Quarkonium production through hard comover scattering

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We propose a qualitatively new mechanism for quarkonium production, motivated by the global features of the experimental data and by the successes or failures of existing models. In QCD, heavy quarks are created in conjunction with a bremsstrahlung color field emitted by the colliding partons. We study the effects of perturbative gluon exchange between the quark pair and a comoving color field. Such scattering can flip the spin and color of the quarks to create a nonvanishing overlap with the wave function of physical quarkonium. Several observed features that are difficult to understand in current models find simple explanations. Transverse gluon exchange produces unpolarized J/ψ's, the χ_{c1} and χ_{c2} states are produced at similar rates, and the anomalous dependence of the J/ψ cross section on the nuclear target size can be qualitatively understood. [S0556-2821(99)02701-0]

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I. INTRODUCTION AND SUMMARY

A. Introduction

Quarkonium production has turned out to be a challenge as well as an inspiration for our understanding of hard QCD processes [1–7]. In the case of standard inclusive processes, the theoretical framework is uniquely defined by the QCD factorization theorem [8]. This theorem allows a physical cross section σ to be expressed as a product of universal parton distribution and fragmentation functions multiplied by a subprocess cross section $\hat{\sigma}$, which is calculable in perturbative QCD (PQCD). The factorization theorem relies on a completeness sum over the final state and does not apply to the quarkonium cross section, which constitutes only a small fraction of the total heavy quark production cross section.

While a theoretical description of quarkonium production is thus more model dependent, it can potentially reveal more about the dynamics of hard processes than can be learned from, e.g., the total heavy quark cross section. In particular, it is intuitively plausible that the quarkonium cross section is sensitive to reinteractions with partons created along with the heavy quark pair. Thus quarkonium production can serve as a ''thermometer'' of the environment, as has been recognized in the search for a quark-gluon plasma in heavy ion collisions [9]. In this paper we wish to explore the possibility that rescattering of the heavy quarks causes the puzzling anomalies seen in quarkonium hadroproduction.

There is independent evidence that the environment in charm hadroproduction is rather ''hot.'' In $\pi^- N \to D\bar{D}+X$ the observed spread in relative azimuthal angle of the D-mesons requires an average intrinsic transverse momentum of the incoming partons $\langle k_\perp^2 \rangle \simeq 1\ \mathrm{GeV}^2$ [10]. The ''leading particle'' asymmetry between D^- and D^+ is larger than expected from PQCD, and persists for D mesons produced with $k_\perp^2 \lesssim 10\ \mathrm{GeV}^2$ [11]. Both effects are weaker for photoproduced charm [10,12].

We shall study the effects of perturbative gluon exchange between the heavy quark pair and a comoving [13] color field. The interaction is assumed to occur at an early stage, before the pair has expanded to the size of physical quarkonium. Hence only comovers which are created (via bremsstrahlung) in the hard process itself are relevant, whereas interactions with beam fragments at typical hadronic distances ~ 1 fm are ignored. In this sense our approach differs from that of the ''color evaporation model'' (CEM) [14,15], which only considers late, non-perturbative interactions of the heavy quarks. On the other hand, similarly to the CEM our quark pairs are produced near threshold. Hence many of the phenomenological successes of the CEM concerning the dependence of quarkonium cross sections on various kinematical variables are incorporated in our model. The ''color octet model'' (COM) [16], also considers late interactions, through an expansion in powers of the relative velocity v of the bound quarks as specified by nonrelativistic QCD (NRQCD) [17]. This expansion is general and should hold for any description of quarkonium production, including ours. The higher order $(v/c)^n$ terms need not, however, give a dominant contribution to the cross section. To our knowledge, the COM assumption that the heavy quark pair is unaffected by earlier reinteractions with its environment has not been proven.

Data on charmonium and bottomonium production is available for a wide variety of beams, targets and kinematical conditions. Comparisons with the COM and CEM approaches have met with some successes, but also with difficulties [1–7]. The data suggests a production dynamics which in some respects differs from the late and soft reinteraction scheme of the CEM and COM. In particular, (A) the heavy quark pair turns color singlet at an early stage, while the pair is still compact (i.e., small compared to the size of the quarkonium wave function), (B) in hadroproduction there is at least one secondary gluon exchange after the primary, heavy quark production vertex, and (C) the ''anomalies'' of quarkonium production depend only weakly on the quark mass m, on the c.m. energy and on the transverse momentum $p_\perp$.

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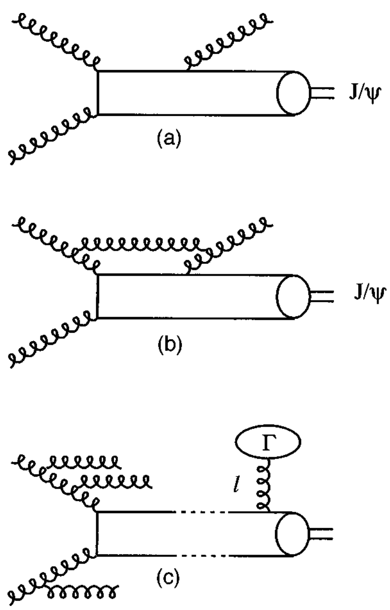

FIG. 1. Basic processes for J/ψ hadroproduction in the CSM [(a) and (b)] and in our model [(c)].

We discuss the experimental basis for these features in Sec. II and then develop our QCD scenario in Sec. III. This scenario applies to quarkonium production at both moderate and large transverse momentum. However, we shall limit our discussion and the calculations presented in Sec. IV to the total quarkonium cross section, i.e., moderate x_F and $p_\perp \sim m$. In the rest of the present section we summarize our results. Conclusions and an outlook are presented in Sec. V.

B. Summary

The basic color singlet mechanism (CSM) [18], which is known to grossly underestimate the J/ψ hadroproduction cross section, is shown in Fig. 1(a). The gluon emission takes place in a (proper) time $\tau \sim 1/m$, simultaneously with the heavy quark production process. This is compatible with feature (A), but since there are no relevant later interactions the CSM does not agree with (B). The situation is qualitatively the same for loop corrections to the CSM [Fig. 1(b)], since the space-time scale of the loop is $1/m$.

Prior to the heavy quark production vertex the colliding partons radiate gluons as part of the normal QCD structure function evolution. The space-time scale of this process is determined by the virtuality k^2 of the partons which couple to the heavy quark line. As is characteristic of evolution processes, the k^2 distribution $\alpha_s(k^2)dk^2/k^2$ is logarithmic between a lower cutoff determined by the (perturbative) factorization scale and an upper limit given by the heavy quark mass m. Thus the effective value of $|k^2|$ is given by a perturbative scale which we denote by μ^2. This scale grows with m^2 but satisfies $\mu^2 \ll m^2$. We will investigate the effects of rescattering at this hardness scale μ.

The approach presented in Sec. III is based on a perturbative reinteraction of momentum transfer $l \sim \mu$ between the heavy quarks and a classical color field Γ [Fig. 1(c)]. This field is assumed to originate from gluon bremsstrahlung in the $gg \to Q\overline{Q}$ subprocess. The reinteraction can occur long after the heavy quarks are created provided the field Γ is comoving with the quark pair. Hence we shall assume Γ to be isotropic in the pair rest frame. The scale μ of the secondary interaction is smaller than the quark mass scale m of the CSM but larger than the bound state scale $\alpha_s m$ of the COM.[1] The existence of the comoving color field Γ in hadroproduction is our main postulate, motivated by the data. There should be no corresponding field in the current fragmentation region of photo- and leptoproduction, since photons do not radiate gluons (at lowest order).

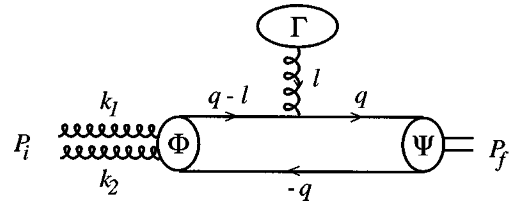

FIG. 2. A perturbative interaction between the quark pair and a gluon from the color field Γ creates an overlap between the $Q\overline{Q}$ wave function Φ from the $gg \to Q\overline{Q}$ process with the physical quarkonium wave function Ψ. There is a second diagram where the gluon attaches to the antiquark.

Our quarkonium hadroproduction amplitude is essentially given by the perturbative diagram of Fig. 2 (together with the diagram corresponding to rescattering of the antiquark). Here Φ is the (color octet) wave function of the heavy quark pair produced in the fusion process $gg \to Q\overline{Q}$ and Ψ is the (color singlet) quarkonium wave function.

As we shall see, there are two production mechanisms for spin triplet S-wave quarkonia such as the J/ψ. In the $gg \to Q\overline{Q}$ subprocess the quark pair can either be produced in an $S=L=0$ state, followed by a spin-flip interaction with a transverse gluon from the color field Γ, or the pair can be produced with $S=L=1$, followed by a spin-conserving interaction with a longitudinal gluon. The first contribution gives unpolarized quarkonia, since the quark pair is produced with total angular momentum $J=0$ and the color field Γ is isotropic. The second contribution turns out to give quarkonia with a transverse polarization. The striking experimental fact [19–21] that the J/ψ and ψ' are produced unpolarized (at moderate $p_\perp$) thus implies that the former mechanism dominates, i.e., that the gluons in Γ are transversely polarized.

