Fading of symmetry nonrestoration at finite temperature

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The fate of symmetries at high temperature determines the dynamics of the very early universe. It is conceivable that temperature effects favor symmetry breaking instead of restoration. Concerning global symmetries, the nonlinear σ model is analyzed in detail. For spontaneously broken gauge symmetries, we propose the gauge boson magnetic mass as a ''flag'' for symmetry (non)restoration. We consider several cases: the standard model with one and two Higgs doublets in the perturbative regime and the case of a strongly interacting Higgs sector. The latter is done in a model-independent way with the tools provided by chiral Lagrangians. Our results clearly point towards restoration, a pattern consistent with recent lattice computations for global symmetries. In addition, we explicitly verify Becchi-Rouet-Stora-Tyutin invariance for gauge theories at finite temperature. [S0556-2821(98)03518-8]

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In which sense does one say that an internal symmetry is restored or broken due to temperature effects? What is the relevant order parameter? Whenever more than one such parameter can be defined, for which physical consequences are their differences relevant? These are the type of questions one is faced with when discussing symmetry (non)restoration.[1]

The vacuum structure of a system remains unchanged when it is heated. In this sense the degree of symmetry of a system is not modified. ''Symmetry restoration'' due to temperature effects is thus a misleading denomination for a very simple effect: the spontaneous breaking of a global or gauge symmetry can be masked for all physical purposes when thermal agitation is present. This suits intuition, as a thermal excitation gives in general a positive energy contribution, allowing particles to ''climb'' barriers between separate minima and finally hiding those barriers for high enough temperatures. Thermal field theory computes these effects and usually synthesizes them in the form of a so-called effective potential whose minimum sits at zero values of the fields. Ferromagnets provide well-known experimental examples of a similar behavior when heated above some critical temperature.

The suggestion that spontaneously broken field theories are restored at high temperature was first made by Kirzhnits and Linde [1]. They gave qualitative arguments to support this idea in the case of global symmetries. In the same direction pointed the results of Dolan and Jackiw [2] and Weinberg [3] for gauge theories (although in this case the choice of the scalar field vacuum expectation value as an order parameter is a delicate one).

Weinberg noticed as well an opposite possibility: global symmetry nonrestoration at high temperatures for scalar potentials with more than one Higgs multiplet. With just one Higgs the scenario is ruled out due to the constraints imposed on the scalar self-coupling by the boundedness of the potential, while models with two (or more) multiplets can easily accommodate it. The same behavior was found in the Schwinger model and in a dynamical model of symmetry violation in four dimensions [2].

An analogous situation has been experimentally observed in nature for the ferroelectric material known as Rochelle salt, which shifts from a disordered phase to a more ordered one when heated, as measured by the spontaneous polarization parameter. In the case of the Rochelle salt the symmetry is restored again for high enough temperatures, though. Common sense suggests that this should be as well the case in field theory, with thermal excitations dominating the free energy unless some finite parameter, such as finite volume, causal domain size, etc., plays a role. Without entering to discuss it, it is clear that even a temporal intermediate period, in which thermal effects enhance the effective symmetry breaking instead of restoring it, could have far reaching cosmological consequences.

It is worth remarking, though, that Weinberg results on symmetry nonrestoration are based on the one-loop approximation to the finite temperature effective potential, which is known to be unreliable for the discussion of many aspects of phase transitions. Different techniques, including nonperturbative ones, are being actively applied to improve the one-loop approximation, mainly for the study of global symmetries. The results are very interesting and quite often contradictory: some studies confirm that symmetry nonrestoration exists, although with a sizable reduction of the parameter space where it occurs [4–7], while other analysis concludes that symmetry is always restored at high temperature when nonperturbative effects are taken into account [8–9]. It

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[1]A related question is the so-called inverse symmetry breaking, describing systems for which the symmetry is exact at zero temperature and broken when heated; all through the paper we will take the liberty of dubbing symmetry nonrestoration both scenarios, unless the contrary is explicitly stated.

has been shown that in a finite lattice no order is possible at sufficiently high temperature [10]. Although the relevance of this result for the continuum limit is unclear, a Monte Carlo simulation in $2+1$ dimensions seems to support this conclusion [11].

Symmetry nonrestoration is indeed being increasingly reconsidered as a candidate way out of many cosmological problems arising in spontaneously broken theories. Examples are the domain wall and axion problems [12] and the monopole problem in grand unified theories [13].

As recalled in Sec. III, in the minimal standard $SU(2)\otimes U(1)$ model the symmetry is necessarily restored, given the simplicity of its Higgs sector. At present, there are two main avenues to explore physics beyond the standard model: theories in which the Higgs particle is a fundamental one, supersymmetry being its most representative example, and those for which it is not, currently dubbed as strongly interacting Higgs scenarios.

Supersymmetry is broken *de facto* at high temperatures, due to the difference in the boson and fermion populations, as dictated by Bose-Einstein versus Fermi-Dirac statistics. The debatable and interesting question is whether the internal symmetries present in supersymmetric theories, and whose fate is fundamental for the existence of topological defects, are restored. It has been proven that such is the case for renormalizable supersymmetric theories [14]. For the latter, a recent analysis for systems involving nonvanishing background charges shows that symmetry nonrestoration could be possible [15]. The consideration of nonrenormalizable terms in the Lagrangians has led as well to a polemics: their mere addition does not lead to symmetry nonrestoration [16].

Here we rather follow the path leading to a nonelementary Higgs scenario. In so doing, we first reanalyze the global $SU(N_f)_R\otimes SU(N_f)_L$ nonlinear σ model, relevant in supergravity and many other scenarios, in Sec. II. Section III is devoted to the analysis of gauge symmetries; after discussing Becchi-Rouet-Stora-Tyutin (BRST) invariance at finite temperature, we study the behavior of the $SU(2)\otimes U(1)$ symmetry in several scenarios. In Sec. III C we analyze both the minimal standard model and the standard model with two Higgs doublets within the perturbative regime, while in Sec. III D we consider a strongly interacting Higgs sector in a model-independent way, using the techniques of chiral Lagrangians, and we discuss the differences with the results of the previous section. These different chapters are preceded by some comments on order parameters, Sec. I, and followed by our conclusions.

I. THE ORDER PARAMETER

The interesting order parameter to consider in a phase transition depends first of all on the question one wants to study. An illustrative example is provided by spin systems in solid state physics. Both in ferromagnets and antiferromagnets, the ground state breaks rotational symmetry: the spins align for the former and display an antiparallel alignment for the latter. The traditional order parameter is the average spontaneous magnetization $\langle\vec{m}\rangle\neq 0$ which plays a crucial role in the description of the response of the system to an external magnetic field: it turns out to be important for ferromagnets, while marginal or even vanishing for antiferromagnets to the extent that the ground state approaches the Néel-type magnetic order. Hence, the spontaneous magnetization is an example of an order parameter whose nonzero value is not necessary for the spontaneous breakdown of the symmetry. Ferrimagnets are yet another scenario: antialignment is present similar to the case of antiferromagnets although $\langle\vec{m}\rangle\neq 0$, as the weight allocated to the two possible spin projections differs.

Analogous questions arise in particle physics: different so-called order parameters can be correlated to different physical effects. The appropriate parameter depends on the aspect of the history of the universe under study, and not all of them necessarily ''bip'' simultaneously.

Already at zero temperature, the relationship among different possible order parameters is not always straightforward. Recall massless QCD at low energies, with pion interactions appropriately described by chiral Lagrangians. The pion decay constant F_π and the condensate $\langle\bar{\Psi}\Psi\rangle$ are not necessarily equivalent order parameters. Although unnatural, $\langle\bar{\Psi}\Psi\rangle=0$ is not theoretically forbidden while a non-null vacuum expectation value (VEV) of some higher dimension operator accompanies F_π as a ''flag'' for dynamical symmetry breaking [17].

In a general way it is clear that when the Lagrangian, at zero temperature, is just a one parameter theory, all putative order parameters should be equivalent. Such is the case with most Lagrangians respecting global symmetries, where the value of the field at the minimum of the effective potential is a well-defined order parameter, commonly used, and any other one is simply related to it.

