Kaon electroweak form factors in the light-front quark model

Ho-Meoyng Choi and Chueng-Ryong Ji
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202

(Received 27 July 1998; published 22 December 1998)

We investigate the form factors and decay rates for the semileptonic decays of the kaon(K_{l3}) using the light-front quark model. The form factors $f_{\pm}(q^2)$ are calculated in the $q^+=0$ frame and analytically continued to the timelike region, $q^2>0$. Our numerical results for the physical observables $f_-/f_+|_{q^2=m_l^2}=-0.38$, $\lambda_+=0.025$ (the slope of f_+ at $q^2=m_l^2$), $\Gamma(K^0_{e3})=(7.30\pm0.12)\times10^6\ \mathrm{s}^{-1}$, and $\Gamma(K^0_{\mu3})=(4.57\pm0.07)\times10^6\ \mathrm{s}^{-1}$ are quite comparable with the experimental data and other theoretical model calculations. The nonvalence contributions from the $q^+\neq0$ frame are also estimated. [S0556-2821(99)03001-5]

PACS number(s): 14.40.Aq, 12.39.Ki, 13.10.+q, 13.20.Eb

I. INTRODUCTION

Even though there have been many analyses of the heavy-to-heavy and heavy-to-light form factors for weak transitions from a pseudoscalar meson to another pseudoscalar meson within the light-front quark model (LFQM) [1–8], the light-to-light weak form factor analysis such as K_{l3} has not yet been studied in the LFQM. However, the analysis of semileptonic K_{l3} decays comparing with the experiment [9] has been provided by many other theoretical models, e.g., chiral perturbation theory (CPT) [10,11], the effective chiral Lagrangian approach [12], vector meson dominance [13], the extended Nambu–Jona-Lasinio model [14], Dyson-Schwinger approach [15], and other quark models [16,17]. Thus, in this work, we use the LFQM to analyze both form factors of the K_{l3} decays, i.e., f_+ and f_-, and compare with the experimental data as well as other theoretical models.

In the LFQM calculations presented in Refs. [4–7], the $q^+\neq0$ frame has been used to calculate the weak decays in the timelike region $m_l^2\leq q^2\leq(M_i-M_f)^2$, with $M_{i[f]}$ and m_l being the initial [final] meson mass and the lepton (l) mass, respectively. However, when the $q^+\neq0$ frame is used, the inclusion of the nonvalence contributions arising from quark-antiquark pair creation (''Z-graph'') is inevitable and this inclusion may be very important for heavy-to-light and light-to-light decays. Nevertheless, the previous analyses [4–7] in $q^+\neq0$ frame considered only valence contributions neglecting nonvalence contributions. In this work, we circumvent this problem by calculating the processes in $q^+=0$ frame and analytically continuing to the timelike region. The $q^+=0$ frame is useful because only valence contributions are needed. However, one needs to calculate the component of the current other than J^+ to obtain the form factor $f_-(q^2)$. Since J^- is not free from the zero-mode contributions even in $q^+=0$ frame [19,20], we use $J_\perp$ instead of J^- to obtain f_-. The previous works in Refs. [1–3] have considered only the ''+'' component of the current which was not sufficient to obtain the form factor $f_-(q^2)$. Furthermore, the light-to-light decays such as K_{l3} have not yet been analyzed, even though the calculation of f_- for heavy-to-heavy and heavy-to-light decays has been made in Ref. [8] using the dispersion formulations. Thus, we analyze both currents of J^+ and $J_\perp$ for K_{l3} decays using $q^+=0$ frame and analytically continue to the timelike region. Our method of changing $q_\perp$ to $iq_\perp$ is not only simple to use in practical calculations for the exclusive processes but also provides the identical results obtained by the dispersion formulations presented in Ref. [8].

The calculation of the form factor $f_-(q^2)$ is especially important for the complete analysis of K_{l3} decays, since the $f_-(q^2)$ is prerequisite for the calculation of the physical observables $\xi_A=f_-/f_+|_{q^2=m_l^2}$ and λ_-, the slope of $f_-(q^2)$ at $q^2=m_l^2$. We also estimate the nonvalence contributions from $q^+\neq0$ frame by calculating only valence contributions from $q^+\neq0$ frame and comparing them with those obtained from $q^+=0$ frame. Including the lepton mass effects for the $d\Gamma/dq^2$ spectrum of K_{l3}, we distinguish the decay rate of $K_{\mu3}$ from that of K_{e3}, where the contribution from f_- is found to be appreciable for μ decays.

Our model parameters summarized in Table I were obtained from our previous analysis of quark potential model [18], which provided a good agreement with the experimental data of various electromagnetic properties of mesons such as f_π, f_K, charge radii of π and K, and rates for radiative meson decays etc. As shown in Ref. [18], the Gaussian radial wave function $\phi(x,\mathbf{k}_\perp)$ for our LF wave function $\Psi^{JJ_z}_{\lambda_q,\lambda_{\bar{q}}}(x,\mathbf{k}_\perp)=\phi(x,\mathbf{k}_\perp)\mathcal{R}^{JJ_z}_{\lambda_q,\lambda_{\bar{q}}}(x,\mathbf{k}_\perp)$ is given by

$$\phi(x,\mathbf{k}_\perp)=\sqrt{\frac{\partial k_z}{\partial x}}\left(\frac{1}{\pi^{3/2}\beta^3}\right)^{1/2}\exp(-k^2/2\beta^2), \tag{1}$$

where $\partial k_z/\partial x$ is the Jacobian of the variable transformation $\{x,\mathbf{k}_\perp\}\to\mathbf{k}=(k_n,\mathbf{k}_\perp)$. The spin-orbit wave function $\mathcal{R}^{JJ_z}_{\lambda_q,\lambda_{\bar{q}}}(x,\mathbf{k}_\perp)$ is obtained by the interaction-independent Melosh transformation. The detailed description for the spin-orbit wave function can also be found in previous literatures [1–3,5–7,18].