The P-wave quarkonium states χ_J are produced from quark pairs with $S=0$, $L=1$ followed by a spin-flip interaction with a transverse gluon from Γ. The calculated cross section ratio $\sigma(\chi_1)/\sigma(\chi_2)=3/5$, in agreement with the ratio

[1]For the charmonium system, some of these scales are numerically similar, but should be distinguished for reasons of principle. The scales do differ for bottomonium.

measured in πN collisions for charmonium, 0.6 ± 0.3 [22,23]. We also find that due to the indirect $\chi_{cJ}\to J/\psi+\gamma$ contributions the total J/ψ polarization is slightly longitudinal, $\lambda=-0.14$ [cf. Eq. (36)]. Taking into account also the CSM mechanism, which dominantly produces χ_{c2}'s with $J_z=\pm2$ that decay into transversely polarized J/ψ's [24], the expected ratio $\sigma(\chi_1)/\sigma(\chi_2)<3/5$ and $\lambda>-0.14$. This is compatible with data as long as the CSM does not dominate the rescattering mechanism (Fig. 2) for χ_2 production.

The χ_J wave function Ψ (Fig. 2) vanishes at the origin, which suppresses its overlap with small sized ($\sim1/m$) heavy quark pairs. Thus in the CSM the relative production rate of *P*- and *S*-wave charmonium states is governed by the small ratio $|R'_\chi(0)/2m|^2/|R_\psi(0)|^2\simeq0.01$ [25]. In our approach the initially compact quark pair expands, with velocity $\sim\mu/m$, before the gluon exchange in Fig. 2 gives the quark pair the bound state quantum numbers. The wave function Φ is thus an expanded version of the quark pair wave function created in the $gg\to Q\bar{Q}$ subprocess. We model this by a scale factor ρ, which we fit to the measured $\sigma(\chi_{cJ})/\sigma_{dir}(J/\psi)$ cross section ratio. We find that we need $\rho\simeq3$, suggesting significant expansion of the quark pair before reinteraction in χ_{cJ} production.

Our approach can also be applied to quarkonium production at high $p_\perp\gg m$, where the dominant production mechanism is gluon fragmentation [26]. The fragmenting gluon is initially highly virtual and radiates gluons with hardness ranging from the factorization scale up to $p_\perp$. The gluons of relatively small hardness $\sim\mu$ can form a color field comoving with the quark pair. We plan to study the detailed predictions of our scenario for high $p_\perp$ quarkonium production in a future publication.

We shall also not discuss here the special features of quarkonium production which appear at high x_F, and may be related to intrinsic charm [27] and scattering from light constituents [28]. Thus our discussion is limited to the bulk of the charmonium cross section only, which (at fixed target energies) originates from partons with $\langle x\rangle\sim0.1$ and $\langle p_\perp\rangle\sim m$.

II. QUALITATIVE FEATURES OF THE DATA

The data on quarkonium production shows many interesting features and regularities. Several of them are left unexplained (some are even contradicted) by the dynamics assumed in the color octet model (COM) and the color evaporation model (CEM). Here we wish to make the phenomenological case for the three general features [(A)–(C)] of the production dynamics that we listed in Sec. I.

A. Early color neutralization

Heavy quarks are produced in $Q\bar{Q}$ pairs of (transverse) size $\sim1/m$, where m is the quark mass. The pair is thus initially much smaller than the Bohr radius of quarkonium bound states, which is of order $1/(\alpha_s m)$. If the quark pair ceases to interact with its surroundings (in particular, turns color singlet) while it is still in such a compact configuration then the production rates of all *nS* states are proportional to $|R_n(0)|^2$, the square of their wave functions at the origin. Analogous proportionality holds for the other $^{2S+1}L_J$ quarkonium states.

The above argument requires no assumption about how the compact pair is produced. The "$R(0)$ proportionality test" is thus a good indication of whether the color neutralization occurs early or late, as measured by the size of the quark pair. This test should moreover be quite sensitive since the higher radial excitations have a mass near open flavor threshold. Late scattering of the $c\bar{c}$ system will thus affect the ψ' (44 MeV below $D\bar{D}$ threshold) more than the ψ (630 MeV below threshold). This is supported by the observation that the $\sigma(\psi')/\sigma(J/\psi)$ ratio is significantly *reduced* in central nucleus-nucleus (*SU*) collisions [29], as would be expected due to late interactions with comoving nuclear fragments (or plasma).

It has been pointed out [30] that due to the moderate mass of the charm quark the wave function of charmonium is probed beyond its origin. In particular, since the diffractive $Q\bar{Q}$ photoproduction amplitude is proportional to the square of the transverse $Q\bar{Q}$ separation, the overlap integral between the quark pair and the bound state wave functions gets a negative contribution beyond the first node in the radial wave function of the ψ'. The predicted [30] ratio $\sigma(\psi')/\sigma(J/\psi)=0.17$ is thus smaller than the $|R(0)|^2$ ratio and agrees with a recent measurement at the DESY *ep* collider $0.150\pm0.027\pm0.018\pm0.011$ [31].

The $\sigma(\psi')/\sigma(J/\psi)$ ratio is remarkably universal in inelastic hadroproduction processes, being nearly independent of the nature of the beam hadron and the target nucleus, and also of the energy and of x_F [24,29,32]. This also holds for $\Upsilon(nS)$ states. The measured ratio for the directly produced J/ψ cross section (from which decay contributions have been subtracted) moreover is consistent with [24]

$$\frac{\sigma(\psi')}{\sigma_{dir}(J/\psi)}=\frac{\Gamma(\psi'\to e^+e^-)}{\Gamma(J/\psi\to e^+e^-)}\frac{M^3_{J/\psi}}{M^3_{\psi'}}\simeq0.24. \tag{1}$$

The hadroproduction ratio (1) is somewhat larger than the one measured in diffractive photoproduction, indicating that in hadroproduction the inelastic cross section is more closely proportional to the wave function at the origin.[2]

Interesting subtleties aside, the data clearly suggests the relevance of the perturbative $Q\bar{Q}$ wave function for quarkonium photo- and hadroproduction. In the COM and CEM approaches, on the other hand, the heavy quark pair turns

[2]In the approach discussed in this paper, the scattering amplitude is proportional only to the *first* power of the $Q\bar{Q}$ separation. There is also high $p_\perp$ data from the Tevatron [3] which indicates that the $\sigma(\psi')/\sigma(J/\psi)$ ratio is still larger than in Eq. (1). This may imply an even more pointlike production dynamics at large $p_\perp$.

into a color singlet only after it has expanded to a size comparable to that of the bound state. There is then no reason to expect the cross section ratio to satisfy Eq. (1) (although this value is also not excluded by those models). We believe that the agreement of the quarkonium cross section ratios with expectations based on the wave function of the quarks created in the hard subprocess is not an accident. This implies that the pair decouples from its environment while it is still compact (except in the presence of nuclear comovers [13]).

B. Reinteraction with a color field

Quarkonium data provides two indications that a rescattering of the heavy quark pair with a comoving color field is important in hadroproduction.

1. Photoproduction

Photons do not radiate gluons.[3] At the early stages of heavy quark creation through the $\gamma g \to Q\bar{Q}$ subprocess we should therefore expect *no* comoving color field in the photon fragmentation region. With the rescattering process of Fig. 1(c) thus eliminated, the production process should be dominated by the color singlet mechanism (CSM) of Fig. 1(a). It is indeed one of the remarkable facts of quarkonium production that the CSM works very well for inelastic J/ψ photoproduction [33–35], whereas the same model underestimates the hadroproduction cross section by an order of magnitude [2–4]. This suggests that hadroproduction dynamics is coupled to initial parton bremsstrahlung.

The COM parameters which fit J/ψ hadroproduction tend to overestimate the photoproduction cross section [7,34,35], although it is possible that the discrepancy could be due to higher order effects [36].

2. Nuclear target dependence

Cross sections of hard incoherent processes on nuclear targets A are expected to scale like the atomic number of the target, $\sigma(A) \propto A^{\alpha}$, with $\alpha \simeq 1$. Modifications due to the A-dependence of the quark structure functions are minor in the presently relevant kinematic range. This is verified by high mass lepton pair production (the Drell-Yan process), for which $\alpha \simeq 1.00$ is observed [29].

Charm quark pairs produced in the beam fragmentation region have large Lorentz factors and expand only after leaving the nucleus. While compact, the uncertainty in the energy of the $c\bar{c}$ pairs is large and they couple both to open charm $(D\bar{D}, \bar{D}\Lambda_c, \dots)$ and to quarkonium $(J/\psi, \psi', \dots)$ channels. In the absence of effects due to partons comoving with the pair one should therefore expect the same A-dependence for open and hidden charm, with $\alpha \simeq 1$.

