String duality and nonsupersymmetric strings

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In recent work Kachru, Kumar, and Silverstein introduced a special class of nonsupersymmetric type II string theories in which the cosmological constant vanishes at the first two orders of perturbation theory. Heuristic arguments suggest the cosmological constant may vanish in these theories to all orders in perturbation theory leading to a flat potential for the dilaton. A slight variant of their model can be described in terms of a dual heterotic theory. The dual theory has a nonzero cosmological constant which is nonperturbative in the coupling of the original type II theory. The dual theory also predicts a mismatch between Bose and Fermi degrees of freedom in the nonperturbative D-brane spectrum of the type II theory. [S0556-2821(98)01724-X]

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I. INTRODUCTION

In spite of great recent progress in the understanding of string theory and quantum gravity the smallness of the cosmological constant remains a great mystery. At present the only obvious explanation for a vanishing cosmological constant is unbroken supersymmetry. Since supersymmetry is broken if it is realized in nature the puzzle becomes why the cosmological constant is so much smaller than the scale set by the supersymmetry breaking scale, that is why $\Lambda \ll (\mathrm{TeV})^4$.

Motivated by the anti-de Sitter space (AdS) conformal field theory (CFT) correspondence [1,2] Kachru and Silverstein have suggested [3] that the cosmological constant Λ might vanish in certain special nonsupersymmetric string theories and together with Kumar have constructed a candidate such theory [4]. The one-loop contribution to Λ vanishes trivially in the model of Ref. [4] due to equality between the number of boson and fermion mass states at each level (in spite of the fact that the model is not supersymmetric). What is more surprising is the claim that the two-loop and perhaps higher loop contributions also vanish since without super-symmetry one would expect the nonsupersymmetric interactions to spoil the cancellation at some order in perturbation theory. Unfortunately the intricacies of higher-loop calculations in fermionic string theory make a direct analysis of this claim difficult. In addition, if the perturbative contribution to Λ does indeed vanish it will be important to investigate nonperturbative contributions and their dependence on the string coupling constant.

In this paper an indirect approach to this problem is taken using string duality. The heuristic arguments of Refs. [3,4] suggest that the model of Ref. [4] has a vanishing dilaton potential and therefore exists at all values of the coupling. If this were the case we should be able to use string duality to study the strongly coupled limit of this theory. As it turns out, we will find evidence that the cosmological constant is nonzero so that there is a potential for the dilaton. Because of this the theory probably does not have a stable vacuum at strong coupling.

In spite of this one might hope to study some features of the theory using string duality. For example, the theory presumably has cosmological solutions in which the dilaton evolves in time and one could try to use string duality to explore the dilaton potential in different regimes. In the models at hand the dilaton potential vanishes in a limit where supersymmetry is restored and one can try to use duality to study the theory near this limit. Of course the use of string duality is on much less firmer ground in theories without spacetime supersymmetry because one loses the Bogomol'nyi-Prasad-Sommerfield (BPS) states and nonrenormalization theorems which provide the most direct evidence for string duality. Nonetheless there are some rough indications that duality can be applied in this context [5]. In addition the string-string duality that will be applied here is well understood in the supersymmetric case [6–9] and the adiabatic argument of Ref. [10] can be applied to the nonsupersymmetric dual pairs which are constructed. These facts plus the nature of the results found here give some indication that string duality can be successfully used to study models of the type discussed in Ref. [4].

II. A NONSUPERSYMMETRIC STRING

Following Ref. [4] consider type IIA string theory compactified on a six-torus T^6. We take the first four components to be a square torus at the self-dual radius $T^4=(S^1_R)^4$ with $R=1/\sqrt{2}$ (the string scale has been set to 1). The last two components we take to be the product of two circles of radius R_5 and R_6. We now consider an asymmetric orbifold of this theory. An element of the space group of the orbifold will be denoted by

$$[(\theta_L),(\theta_R),(v_L),(v_R),\Theta], \tag{2.1}$$

where $\theta_{L,R}$ are rotations by elements of SO(6) acting on the left- or right-moving degrees of freedom and $v_{L,R}$ are shifts acting on the left- or right-moving bosons. In the examples considered here $\theta_{L,R}$ are order 2 and will be denoted by listing their eigenvalues. Θ will either be the identity or $(-1)^{F_{L,R}}$ which are rotations by 2π acting on the left- or right-moving degrees of freedom and are therefore $+1$ on left- or right-moving bosonic states and -1 on left- or right-moving fermion states.