The paper is organized as follows. In Sec. II, we obtain the form factors of K_{l3} decays in $q^+=0$ frame and analyti-

TABLE I. Quark masses m_q (GeV) and Gaussian parameters β (GeV) used in our analysis. $q=u$ and d.

	m_q	m_s	$\beta_{q\bar{q}}$	$\beta_{s\bar{s}}$	$\beta_{q\bar{s}}$
Set 1	0.25	0.48	0.3194	0.3681	0.3419
Set 2	0.22	0.45	0.3659	0.4128	0.3886

cally continue to the timelike $q^2>0$ region by changing $q_\perp$ to $iq_\perp$ in the form factors. In Sec. III, our numerical results of the observables for K_{l3} decays are presented and compared with the experimental data as well as other theoretical results. Summary and discussion of our main results follow in Sec. IV. In the Appendix A, we show the derivation of the matrix element of the weak vector current for K_{l3} decays in the standard $q^+=0$ frame. In the Appendix B, the valence contribution in $q^+\neq 0$ frame is formulated.

II. WEAK FORM FACTORS IN DRELL-YAN FRAME

The matrix element of the hadronic current for K_{l3} can be parametrized in terms of two hadronic form factors as follows:

$$\langle \pi|\bar{u}\gamma^\mu s|K\rangle = f_+(q^2)(P_K+P_\pi)^\mu + f_-(q^2)(P_K-P_\pi)^\mu,$$
$$= f_+(q^2)\left[(P_K+P_\pi)^\mu - \frac{M_K^2-M_\pi^2}{q^2}q^\mu\right]$$
$$+ f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q^\mu, \tag{2}$$

where $q^\mu=(P_K-P_\pi)^\mu$ is the four-momentum transfer to the leptons and $m_l^2\leqslant q^2\leqslant (M_K-M_\pi)^2$. The form factors f_+ and f_0 are related to the exchange of 1^- and 0^+, respectively, and satisfy the following relations:

$$f_+(0)=f_0(0), \quad f_0(q^2)=f_+(q^2)+\frac{q^2}{M_K^2-M_\pi^2}f_-(q^2). \tag{3}$$

Since the lepton mass is small except in the case of the τ lepton, one may safely neglect the lepton mass in the decay rate calculation of the heavy-to-heavy and heavy-to-light transitions. However, for K_{l3} decays, the muon(μ) mass is not negligible, even though electron mass can be neglected. Thus, including nonzero lepton mass, the formula for the decay rate of K_{l3} is given by [21]

$$\frac{d\Gamma(K_{l3})}{dq^2}=\frac{G_F^2}{24\pi^3}|V_{us}|^2K_f(q^2)\left(1-\frac{m_l^2}{q^2}\right)^2$$
$$\times\left\{[K_f(q^2)]^2\left(1+\frac{m_l^2}{2q^2}\right)|f_+(q^2)|^2\right.$$
$$\left.+M_K^2\left(1-\frac{M_\pi^2}{M_K^2}\right)^2\frac{3}{8}\frac{m_l^2}{q^2}|f_0(q^2)|^2\right\}, \tag{4}$$

where G_F is the Fermi constant, $V_{q_1\bar{q}_2}$ is the element of the Cabbibo-Kobayashi-Maskawa mixing matrix and the factor $K_f(q^2)$ is given by

$$K_f(q^2)=\frac{1}{2M_K}[(M_K^2+M_\pi^2-q^2)^2-4M_K^2M_\pi^2]^{1/2}. \tag{5}$$

Since our analysis will be performed in the isospin symmetry ($m_u=m_d$) but $SU_f(3)$ breaking ($m_s\neq m_{u(d)}$) limit, we do not discriminate between the charged and neutral kaon weak decays, i.e., $f_\pm^{K^0}=f_\pm^{K^+}$. For K_{l3} decays, the three form factor parameters, i.e., λ_+, λ_0 and ξ_A, have been measured using the following linear parametrization [9]:

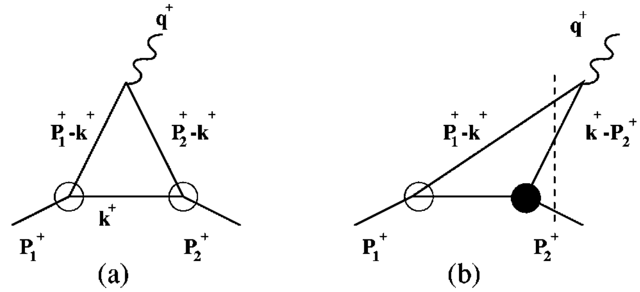

FIG. 1. The form factor calculation in $q^+\neq 0$ frame requires both the usual light-front triangle diagram (a) and the nonvalence (pair-creation) diagram (b). The vertical dashed line in (b) indicates the energy-denominator for the nonvalence contribution. While the white blob represents our LF wave function $\Psi^{JJ_z}_{\lambda_q,\lambda_{\bar{q}}}(x,\mathbf{k}_\perp)$, the modeling of black blob has not yet been made.

$$f_\pm(q^2)=f_\pm(q^2=m_l^2)\left(1+\lambda_\pm\frac{q^2}{M_{\pi^+}^2}\right), \tag{6}$$

where $\lambda_{\pm,0}$ is the slope of $f_{\pm,0}$ evaluated at $q^2=m_l^2$ and $\xi_A=f_-/f_+|_{q^2=m_l^2}$.