Data shows that $\alpha = 1.02 \pm 0.03 \pm 0.02$ for $D/\bar{D}$ production at $\langle x_F \rangle = 0.031$ [37], whereas $\alpha = 0.92 \pm 0.01$ for J/ψ and ψ' [29]. The deviation of α from unity for charmonium appears to be independent of E_{CM}, and increases with the charmonium momentum fraction x_F. This, and the similar A-dependence of J/ψ and ψ', indicates that the nuclear suppression is not due to an expansion of the $c\bar{c}$ inside the target. A plausible explanation is that a further interaction, beyond the nucleus, between the heavy quark pair and comoving gluons is required for charmonium formation.

Based on the measured A-dependence of charmonium production an effective "absorption" cross section $\sigma_{abs} \simeq 7.3 \pm 0.6$ mb was obtained [38] in a Glauber framework. This cross section is too large for a compact $c\bar{c}$ pair [29,39,40] and should, in the present framework, be interpreted as the joint cross section of the $c\bar{c}$ pair and the comoving gluon field. The size of σ_{abs} is then reasonable, since the gluons are at a relatively large distance $\sim 1/\mu$ from the quark pair. An analogous interpretation of the nuclear suppression, albeit in a different dynamical picture, was earlier put forward in Ref. [41].

There is no evidence for a nuclear target suppression of inelastic J/ψ *photo-* and *lepto*production. On the contrary, there is an indication [42] of a slight nuclear *enhancement* at $x_F \simeq 0.7$ $(\alpha = 1.05 \pm 0.03)$, in stark contrast to the strong nuclear suppression seen at this x_F in hadroproduction [29]. Due to the absence of comoving gluons and the validity of the CSM we expect $\alpha \simeq 1$. The enhancement may signal a slight antishadowing of the nuclear gluon distribution [43].

The COM and CEM assume that the process which turns the color octet $Q\bar{Q}$ into physical quarkonium is independent of the nature of the beam and target. Both models expect photo- and hadroproduction of charmonium to have the same target A-dependence, which should moreover equal that of open charm.

C. Dependence on m, E_{CM} and $p_\perp$

The available data shows that the "anomalies" of quarkonium production are rather insensitive to the quark mass $m = m_c$, m_b and to variations in kinematic variables such as the total energy E_{CM} and the quarkonium transverse momentum $p_\perp$.

The measured cross section of $\Upsilon(3S)$, which presumably is directly produced, exceeds the CSM prediction by an order of magnitude [3,44], in analogy to the original "ψ' anomaly." The $\Upsilon(1S)$ and $\Upsilon(2S)$ cross sections are more compatible with the CSM, which predicts them to originate almost exclusively from P-wave decays. The situation is thus similar to that of the charmonium system before the P-wave contributions were experimentally separated and found not to

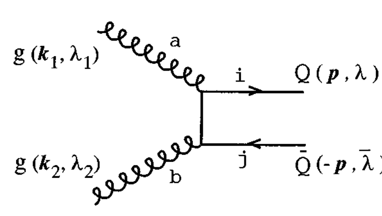

FIG. 3. Notation for the CM amplitude $\Phi(\boldsymbol{p})$. The spin projections $\lambda_{1,2} = \pm 1$ and λ, $\bar{\lambda} = \pm 1/2$ all refer to the z-axis, taken as the direction of $\boldsymbol{k}_1$. Only one of three contributing Feynman diagrams is shown.

[3]Except via higher order resolved processes which are unimportant here.

TABLE I. The amplitude $\Phi^{[8]}$ of the $gg\to Q\overline{Q}$ process to first order in $|\boldsymbol{p}|/m$.

$(\Phi^{[8]})^{SS_z}_{LL_z}$		$S=0$	$S=1$	
			$S_z=0$	$S_z=\pm1$
$L=0$		$\lambda_1 d_{abc}\delta^{-\lambda_2}_{\lambda_1}$	0	0
$L=1$	$L_z=0$	$\frac{\lvert\boldsymbol{p}\rvert}{3m}\lambda_1 i f_{abc}\delta^{-\lambda_2}_{\lambda_1}$	$\frac{\lvert\boldsymbol{p}\rvert}{3m}d_{abc}\delta^{-\lambda_2}_{\lambda_1}$	0
	$L_z=\pm1$	0	0	$-\frac{\lvert\boldsymbol{p}\rvert}{3m}d_{abc}(\delta^{L_z}_{\lambda_1}\delta^{S_z}_{\lambda_2}+\delta^{S_z}_{\lambda_1}\delta^{L_z}_{\lambda_2})$

account for the bulk of the J/ψ cross section. It would obviously be very important to measure the directly produced fractions of the Υ states.

All $\Upsilon(nS)$ states have similar nuclear target A-dependence, with $\alpha=0.962\pm0.014$ [45]. The nuclear suppression is thus smaller than that for charmonium discussed in Sec. II B2, but still significant compared to the Drell-Yan case. In our approach, the smaller suppression for bottomonium is related to the effective distance $\sim 1/\mu$ between the comoving gluon field and the heavy quarks, which decreases as the inverse of the quark mass.

The $\Upsilon(3S)$ total cross section anomaly has been observed at both fixed target [45,46] and collider energies [44], i.e., for $4.2\lesssim E_{CM}/M_\Upsilon\lesssim 190$. Similarly, the nuclear target suppression of charmonium production seems to be independent of the projectile type (π or p) and energy (for 150 GeV $<E_{LAB}<800$ GeV) [19,29]. The discrepancy between the CSM and data on direct J/ψ production is somewhat larger at high transverse momentum ($p_\perp \gg m$) than for the total cross section. The relative contribution of P-wave decays to J/ψ production is roughly independent of $p_\perp$ [3].

The above features suggest that the anomalies observed in quarkonium production are ''leading twist'' in the quark mass m, in the total energy E_{CM} and in $p_\perp$, in the sense that the effects do not vanish as inverse powers of any of those variables.

In the COM, the color octet contributions which account for direct J/ψ and ψ' production scale by a factor v^2 relative to the contributions from P-wave decays, and thus are relatively less important for bottomonia than for charmonia. Thus P-wave decay contributions dominate $\Upsilon(nS)$ production in the COM. In particular, $\Upsilon(3S)$ production can only be understood by assuming [46,47] that it results from the decay of an (as yet undetected) higher lying P-wave state.

III. A HARD RESCATTERING SCENARIO

In this section we address how the features [(A)–(C)] of quarkonium production, mentioned in Sec. I and discussed in more detail in Sec. II, can be understood in a QCD framework. We put forward a scenario which is consistent with those features, and which forms the basis for the explicit model studied in Sec. IV.

A. The basic heavy quark creation process

The hadroproduction of heavy quarks proceeds mainly via the gluon fusion subprocess[4] $gg\to Q\overline{Q}$. In our approach (as well as in that of COM and CEM, but not of CSM) color neutralization occurs at a time scale which is large compared to the time scale $1/m$ of the gluon fusion process. This implies that the heavy quarks are produced (nearly) on their mass shell.

A basic building block of our rescattering process of Fig. 2 is thus the amplitude $\Phi^{[8]}$ of the $gg\to Q\overline{Q}$ fusion process shown in Fig. 3, evaluated at leading order in the heavy quark momentum p in the c.m. frame, and with the quark pair in a color octet state.

This amplitude is given in Table I in terms of the spin (S) and angular momentum (L) of the $Q\overline{Q}$ pair [see Sec. IV, Eqs. (6) and (7) for definitions].

The gluon fusion amplitude $\Phi^{[1]}$ for $Q\overline{Q}$ pairs in a color singlet state, which is relevant for the CSM, can be obtained from Table I by the substitutions $f_{abc}\to 0$, $d_{abc}T^c_{ij}\to\delta_{ab}\delta_{ij}/N_c$. It has two parts, with spin and angular momentum $S=L=0$ and $S=L=1$, respectively. The former can directly form ${}^1S_0(\eta)$ quarkonia, while the latter couples to ${}^3P_{0,2}(\chi_{0,2})$ states. The χ_1 decouples since for $S=L=1$ the amplitude is symmetric in L_z and S_z, whereas the Clebsch-Gordan coefficients for the corresponding ($J=1$, J_z) state are antisymmetric in L_z and S_z (Yang's theorem).

The color octet amplitude $\Phi^{[8]}$ of Table I has contributions from $S=L=0$, $S=L=1$ and $S=0$, $L=1$. The near-threshold heavy quark cross section is dominantly $S=L=0$. In the CEM [14] the $S=0$ quark pair is assumed to turn into physical $S=1$ quarkonia through soft, nonperturbative gluon interactions. This contradicts the conservation of heavy quark spin in soft interactions, which is believed to be a general feature of QCD and follows from the nonperturbative concept of heavy quark symmetry [48].

In the COM [16] the suppression of heavy quark spin-flip appears as extra powers of the velocity $v\sim\alpha_s$ of the bound

[4]We neglect light quark fusion $q\overline{q}\to Q\overline{Q}$, which is unimportant for the total cross section. Similarly, higher order ''gluon fragmentation'' diagrams [26] are irrelevant for quarkonia produced with $p_\perp\lesssim m$.

quarks. COM production of $^3S_1$ states is suppressed by v^4 in the cross section, compared to the CSM.