On the contrary, for spontaneously broken gauge theories the issue is much more subtle. To begin with, the VEV of any non-gauge-invariant operator is necessarily zero [18]. Only gauge-invariant operators, such as $|\phi|^2$, may have a nonvanishing VEV, signaling the Higgs mechanism. Once a gauge-fixing procedure has been performed, a non-gauge-invariant minimum of the effective potential may appear, which may be useful whenever its physical meaning is properly extracted in due respect of general Ward identities. The same applies to the VEV of higher dimension operators.

The effective potential itself is gauge dependent. However, the values of the effective potential at its local minima or maxima are gauge independent. Hence if there is a minimum of $V(\phi)$ with a value lower than $V(0)$ in one gauge, then there will be such minimum in any gauge (although its position will generally be different) and the symmetries will definitely be broken.

In practice, gauge-dependent correlation functions are often used in the study of phase transitions: the physical conclusions are expected to be rather close to those derived with gauge-invariant ones if the fluctuations of the scalar fields are small compared to their vacuum expectation values. This can be safe in the broken phase of the theory, while quite misleading in the symmetric phase, as recently discussed in

Ref. [19], where a detailed description of the zoo of correlation functions can be found as well.

It is worthwhile to briefly specify the ''flags'' for symmetry breaking discussed in the present paper.

For the global symmetries of the nonlinear σ model and its extensions in terms of chiral Lagrangians, we discuss both the pion decay constant F_π and the vacuum expectation value of the condensate. The latter is defined from an effective potential and can be interpreted as the remnant of the disappeared sigma field. Both parameters are essentially equivalent since, at zero temperature, we are dealing with a one parameter theory.

For the gauge symmetry case, specifically the standard electroweak model and its extensions, we concentrate instead on particle masses. In the perturbative regime, both the negative Higgs ''mass'' and the magnetic mass for the gauge bosons are discussed. When the Higgs particle disappears from the spectrum and we enter the nonperturbative regime of the gauged nonlinear σ model, our order parameter will be the gauge boson magnetic mass.

The magnetic mass squared is defined as the temperature-dependent contribution to the transverse part of the gauge boson self-energy $\Pi_T(0,\vec{k})$ for vanishing three-momentum $\vec{k}$. At the order we work in it is gauge invariant. Notice that Weinberg [3] advocates the use of gauge-invariant operators carrying moderate momenta and zero energy as order parameters.

The analogous electric mass, whose square is defined by the longitudinal component of the gauge boson self-energy $\Pi_L(0,\vec{k})$ with $\vec{k}\to 0$ is not a suitable parameter. Indeed, it tends to increase at high temperature even when the symmetry is restored. The intuitive explanation is electric screening: some particles in the theory carry an electric charge. Already at one-loop order, thermal fluctuations pull charged pairs out of the vacuum to screen external charges. However, there are no fundamental particles in any gauge theory which carry a magnetic charge. Magnetic screening can presumably then only arise from nonperturbative fluctuations which carry magnetic charge.

A perturbative computation of the magnetic mass shows that it is exactly equal to zero at one loop in an unbroken gauge theory. For unbroken non-Abelian gauge theories, such as QCD, higher orders in perturbation theory suffer from infrared divergences, and a magnetic mass of order g^2T is expected to be generated nonperturbatively. In spontaneously broken gauge theories, such as the standard electroweak model and its extensions, no such divergences are present. Thus, we propose to use the magnetic mass as a ''flag,'' expecting that even in perturbation theory it will show a tendency to vanish at high enough temperatures whenever symmetry restoration occurs. Of course, it will be a valid flag only when exploring the broken phase of the theory, for the reasons given above.

Another pertinent point to recall is that temperature corrections break Lorentz invariance as the plasma sets a preferred reference frame. Assume for instance a zero temperature Lagrangian based on an internal symmetry. Certainly the mixed states describing the new ''effective vacuum'' may greatly differ from the real vacuum structure. What about the finite temperature effective Lagrangian itself? Up to which point may its functional form differ from the initial one? Nonzero temperature is tantamount to treating space and time differently: internal symmetries at the Lagrangian level, such as chiral symmetry, cannot be explicitly broken due to it. What is to be *a priori* expected is a splitting of any operator into its temporal and spatial components, with differing coefficients. For instance F_π in the chiral nonlinear σ model will generate two different coupling constants at finite temperature, a temporal one $F^t_\pi(T)$ and a spatial one $F^s_\pi(T)$ [20]. We leave the corresponding considerations for gauge theories for the beginning of Sec. III. A necessary condition for symmetry restoration is that all possible order parameters or flags for symmetry restoration do signal it.

II. GLOBAL NONLINEAR σ MODEL: THE $T\neq 0$ EFFECTIVE LAGRANGIAN

The restoration of spontaneously broken global symmetries is discussed in this section within an effective Lagrangian approach. We consider the $SU(N_f)_R\otimes SU(N_f)_L$ nonlinear σ model, which may be defined by the Lagrangian

$$\mathcal{L}=\frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi_a+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma \tag{1}$$

with the constraint

$$F_\pi^2=\sigma^2+\vec{\pi}^2. \tag{2}$$

By convention, we take the scalar condensate in the direction of the σ component, that is, at tree level

$$\langle\sigma^2\rangle=F_\pi^2. \tag{3}$$

Since the global symmetry is broken down to an $SU(N_f)$ symmetry, there are a total of N_f^2-1 Goldstone bosons, which are identified with the pion fields, $\vec{\pi}$. The constraint (2) determines the σ field in terms of the pion fields, so that in the nonlinear Lagrangian only the latter appears. A nonlinear redefinition of the fields is possible without changing the physical content of the theory, leading to different parametrizations. The so-called exponential representation can be described by the Lagrangian

$$\mathcal{L}^{(2)}=\frac{1}{4}F_\pi^2\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger), \tag{4}$$

where U is a $SU(N_f)$ unitary matrix field

$$U=\exp\left(i\frac{\pi_aT_a}{F_\pi}\right), \tag{5}$$

with T_a the generators of $SU(N_f)$, normalized as $\mathrm{Tr}(T_aT_b)=2\delta_{ab}$ and $[T_a,T_b]=2if_{abc}T_c$, f_{abc} being the structure constants of $SU(N_f)$.

As is well known, all realizations of the nonlinear chiral Lagrangian, such as the exponential one (4), square root, Weinberg, etc. [21], with $N_f=2(3)$, are low-energy effective theories for QCD with 2(3) massless quarks, expressed in terms of Goldstone bosons and systematically expanded in powers of the Goldstone bosons momenta. As a consequence of the chiral symmetry, these models possess the remarkable property of *universality*: once the coupling constants have been adjusted (F_π is the only one at lowest order) all physical predictions are the same. Therefore, the chiral Lagrangian not only parametrizes the dynamics of the Goldstone bosons that emerge in QCD but also of any other theory, such as the Higgs model, that follows the same symmetry breaking pattern.

We have analyzed two order parameters: the pion decay constant F_π and the vacuum expectation value of the σ field $\langle\sigma\rangle$. Notice that, while their behavior should be essentially equivalent, their precise variation rate with temperature may differ somewhat. Indeed, the constraint (2) as a thermal average implies $\langle\sigma^2\rangle = F_\pi^2$, while in general $\langle\sigma^2\rangle \neq \langle\sigma\rangle^2$. Our treatment differs from previous ones in that we have considered them as Lagrangian parameters, whose variation with temperature is read from the one-loop effective Lagrangian we derive.

Although we will only discuss below the calculation in the exponential representation, we have explicitly checked that the results of measurable quantities are the same in other parametrizations used in the literature, namely, the square root and Weinberg representations. Of course, for quantities without a physical meaning, the temperature corrections can be representation dependent. We drop all temperature-independent ultraviolet divergent quantities from our expressions, recalling that when a theory is renormalized at zero temperature no more infinities of that type appear at finite temperature.