The asymmetric orbifold is generated by the following two elements f and g:

$$f=[(-1^4,1^2),(1^6),(0^4,v_L^5,v_L^6),(s^4,v_R^5,v_R^6)(-1)^{F_R}],$$

$$g=[(1^6),(-1^4,1^2),(s^4,w_L^5,w_L^6),(0^4,w_R^5,w_R^6)(-1)^{F_L}]. \quad (2.2)$$

Here a power indicates repeated entries (except on w or v) and s is a shift by $R/2=1/2\sqrt{2}$ and modular invariance (level matching) requires that

$$(v_L^5)^2+(v_L^6)^2-(v_R^5)^2-(v_R^6)^2 = (w_L^5)^2+(w_L^6)^2-(w_R^5)^2-(w_R^6)^2=1/2. \quad (2.3)$$

The model considered in Ref. [4] arises by setting $R_5=R_6=1/\sqrt{2}$ and taking $v_L^5=1/\sqrt{2}$, $v_R^5=0$, $v_L^6=v_R^6=1/2\sqrt{2}$ and $w_L^5=w_R^5=1/2\sqrt{2}$, $w_L^6=0$, $w_R^6=1/\sqrt{2}$.

The shifts can be described more transparently in the notation of Ref. [10]. Points in the Narain lattice $\Gamma^{1,1}$ are described by pairs of integers (m,n) with m labeling the momentum and n the winding. There are three choices of shift vector A with $2A\in\Gamma^{1,1}$ modulo vectors in $\Gamma^{1,1}$: $A_1=(1/2,0)$, $A_2=(1/2,1/2)$, and $A_3=(0,1/2)$. The model of Ref. [4] has shifts by A_1 in the first four components of T^6 for both f and g, a shift in the fifth component by A_2 in f and by A_1 in g, and a shift in the sixth component by A_1 in f and by A_2 in g.

Besides satisfying level-matching, the shifts in the model of Ref. [4] are chosen to ensure that there are no massless states in sectors twisted by f, g, or fg. f and g project out the gravitinos coming from the right- and left-movers and without shifts the gravitinos would come back in the sectors twisted by f or g. The sector twisted by fg on the other hand does not lead to gravitinos even without shifts.

Now consider a slight variation of the previous model in which the shift $w_{L,R}^5$ is exchanged with $w_{L,R}^6$ and denote the resulting generators by f' (which is the same as f) and g'. In addition since the radii of the last two components of T^6 are not fixed by the asymmetric orbifold we should allow arbitrary radii R_5 and R_6 in these components. Furthermore, after this modification, the last component of T^6 is irrelevant to the construction, the shift in this circle is by A_1 in both f and g and is not needed for modular invariance or to ensure that there are no massless states in the sectors twisted by f or g. For simplicity we might as well take $R_6\to\infty$ and consider the resulting five-dimensional variant of the previous model. We will also denote R_5 by R' from here on.

This gives a model which like the model of Ref. [4] is not supersymmetric and which has equal numbers of bosons and fermions at each mass level at string tree level. Most of the higher loop analysis of Ref. [4] also appears to be unchanged by this modification although this has not been investigated in detail. Also, in Ref. [4] a heuristic argument for the vanishing of Λ was given using the AdS-CFT correspondence. This argument involves the existence of Reissner-Nordström (RN) black holes with two-dimensional AdS (AdS_2) near horizon geometry. This in turn requires the vanishing of couplings of the form $\int\phi F^2$ with ϕ a modulus and F the gauge field of the U(1) gauge symmetry under which the black hole is charged. In the model of Ref. [4] such black holes can be constructed as linear combinations of D-brane states which are invariant under the orbifold group [11]. These give rise to RN black holes which carry charge under a U(1) which arises in the untwisted, RR sector of the orbifold. The modification made here leads to new moduli in the fg twisted sector of the orbifold. However, these new moduli cannot have couplings of the form ϕF^2 since ϕ arises in a twisted sector while F comes from the untwisted sector. Thus the heuristic argument of Ref. [4] also goes through with this modification.