B. Scenario for perturbative rescattering

For the quarks to form a quarkonium bound state their relative momentum must be of order $vm \ll m$. In current quarkonium production models the quarks are created directly in the gluon fusion process with a relative momentum of this magnitude. It is thus implicitly assumed that all later interactions are soft, commensurate with the bound state momentum scale.

Here we shall consider reinteractions with momentum transfers of $\mathcal{O}(\mu)$, related to the hardness scale of the bremsstrahlung field in the fusion subprocess. Since $v \ll \mu/m \ll 1$ the relative momentum of the quarks created in the fusion process is fairly large. The rescattering is hard and allows perturbative spin-flip interactions.

It might at first appear that the rescattering physics we have in mind is contained in the loop correction [Fig. 1(b)] to the CSM lowest order process of Fig. 1(a). However, the logarithmic enhancements which favor collinear and wee gluon bremsstrahlung are absent in the loop.[5] Consequently, the loop momentum is of order m and the spatial size of the loop is of order $1/m$. The loop should be thought of as a vertex correction to the primary process, not as the rescattering envisioned in point (B) of Sec. I.

In order to have a rescattering which is well separated from the primary fusion process we must assume that the initial state gluon radiation gives rise to a (classical) color field Γ which is comoving with the $Q\bar{Q}$ pair. The compact color octet $Q\bar{Q}$ will dominantly interact with Γ as a color monopole (massive pointlike gluon). In these relatively soft interactions the internal structure of the pair is preserved, in particular it remains a color octet [49].

The color structure and spin of the quark pair can, however, be changed in a harder, color dipole interaction with the comoving field, as depicted in Fig. 2. Here we consider only a single such interaction, and evaluate it perturbatively. The main unknown in Fig. 2 is then the postulated color field $\Gamma^\mu(l)$. Quarkonium production in fact offers an opportunity of detecting whether such fields are created in hard interactions.

The physical picture sketched above implies that the classical color source $\Gamma^\mu(l)$ of gluons with momentum l should have the following properties.

Since the field Γ originates from bremstrahlung in the gluon fusion subprocess it is independent of the beam and target.

Since only those components of the radiated field which are comoving with the quark pair are relevant, the spatial distribution of Γ is isotropic in the rest frame of the $Q\bar{Q}$ pair.

[5]Leading logarithms in hard processes originate from tree diagrams. It is also straightforward to verify the absence of logarithms directly from the loop integral.

The 3-momentum exchange $\boldsymbol{l}$ is of order μ. For the heavy quark propagators in Fig. 2 to be nearly on-shell the energy component satisfies $|l^0| \ll |\boldsymbol{l}|$ in the quarkonium rest frame. The field Γ effectively acts as a time-independent color source in the rescattering process.

By gauge invariance we have $l_\mu \Gamma^\mu = 0$. Hence the field Γ^μ can be expressed in terms of its transverse ($\lambda = \pm 1$) and longitudinal ($\lambda = 0$) components as

$$\Gamma^\mu(l) = \sum_\lambda \varepsilon_\lambda^\mu(l)\Gamma_\lambda(l), \tag{2}$$

where ε_λ^μ is the gluon polarization vector. As we shall see in Sec. IV, the quarkonium data requires that $|\Gamma^0(l)| \ll |\boldsymbol{\Gamma}(l)|$, which is equivalent to $|\Gamma_{\lambda=0}(l)| \ll |\Gamma_{\lambda=\pm 1}(l)|$ since $l_\mu \varepsilon_\lambda^\mu = 0$ and $|l^0| \ll |\boldsymbol{l}|$. Hence data suggests that the transverse components $\Gamma_{\pm 1}$ of the color field dominate.

Quark pairs that in the gluon fusion process are created as color singlets can evolve directly into $^1S_0$ and $^3P_{0,2}$ quarkonia, as in the CSM. The influence of the color field Γ on this process is disruptive. To first order in the dipole interaction the pair turns into a color octet, thus losing its overlap with the quarkonium wave function.

Since the direct CSM process does not require any perturbative rescattering, one might expect that it will dominate the contribution of Fig. 2 for, e.g., χ_2 production. However, direct production of P states is suppressed because the quarkonium wave function vanishes at the origin. In the rescattering process the quark pair is produced with a comparatively high relative momentum of $\mathcal{O}(\mu)$ and the spatial size of its wave function increases before the rescattering. The expansion factor ρ depends on the time interval between the gluon fusion and rescattering processes. Our model can explain the relative rates of χ_{c2}, χ_{c1} and J/ψ production provided $\rho \simeq 3$. The expansion will have a smaller effect on S-wave cross sections as long as the pair stays compact compared to the size of the quarkonium wave function.

In the next section we construct a simple, specific model for our scenario and derive its quantitative predictions. We show how the observed fact that the J/ψ and ψ' are produced unpolarized requires the field Γ to be dominantly transverse. We then have two free parameters, the strength of the transverse field Γ and the spatial expansion parameter ρ, which as mentioned above is related to the time interval between the quark creation and rescattering processes.

IV. QUARKONIUM CROSS SECTIONS AND POLARIZATION

A. The quarkonium production amplitude

As illustrated in Fig. 2, the production amplitude can be viewed as a transition from a color octet $Q\bar{Q}$ state, created in the gluon fusion process and described by the wave function Φ, to a quarkonium state specified by the wave function Ψ. The transition is mediated by a gluon exchange $\mathcal{R}$ with the color field Γ. The transition amplitude can be expressed as an overlap integral,

$$\mathcal{M}=\sum_{L_z,S_z}\langle LL_z;SS_z|JJ_z\rangle\sum_{\lambda\bar{\lambda},\sigma\bar{\sigma}}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{d^3\boldsymbol{q}}{(2\pi)^3}\times\Phi^{[8]}_{\lambda\bar{\lambda}}(\boldsymbol{p})\mathcal{R}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{p},\boldsymbol{q})\Psi^{L_zS_z}_{\sigma\bar{\sigma}}(\boldsymbol{q})^* \quad (3)$$

which can readily be derived starting from the usual Feynman formulation. We work in a non-relativistic approximation where all quark lines are on-shell and have 3-momenta much smaller than the quark mass m. The z-axis is taken along the direction of the initial gluon momentum $\boldsymbol{k}_1$ in Fig. 2. The orbital angular momentum and spin components of the quarkonium state are denoted by L_z and S_z, whereas $\lambda,\bar{\lambda}$ and $\sigma,\bar{\sigma}$ are the spin projections of the Q and $\bar{Q}$ along the z-axis before and after the rescattering, respectively. The relative momentum between the Q and $\bar{Q}$ is $2\boldsymbol{p}$ before and $2\boldsymbol{q}$ after the rescattering, while $\boldsymbol{l}=\boldsymbol{P}_f-\boldsymbol{P}_i$ is the momentum of the exchanged gluon. In the quarkonium rest frame $\boldsymbol{P}_f=0$ and $\boldsymbol{l}=-\boldsymbol{P}_i$.

1. The gluon fusion amplitude Φ

A standard calculation of the gluon fusion process $gg\to Q\bar{Q}$ of Fig. 3 gives the $Q\bar{Q}$ wave function, to first order in the quark c.m. momentum $\boldsymbol{p}$, as

$$\Phi^{[8]}_{\lambda\bar{\lambda}}(\boldsymbol{p})=g^2T^c_{ij}\Big\{i\Big(d_{abc}+\frac{p^z}{m}if_{abc}\Big)(\varepsilon_1\times\varepsilon_2)^z\delta^{-\bar{\lambda}}_{\lambda}+2\lambda d_{abc}\frac{1}{m}[\boldsymbol{\varepsilon}_1\cdot\varepsilon_2\,\boldsymbol{p}\cdot\boldsymbol{e}(0)^*\delta^{-\bar{\lambda}}_{\lambda}-(\varepsilon_1\cdot\boldsymbol{p}\,\varepsilon_2\cdot\boldsymbol{e}(2\lambda)^*+\varepsilon_2\cdot\boldsymbol{p}\,\varepsilon_1\cdot\boldsymbol{e}(2\lambda)^*)\sqrt{2}\delta^{\bar{\lambda}}_{\lambda}]\Big\} \quad (4a)$$

$$\Phi^{[1]}_{\lambda\bar{\lambda}}=\Phi^{[8]}_{\lambda\bar{\lambda}}\{f_{abc}\to 0,\ d_{abc}T^c_{ij}\to\delta_{ab}\delta_{ij}/N_c\}. \quad (4b)$$

As indicated, the amplitude $\Phi^{[1]}$ for a color singlet pair can be obtained from the color octet amplitude $\Phi^{[8]}$ by a trivial substitution. The incoming gluons have colors a,b, momenta $m(1,0_\perp,\pm 1)$ and polarization vectors $\varepsilon_1,\varepsilon_2$. The quarks $Q,\bar{Q}$ of colors i,j have momenta $(m,\pm\boldsymbol{p})$ and spin projections $\lambda,\bar{\lambda}$ along the z-axis. The spin polarization vector $\boldsymbol{e}$ is defined conventionally by

$$e(\pm 1)=(0,\mp 1,-i,0)/\sqrt{2} \quad (5a)$$

$$e(0)=(0,0,0,1). \quad (5b)$$

Lorentz transforming from the $gg\to Q\bar{Q}$ c.m. to the quarkonium rest frame shifts any four-vector by an amount of $\mathcal{O}(|\boldsymbol{l}|/m)$. In particular, $(\boldsymbol{\varepsilon}_1\times\varepsilon_2)^z$ is shifted to $(\varepsilon_1\times\varepsilon_2)^z[1+\mathcal{O}(\boldsymbol{l}^2/m^2)]$. To the accuracy of our calculation we can ignore the boost and use the c.m. amplitude (4a) directly in Eq. (3).

