 + 1/N_c)](\tilde{v} - 4\tilde{g})$:

$$\langle B_q | \bar{b}_i \gamma_\alpha (1 - \gamma_5) q^i \bar{b}_j \gamma_\alpha (1 - \gamma_5) q^j | \bar{B}_q \rangle$$
$$= -2 \left(1 + \frac{1}{N_c} \right) \tilde{B}_B \tilde{f}_B^2 M_B^2. \qquad (C18)$$

The anomalous dimension of this operator equals two times the anomalous dimensions of the $\bar{b}q$ currents, so that $\tilde{B}_B$ is renorm-invariant in one loop [7,8] (all operators above are normalized at the low point, not at m_b). The combination of the operators corresponding to the $\tilde{v}$ structure also renormalizes multiplicatively in one loop; its anomalous dimension was calculated in [9]. Power mixing of these operators is absent.

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[1] M. Voloshin and M. Shifman, Yad. Phys. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)]; Zh. Éksp. Teor. Fiz. **91**, 1180 (1986) [Sov. Phys. JETP **64**, 698 (1986)].

[2] LEP B lifetimes working group, J. Alcaraz *et al.*, http://wwwcn.cern.ch/claires/lepblife.html

[3] N. Uraltsev, Phys. Lett. B **376**, 303 (1996).

[4] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 1995), Vol. 1.

[5] I. Bigi, M. Shifman, N. Uraltsev, and A. Vainshtein, Phys. Rev. D **52**, 196 (1995).

[6] M. Voloshin and M. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)].

[7] M.A. Shifman and M.B. Voloshin, Report No. **ITEP**-54, 1986.

[8] M. Voloshin and M. Shifman, Yad. Phys. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)].

[9] V.A. Khoze, M.A. Shifman, N.G. Uraltsev, and M.B. Voloshin, Yad. Fiz. **46**, 181 (1987); [Sov. J. Nucl. Phys. **46**, 112 (1987)].

[10] M.A. Luty and R. Sundrum, Phys. Rev. D **52**, 1627 (1995).

[11] I.I. Bigi and N.G. Uraltsev, Nucl. Phys. **B423**, 33 (1994).

[12] I.I. Bigi and N.G. Uraltsev, Z. Phys. C **62**, 623 (1994).

[13] A. Czarnecki, Phys. Rev. Lett. **76**, 4124 (1996); A. Czarnecki and K. Melnikov, Nucl. Phys. **B505**, 65 (1997); A. Czarnecki, K. Melnikov, and N. Uraltsev, Phys. Rev. D **57**, 1769 (1998).

[14] C.K. Chow and D. Pirjol, Phys. Rev. D **54**, 2063 (1996).

[15] T.P. Cheng, Phys. Rev. D **13**, 2161 (1976).

[16] E. Jenkins, Phys. Rev. D **55**, R10 (1997).

[17] D. Pirjol and T.M. Yan, Phys. Rev. D **56**, 5483 (1997).

[18] Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[19] M.A. Bég and A. Zepeda, Phys. Rev. D **6**, 2912 (1972).

[20] E. Bagan *et al.*, Phys. Lett. B **278**, 457 (1992); for a review see [44].

[21] G. Barton, *Introduction to Dispersion Techniques in Field Theory* (W.A. Benjamin, New York, 1965), Chap 12.

[22] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. B **334**, 175 (1994); **339**, 151 (1994).

[23] V.M. Beliaev *et al.*, Phys. Rev. D **51**, 6177 (1995).

[24] H.G. Dosch and S. Narison, Phys. Lett. B **368**, 163 (1996).

[25] T.M. Aliev *et al.*, Phys. Lett. B **334**, 169 (1994); J. Phys. G **22**, 1759 (1996).

[26] J. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).

[27] J.F. Donoghue and E. Golowich, Phys. Rev. D **49**, 1513 (1994).

[28] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).

[29] D.I. Dyakonov and M.I. Eides, Sov. Phys. JETP **54**, 232 (1981).

[30] V. Chernyak, Nucl. Phys. **B457**, 96 (1995).

[31] I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994).

[32] C.K. Chow and D. Pirjol, Phys. Rev. D **53**, 3998 (1996).

[33] I. Bigi, M. Shifman, and N.G. Uraltsev, Annu. Rev. Nucl. Part. Sci. **47**, 591 (1997).

[34] A. Czarnecki, K. Melnikov, and N. Uraltsev, Phys. Rev. Lett. **80**, 3189 (1998).

[35] N. Uraltsev, in *Heavy Flavour Physics: A Probe of Nature's Grand Design*, Proceedings of the International School of Physics ''Enrico Fermi,'' Course CXXXVII, Varenna, 1997, edited by I. Bigi and L. Moroni (IOS Press, Amsterdam, in press), hep-ph/9804275.

[36] N. Isgur, M.B. Wise, and M. Youssefmir, Phys. Lett. B **254**, 215 (1991).

[37] M. Voloshin, Phys. Rev. D **46**, 3062 (1992).

[38] I. Bigi, A. Grozin, M. Shifman, N. Uraltsev, and A. Vainshtein, Phys. Lett. B **339**, 160 (1994).

[39] I. Bigi, N.G. Uraltsev, and A. Vainshtein, Phys. Lett. B **293**, 430 (1992); **297**, 477(E) (1993).

[40] P. Colangelo and F. De Fazio, Phys. Lett. B **387**, 371 (1996).

[41] H.Y. Cheng, Phys. Rev. D **56**, 2783 (1997).

[42] M.S. Baek, J. Lee, C. Liu, and H.S. Song, Phys. Rev. D **57**, 4091 (1998).

[43] M. Neubert and C.T. Sachrajda, Nucl. Phys. **B483**, 339 (1997).

[44] P. Ball and V.M. Braun, Phys. Rev. D **58**, 094016 (1998).

[45] B. Blok, R. Dikeman, and M. Shifman, Phys. Rev. D **51**, 6167 (1995).

[46] M. Gremm and A. Kapustin, Phys. Rev. D **55**, 6924 (1997).

[47] S.J. Brodsky, P. Lepage, and P. Mackenzie, Phys. Rev. D **28**, 228 (1983).

[48] I. Bigi and N. Uraltsev, Phys. Lett. B **280**, 271 (1992).

[49] I. Bigi, B. Blok, M. Shifman, N. Uraltsev, and A. Vainshtein, in *B Decays*, 2nd ed., edited by S. Stone (World Scientific, Singapore, 1994), p. 132.

[50] H.Y. Cheng and K.-C. Yang, Phys. Rev. D **59**, 014011 (1999).

[51] S.L. Adler, Phys. Rev. **177**, 2426 (1969).