As shown in Fig. 1, the quark momentum variables for $q_1\bar{q}\to q_2\bar{q}$ transitions in the standard $q^+=0$ frame are given by

$$p_1^+=(1-x)P_1^+, \quad p_{\bar{q}}^+=xP_1^+,$$
$$\mathbf{p}_{1\perp}=(1-x)\mathbf{P}_{1\perp}+\mathbf{k}_\perp, \quad \mathbf{p}_{\bar{q}\perp}=x\mathbf{P}_{1\perp}-\mathbf{k}_\perp,$$
$$p_2^+=(1-x)P_2^+, \quad p'^+_{\bar{q}}=xP_2^+,$$
$$\mathbf{p}_{2\perp}=(1-x)\mathbf{P}_{2\perp}+\mathbf{k}'_\perp, \quad \mathbf{p}'_{\bar{q}\perp}=x\mathbf{P}_{2\perp}-\mathbf{k}'_\perp, \tag{7}$$

which requires that $p_{\bar{q}}^+=p'^+_{\bar{q}}$ and $\mathbf{p}_{\bar{q}\perp}=\mathbf{p}'_{\bar{q}\perp}$. Our analysis for K_{l3} decays will be carried out using the frame where the decaying hadron (kaon) is at rest and $q^+=0$. Using the matrix element of the ''+'' component of the current, J^+, given by Eq. (2), we obtain the form factor $f_+(q_\perp^2)$ as follows:

$$f_+(q_\perp^2)=\int_0^1 dx\int d^2\mathbf{k}_\perp\,\phi_2(x,\mathbf{k}'_\perp)\phi_1(x,\mathbf{k}_\perp)$$
$$\times\frac{\mathcal{A}_1\mathcal{A}_2+\mathbf{k}_\perp\cdot\mathbf{k}'_\perp}{\sqrt{\mathcal{A}_1^2+k_\perp^2}\sqrt{\mathcal{A}_2^2+k'^2_\perp}}, \tag{8}$$

where $q_\perp^2=-q^2$, $\mathcal{A}_i=m_ix+m_{\bar{q}}(1-x)$ and $\mathbf{k}'_\perp=\mathbf{k}_\perp-x\mathbf{q}_\perp$. As we discussed in the introduction, we need the ''$\perp$'' component of the current, $J_\perp$, to obtain the form factor $f_-(q_\perp^2)$ in Eq. (2), viz.,

$$\langle P_2|\bar{q}_2(\mathbf{q}_\perp\cdot\vec{\gamma}_\perp)q_1|P_1\rangle=q_\perp^2[f_-(q_\perp^2)-f_+(q_\perp^2)], \tag{9}$$

after multiplying $\mathbf{q}_\perp$ on both sides of Eq. (2). The left-hand side (LHS) of Eq. (9) is given by

$$\langle P_2|\overline{q_2}(\mathbf{q}_\perp\cdot\vec{\gamma}_\perp)q_1|P_1\rangle = -\int dx d^2\mathbf{k}_\perp \frac{x\phi_2(x,\mathbf{k'}_\perp)\phi_1(x,\mathbf{k}_\perp)}{2\sqrt{\mathcal{A}_1^2+k_\perp^2}\sqrt{\mathcal{A}_2^2+k_\perp'^2}} \times \mathrm{Tr}[\gamma_5(\not{p}_2+m_2)(\mathbf{q}_\perp\cdot\vec{\gamma}_\perp)(\not{p}_1+m_1)\gamma_5(\not{p}_{\bar{q}}-m_{\bar{q}})]. \tag{10}$$

Using the quark momentum variables given in Eq. (7), we obtain the trace term in Eq. (10) as follows:

$$\mathrm{Tr}[\gamma_5(\not{p}_2+m_2)(\mathbf{q}_\perp\cdot\vec{\gamma}_\perp)(\not{p}_1+m_1)\gamma_5(\not{p}_{\bar{q}}-m_{\bar{q}})] = -2\left\{\frac{(\mathcal{A}_1^2+k_\perp^2)}{x(1-x)}(\mathbf{k}_\perp-\mathbf{q}_\perp)\cdot\mathbf{q}_\perp+\frac{(\mathcal{A}_2^2+k_\perp'^2)}{x(1-x)}\mathbf{k}_\perp\cdot\mathbf{q}_\perp + [(m_1-m_2)^2+q_\perp^2]\mathbf{k}_\perp\cdot\mathbf{q}_\perp\right\}. \tag{11}$$

The more detailed derivation of Eqs. (8) and (10) are presented in Appendix A. Since both sides of Eq. (9) vanish as $q^2\to 0$, one has to be cautious for the numerical computation of f_- at $q^2=0$. Thus, for the numerical computation at $q^2=0$, we need to find an analytic formula for f_0. In order to obtain the analytic formula for the form factor $f_-(0)$, we make a low $q_\perp^2$ expansion to extract the overall $q_\perp^2$ from Eq. (10). Then, the form factor $f_-(0)$ is obtained as follows:

$$f_-(0)=f_+(0)+\int_0^1 dx\int d^2\mathbf{k}_\perp \frac{x\phi_2(x,\mathbf{k}_\perp)\phi_1(x,\mathbf{k}_\perp)}{\sqrt{\mathcal{A}_1^2+k_\perp^2}\sqrt{\mathcal{A}_2^2+k_\perp^2}} \times\{[C_{T1}(C_{J1}-C_{J2}+C_M+C_R) + C_{T2}]k_\perp^2\cos^2\phi+C_{T3}\}, \tag{12}$$

where the angle ϕ is defined by $\mathbf{k}_\perp\cdot\mathbf{q}_\perp=|\mathbf{k}_\perp||\mathbf{q}_\perp|\cos\phi$ and the terms of $C's$ are given by

$$C_{Ji}=\frac{2\beta_i^2}{(1-x)(\beta_1^2+\beta_2^2)M_{i0}^2}\left[\frac{1}{1-[(m_i^2-m_{\bar{q}}^2)/M_{i0}^2]^2}-\frac{3}{4}\right],$$

$$C_M=\frac{1}{(1-x)(\beta_1^2+\beta_2^2)} \times\left[\frac{\beta_2^2}{M_{20}^2-(m_2-m_{\bar{q}})^2}-\frac{\beta_1^2}{M_{10}^2-(m_1-m_{\bar{q}})^2}\right],$$

$$C_R=\frac{-1}{4(1-x)(\beta_1^2+\beta_2^2)}\left[\left(\frac{m_2^2-m_{\bar{q}}^2}{M_{20}^2}\right)^2-\left(\frac{m_1^2-m_{\bar{q}}^2}{M_{10}^2}\right)^2\right],$$

$$C_{T1}=\frac{1}{x(1-x)}(\mathcal{A}_1^2+\mathcal{A}_2^2+2k_\perp^2)+(m_1-m_2)^2,$$

$$C_{T2}=\frac{2(\beta_1^2-\beta_2^2)}{(1-x)(\beta_1^2+\beta_2^2)},\quad C_{T3}=\frac{x\beta_1^2}{\beta_1^2+\beta_2^2}C_{T1}-\frac{\mathcal{A}_1^2+k_\perp^2}{x(1-x)}, \tag{13}$$

with

$$M_{i0}^2=\frac{k_\perp^2+m_i^2}{1-x}+\frac{k_\perp^2+m_{\bar{q}}^2}{x}. \tag{14}$$