It is instructive to express the amplitudes (4a),(4b) in terms of the spin S and orbital angular momentum L of the $Q\bar{Q}$ pair. The amplitudes Φ^{SS_z} of definite $Q\bar{Q}$ spin are defined by

$$\Phi^{00}=-(\Phi_{1/2,-1/2}+\Phi_{-1/2,1/2})/\sqrt{2} \quad (6a)$$

$$\Phi^{10}=-(\Phi_{1/2,-1/2}-\Phi_{-1/2,1/2})/\sqrt{2} \quad (6b)$$

$$\Phi^{1,\pm 1}=\pm\Phi_{\pm 1/2,\pm 1/2}. \quad (6c)$$

The angular momentum components are defined via a partial wave expansion,

$$\Phi^{SS_z}=\sum_{LL_z}\sqrt{4\pi(2L+1)}Y^{L_z}_L(\theta,\phi)\sqrt{2}g^2T^c_{ij}\Phi^{SS_z}_{LL_z}, \quad (7)$$

where $Y^{L_z}_L$ are standard spherical harmonics [50]. The amplitudes $\Phi^{SS_z}_{LL_z}$ are given in Table I (see also [47]).

2. The rescattering kernel $\mathcal{R}$

The rescattering kernel $\mathcal{R}$ has two terms, describing gluon scattering from the quark and from the antiquark,

$$\mathcal{R}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{p},\boldsymbol{q})=(2\pi)^3\delta^3\Big(\boldsymbol{p}-\boldsymbol{q}+\frac{\boldsymbol{l}}{2}\Big)\times\mathcal{R}^{Q}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q})+(2\pi)^3\delta^3\Big(\boldsymbol{p}-\boldsymbol{q}-\frac{\boldsymbol{l}}{2}\Big)\times\mathcal{R}^{\bar{Q}}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q}), \quad (8)$$

where

$$\mathcal{R}^{Q}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q})=-ig\frac{\Gamma_\mu(\boldsymbol{l})}{\boldsymbol{l}^2}\delta^{\bar{\sigma}}_{\bar{\lambda}}\frac{1}{2m}\bar{u}(\boldsymbol{q},\sigma)\times\gamma^\mu u(\boldsymbol{q}-\boldsymbol{l},\lambda)$$

$$\mathcal{R}^{\bar{Q}}_{\lambda\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q})=+ig\frac{\Gamma_\mu(\boldsymbol{l})}{\boldsymbol{l}^2}\delta^{\sigma}_{\lambda}\frac{1}{2m}\bar{v}(-\boldsymbol{q}-\boldsymbol{l},\bar{\lambda})\times\gamma^\mu v(-\boldsymbol{q},\bar{\sigma}). \quad (9)$$

To first order in $\boldsymbol{l}/m,\boldsymbol{q}/m$ this reduces to

$$\mathcal{R}^{Q}_{\lambda\bar{\lambda},\sigma,\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q})=ig\frac{\Gamma_\mu(\boldsymbol{l})}{\boldsymbol{l}^2}\delta^{\bar{\sigma}}_{\bar{\lambda}}\Big\{-\delta^\mu_0\delta^\sigma_\lambda+\delta^\mu_i\frac{1}{2m}\times[\delta^\sigma_\lambda(l-2q)^i+i\epsilon_{ijk}l^j\chi^\dagger_\sigma\sigma_k\chi_\lambda]\Big\}$$

$$\mathcal{R}^{\bar{Q}}_{\lambda,\bar{\lambda},\sigma\bar{\sigma}}(\boldsymbol{l},\boldsymbol{q})=ig\frac{\Gamma_\mu(\boldsymbol{l})}{\boldsymbol{l}^2}\delta^{\sigma}_{\lambda}\Big\{\delta^\mu_0\delta^{\bar{\sigma}}_{\bar{\lambda}}+\delta^\mu_i\frac{1}{2m}\times[\delta^{\bar{\sigma}}_{\bar{\lambda}}(-l-2q)^i+i\epsilon_{ijk}l^j\chi^\dagger_{-\bar{\lambda}}\sigma_k\chi_{-\bar{\sigma}}]\Big\}. \quad (10)$$

Here σ_k are the Pauli matrices and χ_λ the spinors $\chi_+^\dagger=(1\ 0)$, $\chi_-^\dagger=(0\ 1)$.

We note that the rescattering kernel $\mathcal{R}$ consists of two parts. A $\mu=0$ spin-conserving part which is of $\mathcal{O}(1)$, and a $\mu=i$ part which also contains spin-flip and is of $\mathcal{O}(\boldsymbol{l}/m,\boldsymbol{q}/m)$.

3. The quarkonium wave function Ψ

The quarkonium wave function Ψ in Eq. (3) is

$$\Psi_{\sigma\bar{\sigma}}^{L_zS_z}(\boldsymbol{q})^*=\Psi_{LL_z}^*(\boldsymbol{q})\ \frac{1}{2m}\ \overline{v(-\boldsymbol{q},\bar{\sigma})}\overline{\mathcal{P}}_{SS_z}(\boldsymbol{q},-\boldsymbol{q})u(\boldsymbol{q},\sigma), \tag{11}$$

where $\Psi_{LL_z}(\boldsymbol{q})$ is the usual non-relativistic bound state wave function and the spin-projection operator $\overline{\mathcal{P}}_{SS_z}$ is given by [51]

$$\begin{aligned}\overline{\mathcal{P}}_{SS_z}(\boldsymbol{p}_1,\boldsymbol{p}_2)&=\frac{1}{2m}\sum_{\lambda_1,\lambda_2}\langle \tfrac{1}{2}\ \lambda_1,\ \tfrac{1}{2}\ \lambda_2|SS_z\rangle\\ &\quad\times v(\boldsymbol{p}_2,\lambda_2)\overline{u(\boldsymbol{p}_1,\lambda_1)}\\ &=\frac{1}{2m}\ \frac{\not{p}_2-m}{\sqrt{p_2^0+m}}\ \overline{\Pi}_{SS_z}\frac{\gamma^0+1}{2\sqrt{2}}\ \frac{\not{p}_1+m}{\sqrt{p_1^0+m}}\\ &\simeq\frac{1}{(2m)^2\sqrt{2}}\ (\not{p}_2-m)\overline{\Pi}_{SS_z}(\not{p}_1+m)\end{aligned} \tag{12}$$

$$\overline{\Pi}_{SS_z}=\begin{cases}-\gamma_5 & (S=0)\\ \not{e}(S_z)^* & (S=1).\end{cases} \tag{13}$$

Here we used $p_1=P_f/2+q=(m,\boldsymbol{q})$, $p_2=P_f/2-q=(m,-\boldsymbol{q})$ and terms of order q^2 were neglected in the last line of Eq. (12). A simple calculation gives for the $S=1$ case

$$\Psi_{\sigma\bar{\sigma}}^{L_zS_z}(\boldsymbol{q})^*=\Psi_{LL_z}^*(\boldsymbol{q})\ \frac{1}{\sqrt{2}}\ \boldsymbol{e}(S_z)^*\chi_{-\bar{\sigma}}^\dagger\sigma\chi_\sigma. \tag{14}$$

It is now straightforward to use Eqs. (4), (10) and (14) in Eq. (3). We contract over both pairs of indices $\lambda,\bar{\lambda}$ and $\sigma,\bar{\sigma}$, and work to first order in $\boldsymbol{l}/m,\boldsymbol{q}/m$. The color structure of the field Γ in Eq. (10) is made explicit by

$$\Gamma_\mu(\boldsymbol{l})\to\frac{1}{\sqrt{3}}\ T_{j'i'}^d\Gamma_\mu^d(\boldsymbol{l}) \tag{15}$$

where d is the color index of the rescattering gluon and i',j' are the colors of the quark and antiquark just before the rescattering. The factor $1/\sqrt{3}$ is from the color singlet bound state wave function.

Finally, we use for S-wave states

$$\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\ \Psi_{00}^*(\boldsymbol{q})=\frac{R_0}{\sqrt{4\pi m}}$$

$$\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\ \boldsymbol{q}\Psi_{00}^*(\boldsymbol{q})=0, \tag{16}$$

where R_0 is the value of the S-wave function at the origin. For P-wave states

$$\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\ \Psi_{1L_z}^*(\boldsymbol{q})=0$$

$$\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\ \boldsymbol{q}\Psi_{1L_z}^*(\boldsymbol{q})=i\sqrt{\frac{3}{4\pi m}}\ R_1'e(L_z)^*, \tag{17}$$

where R_1' is the derivative of the P-wave function at the origin.