A. Temperature corrections to F_π

Temperature corrections to F_π are obtained from an effective Lagrangian approach. The chiral Lagrangian (4) can be expanded in powers of $(\pi/F_\pi)^2$ up to a certain order,

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{6F_\pi^2}[(\vec{\pi}\partial_\mu\vec{\pi})(\vec{\pi}\partial^\mu\vec{\pi}) - (\vec{\pi}\vec{\pi})\partial_\mu\vec{\pi}\partial^\mu\vec{\pi}] + \cdots . \tag{6}$$

We have computed the one-loop temperature corrections to this Lagrangian to leading order T^2/F_π^2, and proved that they lead to an effective Lagrangian with the same structure as the tree-level one, albeit with two F_π's: a temporal one F_π^t and a spatial one F_π^s. It encloses the full temperature effects in the renormalized (temperature-dependent) parameters.

Due to the derivative character of the interactions, a contribution to the kinetic energy term, at leading order T^2/F_π^2, is obtained when computing the one particle irreducible (1PI) two-point Green function at one loop (Fig. 1). This term is absorbed by pion field renormalization. In the exponential representation used here, we find

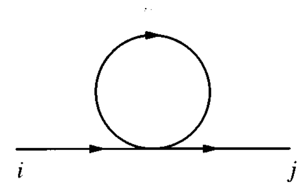

FIG. 1. One-loop self-energy diagram for the pions.

$$\pi^2(T) = \pi^2\left[1 - \frac{(N-1)T^2}{36F_\pi^2}\right]. \tag{7}$$

The diagrams in Fig. 2 contribute to the 1PI four-point function at one loop, leading to different thermal corrections for the spatial and the temporal coupling constants in which F_π splits at finite temperature, as mentioned above. To this order

$$F_\pi^s(T) = F_\pi\left[1 - \frac{(N-1)T^2}{24F_\pi^2}\right], \tag{8}$$

$$F_\pi^t(T) = F_\pi\left[1 - \frac{(N+1)T^2}{24F_\pi^2}\right], \tag{9}$$

where $N = N_f^2 - 1$ represents the number of pions. Both temperature-dependent renormalized parameters $F_\pi^s(T)$ and $F_\pi^t(T)$ show a clear tendency to vanish at high enough temperatures, pointing towards chiral symmetry restoration. We have also explicitly checked that F_π^s so derived is representation independent.

Thermal corrections to the pion decay constant have been computed in the literature following different approaches [22,23,21,25,26]. Our result for the effective spatial coupling $F_\pi^s(T)$ is in agreement with those calculations of $F_\pi(T)$. In most of them, $F_\pi(T)$ is obtained from its usual definition (slightly modified at finite temperature [21]) through the two-point function of the axial vector current, and there is no splitting between temporal and spatial couplings at one loop; it appears at two loops [27]. Notice that since we consider F_π just as a parameter in the Lagrangian, it does not necessarily coincide with the pion decay constant as usually defined. To avoid technical complications, we have computed the pion field and F_π renormalization from the lowest order terms in the expansion of the Lagrangian $\mathcal{L}^{(2)}$ in powers of

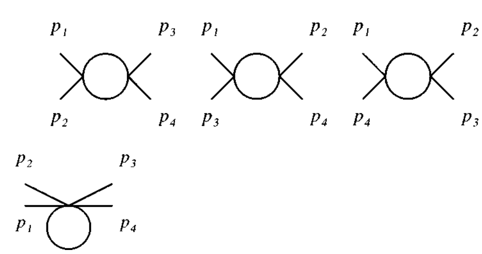

FIG. 2. One-loop diagrams contributing to the 1PI four-point Green function for the pions.

the pion fields; chiral symmetry ensures that all higher terms in the field expansion are consistently renormalized once $\vec{\pi}$ and F_π have been renormalized from these lowest order terms.

B. Temperature corrections to the condensate at one loop

Temperature corrections to $\langle\sigma\rangle$ are computed through the addition of a small chirality breaking term, which makes the Lagrangian slightly asymmetric,[2] that is, we consider

$$\mathcal{L}=\mathcal{L}^{(2)}+\mathcal{L}_B, \tag{10}$$

with

$$\mathcal{L}_B=c\sigma=\frac{cF_\pi}{4}\,\mathrm{Tr}(U+U^\dagger). \tag{11}$$

Expanding the last term in powers of π^2/F_π^2 in the exponential representation it is found that

$$\mathcal{L}_B=cF_\pi-\frac{c}{2F_\pi}\vec{\pi}\vec{\pi}+\frac{c}{24F_\pi^3}(\vec{\pi}\vec{\pi})^2+\cdots. \tag{12}$$

Following the effective Lagrangian approach, we compute the one-loop order T^2 corrections to $\mathcal{L}$ through the 1PI zero, two- and four-point Green functions. The kind of diagrams involved in the calculation are vacuum energy ones for the zero-point 1PI Green function and the same as in the previous section (see Figs. 1 and 2), although with modified couplings, for the two- and four-point 1PI functions. Now, besides F_π and the pion field, also the parameter c is renormalized. Since it only appears in the product cF_π, there is an ambiguity, depending on which (spatial or temporal) $F_\pi(T)$ we consider, leading to

$$c^s(T)=c\left(1-\frac{T^2}{24F_\pi^2}\right), \tag{13}$$

$$c^t(T)=c\left(1+\frac{T^2}{24F_\pi^2}\right). \tag{14}$$

Again, the one-loop effective Lagrangian written in terms of the temperature-dependent parameters has the same structure as the tree-level one, space-time splitted, though.

Notice that (minus) the first term in the expansion of $\mathcal{L}_B$, $-cF_\pi$, can be interpreted as the vacuum energy density of the system. That is, the free energy of a system of free bosons, given by

$$c^s(T)\;F_\pi^s(T)=c^t(T)\;F_\pi^t(T). \tag{15}$$

Since the operator σ can be obtained by deriving the bare Lagrangian with respect to the parameter c [see Eq. (11)], we can also interpret the result as a thermal correction to the scalar condensate.[3] Taking the derivative with respect to (bare) c of the one-loop effective Lagrangian, the temperature corrections to the condensate are found. Explicit chiral symmetry is recovered by fixing $c=0$ at the end of the computation. The final result is satisfactorily the same whether either the spatial set F_π^s, c^s or the temporal set F_π^t, c^t is used, leading to

$$\langle\sigma\rangle_T=\langle\sigma\rangle\left(1-N\frac{T^2}{24F_\pi^2}\right), \tag{17}$$

in agreement with Refs. [24,22,21]. As can be seen in Eq. (17), the temperature correction to the condensate also points towards chiral symmetry restoration.

Notice that $\mathcal{L}_B$ is just the well known classical potential up to a minus sign. However, it was not possible to use the standard method for computing effective potentials [2] due to the presence of derivative couplings. Using a generalization of this method [24] the same result is recovered, as already mentioned.

III. GAUGE SYMMETRY: SU(2)⊗U(1)

In this section we study theories with gauge group SU(2)⊗U(1). We consider the cases with one light Higgs doublet, two light Higgs doublets, and the generic one where the Higgs sector becomes strongly interacting, the latter done in a model-independent way. Before entering into such details, we dwell again into the delicate issue of the flag for symmetry (non)restoration for gauge theories, and in the fate of gauge invariance itself when a system is heated.

A. The magnetic mass

In a gauge theory, the pseudo-Goldstone bosons of the Lagrangian are unphysical fields, unlike the gauge bosons. As stated in Sec. I, we choose the gauge boson magnetic mass as our flag or indicator for symmetry (non)restoration.

At nonzero temperature, the self-energy tensor of the gauge boson may depend on the four-velocity of the plasma u_μ. Consequently, the gauge boson self-energy can be ex-

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[2]We recall that the QCD scalar density $\bar{\Psi}\Psi$ whose vacuum expectation value represents the familiar QCD condensate, is equivalent to $\sigma\equiv(F_\pi/4)\,\mathrm{Tr}(U+U^+)$ since $\bar{\Psi}\Psi$ and $\mathrm{Tr}(U+U^+)$ can be shown to transform in the same way under the chiral group. $\mathcal{L}_B$ is thus equivalent to a quark mass term.