The form factors f_+ and f_- can be analytically continued to the timelike $q^2>0$ region[1] by replacing $q_\perp$ by $iq_\perp$ in Eqs. (8) and (9). Since $f_-(0)$ in Eq. (12) is exactly zero in the $SU_f(3)$ symmetry [15], i.e., $m_{u(d)}=m_s$ and $\beta_{u\bar{d}}=\beta_{u\bar{s}}=\beta_{s\bar{s}}$, one can get $f_+(q_\perp^2)=F_\pi(q_\perp^2)$ for the $\pi^+\to\pi^0$ weak decay (π_{e3}), where $F_\pi(q_\perp^2)$ is the electromagnetic form factor of pion, and $f_-(q^2)=0$ because of the isospin symmetry. For comparison, we briefly discuss in Appendix B the form factors in $q^+\neq 0$ frame.

III. NUMERICAL RESULTS

As we discussed in the Introduction, we used the same quark model parameters ($m_{u(d)},m_s,\beta_{u\bar{d}},\beta_{u\bar{s}}$) as in Ref. [18] to predict various observables for K_{l3} decays. These parameters are summarized in Table I. Sets 1 and 2 in this table represent the model parameters obtained by the harmonic oscillator and linear confinement potentials, respectively, in LFQM [18].

Our predictions of the parameters for K_{l3} decays in $q^+=0$ frame, i.e., $f_+(0)$, λ_+, λ_0, $\langle r^2\rangle_{K\pi}=6f'_+(0)/f_+(0)=6\lambda_+/M_{\pi^+}^2$, and $\xi_A=f_-/f_+|_{q^2=m_l^2}$, are summarized in Table II. We do not distinguish K_{e3} from $K_{\mu 3}$ in the calculation of the above parameters since the slopes of $f_\pm$ are almost constant in the range of $m_e^2\leqslant q^2\leqslant m_\mu^2$. However, the decay rates should be different due to the phase space factors given by Eq. (4) and our numerical results for $\Gamma(K_{e3})$ and $\Gamma(K_{\mu 3})$ in $q^+=0$ frame are also presented in Table II. Our results for the form factor f_+ at zero momentum transfer, $f_+(0)=0.961[0.962]$ for set 1[set 2], are consistent with the Ademollo-Gatto theorem [22] and also coincides with the result of chiral perturbation theory [10], $f_+(0)=0.961\pm 0.008$. Our results for other observables such as λ_+, ξ_A, and $\Gamma(K_{l3})$ are overall in a good agreement with the experimental data [9]. We have also investigated the sensitivity of our results by varying quark masses. For instance, the results[2] obtained by changing the strange quark mass from

[1]We note that our numerical results of f_+ obtained by the method of replacing $q_\perp$ by $iq_\perp$ in Eq. (8) for any $P\to P$(P=Pseudoscalar) semileptonic decays are identical to those obtained from dispersion formulation in Ref. [8].

[2]Even though we show the results only for the set 1, we find the similar variations for the set 2; i.e., the positive sign of λ_0 can be obtained when $m_s/m_u\leqslant 1.8$ for both sets 1 and 2. In addition to the observables in this work, our predictions for f_K, $r_{K^+}^2$, and $r_{K^0}^2$ in [18] are changed to 108 MeV (1% change), 0.385 fm^2 (0.3% change), and -0.077 fm^2 (15%), respectively.

TABLE II. Model predictions for the parameters of K_{l3} decay form factors obtained from $q^+=0$ frame. The charge radius $r_{\pi K}$ is obtained by $\langle r^2\rangle_{\pi K}=6f'_+(q^2=0)/f_+(0)$. As a sensitivity check, we include the results in square brackets by changing $m_s=0.48$ to 0.43 GeV for the parameter set 1. The CKM matrix used in the calculation of the decay width (in units of $10^6\ \mathrm{s}^{-1}$) is $|V_{us}|=0.2205\pm0.0018$ [9].

Observables	Set 1 $[m_s=0.48\to0.43]$	Set 2	Other models	Experiment
$f_+(0)$	0.961[0.974]	0.962	0.961 ± 0.008,[a] 0.952,[e] 0.98,[f] 0.93[g]	
λ_+	0.025[0.029]	0.026	0.031,[b] 0.033,[c] 0.025[d]	$0.0286\pm0.0022[K^+_{e3}]$
			0.028,[e] 0.018,[f] 0.019[g]	
				$0.0300\pm0.0016[K^0_{e3}]$
λ_0	$-0.007[+0.0027]$	-0.009	0.017 ± 0.004,[b] 0.013,[c] 0.0[d]	$0.004\pm0.007[K^+_{\mu3}]$
			0.0026,[e] -0.0024,[f] -0.005[g]	$0.025\pm0.006[K^0_{\mu3}]$
ξ_A	$-0.38[-0.31]$	-0.41	-0.164 ± 0.047,[b] -0.24,[c] -0.28[d]	$-0.35\pm0.15[K^+_{\mu3}]$
			-0.28,[e] -0.25,[f] -0.28[g]	$-0.11\pm0.09[K^0_{\mu3}]$
$\langle r\rangle_{\pi K}$(fm)	0.55[0.59]	0.56	0.61,[b] 0.57,[e] 0.47,[f] 0.48[g]	
$\Gamma(K^0_{e3})$	$7.30\pm0.12[7.60\pm0.12]$	7.36 ± 0.12		$7.7\pm0.5[K^0_{e3}]$
$\Gamma(K^0_{\mu3})$	$4.57\pm0.07[4.84\pm0.08]$	4.56 ± 0.07		$5.25\pm0.07[K^0_{\mu3}]$

[a]Reference [10].
[b]Reference [11].
[c]Reference [12].
[d]Reference [13].
[e]Reference [14].
[f]Reference [15].
[g]Reference [16].