Finally, it is important to note that f and g do not commute as elements of the space group S either in the original model or in the modification considered here. However, in constructing a string orbifold we can first mod out the theory in R^d by the lattice Λ, which is the normal subgroup of S consisting of all pure translations in the space group. We can then mod out the resulting theory by the point group $\overline{P}=S/\Lambda$. In the one-loop string path integral it is necessary to sum over twist structures on the world-sheet torus by commuting pairs elements of the orbifold group. Thus from the space group point of view there are no sectors with boundary conditions (f,g). On the other hand, f and g do commute as elements of $\overline{P}$ so that if we first mod out by Λ and then mod out by $\overline{P}$ we do expect to have sectors with boundary conditions (f,g). These two points of view are reconciled by the fact that the contribution from the sector (f,g) vanishes due to the fact that the trace of f is zero in the Hilbert space twisted by g and vice versa. The fact that f and g commute in the point group also means that we can first consider the theory twisted by the product fg and then mod out this theory by f.

III. ANALYSIS OF THE MODIFIED MODEL

Now consider the modified model constructed by first modding out by the product $f'g'$ and the modding out by f'. The product $f'g'$ is given by

$$f'g'=[(-1^4,1),(-1^4,1),(0^5),(0^5),(-1)^{F_L+F_R}]. \quad (3.1)$$

The shifts in the product $f'g'$ are shifts by elements of the lattice and can thus be taken to be zero in the point group. The $(-1)^4$ action is a twist by an element in the center of one of the SU(2) factors in the decomposition SO(4)=SU(2)×SU(2) of the rotation acting on the first four coordinates. Furthermore, the factor of $(-1)^{F_L+F_R}$ can be dropped since it is simply a 2π rotation on both left and right coordinates and so can be absorbed by a choice of the $(-1)^4$ action.

Thus the theory twisted by $f'g'$ is just type IIA theory on $T^4/Z^2\times S^1$ which is an orbifold limit of type IIA theory on $K3\times S^1$. The massless spectrum of this theory is just the naive dimensional reduction on the S^1 of type IIA theory on T^4/Z_2. In terms of representations of $D=6$, (1,1) supersymmetry the massless spectrum on T^4/Z_2 consists of the graviton supermultiplet $G_{(1,1)}$ and four copies of the matter multiplet $\Phi_{(1,1)}$ from the untwisted sector and sixteen copies of the matter multiplet $\Phi_{1,1}$ coming from the twisted sector. In terms of the massless little group SO(4)=SU(2)×SU(2) these representations decompose as

$$\Phi_{(1,1)}=[(2,2)+2(2,1)+2(1,2)+4(1,1)],$$

$$G_{(1,1)}=[(3,3)+2(3,2)+2(2,3)+(3,1)+(1,3)+4(2,2)+2(1,2)+2(2,1)+(1,1)]=[(2,2)]\times\Phi_{(1,1)}. \tag{3.2}$$

We now twist this theory by f' which acts as a twist by -1 on each of the four left-moving coordinates of T^4, as a shift by A_1 on each of the four components of T^4, as a shift by A_2 on the S^1 and as $(-1)^{F_R}$ on the right-moving degrees of freedom. In the untwisted sector this implies that f' has eigenvalue ±1 on the bosons (fermions) in $G_{(1,1)}$ and eigenvalue ±1 on the fermions (bosons) in $4\Phi_{(1,1)}$ coming from the untwisted sector. Acting on the states in the sector twisted by $f'g'$, the shifts in f' permute the 16 fixed points so that the trace of f' vanishes. Taking this and the $(-1)^4_L(-1)^{(F_R)}$ action into account one sees that f' acts as ±1 on the bosons (fermions) for eight linear combinations of the $16\Phi_{(1,1)}$ and as ∓1 on the bosons (fermions) for the other eight orthogonal linear combinations. Thus projecting onto f' invariant states leads to equal numbers of massless (and massive) bosons and fermions in the sector twisted by $f'g'$.