[3]Recall that the thermal average of the operator σ is defined as

$$\langle\sigma\rangle_T=\frac{\mathrm{Tr}(\sigma e^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} \tag{16}$$

where $\mathrm{Tr}\,e^{-\beta H}=\int[dU]e^{-\int d^4x\mathcal{L}}$ is the partition function and $\beta=1/T$. Thus, one can compute $\langle\sigma\rangle_T$ as the derivative of the partition function with respect to the parameter c, at $c=0$. This is analogous to the extraction of $\langle\bar{\Psi}\Psi\rangle_T$ in QCD, by first adding an explicitly chiral symmetry breaking term $m\bar{\Psi}\Psi$ to the bare Lagrangian, computing the temperature corrections, and deriving then with respect to m [22].

pressed as a linear combination of four possible tensors: $g_{\mu\nu}$, $k_\mu k_\nu$, $u_\mu u_\nu$, and $k_\mu u_\nu + k_\nu u_\mu$. Some linear combinations of these tensors are usually chosen as the standard basis set [28], denoted $A_{\mu\nu}$, $B_{\mu\nu}$, $C_{\mu\nu}$, and $D_{\mu\nu}$ and defined in Appendix A. In this basis, the one-loop gauge boson self-energy is written as

$$\Pi^{\mu\nu} = \Pi_T A^{\mu\nu} + \Pi_L B^{\mu\nu} + \Pi_D D^{\mu\nu}, \tag{18}$$

where the subscripts T and L denote transverse and longitudinal with respect to the spatial component $\vec{k}$ of the wave vector.

The magnetic mass is defined as $\Pi_T(0,\vec{k})$, with vanishing $\vec{k}$. At one loop and leading order $[O(gT)]$ it will be shown to be gauge invariant both for the standard model and for its extensions considered below. The explicit computations will be focused in the W gauge boson mass.

B. Checking gauge invariance: BRST identities

To the best of our knowledge, the Slavnov-Taylor identities at finite temperature have never been explicitly verified in the literature for the electroweak theory. We explicitly perform such a task in the present work, for the two-point functions of the theory.

Indeed, one expects gauge invariance to be preserved at nonzero temperature. A simple reasoning can be developed in the imaginary time formalism, where finite temperature just amounts to compactifying the time direction, that is, to perform a global ''distortion'' of the system. Gauge transformations are local ones by definition, and thus they should not be affected by global topological conditions. Once the gauge-fixing procedure has been implemented, BRST invariance remains, and the corresponding Slavnov-Taylor identities are to be proven.

One should realize that the proof is much more juicy than at zero temperature: there, quadratic divergences are disposed of by counterterms from the start, and the Slavnov-Taylor identities for such nonphysical quadratically divergent terms are not even considered. At finite temperature those quadratic divergences are the source of the T^2 dependence. It is then mandatory, and new, to check the BRST identities on them.

Both in the linear and the nonlinear realizations of the SU(2)⊗U(1) gauge symmetry, the Ward identities relating the two-point Green functions at one loop are given by

$$\begin{aligned}
k^2(\Pi_D^W + 2M_W \Pi^{W\pi^\pm}) - M_W^2 \Pi^{\pi^\pm} &= 0,\\
k^2(\Pi_D^Z - 2iM_Z \Pi^{Z\pi^0}) - M_Z^2 \Pi^{\pi^0} &= 0,\\
k^2 \Pi_D^\gamma &= 0,\\
k^2(\Pi_D^{\gamma Z} - iM_Z \Pi^{\gamma\pi^0}) &= 0,\\
\xi \Pi_D^Z - iM_Z \xi \Pi^{Z\pi^0} - \Pi^{c^0} &= 0,\\
\xi \Pi_D^W + M_W \xi \Pi^{W\pi^\pm} - \Pi^{c^\pm} &= 0,
\end{aligned} \tag{19}$$

where ξ is the gauge-fixing parameter in R_ξ gauges. Π_D^W, Π_D^Z, Π_D^γ, and $\Pi_D^{\gamma Z}$, are the form factors introduced in Eq. (18) for the $W^\pm - W^\pm$, $Z-Z$, $\gamma-\gamma$, and $Z-\gamma$ self-energies, respectively. $\Pi^{W\pi^\pm}$ is defined from the two-point Green function with external legs $W^\pm - \pi^\pm$ as

$$\Pi_\mu^{W\pi^\pm} = k_\mu \Pi^{W\pi^\pm}, \tag{20}$$

and analogously for $\Pi^{Z\pi^0}$ and $\Pi^{\gamma\pi^0}$, while $\Pi^{c^\pm}$ and Π^{c^0} represent the charged and neutral Faddeev-Popov ghosts self-energies.

C. Perturbative Higgs sector

1. Minimal standard model

Consider the minimal electroweak standard model, that is, with just one light Higgs doublet. Here, all the couplings of the theory are in the perturbative range, and we can rely on the one-loop approximation to the effective potential at nonzero temperature.

It is well known that with just one Higgs doublet the gauge symmetry is always restored at high temperature. Indeed, given the simplicity of the potential,

$$V(\phi)_{T=0} = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2, \tag{21}$$

the condition that it has to be bounded from below forces the sign of λ to be positive. The one-loop thermal corrections to the above potential can be readily computed in R_ξ gauges by the usual methods [2]. In the high-temperature limit ($T \gg m_i$, with m_i the masses of all standard model particles) the leading order T^2 corrections are gauge invariant and read

$$\delta\mu^2 = -\frac{T^2}{12}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 3h_t^2 + 3h_b^2 + h_\tau^2\right), \tag{22}$$

leading to the temperature-dependent Higgs vacuum expectation value (VEV)

$$v(T)^2 = v^2 - \frac{T^2}{2}\left[1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} + \frac{h_t^2}{2\lambda} + \frac{h_b^2}{2\lambda} + \frac{h_\tau^2}{6\lambda}\right], \tag{23}$$

where $v^2 = \mu^2/\lambda$ denotes the Higgs VEV at tree level and h_t, h_b, and h_τ are the Yukawa coupling constants of the quarks t and b, and the τ lepton, respectively.

As expected, exactly the same behavior is seen from the Z and W gauge boson magnetic mass. The set of diagrams contributing to the Z self-energy at one loop are shown in Fig. 3. The coupling constants are not renormalized at one loop (at order T^2), which allows us to write

$$\begin{aligned}
M_{W,\mathrm{mag}}^2 &= \frac{g^2}{4} v(T)^2,\\
M_{Z,\mathrm{mag}}^2 &= \frac{g^2 + g'^2}{4} v(T)^2,
\end{aligned} \tag{24}$$

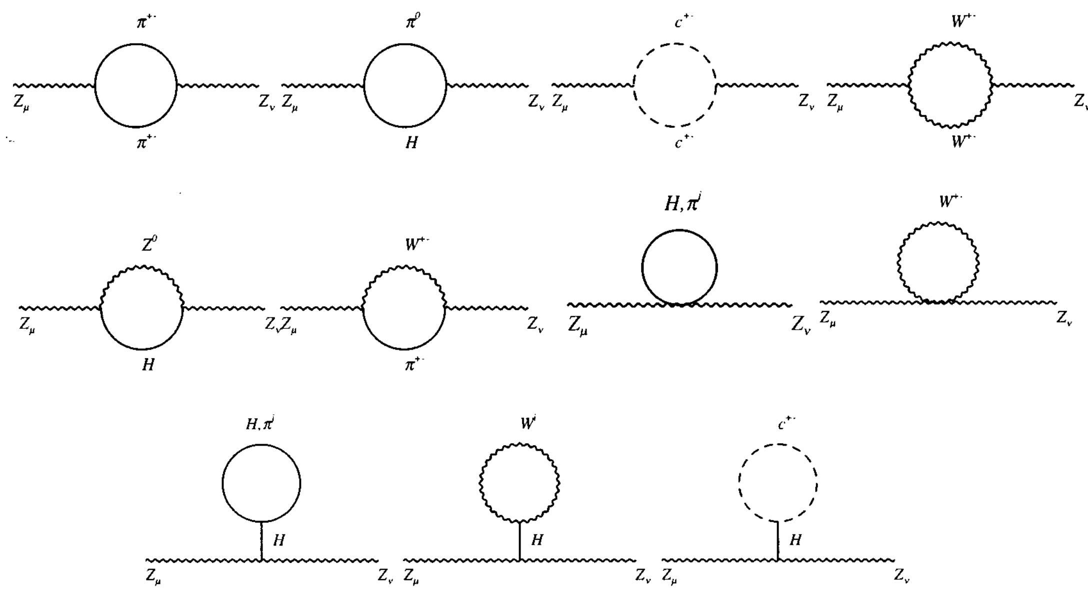

FIG. 3. One-loop self-energy diagrams for the Z gauge boson in the minimal standard model.

$$v(T)^2 = v^2 - \frac{T^2}{2}\left[1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} + \frac{h_t^2}{2\lambda} + \frac{h_b^2}{2\lambda} + \frac{h_\tau^2}{6\lambda}\right].$$

The result in Eq. (23) is then recovered, pointing towards restoration in an inescapable way. Finally, we have checked all the Ward identities in Eq. (19); the explicit results for the diagrams involved can be found in Appendix C.