$m_s=0.48$ GeV to 0.43 GeV (10% change) for the set 1 are included in Table II. As one can see in Table II, our model predictions are quite stable for the variation of m_s except λ_0, which changes its sign from -0.007 to $+0.0027$. The large variation of λ_0 is mainly due to the rather large sensitivity of $f_-(0)$ (18% change) to the variation of m_s. Similar observation regarding on the large sensitivity for λ_0 compared to other observables has also been reported in Ref. [14] for the variation of quark masses. As discussed in Refs. [15] and [17], $f_-(0)$ is sensitive to the nonperturbative enhancement of the SU(3) symmetry breaking mass difference $m_s-m_{u(d)}$ since $f_-(0)$ depends on the ratio of m_s and $m_{u(d)}$.

Of special interest, we also observed that the nonvalence contributions from $q^+\neq0$ frame are clearly visible for λ_+, λ_0 and ξ_A even though it may not be quite significant for the decay rate $\Gamma(K_{l3})$. Our predictions with only the valence contributions in $q^+\neq0$ frame are $f_+(0)=0.961[0.962]$, $\lambda_+=0.081[0.083]$, $\lambda_0=-0.014[-0.017]$, $\xi_A=-1.12[-1.10]$, $\Gamma(K_{e3})=(8.02[7.83]\pm0.13)\times10^6\ \mathrm{s}^{-1}$ and $\Gamma(K_{\mu3})=(4.49[4.36]\pm0.13)\times10^6\ \mathrm{s}^{-1}$ for the set 1[set 2]. Even though the form factor $f_+(0)$ in $q^+\neq0$ frame is free from the nonvalence contributions, its derivative at $q^2=0$, i.e., λ_+, receives the nonvalence contributions. Moreover, the form factor $f_-(q^2)$ in $q^+\neq0$ frame is not immune to the nonvalence contributions even at $q^2=0$ [19]. Unless one includes the nonvalence contributions in the $q^+\neq0$ frame, one cannot really obtain reliable predictions for the observables such as λ_+, λ_0 and ξ_A for K_{l3} decays.

In Fig. 2, we show the form factors f_+ obtained from both $q^+=0$ and $q^+\neq0$ frames for $0\leq q^2\leq(M_K-M_\pi)^2$ region. As one can see in Fig. 2, the form factors f_+ obtained from $q^+=0$ frame (solid lines) for both parameter sets 1 and 2 appear to be linear functions of q^2 justifying Eq. (6) usually employed in the analysis of experimental data [9]. Note, however, that the curves without the nonvalence contributions in $q^+\neq0$ frame (dotted lines) do not exhibit the same behavior. In Fig. 3, we show $d\Gamma/dq^2$ spectra for K_{e3} (solid line) and $K_{\mu3}$ (dotted line) obtained from $q^+=0$ frame. While the term proportional to f_0 in Eq. (4) is negligible for K_{e3} decay rate, its contribution for $K_{\mu3}$ decay rate is quite substantial (dot-dashed line). Also, we show in Fig. 4 the form factors $f_+(q^2)$ (solid and dotted lines for the sets 1 and

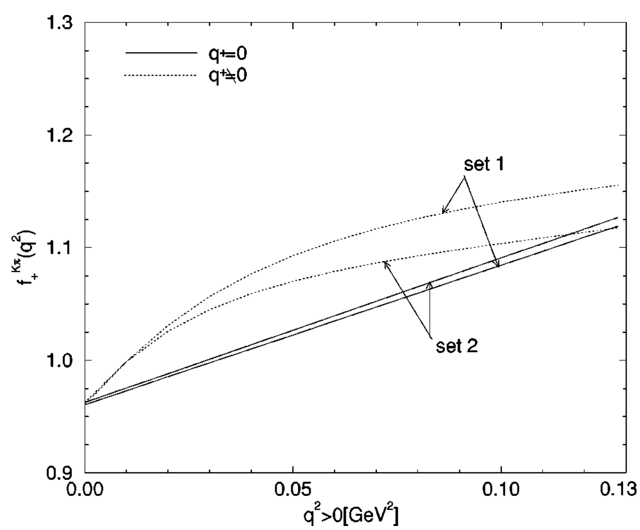

FIG. 2. The form factors $f_+(q^2)$ for the $K\to\pi$ transition in timelike momentum transfer $q^2>0$. The solid and dotted lines are the results from the $q^+=0$ and $q^+\neq0$ frames for the parameter sets 1 and 2 given in Table I, respectively. The differences of the results between the two frames are the measure of the nonvalence contributions from $q^+\neq0$ frame.

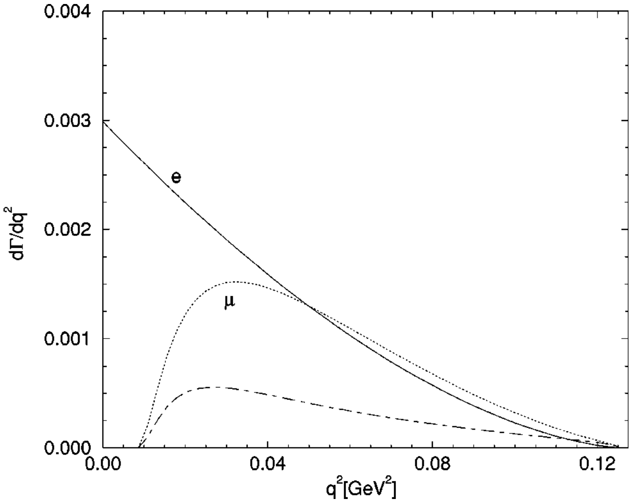

FIG. 3. The decay rates $d\Gamma/dq^2$ of K_{e3} (solid line) and $K_{\mu 3}$ (dotted line) for the parameter set 1 in $q^+=0$ frame. The dot-dashed line is the contribution from the term proportional to f_0 in Eq. (4) for $K_{\mu 3}$ decay. The results for the set 2 are not much different from those for the set 1.