Because of the asymmetric shifts in f' and g' there are no massless states in the sectors twisted by f' and g' separately. As in the model of Ref. [4] the massive states are Bose-Fermi degenerate in these sectors as well.

IV. THE HETEROTIC DUAL

We have seen that the type IIA theory twisted by $f'g'$ is an orbifold limit of type IIA on $K3\times S^1$ and this is known to be dual to heterotic string on $T^4\times S^1$ [6–9]. It will be useful in what follows to recall some facts from the detailed discussion in Ref. [7]. Consider the heterotic string compactified on a fixed T^4 and a circle S^1_R of radius R and with coupling λ. This is dual to a type IIA string with coupling λ' on a fixed $K3$ and a circle of radius R' or after T-duality to a type IIB theory with coupling λ'' on a fixed $K3$ and a circle of radius R''. The relation between the parameters is

$$\lambda'=1/\lambda,\quad R'=R/\lambda,$$

$$\lambda''=1/R,\quad R''=\lambda/R. \tag{4.1}$$

The mapping between charged states in the heterotic and type IIA theories is such that states with momentum on the S^1 in the heterotic string map to momentum states in type IIA, winding heterotic states map to wrapped five-branes in type IIA, wrapped heterotic five-branes map to winding states in type IIA and perturbative states charged under the original ten-dimensional gauge fields of the heterotic string map to D0-brane states in the five-dimensional IIA theory (some of which arise from wrapping D2- and D4-branes on the $K3$). The mapping of states in the type IIB description follows from the standard action of T duality on branes.

Now given a pair of dual theories it is often possible to construct further dual pairs by orbifolding [12]. This procedure is most reliable when the orbifold symmetry acts freely on an S^1 so that the adiabatic argument of Ref. [10] can be used. This is true in the case at hand. We mod out the type II theory on $T^4/Z_2\times S^1$ by the action of f' in order to obtain a nonsupersymmetric string. Since f' acts as a shift by A_2 on the S^1 we can apply the adiabatic argument.

In order to construct the heterotic dual we need to know the image f_H of f' under duality. Given the action of f' on the $U(1)^{26}$ gauge bosons of the type II theory on $T^4/Z_2\times S^1$ it is easy to see that up to shifts f_H must have twelve $+1$ eigenvalues and twelve -1 eigenvalues when acting on the left-moving degrees of freedom of the heterotic string and act as $(-1)^{F_R}$ on the right-moving degrees of freedom. On the type II side f' exchanges the 16 fixed points in the $f'g'$ twisted sector with each other. On the heterotic side this maps to an action of f_H which exchanges two E_8 lattices on the left. The remaining four -1 eigenvalues must then come from a -1^4 action on four left-moving coordinates of a $\Gamma^{4,4}$ lattice. String-string duality does not give a unique prescription for the shifts, in part because perturbative shifts on the heterotic side map to Ramond-Ramond (RR) gauge transformations on the type II side which are not visible in perturbation theory [12]. The shifts on the heterotic side must then be determined by demanding level matching. In this case f_H acts as -1 on 12 left-moving bosons and this raises the vacuum energy by $\frac{12}{16}=\frac{3}{4}$. This can be compensated by a shift by A_1 in each of the four components of the $\Gamma^{4,4}$ lattice.[1]

Finally, modular invariance is consistent with a shift in the remaining $\Gamma^{1,1}$ lattice by either A_1 or A_3. We choose the shift to be A_3 so that supersymmetry is restored at large radius. The T-dual theory would have a shift by A_1 and have supersymmetry restored at small R.

To summarize, we consider a point in the Narain moduli space of the heterotic string on $T^4\times S^1$ where the Narain lattice can be decomposed as[2]

$$\Gamma^{21,5}=\Gamma^{8,0}\oplus\Gamma^{8,0}\oplus\Gamma^{4,4}(D_4)\oplus\Gamma^{1,1}(R). \tag{4.2}$$

Here $\Gamma^{8,0}$ is the E_8 lattice, $\Gamma^{4,4}(D_4)$ is the (4,4) Narain lattice at the $D_4=SO(8)$ enhanced symmetry point, and $\Gamma^{1,1}$ is the Narain lattice for compactification on a S^1 of radius R. Then in terms of this decomposition f_H acts as an interchange of the two $\Gamma^{8,0}$ factors, as -1^4 on the left-moving degrees of freedom and a shift by A_1^4 on the right-moving degrees of freedom of the third component and as a shift by A_3 on the fourth component in addition to the $(-1)^{F_R}$ action on the right movers.

