Topological mass mechanism from dimensional reduction

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We show that the Abelian topological mass mechanism in four dimensions, described by the Cremmer-Sherk action, can be obtained from dimensional reduction in five dimensions. Starting from a gauge invariant action in five dimensions, where the dual equivalence between a massless vector field and a massless second-rank antisymmetric field is established, the dimensional reduction is performed keeping only one massive mode. [S0556-2821(98)03324-4]

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Several alternatives of the Higgs mechanism, based on the coupling of vector fields with antisymmetric fields through topological terms, have been developed in the last years. In particular, the Abelian Cremmer-Sherk theory [1] has been studied extensively [2] as an appropriate model for this proposal. Its non-Abelian extension is a Freedman-Townsend theory [3] that can be derived using the self-interaction mechanism [4]. Also, other attempts to search a non-Abelian generalization have been formulated [5]; however, serious problems with renormalizability in all these non-Abelian generalizations has been pointed out in Ref. [6]. An interesting aspect is the fact that the Cremmer-Sherk theory is related by duality [7,4] with the Kalb-Ramond theory [8], where the latter can be obtained by dimensional reduction of the second-rank antisymmetric theory in five dimensions keeping only one massive mode [9,10]. It is worth recalling that the Kalb-Ramond theory provides mass in a nontopological way and can be understood as the resulting theory after condensation of magnetic monopoles in four dimensions QED [11]. In this Brief Report, we show that it is possible to obtain the Cremmer-Sherk theory in four dimensions by dimensional reduction, despite Barcelos-Neto's claim that the Cremmer-Sherk theory cannot come from dimensional reduction of any gauge theory in five dimensions [10].

Let us describe briefly the dual equivalence between the Cremmer-Sherk and Kalb-Ramond theories in four dimensions. The Cremmer-Sherk action, which provides mass to spin-1 fields without spoil gauge invariance, is written down as

$$I_{\mathrm{CrSc}}=\int d^4x\left[-\frac{1}{4}F_{mn}F^{mn}-\frac{1}{12}H_{mnp}H^{mnp}-\frac{1}{4}\mu\epsilon^{mnpq}B_{mn}F_{pq}\right], \qquad (1)$$

where $F_{mn}\equiv\partial_m A_n-\partial_n A_m$ and $H_{mnp}\equiv\partial_m B_{np}+\partial_n B_{pm}+\partial_p B_{mn}$ are the field strengths associated with the A_m and B_{mn} fields. This action is invariant under gauge transformations: $\delta A_m=\partial_m\lambda$ and $\delta B_{mn}=\partial_m\zeta_n-\partial_n\zeta_m$. We observe that this action has global symmetries, for instance, $A_m\to A_m-\epsilon_m$, with ϵ_m the global parameter. A useful way to obtain the dual theory relies on gauging this local symmetry [12] by introducing an antisymmetric field a_{mn}, such that the field strength F_{mn} is modified by $F_{mn}\equiv\partial_m A_n-\partial_n A_m+\frac{1}{2}(a_{mn}-a_{nm})$ and adding a BF term, which assures the nonpropagation of a_{mn}. Then, we have the following action:

$$I_M=\int d^4x\left\{-\frac{1}{4}\left[\partial_m A_n-\partial_n A_m+\frac{1}{2}(a_{mn}-a_{nm})\right]^2-\frac{1}{12}H_{mnp}H^{mnp}-\frac{1}{4}\mu\epsilon^{mnpq}B_{mn}\times\left[\partial_p A_q-\partial_q A_p+\frac{1}{2}(a_{pq}-a_{qp})\right]+\frac{1}{2}a_m\epsilon^{mnpq}\partial_n a_{pq}\right\} \qquad (2)$$

and we now have new gauge symmetries, given by $\delta A_m=-\epsilon_{m(x)}$, $\delta a_m=\partial_m\alpha$, and $\delta a_{mn}=\partial_m\epsilon_n-\partial_n\epsilon_m$, which allow us to fix the gauge $A_m=0$. After fixing this gauge, the action becomes

$$I_M=\int d^4x\left[-\frac{1}{4}a_{mn}a^{mn}-\frac{1}{12}H_{mnp}H^{mnp}-\frac{1}{4}\mu\epsilon^{mnpq}B_{mn}a_{pq}+\frac{1}{2}\epsilon^{mnpq}a_{mn}\partial_p a_q\right]. \qquad (3)$$

Integrating out the field $a^{mn}\{=-\frac{1}{2}\epsilon^{mnpq}[\mu B_{pq}-(\partial_p a_q-\partial_q a_p)]\}$, the Kalb-Ramond action is obtained:

$$I_{KR}=\int d^4x\left[-\frac{1}{12}H_{mnp}H^{mnp}-\frac{1}{4}[(\partial_m a_n-\partial_n a_m)-\mu B_{mn}][(\partial^m a^n-\partial^n a^m)-\mu B^{mn}]\right]. \qquad (4)$$