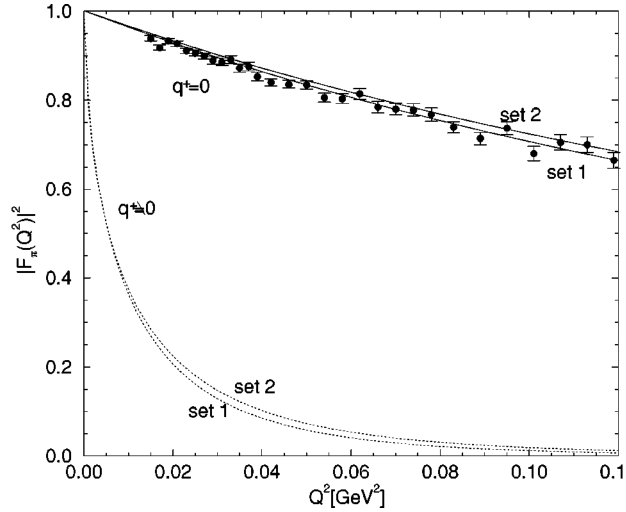

FIG. 5. The EM form factor of pion for low $Q^2=-q^2$ compared with data [23]. The solid and dotted lines are the results from the $q^+=0$ and $q^+\neq 0$ frames for the parameter sets 1 and 2, respectively.

2, respectively) at spacelike momentum transfer region and compare with the theoretical prediction from Ref. [14] (dot-dashed line). The measurement of this observable in $q^2<0$ region is anticipated from TJNAF [14].

In addition, we calculated the electromagnetic form factors $F_\pi(q^2)$ and $F_K(q^2)$ in the spacelike region using both $q^+=0$ and $q^+\neq 0$ frames to estimate the nonvalence contributions in $q^+\neq 0$ frame. As shown in Figs. 5 and 6 for $F_\pi(q^2)$ and $F_K(q^2)$, respectively, our predictions in $q^+=0$ frame are in a very good agreement with the available data [23,24] while the results for $q^+\neq 0$ frame deviate from the data significantly. The deviations represent the nonvalence contributions in $q^+\neq 0$ frame [see Fig. 1(b)]. However, the deviations are clearly reduced for $F_K(q^2)$ (see Fig. 6) because of the large suppression from the energy denominator shown in Fig. 1 for the nonvalence contribution. The suppressions are much bigger for the heavier mesons such as D and B. Especially, for the B meson case, the nonvalence contribution is almost negligible up to $Q^2=-q^2 \sim 10$ GeV^2.

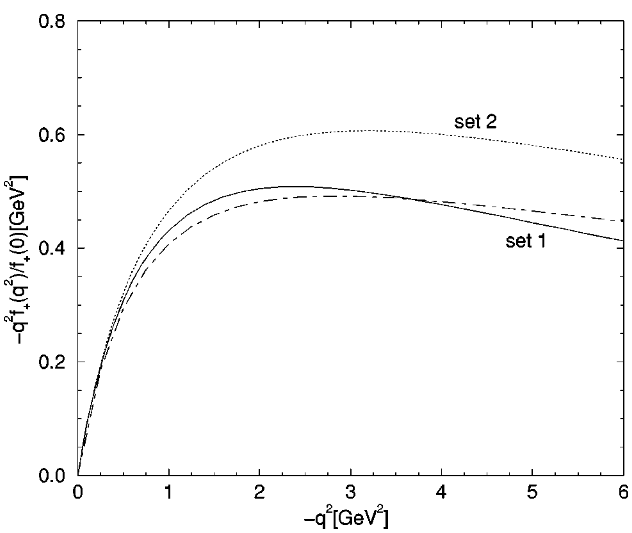

FIG. 4. The form factors $f_+(q^2)$ for the $K\to\pi$ transition in spacelike momentum transfer $-q^2<0$. The solid and dotted lines are the results from the sets 1 and 2, respectively. The dot-dashed line is the result from Ref. [14].

IV. SUMMARY AND CONCLUSION

In this work, we investigated the weak decays of K_{l3} using the light-front quark model. The form factors $f_\pm$ are obtained in $q^+=0$ frame and then analytically continued to the timelike region by changing $q_\perp$ to $iq_\perp$ in the form factors. The matrix element of the ''⊥'' component of the current J^μ is used to obtain the form factor f_-, which is nec-

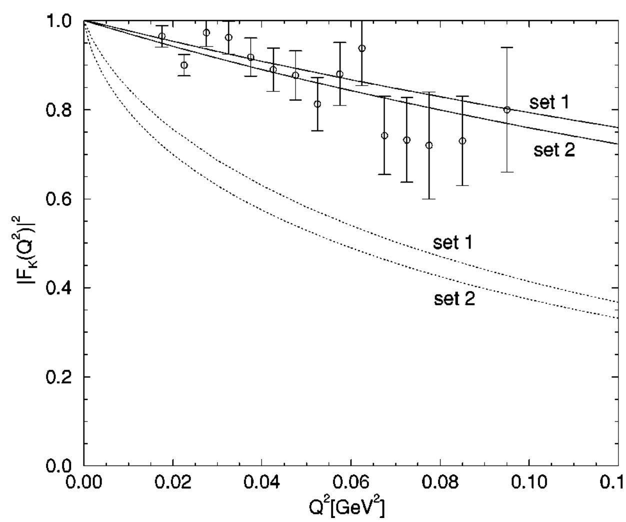

FIG. 6. The EM form factor of kaon compared with data [24]. The same line code as in Fig. 5 is used.

essary for the complete analysis of K_{l3} decays. Using the nonzero lepton mass formula [Eq. (4)] for the decay rate of K_{l3}, we also distinguish $K_{\mu 3}$ from K_{e3} decay. Especially, for $K_{\mu 3}$ decay, the contribution from f_0 [or f_-] form factor is not negligible in the calculation of the decay rate. Our theoretical predictions for K_{l3} weak decays are overall in a good agreement with the experimental data. We also confirmed that our analytic continuation method is equivalent to that of Ref. [8] where the form factors are obtained by the dispersion representations through the (Gaussian) wave functions of the initial and final mesons. In all of these analyses, it was crucial to include the nonvalence contributions in $q^+\neq 0$ frame. As we have estimated these contributions in various observables, their magnitudes are not at all negligible in the light-to-light electroweak form factors. In fact, the nonvalence contributions were very large for the most of observables such as λ_+, λ_0, ξ_A, $F_\pi(Q^2)$ and $F_K(Q^2)$.