B. The ${}^3S_1$ quarkonium cross section

We find for the ${}^3S_1$ quarkonium production amplitude

$$\begin{aligned}\mathcal{M}({}^3S_1,S_z)=D^d\ \frac{2ig^3R_0}{\sqrt{6\pi m^3}}\ \frac{1}{\boldsymbol{l}^2}\ \{&i\lambda_1\delta_{\lambda_1}^{-\lambda_2}\Gamma^d(\boldsymbol{l})\times\boldsymbol{l}\cdot\boldsymbol{e}(S_z)^*\\ &+\Gamma_0^d(\boldsymbol{l})[-\delta_{\lambda_1}^{-\lambda_2}l^z\delta_{S_z}^0+\boldsymbol{e}(\lambda_1)\cdot\boldsymbol{l}\delta_{S_z}^{\lambda_2}\\ &+\boldsymbol{e}(\lambda_2)\cdot\boldsymbol{l}\delta_{S_z}^{\lambda_1}]\},\end{aligned} \tag{18}$$

where the color factor is proportional to d_{abc},

$$D^d=\frac{1}{2}\ d_{abc}T_{ij}^cT_{j'i'}^d, \tag{19}$$

and λ_1,λ_2 are the z-components of the spins of the incoming gluons ($\lambda_{1,2}=\pm1$). As can be readily inferred from the form of Eqs. (4) and (10) the 1st and 2nd term of Eq. (18) correspond, respectively, to the production of the $Q\bar{Q}$ pair in an $S=L=0$ state, followed by a spin-flip interaction with a gluon from the color field $\boldsymbol{\Gamma}$, and the production of the pair in an $S=L=1$ state, followed by a spin-conserving interaction with a gluon from the color field Γ^0.

The result for the amplitude squared, summed over the quark color indices i,j and averaged over i',j' and the gluon spin components $\lambda_1,\lambda_2=\pm1$, is for a given spin component S_z of the quarkonium

$$\begin{aligned}\sum\overline{|\mathcal{M}({}^3S_1,S_z)|^2}=\frac{2}{3}\ \frac{N^2-4}{2N^3}\ \frac{g^6R_0^2}{6\pi m^3}\ \frac{1}{\boldsymbol{l}^4}\\ \times\{|\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}|^2+c(S_z)\boldsymbol{l}^2|\Gamma_0^d(\boldsymbol{l})|^2\},\end{aligned} \tag{20}$$

where $c(S_z=0)=1$ and $c(S_z=\pm1)=3$. We have used $D^dD^{d'}=\delta_d^{d'}(N^2-4)/2N$ and the fact that the color field Γ is isotropically distributed. Thus, for instance,

$$(\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l})^x(\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l})^y\to0$$

$$|(\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l})^z|^2\to\frac{1}{3}\ |\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}|^2. \tag{21}$$

The fixed target data [19–21] shows that the polarization of both the J/ψ and the ψ' is small and consistent with zero. We see from Eq. (20) that the condition for the (directly produced) $^3S_1$ quarkonium states to be unpolarized is

$$|\Gamma_0^d(\boldsymbol{l})| \ll |\boldsymbol{\Gamma}^d(\boldsymbol{l}) \times \hat{\mathbf{l}}|. \tag{22}$$

In the following we shall therefore take

$$\Gamma_0^d(\boldsymbol{l}) = 0 \tag{23}$$

which implies [cf. Eq. (2)] that the color field Γ is made of transversely polarized gluons (in the $Q\overline{Q}$ rest frame), and that $Q\overline{Q}$ pairs that form $^3S_1$ quarkonia are created with $S = L = 0$.

We thus have, for $^3S_1$ production via gluon fusion,

$$\sum \overline{|\mathcal{M}(^3S_1,S_z)|^2} = \frac{160}{243}\,\pi^2\alpha_s^3\,\frac{R_0^2}{m^3}\,\frac{|\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}|^2}{\boldsymbol{l}^4}. \tag{24}$$

The cross section will involve an integral of this expression over $\boldsymbol{l}$. Hence the weighted average

$$\overline{\boldsymbol{\Gamma}^2} \equiv \int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\,\frac{|\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}|^2}{\boldsymbol{l}^4} \tag{25}$$

is the main free parameter of our model, which can be determined, e.g., using the measured (direct) J/ψ cross section. The ψ' cross section then satisfies Eq. (1), as discussed in Sec. II. As we shall see below, the P-wave production cross sections are also proportional to $\overline{\boldsymbol{\Gamma}^2}$, but in that case an additional parameter enters, which is related to the length of the time interval between the gluon fusion and rescattering processes.

Since the color field Γ arises through gluon radiation in the primary gluon fusion process it should be the same for all charmonium (as well as open charm) amplitudes. In the production of $b\bar{b}$ pairs, on the other hand, the gluon radiation and thus also the color field parameter (25) will be different.

C. The *P*-wave quarkonium cross sections

The P-wave production amplitudes are obtained similarly to the S-wave ones, by substituting Eqs. (4a), (4b), (10) and (14) into Eq. (3). However, since the P-wave function vanishes at the origin it is now important to take into account the spatial expansion of the $Q\overline{Q}$ pair between its creation in the gluon fusion process and its reinteraction with the color field Γ.

The production amplitude (3) can equivalently be expressed in coordinate space as an overlap between the wave function of the quark pair (Φ) and that of the quarkonium bound state (Ψ) at the *same* spatial separation,

$$\begin{aligned}\mathcal{M} = \int & d^3\boldsymbol{x}_1 d^3\boldsymbol{x}_2 d^3\boldsymbol{x}_3 \Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)\\ &\times \Gamma(\boldsymbol{x}_3)\Psi^*(\boldsymbol{x}_1-\boldsymbol{x}_2)[D(\boldsymbol{x}_1-\boldsymbol{x}_3)+D(\boldsymbol{x}_2-\boldsymbol{x}_3)]\\ &\times \exp\left[-i(\boldsymbol{P}_f-\boldsymbol{P}_i)\cdot\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}\right].\end{aligned} \tag{26}$$

Here $\boldsymbol{x}_1,\boldsymbol{x}_2$ are the positions of the heavy quarks at the reinteraction time, and D stands for the exchanged gluon propagator. Since $\Psi(\mathbf{0})=0$ for P-wave quarkonia the amplitude (26) is sensitive to the spatial extent of the quark wave function $\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$ at the time of reinteraction.

According to our discussion in Sec. III B, the $Q\overline{Q}$ pair is created in the gluon fusion process with a size $\sim 1/m$ and a relative momentum of $\mathcal{O}(\mu)$, which is large compared to the relative momentum $\boldsymbol{q}$ of the quarkonium bound state, $\mu \sim |\boldsymbol{l}| \gg |\boldsymbol{q}|$. In the (proper) time interval τ before the pair reinteracts with Γ it thus expands a distance

$$\Delta|\boldsymbol{x}| \simeq \frac{\mu}{m}\,\tau. \tag{27}$$

The expansion time must be at least of the order of the spatial size of the comoving field, $\tau \gtrsim 1/\mu$, hence $\Delta|\boldsymbol{x}| \gtrsim 1/m$ is comparable to (or even larger than) the initial size of the pair. We shall parametrize the expansion by a factor $\rho > 1$, and rescale the initial coordinate space gluon fusion amplitude accordingly,

$$\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2) \to \frac{1}{\rho^3}\,\Phi\left(\frac{1}{\rho}\,(\boldsymbol{x}_1-\boldsymbol{x}_2)\right) \tag{28}$$

where the factor $1/\rho^3$ preserves the normalization of the squared wave function. In momentum space, the rescaling (28) implies

$$\Phi_{\lambda\bar{\lambda}}(\boldsymbol{p}) \to \Phi_{\lambda\bar{\lambda}}(\rho\boldsymbol{p}) \tag{29}$$

in Eqs. (4a),(4b).

Intuitively it is clear that the rescaling of the argument of Φ by ρ will increase the overlap with the P-wave quarkonium wave function by the same factor (and hence result in a ρ^2 enhancement of the cross section). Conversely, the effect for S-waves vanishes in the limit where the quarkonium bound state radius $\gg \rho/m$. Formally, the enhancement of the P-wave amplitude can be seen from Eq. (4a), where the f_{abc} part (which due to charge conjugation invariance contributes to P-wave production) is *linear* in $\boldsymbol{p}$, and thus gets multiplied by ρ in the rescaling (29).

We find for the spin triplet P-wave production amplitude, taking into account Eq. (23),[6]

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[6]When Eq. (23) is satisfied, the rescattering kernel $\mathcal{R}$ is linear in the small quantities $\boldsymbol{l}/m,\boldsymbol{q}/m$ [see Eq. (10) with the $\mu=0$ term removed]. For ^3P_J production, the only contributing part in $\Phi^{[8]}$ of Eq. (4a) is the term $\propto f_{abc}$, which is also linear. The amplitude (30) thus arises from a *quadratic* term $\sim \boldsymbol{q}\cdot\boldsymbol{l}/m^2$, however our *linear* approximation in $\boldsymbol{l}/m,\boldsymbol{q}/m$ is perfectly justified.

TABLE II. Relative ^3P_J cross sections and induced J/ψ polarizations.