[1]If we take the $\Gamma^{4,4}$ lattice to be at the SU(2) self-dual point in all four coordinates then f_H is order 4. Its square is a translation and orbifolding by this translation generates a $\Gamma^{4,4}$ lattice at the SO(8) enhanced symmetry point. Acting on this new lattice f_H is then order 2.

[2]This is not the same point in moduli space as the type II dual and is chosen to simplify the presentation.

Twisting the heterotic theory by f_H breaks all the supersymmetry since it projects out the gravitino coming from the right-movers. As in the type II construction, supersymmetry is restored in the large radius limit. The massless spectrum in the untwisted sector has equal numbers of bosons and fermions. To see this note that since f_H is ± 1 on right moving bosons (fermions) and has an equal number of ± 1 eigenvalues on the left the massless states with one left-moving oscillator state excited come in Bose-Fermi pairs. Similarly, f_H has 252 eigenvalues $+1$ and 252 eigenvalues -1 acting on $P_L^2=2$ states in the $\Gamma^{8,0}\times\Gamma^{8,0}\times\Gamma^{4,4}(D_4)$ lattice so these states also contribute equal numbers of bosons and fermions. Note that there is a low-energy non-Abelian E_8 gauge theory with the same field content as that of $N=4$ supersymmetric gauge theory. In the heterotic theory there are states with momentum m/R which become massless as $R\to\infty$. From the previous argument it is clear these states are also Bose-Fermi degenerate. Since these states map to perturbative momentum states in the type II theory which are Bose-Fermi degenerate this is required for the duality to act correctly.

Now let us study the spectrum of states of this asymmetric heterotic orbifold which are massive at large R. First consider the twisted sector. In the twisted sector all states are massive at a generic radius R as a result of the shift by A_3. Since there are 12 antiperiodic bosons on the left the left-moving vacuum energy is $E_L=-\frac{1}{4}+(A_{3L})^2/2$. The right-moving fields are untwisted but have a shift in the $\Gamma^{4,4}$ which raises the vacuum energy by $\frac{1}{4}$ so the right-moving vacuum energy is $E_R=-1/4+(A_{3R})^2/2$ in the Neveu-Schwarz (NS) sector and $E_R=1/4+(A_{1R})^2/2$ in the Ramond sector. Because of the $(-1)^{F_R}$ action we must also change the Gliozzi-Scherk-Olive (GSO) projection so that the NS vacuum now survives the GSO projection. We thus see that this theory has a tachyon for $R<\frac{1}{2}\sqrt{2}$. In the Ramond sector there are of course no tachyons, so there is a mismatch between bosons and fermions in the twisted sector. This mismatch clearly continues to exist at large values of R where there is no tachyon.

There is also a mismatch between bosons and fermions in the untwisted sector of the orbifold for states which are massive at large R. This can be seen just be writing down the first few massive states or can be summarized by the one-loop partition function in the untwisted sector. This is given by $(Z_{1,1}+Z_{f_H,1})/2$ where $Z_{a,b}$ is the one-loop torus amplitude with boundary condition twisted by a in the time direction and by b in the space direction and the usual sum over fermion spin structures has been suppressed. $Z_{1,1}=0$ since supersymmetry is only broken by the f_H projection. On the other hand

$$Z_{f_H,1}(\tau)=\frac{\Theta_{2E_8}(q)}{\eta(q^2)^{12}\eta(\overline{q})^8}\,\theta_4^4(\overline{q}^2)\times\left(\sum_{p\in\Gamma^{1,1}} q^{p_L^2/2}\overline{q}^{p_R^2/2}e^{2\pi i p\cdot A_1}\right)\times\left(\frac{\theta_3^4(\overline{q})}{\overline{\eta}^4}-\frac{\theta_4^4(\overline{q})}{\overline{\eta}^4}+\frac{\theta_2^4(\overline{q})}{\overline{\eta}^4}\right), \tag{4.3}$$

where $q=e^{2\pi i\tau}$ with τ the modular parameter of the world-sheet torus. Θ_{2E_8} is the theta function for the E_8 lattice with norm rescaled by 2, and standard notation has been used for the other theta functions and for the Dedekind η function. Writing out the first few terms in the $q,\overline{q}$ expansion of $Z_{f_H,1}$ shows a mismatch between bosons and fermions at massive levels.