Finally, we have also estimated the zero-mode contribution by calculating the ''$-$'' component of the current. Our observation in an exactly solvable scalar field theory was presented in Ref. [19]. Using the light-front bad current J^- in $q^+=0$ frame, we obtained $f_-(0)=12.6[18.6]$ for the set 1[set 2]. The huge ratio of $f_-(0)|_{J^-}/f_-(0)|_{J_\perp}\approx -36[-48]$ for the set 1[set 2] is consistent with our observation in Ref. [19]. We also found that the zero-mode contribution is highly suppressed as the quark mass increases. The detailed analysis of heavy-to-heavy and heavy-to-light semileptonic decays is currently underway.

ACKNOWLEDGMENTS

This work was supported by the U.S. DOE under contracts DE-FG02-96ER40947. The North Carolina Supercomputing Center and the National Energy Research Scientific Computer Center are also acknowledged for the grant of supercomputer time. We would like to acknowledge Andrei Afanasev for giving us their data for K_{l3} so that we can compare our results with theirs.

APPENDIX A: DERIVATION OF THE MATRIX ELEMENT OF THE WEAK VECTOR CURRENT $\langle P_2|\bar{q}_2\gamma^{\mu(=+,\perp)}q_1|P_1\rangle$ IN $q^+=0$ FRAME

In this appendix, we show the derivation of the matrix element of the weak vector current $\langle P_2|\bar{q}_2\gamma^\mu q_1|P_1\rangle$ given in Eq. (2) for $\mu=+$ and $\perp$, respectively.

In the light-front quark model, the matrix element of the weak vector current can be calculated by the convolution of initial and final light-front wave function of a meson as follows:

$$\langle P_2|\bar{q}_2\gamma^\mu q_1|P_1\rangle = \sum_{\lambda_1,\lambda_2,\bar{\lambda}}\int dp^{\pm}_{\bar{q}}d^2\mathbf{k}_\perp \phi_2^\dagger(x,\mathbf{k'}_\perp)\phi_1(x,\mathbf{k}_\perp)\mathcal{R}^{00\dagger}_{\lambda_2\bar{\lambda}} \times(x,\mathbf{k'}_\perp)\frac{\overline{u}(p_2,\lambda_2)}{\sqrt{p_2^+}}\gamma^\mu\frac{u(p_1,\lambda_1)}{\sqrt{p_1^+}}\mathcal{R}^{00}_{\lambda_1\bar{\lambda}}(x,\mathbf{k}_\perp), \quad \text{(A1)}$$

where the spin-orbit wave function $\mathcal{R}^{JJ_z}(x,\mathbf{k}_\perp)$ for pseudoscalar meson ($J^{PC}=0^{-+}$) obtained from Melosh transformation is given by

$$\mathcal{R}^{00}_{\lambda_i\bar{\lambda}}=\frac{1}{\sqrt{2}\sqrt{M_{i0}^2-(m_i-m_{\bar{q}})^2}}\overline{u}(p_i,\lambda_i)\gamma^5 v(p_{\bar{q}},\bar{\lambda}), \quad \text{(A2)}$$

and

$$M_{i0}^2=\frac{k_\perp^2+m_i^2}{1-x}+\frac{k_\perp^2+m_{\bar{q}}^2}{x}. \quad \text{(A3)}$$

Substituting Eq. (A2) into Eq. (A1) and using the quark momentum variables given in Eq. (7), one can easily obtain

$$\langle P_2|\bar{q}_2\gamma^\mu q_1|P_1\rangle =-\int dx d^2\mathbf{k}_\perp\frac{\phi_2^\dagger(x,\mathbf{k'}_\perp)\phi_1(x,\mathbf{k}_\perp)}{2(1-x)\Pi_i^2\sqrt{M_{i0}^2-(m_i-m_{\bar{q}})^2}} \times\mathrm{Tr}[\gamma_5(\not{p}_2+m_2)\gamma^\mu(\not{p}_1+m_1)\gamma_5(\not{p}_{\bar{q}}-m_{\bar{q}})], \quad \text{(A4)}$$

where we used the following completeness relations of the Dirac spinors

$$\sum_{\lambda_{1,2}}u(p,\lambda)\overline{u}(p,\lambda)=\not{p}+m, \quad \sum_{\lambda_{1,2}}v(p,\lambda)\overline{v}(p,\lambda)=\not{p}-m. \quad \text{(A5)}$$

In the frame where the decaying hadron is at rest and $q^+=0$, the trace terms in Eq. (A4) for the ''+'' and ''$\perp$'' components of the vector current $J^\mu=\bar{q}_2\gamma^\mu q_1$, respectively, are obtained as follows:

$$\mathrm{Tr}[\gamma_5(\not{p}_2+m_2)\gamma^\mu(\not{p}_1+m_1)\gamma_5(\not{p}_{\bar{q}}-m_{\bar{q}})] =-4[p_1^\mu(p_2\cdot p_{\bar{q}}+m_2m_{\bar{q}})+p_2^\mu(p_1\cdot p_{\bar{q}}+m_1m_{\bar{q}})+p_{\bar{q}}^\mu(-p_1\cdot p_2+m_1m_2)] =-\frac{4P^+}{x}[\mathcal{A}_1\mathcal{A}_2+\mathbf{k}_\perp\cdot\mathbf{k'}_\perp], \quad \text{for } \mu=+ \quad \text{(A6)}$$

$$=-2\left[\frac{(\mathcal{A}_1^2+k_\perp^2)}{x(1-x)}(\mathbf{k}_\perp-\mathbf{q}_\perp)+\frac{(\mathcal{A}_2^2+k'^2_\perp)}{x(1-x)}\mathbf{k}_\perp +[(m_1-m_2)^2+q_\perp^2]\mathbf{k}_\perp\right], \quad \text{for } \mu=\perp \quad \text{(A7)}$$

where $\mathcal{A}_i=m_ix+m_{\bar{q}}(1-x)$ and $\mathbf{k'}_\perp=\mathbf{k}_\perp-x\mathbf{q}_\perp$. Our convention of the scalar product, $p_1\cdot p_2=(p_1^+p_2^-+p_1^-p_2^+)/2-\mathbf{p}_{1\perp}\cdot\mathbf{p}_{2\perp}$ were used to derive Eqs. (A6) and (A7) from the second line of the above equation. Substituting Eqs. (A6) and