2. Two Higgs doublets

Models with a richer Higgs structure have several scalar couplings. In order to explore symmetry nonrestoration, the rule of the game is then to play with the freedom in the sign of some of those couplings, while respecting the boundedness condition.

The simplest extension, i.e., the standard model with two Higgs doublets, is considered now. We make the usual assumption that the down quarks and charged leptons only couple to the Higgs doublet ϕ_1 and the up quarks to ϕ_2, ensuring tree-level flavor conservation of scalar mediated neutral currents. In order to avoid radiatively induced flavor changing neutral current (FCNC) terms, we also impose the discrete symmetry $\phi_1 \to -\phi_1$. The most general, renormalizable, scalar potential consistent with the above symmetry and with gauge invariance is

$$\begin{aligned} V(\phi_1,\phi_2) = & -m_1^2\phi_1^\dagger\phi_1 - m_2^2\phi_2^\dagger\phi_2 + \lambda_1(\phi_1^\dagger\phi_1)^2 \\ & + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4|\phi_1^\dagger\phi_2|^2 \\ & + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}]. \end{aligned} \tag{25}$$

The condition for the potential to be bounded from below leads to the constraints

$$\begin{gathered} \lambda_1 > 0, \quad \lambda_2 > 0, \quad 4\lambda_1\lambda_2 > \lambda_3^2, \\ 4\lambda_1\lambda_2 > (\lambda_3+\lambda_4+\lambda_5)^2 \quad (\text{for } \lambda_5 < 0). \end{gathered} \tag{26}$$

The leading one-loop thermal corrections give the following thermal masses for the fields ϕ_1, ϕ_2:

$$\begin{aligned} \Delta V_T(\phi_1,\phi_2) & \\ \simeq \frac{T^2}{12}\Bigg[& \left(6\lambda_1 + 2\lambda_3 + \lambda_4 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 3h_b^2 + h_\tau^2\right)|\phi_1|^2 \\ & + \left(6\lambda_2 + 2\lambda_3 + \lambda_4 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 3h_t^2\right)|\phi_2|^2\Bigg] \\ \equiv & \, m_1^2(T)|\phi_1|^2 + m_2^2(T)|\phi_2|^2. \end{aligned} \tag{27}$$

Although the contributions from both fermions and gauge bosons are positive, the scalar couplings λ_3 and λ_4 may be negative, and therefore it is not possible to make any *a priori* statement about the signs of the mass terms above. What can be stated is that the stability conditions in Eq. (26) do not allow both mass terms in Eq. (27) to be negative. Since ϕ_2 receives a large positive contribution from the top Yukawa coupling, it is easier to get a negative thermal mass for the field ϕ_1. Then, its vacuum expectation value would remain nonzero at high temperature and the $SU(2)\otimes U(1)$ symmetry would never be restored.

According to Eq. (27), $m_1^2(T) < 0$ requires

$$6\lambda_1 + 2\lambda_3 + \lambda_4 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 3h_b^2 + h_\tau^2 < 0. \tag{28}$$

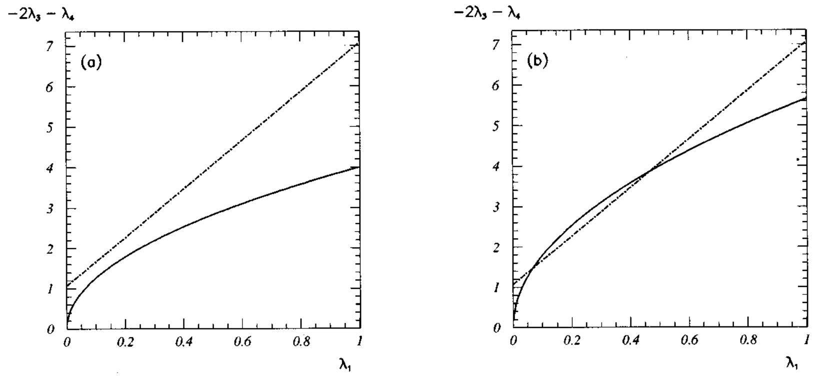

FIG. 4. Stability bound of the tree-level potential (solid line) and parameter space leading to symmetry nonrestoration (above the dashed-dotted line), for $\lambda_2=1$ (a) and $\lambda_2=2$ (b), with $\lambda_5=0$ in both.

Notice that the term $\frac{9}{4}g^2=0.99$ is already of order one at the electroweak scale, which makes it difficult to attain $m_1^2(T)<0$ within the perturbative regime.[4] We have checked numerically that the condition (28) and the stability bounds (26) are incompatible for scalar couplings in the range $[-1<\lambda_i<1]$, where the weak coupling expression (27) is justified. As an example, in Fig. 4 we plot the stability bound of the tree-level potential (the allowed range is below the solid line) and the curve corresponding to $m_1^2(T)=0$ (symmetry nonrestoration occurs above the dashed-dotted line), for $\lambda_5=0$ and $\lambda_2=1,2$.

Heading outside the above mentioned range, the numerical results taken at face value seem to point towards the possibility of symmetry nonrestoration as the scalar sector enters the nonperturbative regime, as can be seen in Fig. 4(b). Of course, the above computation is meaningless outside that range.

On the above, we have used as a flag for symmetry (non)restoration the negative scalar ''masses,'' that is, the location of the minimum of the potential. As discussed in the previous section, one could calculate instead the induced gauge boson magnetic masses, as an alternative analysis of the fate of the symmetry. In the linear realization of the symmetry breaking sector of the minimal standard model, the magnetic mass has been computed and proved to show a tendency towards vanishing. The temperature corrections to the vacuum expectation value of the Higgs field indirectly computed through this procedure agree with the result obtained from the effective potential approach. The magnetic mass squared is given by $g^2\,(v^2(T)/4)$. In the two doublet case, the magnetic mass squared would be given by $g^2\,[v_1^2(T)+v_2^2(T)]/4$, thus if there is a region of parameter space for which one of the VEVs remains nonzero the magnetic mass will not show a tendency towards vanishing and symmetry nonrestoration becomes possible.

From our study of the two doublet model, we conclude that the requirement of the validity of perturbation theory points towards the usual assumption of restoration of the $SU(2)\otimes U(1)$ gauge symmetry. In the next section, we extend the analysis outside the perturbative regime.

D. Strongly interacting Higgs sector

We now study the behavior of the $SU(2)_L\otimes U(1)_Y$ symmetry in the standard model with a strongly coupled Higgs sector. Strong coupling implies (at least naively) heavy physical scalar particles, which can be effectively removed from the physical low-energy spectrum. An effective Lagrangian approach is the natural technique to use when all the physical degrees of freedom in the symmetry breaking sector are heavy. We then consider the most general effective Lagrangian which employs a nonlinear realization of the spontaneously broken $SU(2)_L\otimes U(1)_Y$ gauge symmetry [29]. The resulting chiral Lagrangian is a nonrenormalizable nonlinear σ model coupled in a gauge-invariant way to the Yang-Mills theory. Chiral Lagrangians have been widely used in the last few years as low-energy effective theories for electroweak interactions [34].

The Lagrangian keeps only the light degrees of freedom, namely, the gauge and Goldstone bosons. The latter are collected in a unitary matrix $U=\exp(i\pi_a\tau_a/v)$, where v is the vacuum expectation value that gives the W and Z gauge bosons a mass, π_a are the would-be Goldstone fields and τ_a the Pauli matrices.