This action is invariant under gauge transformations given by $\delta B_{mn}=\partial_m\zeta_n-\partial_n\zeta_m$ and $\delta a_m=\partial_m\alpha-\mu\zeta_m$, which allow us to gauge away the a_m field, leading to the massive B_{mn} antisymmetric action, describing three massive degrees of freedom similar to Abelian massive vector theories in four dimensions. Having seen that in four dimensions the Cremmer-Sherk and Kalb-Ramond theories are equivalent by duality and that the Kalb-Ramond action can be obtained

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from dimensional reduction of a two-form in five dimensions, it must be possible to obtain the Cremmer-Sherk action from dimensional reduction of some gauge theory in five dimensions. Since the dual of a two-form in five dimensions is a one-form, we will consider the following master action in five dimensions (the indices M,N run over 0,1, . . . ,4):

$$I=\int d^5x\left[\frac{1}{4}V^{MN}V_{MN}-\frac{1}{2}V^{MN}[\partial_M A_N-\partial_N A_M]\right], \quad (5)$$

which is just the first order formulation of the Maxwell action and where V^{MN} and A_M are considered as independent variables. Indeed, the Maxwell action is recovered after eliminating V^{MN} through its equation of motion ($V_{MN}=[\partial_M A_N-\partial_N A_M]$). On the other hand, the equations of motion after independent variations in A_M are

$$\partial_M V^{MN}=0, \quad (6)$$

whose solution in five dimensions can be written (locally) as

$$V^{MN}=\frac{1}{2}\epsilon^{MNPQR}\partial_P B_{QR} \equiv \frac{1}{6}\epsilon^{MNPQR}H_{PQR}. \quad (7)$$

Substituting this expression for V^{MN} in Eq. (5), we obtain the action for the second-rank antisymmetric field $I_B=-\frac{1}{12}H_{MNP}H^{MNP}$. In general, for $D>2$ dimensions, the solution of Eq. (6) is

$$V^{MN}=\frac{1}{(D-3)!}\epsilon^{MNPQ_1\cdots Q_{D-3}}\partial_P B_{Q_1\cdots Q_{D-3}} \quad (8)$$

and going back into Eq. (5), the usual duality between one- and zero-forms in three dimensions, one- and one-forms in four dimensions, and so on, is achieved.

Now, we are going to perform the dimensional reduction. Let us deal with complex fields in five dimensions, then we consider the following action:

$$I=\int d^5x\left[\frac{1}{4}V^{MN}V^*_{MN}-\frac{1}{4}V^{MN*}\times[\partial_M A_N-\partial_N A_M]-\frac{1}{4}V^{MN}[\partial_M A^*_N-\partial_N A^*_M]\right]. \quad (9)$$

Using the method of dimensional reduction developed in Ref. [13] for generating mass, we write, in the limit $R\to 0$ (R being the radius of the small circle in the coordinate x^4 where we are compactifying),

$$V_{mn(x,x^4)}=F_{mn(x)}e^{i\mu x^4}, \quad A_{m(x,x^4)}=A_{m(x)}e^{i\mu x^4},$$
$$V_{m4(x,x^4)}=iV_{m(x)}e^{i\mu x^4}, \quad A_{4(x,x^4)}=i\phi_{(x)}e^{i\mu x^4}. \quad (10)$$

As a consequence, we obtain the following reduced action to four dimensions:

$$I_{\text{red4D}}=\int d^4x\left[\frac{1}{4}F^{mn}F_{mn}-\frac{1}{2}F^{mn}[\partial_m A_n-\partial_n A_m]+\frac{1}{2}V^mV_m-V^m[\partial_m\phi-\mu A_m]\right]. \quad (11)$$

From this reduced action, we can eliminate the F_{mn} and ϕ fields through its equations of motion:

$$F_{mn}=\partial_m A_n-\partial_n A_m, \quad \partial_m V^m=0 \quad (12)$$

and we have the following (local) solution for V^m in four dimensions:

$$V^m=-\frac{1}{2}\epsilon^{mnpq}\partial_n B_{pq}=-\frac{1}{6}\epsilon^{mnpq}H_{npq}. \quad (13)$$

Putting this back into the reduced action, the Cremmer-Sherk action is obtained.

Furthermore, the field equation after varying the A_m field is

$$\partial_n F^{nm}=-\mu V^m, \quad (14)$$

which combined with Eq. (13), lead us to following solution for F^{mn}:

$$F^{mn}=\frac{1}{2}\epsilon^{mnpq}[-\mu B_{pq}+(\partial_p a_q-\partial_q a_p)]. \quad (15)$$

Then, we can eliminate the ϕ and A_m fields and substituting Eqs. (13) and (15) into the reduced action, the Kalb-Ramond action is obtained. Moreover, from the reduced action we can integrate out the V_m field ($V_m=\partial_m\phi-\mu A_m$), in order to reach the Stückelberg formulation for massive spin-one fields: $I_{\text{Stuck}}=\int d^4x\{-\frac{1}{4}F^{mn}F^{mn}-\frac{1}{2}\mu^2[A_m-(1/\mu)\,\partial_m\phi]^2\}$.

Summarizing, we have shown that the Cremmer-Sherk action, which provides mass for vector fields compatible with gauge symmetry, as well as the Kalb-Ramond and Stückelberg actions, can be obtained from dimensional reduction of a gauge theory, which reflects the dual equivalence between the Maxwell field and the second-rank antisymmetric field in five dimensions.

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