(A7) into Eq. (A4), we now obtain the matrix element of the weak vector current $\langle P_2|\overline{q_2}\gamma^\mu q_1|P_1\rangle$ for $\mu=+$ [see Eq. (8)] and $\perp$ [see Eq. (10)] in $q^+=0$ frame, respectively.

APPENDIX B: VALENCE CONTRIBUTIONS IN $q^+\neq 0$ FRAME

For the purely longitudinal momentum transfer, i.e., $\mathbf{q}_\perp=0$ and $q^2=q^+q^-$, the relevant quark momentum variables are

$$
\begin{aligned}
&p_1^+=(1-x)P_1^+, \quad p_{\bar{q}}^{\pm}=xP_1^+,\\
&\mathbf{p}_{1\perp}=(1-x)\mathbf{P}_{1\perp}+\mathbf{k}_\perp, \quad \mathbf{p}_{\bar{q}\perp}=x\mathbf{P}_{1\perp}-\mathbf{k}_\perp,\\
&p_2^+=(1-x')P_2^+, \quad p_{\bar{q}}'^{+}=x'P_2^+,\\
&\mathbf{p}_{2\perp}=(1-x')\mathbf{P}_{2\perp}+\mathbf{k}'_\perp, \quad \mathbf{p}'_{\bar{q}\perp}=x'\mathbf{P}_{2\perp}-\mathbf{k}'_\perp,
\end{aligned}
\tag{B1}
$$

where $x(x'=x/r)$ is the momentum fraction carried by the spectator $\bar{q}$ in the initial(final) state. The fraction r is given in terms of q^2 as follows [4–7]:

$$
r_\pm=\frac{M_2}{M_1}\left[\left(\frac{M_1^2+M_2^2-q^2}{2M_1M_2}\right)\pm\sqrt{\left(\frac{M_1^2+M_2^2-q^2}{2M_1M_2}\right)^2-1}\right],
\tag{B2}
$$

where the $+(-)$ signs in Eq. (B2) correspond to the daughter meson recoiling in the positive(negative) z-direction relative to the parent meson. In this $q^+\neq 0$ frame, one obtains [4–7]

$$
f_\pm(q^2)=\pm\frac{(1\mp r_-)H(r_+)-(1\mp r_+)H(r_-)}{r_+-r_-},
\tag{B3}
$$

where

$$
H(r)=\int_0^r dx\int d^2\mathbf{k}_\perp\,\phi_2(x',\mathbf{k}_\perp)\,\phi_1(x,\mathbf{k}_\perp)\times\frac{\mathcal{A}_1\mathcal{A}_2'+k_\perp^2}{\sqrt{\mathcal{A}_1^2+k_\perp^2}\sqrt{\mathcal{A}'^2_2+k_\perp^2}},
\tag{B4}
$$

and $\mathcal{A}_i'=m_ix'+m_{\bar{q}}(1-x')$.

[1] P. J. O'Donnell and Q. P. Xu, Phys. Lett. B **325**, 219 (1994); **336**, 113 (1994).
[2] P. J. O'Donnell, Q. P. Xu, and H. K. K. Tung, Phys. Rev. D **52**, 3966 (1995).
[3] W. Jaus, Phys. Rev. D **53**, 1349 (1996).
[4] N. B. Demchuk, I. L. Grach, I. M. Narodetskii, and S. Simula, Phys. At. Nucl. **59**, 2152 (1996).
[5] I. L. Grach, I. M. Narodetskii, and S. Simula, Phys. Lett. B **385**, 317 (1996).
[6] S. Simula, Phys. Lett. B **373**, 193 (1996).
[7] H.-Y. Cheng, C.-Y. Cheng, and C.-W. Hwang, Phys. Rev. D **55**, 1559 (1997).
[8] D. Melikhov, Phys. Lett. B **394**, 385 (1997); **380**, 363 (1996); Phys. Rev. D **53**, 2460 (1996).
[9] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[10] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
[11] J. Bijnens, G. Colangelo, G. Ecker, and J. Gasser, in *The Second DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Rome, Italy, 1995), p. 315.
[12] E. P. Shabalin, Sov. J. Nucl. Phys. **51**, 296 (1990).
[13] L.-M. Chounet, J.-M. Gailard, and M. K. Gailard, Phys. Rep. **4**, 199 (1974).
[14] A. Afanasev and W. W. Buck, Phys. Rev. D **55**, 4380 (1997).
[15] Yu. Kalinovsky, K. L. Mitchell, and C. D. Roberts, Phys. Lett. B **399**, 22 (1997).
[16] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).
[17] N. Isgur, Phys. Rev. D **12**, 3666 (1975).
[18] H.-M. Choi and C.-R. Ji, hep-ph/9711450.
[19] H.-M. Choi and C.-R. Ji, Phys. Rev. D **58**, 071901 (1998).
[20] S. J. Brodsky and D. S. Hwang, hep-ph/9806358.
[21] D. S. Hwang and D.-W. Kim, hep-ph/9806242.
[22] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[23] S. R. Amendolia *et al.*, Phys. Lett. **146B**, 116 (1984).
[24] S. R. Amendolia *et al.*, Phys. Lett. B **178**, 435 (1986).