1. The lowest order Lagrangian

The lowest order terms in a derivative expansion of the effective Lagrangian are

$$\mathcal{L}_{\rm GChL}=\frac{v^2}{4}\,{\rm Tr}[D_\mu U^\dagger D^\mu U]+\mathcal{L}_{\rm YM}+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}, \quad (29)$$

where

$$D_\mu U=\partial_\mu U+i\frac{g}{2}(\vec{W}_\mu\vec{\tau})U-i\frac{g'}{2}U(B_\mu\tau^3). \quad (30)$$

$\mathcal{L}_{\rm YM}$ is the pure Yang-Mills piece

$$\mathcal{L}_{\rm YM}=-\frac{1}{2}\,{\rm Tr}(W_{\mu\nu}W^{\mu\nu})-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \quad (31)$$

and we consider the following gauge-fixing term:

[4] We neglect small finite temperature renormalization of the couplings.

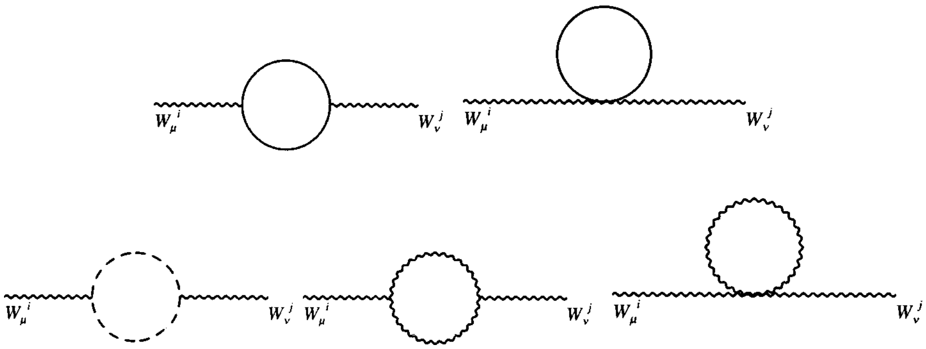

FIG. 5. One-loop self-energy diagrams for the gauge bosons in the standard model with strongly coupled Higgs sector. Solid lines represent would-be Goldstone bosons, wavy lines gauge bosons, and dashed lines Faddeev-Popov ghosts.

$$\mathcal{L}_{\mathrm{GF}}=-\frac{1}{2}\left(\frac{1}{\sqrt{\xi_1}}\partial_\mu W_i^\mu - g\frac{v}{2}\sqrt{\xi_2}\pi_i\right)^2 - \frac{1}{2}\left(\frac{1}{\sqrt{\xi_1}}\partial_\mu B^\mu - g'\frac{v}{2}\sqrt{\xi_2}\pi_3\right)^2, \tag{32}$$

from which the Faddeev-Popov term $\mathcal{L}_{\mathrm{FP}}$ can be computed in the usual way. The part relevant for our calculation is given in Appendix B. At tree level we take $\xi_1=\xi_2$, so that the gauge-boson–Goldstone-boson mixing term is canceled.

Expanding U, we obtain the interaction vertices, in particular the tree-level masses are given by

$$M_Z^2=(g^2+g'^2)\,\frac{v^2}{4},\quad M_W^2=g^2\,\frac{v^2}{4}, \tag{33}$$

$$m_{\pi^0}^2=\xi_2 M_Z^2,\quad m_{\pi^\pm}^2=\xi_2 M_W^2, \tag{34}$$

$$m_{c^0}^2=\sqrt{\xi_1\xi_2}\,M_Z^2,\quad m_{c^\pm}^2=\sqrt{\xi_1\xi_2}\,M_W^2, \tag{35}$$

where $\pi^0,\pi^\pm$ are the longitudinal components of the gauge bosons $Z,W^\pm$, respectively, and $c^0,c^\pm$ are the corresponding ghost fields.

We have computed the one loop temperature corrections to the effective Lagrangian (29) at leading order, i.e., $\mathcal{O}(T^2)$. Before entering the discussion of the results, it is worth recalling the range of validity of the calculation and the approximations involved. Unitarity implies that this low-energy effective theory should be valid for an energy scale much smaller than $4\pi v\sim 3$ TeV. Furthermore, as we shall see in Eq. (36), our approximation cannot be valid unless $T<\sqrt{6}v$, where the latter limit would give the naively extrapolated critical temperature. In the vicinity of it, the loop expansion performed here is not appropriate. Therefore, our conclusions will be reliable up to $T\simeq 200$ GeV, and expected to be an acceptable guideline up to 500 GeV. In order to obtain analytic expressions we work in the limit $T\gg m_i$, with m_i the masses of the low-energy spectrum (which means $T\gg gv$), and $T\gg k$, where k are the external momenta. This approximation is known in the literature as the hard thermal loop (HTL) approximation [30].

We are doing an expansion in both T/v and the small coupling constants g,g'. The corrections of order $g^2\,(T^2/v^2)$ are smaller than T^4/v^4, which will only appear at higher order in perturbation theory since, in the HTL approximation, we assume $gv\ll T$. Indeed, notice that already at $T=0$ the gauge coupling constants g,g' are not corrected by quadratically divergent diagrams, as can be seen from simple power counting arguments on the one-loop diagrams contributing to vertex functions: only logarithmic divergences appear. Hence no T^2 renormalization of g,g' may appear.

Let us start with the thermal corrections to the gauge boson masses, obtained from the corresponding self-energy tensor (Fig. 5). It is well known that the magnetic mass of the gauge bosons in an unbroken gauge theory vanishes at one loop [31], so that only diagrams involving would-be Goldstone boson loops will give a nonzero contribution to the magnetic masses in the broken phase, and we find[5]

$$M_{W,\mathrm{mag}}^2=g^2\frac{v^2}{4}\left(1-\frac{N_fT^2}{12v^2}\right),$$
$$M_{Z,\mathrm{mag}}^2=(g^2+g'^2)\frac{v^2}{4}\left(1-\frac{N_fT^2}{12v^2}\right), \tag{36}$$
$$M_{\gamma,\mathrm{mag}}^2=0,$$

where $N_f=2$ in SU(2). For the electric masses, defined as $\Pi_L(0,\vec{k})$, we get[6]

$$M_{W,\mathrm{el}}^2=g^2\frac{v^2}{4}\left(1+\frac{17N_fT^2}{12v^2}\right),$$
$$M_{Z,\mathrm{el}}^2=(g^2+g'^2)\frac{v^2}{4}\left(1-\frac{N_fT^2}{12v^2}\right)+\frac{N_fT^2}{24}[(g^2+g'^2)(c_W^2-s_W^2)^2+8g^2c_W^2], \tag{37}$$

[5]Although we have not included fermions in our calculation, it is easy to see that they do not contribute to the magnetic masses either, at leading order.

[6]We acknowledge C. Manuel for pointing out two misprints in these formulas, in an earlier version.

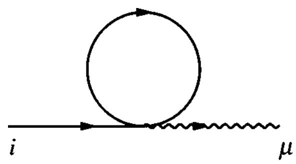

FIG. 6. One-loop diagram which generates the π-W mixing term.

$$M^2_{\gamma,\mathrm{el}}=e^2\frac{N_f T^2}{2},$$

where c_W (s_W) is the cosine (sine) of the weak mixing angle at zero temperature.

Focusing on the magnetic masses, we can then rewrite them as

$$M^2_{W,\mathrm{mag}}=g^2\,\frac{v(T)^2}{4},\tag{38}$$

$$M^2_{Z,\mathrm{mag}}=(g^2+g'^2)\,\frac{v(T)^2}{4},\tag{39}$$

with $v(T)^2$ given by

$$v(T)^2=v^2\left[1-\frac{(N-1)T^2}{12v^2}\right],\tag{40}$$

where $N=3$ in SU(2).

It is worth remarking that the would-be Goldstone boson field renormalization is the same as the one for the Goldstone bosons in the global case [Eq. (7)], while the temperature corrections to v^2 coincide with those for $F^{s\,2}_{\pi}$.