Massive charged states in the heterotic theory map to charged non-perturbative wrapped brane states in the type II theory. Although there is no BPS formula protecting the mass, the lightest state of a given charge must be stable even without supersymmetry and so we should be able to compare these states in the type II and heterotic descriptions. Thus the mismatch in the heterotic theory in the untwisted sector predicts a mismatch in the D0-brane spectrum of the type II theory and the mismatch in the twisted sector implies a mismatch in the wrapped five-brane states of the type IIB theory.

It is also interesting to compute the cosmological constant in the heterotic theory in order to compare with the perhaps vanishing perturbative contribution in the type II theory. The cosmological constant is proportional to the vacuum amplitude

$$\Lambda\sim\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\,(\alpha'\tau_2)^{-5/2}[Z_{1,1}(\tau)+Z_{f_H,1}(\tau)+Z_{1,f_H}(\tau)+Z_{f_H,f_H}(\tau)], \tag{4.4}$$

where $\mathcal{F}$ is the fundamental domain for the modular group $|\tau|>1,|\tau_1|<1/2$. As discussed above $Z_{1,1}=0$ by supersymmetry. The latter two terms in Eq. (4.4) can be determined from Eq. (4.3) using the modular transformations $\tau\to-1/\tau$ and $\tau\to\tau+1$.

The analysis of the cosmological constant in this theory is mathematically very similar to the analysis of the free energy of superstrings at temperature $T\sim 1/R$ [13]. At small R there is a divergence in Λ coming from the tachyon. We can however examine the large R behavior of Λ and compare to type II theory using the duality relations (4.1).

Using the fact that the three terms contributing to Λ are related by the modular transformations $\tau\to-1/\tau$ and $\tau\to\tau+1$ we can write Eq. (4.4) as an integral of $Z_{f_H,1}$ over the fundamental domain of the $\Gamma_0(2)$ subgroup of the modular group Γ. Denoting this by $\mathcal{F}_2$ we then have

$$\Lambda\sim\int_{\mathcal{F}_2}\frac{d^2\tau}{\tau_2^{7/2}}F(q,\overline{q})\sum_{m,n}(-1)^m e^{2\pi i\tau_1 mn}e^{-\pi\tau_2(m^2/2R^2+2n^2R^2)}, \tag{4.5}$$

where $F(q,\overline{q})$ stands for the terms in Eq. (4.3) other than the sum over the $\Gamma^{1,1}$ lattice. From the previous comments we know that F takes the form

$$F(q,\overline{q})=\sum_{i,j}d(i,j)q^i\overline{q}^j=16(q^{-1}+252q+\cdots)(1+8\overline{q}+\cdots). \tag{4.6}$$

Note the absence of a q^0 term in the q expansion of F which indicates Bose-Fermi degeneracy for massless states. In this form the behavior of the integral at large R is not evident because many terms contribute at large R. However we can use Poisson resummation on m to rewrite the double sum in Eq. (4.5) in the form

$$\sum_{m,n}(-1)^m e^{2\pi i\tau_1 mn}e^{-\pi\tau_2(m^2/2R^2+2n^2R^2)}$$
$$=R\sqrt{2/\tau_2}\sum_{n,m'}e^{-2\pi R^2[\tau_2 n^2+(m'-1/2-n\tau_1)^2/\tau_2]}. \quad (4.7)$$

We then need to evaluate the integral

$$\int_{\mathcal{F}_2}\frac{d^2\tau}{\tau_2^{7/2}}R\sqrt{2/\tau_2}\times\sum_{i,j}\sum_{n,m'}d(i,j)q^i\bar{q}^j e^{-2\pi R^2[\tau_2 n^2+(m'-1/2-n\tau_1)^2/\tau_2]}. \quad (4.8)$$