	$^3P_0$	$^3P_1$		$^3P_2$		
J_z	0	1	0	2	1	0
$\dfrac{\sigma(^3P_J,J_z)}{\sigma_{dir}(^3S_1)}$	r	$\frac{3}{2}\,r$	0	0	$\frac{3}{2}\,r$	$2r$
$\lambda_{indir}(^3S_1)$	0	$-\dfrac{1}{3}$	1	1	$-\dfrac{1}{3}$	$-\dfrac{3}{5}$

$$\mathcal{M}(^3P_J,J_z)=\rho F^d\,\frac{-2ig^3\sqrt{3}R_1'/m}{\sqrt{6\pi m^3}}\,\frac{1}{\boldsymbol{l}^2}\,i\lambda_1\delta_{\lambda_1}^{-\lambda_2}[\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}]^i \times\sum_{L_zS_z}\langle LL_z;SS_z|JJ_z\rangle e^z(L_z)^*e^i(S_z)^*, \tag{30}$$

where

$$F^d=\frac{1}{2}\,f_{abc}T^c_{ij}T^d_{j'i'}\,. \tag{31}$$

Here the $Q\bar{Q}$ pair is created with $S=0$, $L=1$ and experiences a spin-flip interaction with the color field Γ. An explicit expression for the spin sum in the last factor of Eq. (30) may be found in Ref. [51]. Thus,

$$\mathcal{M}(^3P_J,J_z)=\rho F^d\,\frac{2ig^3R_1'/m}{\sqrt{6\pi m^3}}\,\frac{1}{\boldsymbol{l}^2}\,i\lambda_1\delta_{\lambda_1}^{-\lambda_2}\times\begin{cases}[\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}]^z & (J=0)\\ -i\sqrt{\dfrac{3}{2}}\,[\boldsymbol{e}^*(J_z)\times(\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l})]^z & (J=1)\\ -\sqrt{3}e^*_{3i}(J_z)[\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}]^i & (J=2).\end{cases} \tag{32}$$

The polarization tensor $e_{\mu\nu}$ for a $J=2$ system is given in Ref. [47].

The P-wave amplitude $\mathcal{M}(^3P_J,J_z)$ depends on the (isotropic) color field Γ through the vector $\boldsymbol{\Gamma}^d(\boldsymbol{l})\times\boldsymbol{l}$. In the amplitude squared, averaged over the incoming gluon spins and over the momentum transfer l, the Γ-dependence thus enters through the same parameter as for S-waves, $\overline{\boldsymbol{\Gamma}^2}$ of Eq. (25). Hence the relative production rates and polarizations of all ^3P_J quarkonia are predicted, and their cross sections can be compared to those of the $^3S_1$ states in terms of the expansion parameter ρ. The ratios $\sigma(^3P_J,J_z)/\sigma_{dir}(^3S_1)$ are given in Table II, where $\sigma_{dir}(^3S_1)$ is the total (unpolarized) $^3S_1$ cross section calculated in Sec. IV B [cf. Eq. (24)]. All cross sections satisfy $\sigma(^3P_J,J_z)=\sigma(^3P_J,-J_z)$. The effective parameter r of Table II is defined as

$$r=\frac{3}{5}\,\rho^2\left(\frac{R_1'/m}{R_0}\right)^2\simeq\begin{cases}2.5\;10^{-2}\;\rho^2 & (c\bar{c}),\\ 6.5\;10^{-3}\;\rho^2 & (b\bar{b}),\end{cases} \tag{33}$$

where the numerical values are from Ref. [25], with $m_c=1.5$ GeV and $m_b=4.5$ GeV. Note that Table II refers only to quarkonium hadroproduction through our rescattering process (cf. Fig. 2). The CSM mechanism may contribute significantly to $^3P_2$ production (see below). Table II is not relevant for quarkonium photoproduction, to which our process does not contribute since $\Gamma=0$ (in the photon fragmentation region) due to the absence of gluon radiation from the beam photon.

As can be seen from Table II, the total P-wave rates satisfy

$$\sigma(\chi_0):\sigma(\chi_1):\sigma(\chi_2)=1:3:5. \tag{34}$$

There is no data on $\sigma(\chi_{c0})$, but our χ_{c1}/χ_{c2} ratio is consistent with the value 0.6 ± 0.3 measured in πN collisions [22,23]. The experimental ratio allows a CSM contribution to χ_{c2} production which is about equal to the one given in Table II [$\sigma(\chi_{c1})$ is small in the CSM]. There is no experimental information on the polarization of the ^3P_J states. Nevertheless, it is interesting to note that we find $\sigma(^3P_2,J_z=\pm2)=0$, contrary to the CSM where only this polarization is produced [24].

The measured [22] cross section ratio[7]

$$\frac{\sigma(\chi_{c2})}{\sigma_{dir}(J/\psi)}=5r\simeq1.8\pm0.4 \tag{35}$$

implies using Eq. (33) a value $\rho=2.7\ldots3.8$, where the lower value corresponds to the CSM contributing 50% of the χ_{c2} rate. The rather large value $\rho\sim3$ of the expansion parameter is a consequence of the theoretical suppression of ^3P_J production due to the vanishing of the P-wave function at the origin [cf. Eq. (33)] and the fact that the measured $\sigma(\chi_{c1,2})$ nevertheless are similar to $\sigma_{\rm dir}(J/\psi)$.

The radiative decays $\chi_{c1,2}\to J/\psi+\gamma$ contribute [22,23] $\sim$40% of the total J/ψ cross section. Since the χ_c states are polarized the indirectly produced J/ψ's will in general be polarized as well. The polarization is conventionally parametrized in terms of a parameter λ in the $J/\psi\to\mu^+\mu^-$ decay angular distribution (we use the Gottfried-Jackson frame, where the z-axis in the J/ψ rest frame is taken along the beam direction):

$$\frac{d\sigma}{d\cos\theta_\mu}\propto1+\lambda\cos^2\theta$$

$$\lambda=\frac{\sigma(S_z=+1)-\sigma(S_z=0)}{\sigma(S_z=+1)+\sigma(S_z=0)}. \tag{36}$$

As discussed above, the condition (23) implies that the directly produced J/ψ's are unpolarized, $\lambda_{dir}=0$. A radiative decay contributes to the indirect $^3S_1$ cross section according to [52]

[7]Reference [23] gives a larger value $3.4\pm0.9\pm0.5$ for this ratio.

$$\sigma_{indir}(^3S_1,S_z) = Br(^3P_J \to {}^3S_1+\gamma)\sum_{J_z} |\langle JJ_z|1(J_z-S_z);1S_z\rangle|^2 \times \sigma(^3P_J,J_z). \quad (37)$$

We show in Table II the J/ψ polarization parameter $\lambda_{indir}(^3S_1)$ induced by the radiative decay of a ^3P_J state of given $|J_z|$. It happens that the induced J/ψ polarization is longitudinal ($\lambda<0$) for all the ^3P_J states which are produced by our mechanism. Using branching fractions $Br(\chi_{cJ}\to J/\psi+\gamma)$ of 27.3% and 13.5% for the χ_{c1} and χ_{c2}, respectively [50], we estimate an overall polarization parameter[8] $\lambda(J/\psi)\simeq-0.14$. The measurements [19,20] tend to prefer a value closer to zero. There is thus room for some χ_{c2} production via the CSM mechanism, which gives rise to fully transverse ($\lambda=1$) J/ψ's. For $\sigma_{\rm CSM}(\chi_{c2}) \simeq(1/2)\sigma_{\rm tot}(\chi_{c2})$ the total $\lambda(J/\psi)\simeq-0.02$.

The ψ' is only produced directly and in our mechanism is unpolarized for a color field Γ satisfying Eq. (23). This is consistent with the experimental value [21] $\lambda(\psi')=0.02 \pm0.14$.

V. CONCLUSIONS

Our motivation for investigating the reinteraction scenario of Fig. 2 for quarkonium hadroproduction was due both to regularities in the data and to shortcomings of alternative mechanisms, as explained in Sec. II. Here we shall briefly comment on some aspects of our results.

We made several simplifying assumptions, some of which may need to be modified in future studies. In particular, (1) we considered only a single hard reinteraction with the comoving field Γ, (2) we assumed Γ to be isotropic and independent of time, and, (3) we assumed that only the origin of the quarkonium wave function is relevant in the overlap integral (26).

These assumptions seem reasonable in a first attempt to consider the effects of reinteractions. We believe that further systematic studies of the environment of partons created in hard collisions are called for. Besides quarkonium production, the flavor and azimuthal angle correlations as well as the spin dependence of open heavy flavor production should be informative.

Our scenario for quarkonium production was based on several striking features of the data, which we interpreted as the properties (A)–(C) listed in Sec. I A. We also built on the extensive experience gained from previous model studies of quarkonium production. Many of the ‘‘successes’’ of the present approach were thus built in from the start. Nevertheless, we find it non-trivial and interesting that so many observed features can be qualitatively understood in a simple theoretical framework. We also found more detailed consequences of our model which could not be anticipated. Let us mention two successes and one difficulty.