The diagram in Fig. 6 generates a gauge-boson–Goldstone-boson mixing term proportional to T^2, which is absorbed by a renormalization of the gauge-fixing parameter ξ_2:

$$\xi_2(T)=\xi_2\left(1+\frac{2T^2}{9v^2}\right).\tag{41}$$

The remaining parameter in the effective Lagrangian in Eq. (29), the gauge-fixing one ξ_1, is not renormalized at $\mathcal{O}(T^2)$. The same applies to the gauge boson and ghost fields. Naive dimensional counting shows that the 1PI one-loop diagrams that will renormalize those entities are at most logarithmically divergent, similar to the situation for g and g' discussed earlier, and thus not able to produce T^2 corrections.

The consistency of our results has been verified by computing the leading order corrections to the masses of ghost and would-be Goldstone bosons through the corresponding one loop self-energies (Figs. 7 and 8). Once the would-be Goldstone boson field renormalization has been taken into account, the temperature-dependent masses depend on the renormalized parameters in the same way as at tree level, i.e.,

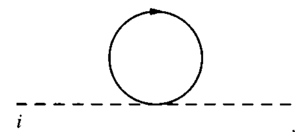

FIG. 7. One-loop self-energy diagram for the Faddeev-Popov ghosts.

$$m^2_{\pi^\pm}(T)=\xi_2(T)M^2_{W,\mathrm{mag}},\tag{42}$$

$$m^2_{c^\pm}(T)=\sqrt{\xi_1\xi_2(T)}\;M^2_{W,\mathrm{mag}}.\tag{43}$$

The one-loop effective Lagrangian does not have exactly the same functional form as the original bare one, Eq. (29): it splits, as exemplified by the differing electric and magnetic masses. Equations (40) and (41) allow us to connect several important finite T quantities in a compact notation, though. Equations (42) and (43) are an example of this.

We have also checked all the Ward identities in Eq. (19). The explicit results for the different diagrams involved are given in Appendix C.

A natural question concerns the relationship with the linear case discussed in Sec. III C 1, one expects that taking there the Higgs boson mass to infinity the results of the present section should be recovered. A superficial look does not show this. For instance, taking the limit $\lambda\to\infty$ in Eq. (25) for the magnetic mass in the linear case, Eq. (36) is not recovered. There is no inconsistency, though: the high-temperature and heavy Higgs mass limits are not interchangeable. We have indeed checked that Eq. (36) is obtained by taking the limit $m\to\infty$ (m being the physical Higgs boson mass), before doing the one-loop computation in the linear case (which implies not considering the following diagrams of Fig. 3: 2, 5, 10, 11 and 7, 9 when the physical Higgs boson is in the loop). The same argumentation is valid for the rest of the physical parameters.

Regarding the question of the SU(2)$\otimes$U(1) symmetry nonrestoration which initially motivated our study, we conclude from Eq. (36) that thermal effects tend to restore the symmetry also in the nonperturbative regime. Notice that although the magnetic mass is nonvanishing in the symmetric phase beyond one-loop order, it is expected to be of order g^2T, and therefore much smaller than the magnetic mass in the broken phase, of order gv (recall that $g^2T\ll gT<gv$). Thus we can interpret the decreasing of the magnetic mass

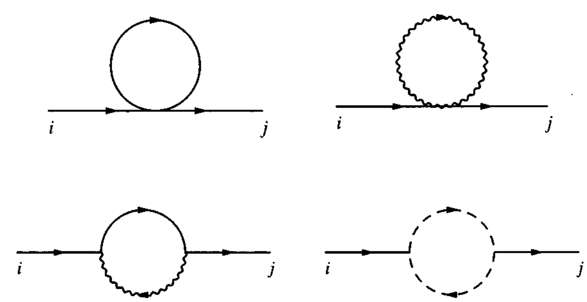

FIG. 8. One-loop self-energy diagrams for the would-be Goldstone bosons.

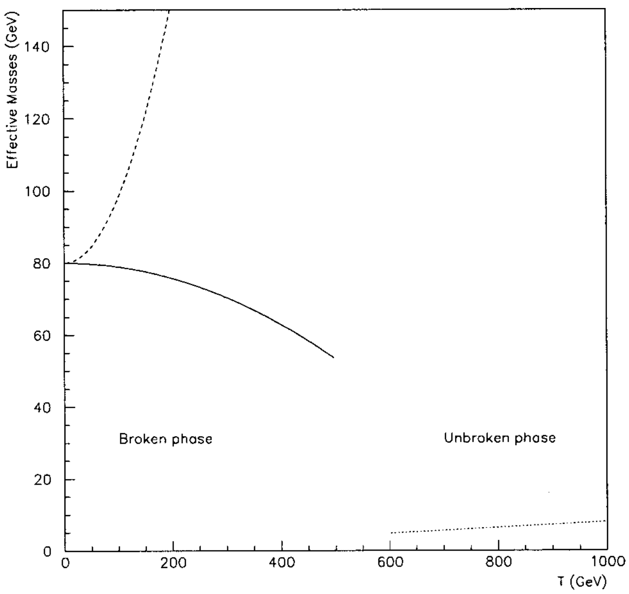

FIG. 9. Electric mass (dashed line) and magnetic mass (solid line) for the W gauge boson in the broken phase, and nonperturbative estimate of the magnetic mass (dotted line) in the symmetric phase $M^{\rm sym}_{W,\rm mag}=0.28g^2T$.

with the temperature as a flag for symmetry restoration. On the contrary, for instance the W electric mass in Eq. (37) can be written as

$$M^2_{W,\rm el}=g^2\,\frac{v^2(T)}{4}+g^2\,\frac{3N_fT^2}{8}, \tag{44}$$

and in the symmetric phase, while $v(T)=0$, it is nevertheless nonvanishing and of order gT, as anticipated.

These results are shown qualitatively in Fig. 9. We plot both the magnetic and the electric masses of the W gauge boson as functions of the temperature, at leading order T^2. The solid and dashed lines correspond to our one-loop calculation, and the dotted line to the nonperturbative estimate of the magnetic mass in the symmetric phase, $M^{\rm sym}_{W,\rm mag}=0.28g^2T$, which is taken from Ref. [19].

2. Model dependence

As already mentioned, the lowest order term in the derivative expansion of the effective Lagrangian $\mathcal{L}_{\rm GChL}$ has a universal character. The next term in the expansion $\mathcal{L}^{(4)}$ is model dependent, namely, it depends on the specific dynamics of the symmetry breaking sector through the different values of the various constants. The logarithmic divergences generated at one loop by $\mathcal{L}_{\rm GChL}$ are consistently absorbed by the renormalization of those constants. As stated before, we have neglected all the zero temperature renormalization effects, and therefore the model dependence contained on them, as well as in the matching conditions.[7] This is justified, since we are looking for temperature effects.

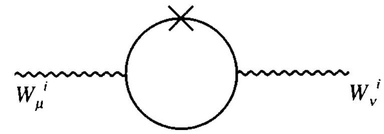

FIG. 10. Contribution of the model-dependent term $\mathcal{L}_\beta$ to the one-loop self-energy of the W and Z gauge bosons.