The $n=0$ term in Eq. (4.8) can be evaluated by saddle point approximation. The saddle point is at large $\tau_2\sim|m'-1/2|R/M_i$ where the integral over τ_1 restricts to states with mass $M_i\sim\sqrt{i}=\sqrt{j}$. Thus the contribution from states of mass M_i is of order e^{-M_iR}. Heuristically this can be thought of as the contribution from a world-line instanton where a particle of mass M_i has its world-line wrapped around the circle of radius R in spacetime. This represents the contribution of a single state and one might worry that the exponential degeneracy of states in string theory might overwhelm the e^{-R} suppression. This is equivalent to finding a tachyon in the spectrum and so does not happen for sufficiently large R. For $n\neq 0$ the saddle point is not necessarily in $\mathcal{F}_2$ and in order to evaluate the integral it is necessary to use the unfolding technique of Ref. [13] to rewrite Eq. (4.8) as an integral over the strip $\tau_2>0,|\tau_1|<1/2$. This again leads to an asymptotic behavior $\Lambda\sim R^{-5/2}e^{-R}$ at large R.[3]

Because of the Bose-Fermi degeneracy among massless states and states which become massless as $R\to\infty$ the usual power law behavior of Λ at large R cancels out and we are left only with contributions decreasing as $R^{-5/2}e^{-R}$. This exponential suppression of Λ at large R in models with equal numbers of massless fermions and bosons and supersymmetry broken by twisted boundary conditions was noted previously in Ref. [14].

To compare to the type II theory we can take large R in the heterotic theory. Since this takes us to a six-dimensional theory we should hold the six-dimensional heterotic coupling small and fixed. In type IIB language this corresponds to very small R'' so that it is more natural to compare to the type IIA theory with radius R' which is becoming large. The e^{-R} behavior we have found in the heterotic theory then becomes $e^{-R'/\lambda'}$ in type IIA variables. This result is compatible with the vanishing of Λ to all orders in perturbation theory conjectured in Ref. [4]. It is natural to interpret an $e^{-R'/\lambda'}$ effect in the type IIA theory as coming from a world-line instanton where a D0-brane world-line wraps the S^1. It would be interesting to investigate such effects directly in the type II theory.

[3]I thank D. Kutasov and G. Moore for help in the evaluation of this integral.

V. COMMENTS AND CONCLUSIONS

By considering a slight variant of the model considered in Ref. [4] it is possible to construct a heterotic dual theory to a nonsupersymmetric string with many if not all of the features of the model of Ref. [4]. The heterotic dual has a mismatch between Bose and Fermi degrees of freedom at the massive level. If duality is a reliable guide to the physics of the type II nonsupersymmetric theory then this mismatch implies a similar mismatch in the type II theory for nonperturbative states which arise either as D0-branes in $D=5$ (including wrapped D4 branes and D-brane states arising from the twisted sector of the asymmetric orbifold) or from wrapped five-branes. It is clearly of some interest to develop brane technology on asymmetric orbifolds in order to test whether this is indeed true. Conversely, if a mismatch is found among these states in the type II theory it will provide evidence for the reliability of duality in the absence of spacetime supersymmetry.

The model of Ref. [4] was motivated by the AdS-CFT correspondence applied to Reissner-Nordström black holes with AdS_2 near horizon geometry. It would also be interesting to see whether a more detailed analysis of the correspondence in this situation sheds some light on the presence of nonperturbative corrections. Such an analysis might also suggest models in which these contributions are eliminated or where the dilaton is stabilized so that the contributions are exponentially small.

In addition to the radius R the heterotic theory also has moduli obtained by putting equal Wilson lines in the two E_8 factors or by turning on equal constant metric and antisymmetric tensor fields $g_{i5}=B_{i5}$ with $i=1,...,4$ labeling the $\Gamma^{4,4}$ directions and 5 labeling the $\Gamma^{1,1}$ direction. It would be interesting to explore Λ and its stationary points as a function of these moduli as in Ref. [15].

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