The $\sigma(\chi_{c1})/\sigma(\chi_{c2})$ ratio is consistent with data. This ratio is found to be much too low, compared to data, in both the color singlet (CSM) [24] and color octet (COM) [46] approaches.

The (non-)polarization of the J/ψ and ψ'. We find, in agreement with data, that the directly produced $^3S_1$ states are unpolarized (at moderate x_F), provided that the reinteraction is dominated by transverse gluon exchange; cf. Eq. (22). In both the CSM and COM the J/ψ's are produced with transverse polarization [24,46,52]. The color evaporation model (CEM) postulates that soft interactions flip the heavy quark spin, in violation of heavy quark symmetry [48].

Spatial expansion of the heavy quark pair. The initially compact pair expands by a factor ρ before its reinteraction with the color field creates an overlap with the quarkonium wave function. We need $\rho\simeq3$ to fit the observed relative rates of J/ψ and χ_{c2}. Such a large expansion of the quark pair may be inconsistent with our approximation, which considers only the quarkonium wave function at the origin.

[8]The inclusion of the unpolarized $\psi'\to J/\psi$ contribution does not change the numerical value of λ significantly.

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[1] G. A. Schuler, CERN-TH.7170/94, hep-ph/9403387; Z. Phys. C **71**, 317 (1996).
[2] M. L. Mangano, ‘‘Proceedings of Batavia Collider Workshop 1995,’’ p. 120, hep-ph/9507353.
[3] A. Sansoni, Nuovo Cimento A **109**, 827 (1996); Nucl. Phys. **A610**, 373c (1996).
[4] E. Braaten, S. Fleming, and T. C. Yuan, Annu. Rev. Nucl. Part. Sci. **46**, 197 (1996).
[5] M. Beneke, ‘‘Lectures given at 24th Annual SLAC Summer Institute on Particle Physics,’’ CERN-TH-97-055, hep-ph/9703429.
[6] P. Hoyer, Nucl. Phys. **A622**, 284c (1997).
[7] M. Cacciari, Report No. DESY 97-091, hep-ph/9706374.
[8] J. C. Collins, D. E. Soper, and G. Sterman, in *Perturbative QCD*, edited by A. H. Mueller (World Scientific, Singapore, 1989); G. Bodwin, Phys. Rev. D **31**, 2616 (1985); **34**, 3932(E) (1986); J. Qiu and G. Sterman, Nucl. Phys. **B353**, 105 (1991); **B353**, 137 (1991).
[9] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
[10] S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, Nucl. Phys. **B431**, 453 (1994); in *Heavy Flavours II*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1998).

[11] E791 Collaboration, E. M. Aitala *et al.*, Phys. Lett. B **371**, 157 (1996).
[12] E687 Collaboration, P. L. Frabetti *et al.*, Phys. Lett. B **370**, 222 (1996).
[13] S. J. Brodsky and A. H. Mueller, Phys. Lett. B **206**, 685 (1988); R. Vogt and S. Gavin, Nucl. Phys. **B345**, 104 (1990).
[14] H. Fritzsch, Phys. Lett. **67B**, 217 (1977); F. Halzen, *ibid.* **69B**, 105 (1977).
[15] R. Gavai, D. Kharzeev, H. Satz, G. A. Shuler, K. Sridhar, and R. Vogt, Int. J. Mod. Phys. A **10**, 3043 (1995); G. A. Schuler, Z. Phys. C **71**, 317 (1996); J. A. Amundson, O. Éboli, E. Gregores, and F. Halzen, Phys. Lett. B **372**, 127 (1996); **390**, 323 (1997); G. A. Schuler and R. Vogt, *ibid.* **387**, 181 (1996); O. Éboli, E. Gregores, and F. Halzen, Report No. MADPH-98-1045, hep-ph/9802421.
[16] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995); M. Cacciari, M. Greco, M. L. Mangano, and A. Petrelli, Phys. Lett. B **356**, 553 (1995); P. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996); **53**, 6203 (1996).
[17] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[18] J. H. Kühn, Phys. Lett. **89B**, 385 (1980); C. H. Chang, Nucl. Phys. **B172**, 425 (1980); E. L. Berger and D. Jones, Phys. Rev. D **23**, 1521 (1981); R. Baier and R. Rückl, Phys. Lett. **102B**, 364 (1981); Z. Phys. C **19**, 251 (1983); J. G. Körner, J. Cleymans, M. Karoda, and G. J. Gounaris, Nucl. Phys. **B204**, 6 (1982).
[19] NA3 Collaboration, J. Badier *et al.*, Z. Phys. C **20**, 101 (1983).
[20] C. Biino *et al.*, Phys. Rev. Lett. **58**, 2523 (1987); E537 Collaboration, C. Akerlof *et al.*, Phys. Rev. D **48**, 5067 (1993); E672/E706 Collaborations, A. Gribushin *et al.*, *ibid.* **53**, 4723 (1996); E771 Collaboration, T. Alexopoulos *et al.*, *ibid.* **55**, 3927 (1997).
[21] J. G. Heinrich *et al.*, Phys. Rev. D **44**, 1909 (1991).
[22] E705 Collaboration, L. Antoniazzi *et al.*, Phys. Rev. Lett. **70**, 383 (1993); Phys. Rev. D **49**, 543 (1994).
[23] E672/E706 Collaborations, V. Koreshev *et al.*, Phys. Rev. Lett. **77**, 4294 (1996).
[24] M. Vänttinen, P. Hoyer, S. J. Brodsky, and W.-K. Tang, Phys. Rev. D **51**, 3332 (1995).
[25] E. J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).
[26] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[27] S. J. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, Phys. Lett. **93B**, 451 (1980); S. J. Brodsky, C. Peterson, and N. Sakai, Phys. Rev. D **23**, 2745 (1981).
[28] S. J. Brodsky, P. Hoyer, A. H. Mueller, and W.-K. Tang, Nucl. Phys. **B369**, 519 (1992).
[29] C. Lourenço, Nucl. Phys. **A610**, 552c (1996).
[30] B. Z. Kopeliovich and B. G. Zakharov, Phys. Rev. D **44**, 3466 (1991); B. Z. Kopeliovich, J. Nemchik, N. N. Nikolaev, and B. G. Zakharov, Phys. Lett. B **309**, 179 (1993).
[31] H1 Collaboration, C. Adloff *et al.*, Phys. Lett. B **421**, 385 (1998).
[32] R. Gavai, D. Kharzeev, H. Satz, G. A. Schuler, K. Sridhar, and R. Vogt, Int. J. Mod. Phys. A **10**, 3043 (1995).
[33] M. Krämer, J. Zunft, J. Steegborn, and P. M. Zerwas, Phys. Lett. B **348**, 657 (1995); M. Krämer, Nucl. Phys. **B459**, 3 (1996).
[34] H1 Collaboration, S. Aid *et al.*, Nucl. Phys. **B472**, 3 (1996).
[35] ZEUS Collaboration, J. Breitweg *et al.*, Z. Phys. C **76**, 599 (1997).
[36] M. Beneke, I. Z. Rothstein, and M. B. Wise, Phys. Lett. B **408**, 373 (1997); B. A. Kniehl and G. Kramer, Report No. DESY 98-023, hep-ph/9803256.
[37] G. A. Alves *et al.*, Phys. Rev. Lett. **70**, 722 (1993); M. J. Leitch *et al.*, *ibid.* **72**, 2542 (1994).
[38] D. Kharzeev, C. Lourenço, M. Nardi, and H. Satz, Z. Phys. C **74**, 307 (1997).
[39] B. G. Kopeliovich, in *Proceedings of the Workshop on QCD Phase Transitions*, Hirschegg, Austria, 1997, edited by H. Feldmeier, J. Knoll, W. Norenberg, and J. Wambach (GSI, Darmstadt, Germany, 1997), p. 458.
[40] D. Kharzeev, ''Talk at the Quark Matter '97 Conference,'' nucl-th/9802037.
[41] D. Kharzeev and H. Satz, Phys. Lett. B **366**, 316 (1996).
[42] NMC Collaboration, P. Amaudruz *et al.*, Nucl. Phys. **B371**, 553 (1992).
[43] T. Gousset and H. J. Pirner, Phys. Lett. B **375**, 349 (1996).
[44] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **75**, 4358 (1995); E771 Collaboration, T. Alexopoulos *et al.*, Phys. Lett. B **374**, 271 (1996).
[45] E772 Collaboration, D. M. Alde *et al.*, Phys. Rev. Lett. **66**, 2285 (1991); E789/E772 Collaborations, M. J. Leitch *et al.*, Nucl. Phys. **A544**, 197c (1992).
[46] M. Beneke and I. Z. Rothstein, Phys. Rev. D **54**, 2005 (1996); **54**, 7082(E) (1996).
[47] P. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996); **53**, 6203 (1996).
[48] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[49] P. Hoyer and S. Peigné, Phys. Rev. D **57**, 1864 (1998).
[50] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[51] J. H. Kühn, J. Kaplan, and E. G. O. Safiani, Nucl. Phys. **B157**, 125 (1979).
[52] W.-K. Tang and M. Vänttinen, Phys. Rev. D **53**, 4851 (1996); **54**, 4349 (1996).