The only model-dependent contribution to the pure thermal corrections at one loop is due to the dimension 2 term

$$\mathcal{L}_\beta=\frac{1}{4}\beta v^2\{{\rm Tr}[U\tau_3U^\dagger(D_\mu U)U^\dagger]\}^2, \tag{45}$$

which explicitly breaks the custodial $SU(2)_C$ symmetry. This term contributes to $\Delta\rho$ at tree level, and is thus strongly constrained by experimental data. The contribution of $\mathcal{L}_\beta$ to the magnetic mass of the gauge bosons is given by (see Fig. 10)

$$\delta M^2_{W,\rm mag}=-g^2\beta\frac{T^2}{12}, \tag{46}$$

$$\delta M^2_{Z,\rm mag}=g^2\beta\frac{T^2}{3}. \tag{47}$$

The sign of the parameter β can either be positive or negative. Whatever the case for a given theory, Eqs. (46) and (47) show an opposite behavior for the W and Z magnetic masses, which are no more forced to behave in a similar way since the operator under study breaks the custodial $SU(2)_C$ symmetry. When combined with the universal leading contribution found in Eqs. (36), the total correction reads

$$M^2_{W,\rm mag}=\frac{g^2}{4}v^2\left(1-\frac{T^2}{6v^2}-\beta\frac{T^2}{3v^2}\right), \tag{48}$$

$$M^2_{Z,\rm mag}=\frac{(g^2+g'^2)}{4}v^2\left(1-\frac{T^2}{6v^2}+\beta\frac{4g^2T^2}{3(g^2+g'^2)v^2}\right). \tag{49}$$

The $SU(2)\otimes U(1)$ symmetry can be considered effectively restored only when all possible flags have signaled it. The above result, taken at face value, would indicate that the $SU(2)\otimes U(1)$ symmetry may never be restored at high temperature for theories where a large enough value of the coefficient of the operator $\mathcal{L}_\beta$ is generated. Such a strong statement has to be tempered by recalling that, if the chiral expansion is valid, we expect β to be small (in typical models it is of order of a coupling constant squared), and the total correction in Eq. (49) would be dominated by the leading one, pointing in a natural way towards restoration. Moreover, low-energy constraints on new physics give an experimentally allowed value of β of order 10^{-3} [32], which implies that in phenomenologically acceptable models the contribution of the operator $\mathcal{L}_\beta$ is indeed negligible. It is interesting

[7]We thank J. Matias for pointing out this fact to us.

to note that the tendency to symmetry restoration can be reversed for values of β which are not outrageously large, as seen from Eqs. (49).

IV. CONCLUSIONS

We have shown that the spontaneously broken $SU(2)\otimes U(1)$ gauge theory in models where the Higgs boson sector becomes strongly interacting (such as composite Higgs boson models and technicolorlike ones) tends to be restored when the system is heated. This conclusion is obtained in a model-independent way using the techniques of the electroweak chiral Lagrangian. Specific models will only affect the sharpness of such a tendency, unless the natural chiral expansion is not respected. We quantify such model dependence computing the generic contribution of the leading effective operator whose coefficient is model sensitive $\mathcal{L}_\beta$; its one-loop contribution to the W and Z magnetic masses is found to have opposite sign. The technique, while valid only for temperatures lower than the electroweak scale, has the advantage of its nonperturbative character. The physical conclusion reached here parallels the corresponding one for the other main avenue of beyond the standard model physics, supersymmetry, where perturbative treatments show a tendency towards restoration.

In this work we have also explored the $SU(2)\otimes U(1)$ gauge symmetry in a perturbative regime: the cases of one and two light Higgs doublets. Again, the results show symmetry restoration at high temperatures when the full scalar, gauge boson, and fermion corrections are taken into account.

The above conclusions have been obtained mainly through the study of the temperature-dependent magnetic mass for the gauge bosons, which we propose as an appropriate flag in the broken phase. In addition, BRST invariance has been explicitly checked for gauge theories at finite temperature, a novel result.

Finally, it is worth remarking that global symmetries have been studied as well for the nonlinear σ model at finite temperature. While this latter subject and the corresponding results are not new, the technical approach we used is: we derive first the temperature corrected one-loop effective Lagrangian, from which the physical conclusions are then extracted.

We have disregarded the putative role of finite parameters such as finite volumes or causal domain sizes in the history of the universe. Their effect could constitute an interesting topic to study.

ACKNOWLEDGMENTS

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APPENDIX A: TENSOR BASIS

The tensor basis in terms of which we have expressed the gauge boson self-energy is given by

$$A^{\mu\nu}=g^{\mu\nu}-B^{\mu\nu}-D^{\mu\nu}, \tag{A1}$$

$$B^{\mu\nu}=-\frac{\bar{K}^\mu\bar{K}^\nu}{K^2}, \tag{A2}$$

$$C^{\mu\nu}=\frac{K^\mu\bar{K}^\nu+\bar{K}^\mu K^\nu}{K^2}, \tag{A3}$$

$$D^{\mu\nu}=\frac{K^\mu K^\nu}{K^2}, \tag{A4}$$

where $\bar{K}^\mu=(K\cdot u\, K^\mu-K^2u^\mu)/k$ and k is such that $K_\mu K^\mu=\omega^2-k^2$ with $\omega=K_\mu u^\mu$.

APPENDIX B: FADDEEV-POPOV LAGRANGIAN

The Faddeev-Popov Lagrangian which corresponds to the non-linear realization of the $SU(2)\otimes U(1)$ gauge symmetry is different than the one derived for the minimal standard model for which the gauge symmetry is linearly realized. Here we present the Faddeev-Popov Lagrangian terms which are relevant for our purposes. More general results can be found in Ref. [33]:

$$\begin{aligned}\mathcal{L}_{\rm FP}=&c_0^+\left\{-\nabla^2-\left(\frac{g'^2v\xi}{2}\right)\left[\frac{v}{2}-\frac{1}{6v}(\pi_1^2+\pi_2^2)+\cdots\right]\right\}c_0+\sum_{i\neq j\neq k=1}^{3}c_i^+\left\{-\nabla^2-\left(\frac{g^2v\xi}{2}\right)\left[\frac{v}{2}-\frac{1}{6v}(\pi_j^2+\pi_k^2)+\cdots\right]\right\}c_i\\&+(c_1^+c_2-c_2^+c_1)\left[-g\partial^\mu W_\mu^3+\left(\frac{g^2v\xi}{2}\right)\frac{\pi_3}{2}\right]+(c_1^+c_3-c_3^+c_1)\left[g\partial^\mu W_\mu^2+\left(\frac{g^2v\xi}{2}\right)\frac{\pi_2}{2}\right]+(c_2^+c_3-c_3^+c_2)\\&\times\left[g\partial^\mu W_\mu^2+\left(\frac{g^2v\xi}{2}\right)\frac{\pi_1}{2}\right]+gg'\frac{v\xi}{4}(c_0^\dagger c_3+c_3^\dagger c_0)\left[v-\frac{1}{3v}(\pi_1^2+\pi_2^2)+\cdots\right]+\cdots,\end{aligned} \tag{B1}$$

where $\xi_1=\xi_2=\xi$.

APPENDIX C: WARD IDENTITIES AT ONE LOOP

We have verified the Ward identities at one loop, both for the linear and nonlinear realization of the gauge symmetry, for the two-point Green functions, computing the $W_\mu^\pm$, Z_μ, A_μ gauge boson, Goldstone boson, and ghost self-energies, together with the one-loop Goldstone boson-gauge boson mixing term. At leading order $[\mathcal{O}(T^2)]$ and for small external momenta the results for the standard model case are

$$\begin{aligned}
\Pi_D^Z&=-(g^2+g'^2)\frac{T^2}{8}-(g^2+g'^2)\frac{3M_Z^2T^2}{8m^2},\\
\Pi_D^W&=-g^2\frac{T^2}{8}-g^2\frac{3M_W^2T^2}{8m^2},\\
\Pi_D^\gamma&=0,\\
\Pi_D^{Z\gamma}&=0,\\
\Pi^\pi&=0,\\
\Pi^{\pi^0\gamma}&=0,\\
\Pi^{\pi^\pm W}&=-g^2\frac{T^2}{16M_W}-g^2\frac{3M_WT^2}{16m^2},\\
\Pi^{\pi^0 Z}&=i(g^2+g'^2)\frac{T^2}{16M_Z}+i(g^2+g'^2)\frac{3M_ZT^2}{16m^2},
\end{aligned}\tag{C1}$$

where $m^2=2\mu^2$ represents the Higgs boson mass squared. Concerning the electroweak chiral Lagrangian, the results for the two-point Green functions are

$$\begin{aligned}
\Pi_D^Z&=-(g^2+g'^2)\frac{T^2}{24},\\
\Pi_D^W&=-g^2\frac{T^2}{24},\\
\Pi_D^\gamma&=0,\\
\Pi_D^{Z\gamma}&=0,\\
\Pi^\pi&=k^2\frac{T^2}{18v^2},\\
\Pi^{\pi\gamma}&=0,\\
\Pi^{\pi^0 Z}&=ig\frac{T^2}{18v},\\
\Pi^{c^\pm}&=-\xi g^2\frac{T^2}{72}.
\end{aligned}\tag{C2}$$

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