Dynamical Lorentz symmetry breaking from a (3+1)-dimensional axion-Wess-Zumino model

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We study renormalizable Abelian vector-field models in the presence of the Wess-Zumino interaction with pseudoscalar matter. Renormalizability is achieved by supplementing the standard kinetic term of vector fields with higher derivatives. The appearance of a fourth power of momentum in the vector-field propagator leads to the superrenormalizable theory in which the β function, the vector-field renormalization constant, and the anomalous mass dimension are calculated exactly. It is shown that this model has the infrared stable fixed point and its low-energy limit is nontrivial. The modified effective potential for the pseudoscalar matter leads to the possible occurrence of dynamical breaking of Lorentz symmetry. This phenomenon is related to the modification of electrodynamics by means of the Chern-Simons (CS) interaction polarized along a constant CS vector. Its presence makes the vacuum that has been recently estimated from astrophysical data optically active. We examine two possibilities for the CS vector to be timelike or spacelike, under the assumption that it originates from VEV of some pseudoscalar matter and show that only the latter one naturally arises in the framework of the present model. [S0556-2821(98)02724-6]

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I. INTRODUCTION

The possible occurrence of a very small deviation from Lorentz covariance was considered and discussed some time ago [1], within the context of the Higgs sector of spontaneously broken gauge theories. There, some ''background'' or ''cosmological'' field is generated, leading to the above-mentioned possible small deviations from Lorentz covariance, within the present experimental limits.

Later on, another possibility was explored to obtain a Lorentz- and parity-violating modification of quantum electrodynamics, by means of the addition of a Chern-Simons Lagrangian [2]. Quite recently, Coleman and Glashow [3] have discussed how Lorentz-noninvariant velocity differences among neutrinos could produce characteristic flavor oscillations in accelerators and solar neutrino fluxes.

In all the above investigations, Lorentz symmetry breaking (LSB) has been treated phenomenologically by means of some very small but explicit Lorentz-noninvariant terms which have a clear physical meaning in a privileged frame. Then, of course, the dynamical (and presumably quantum) origin of possible LSB represents an interesting problem to be tackled.

One of the possible ways to induce LSB by a dynamical mechanism has been recently argued in the (3+1)-dimensional case [4]. Namely, the spontaneous breaking of the Lorentz symmetry via the Coleman-Weinberg mechanism [5] has been revealed for a class of models with the Wess-Zumino interaction between Abelian gauge fields and pseudoscalar axion [axion-Wess-Zumino (AWZ) models].

The original motivation for studying AWZ models was to use them in resolving the old-standing conflict between perturbative renormalizability and unitarity, within the context of the gauge-invariant quantization of (3+1)-dimensional Abelian gauge models in the presence of the U(1)-chiral anomaly [6].

As a matter of fact, it was suggested some time ago [7] that gauge theories in the presence of chiral anomalies could be consistently quantized after integration over the gauge orbits and the introduction of suitable Wess-Zumino fields. Although this idea has been successfully implemented in low dimensions [8,9], its application to the (3+1)-dimensional case is still to be achieved, even within the standard covariant perturbative approach [6,10].

For lower-dimensional theories the LSB phenomenon has been observed by Hosotani in a series of papers [11]. He has found that in (2+1)-dimensional Chern-Simons gauge field theories coupled to Dirac fermions a spontaneous magnetization arises, leading to the breaking of O(2,1) symmetry. This remarkable effect might be also related to the breaking of chiral symmetry, i.e., to the generation of a dynamical mass for fermions [12,13].

In the present paper we continue our exploration of LSB by dynamical mechanisms and study it in more detail in the

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renormalizable model for the Abelian vector field interacting with pseudoscalar axion matter.

The Wess-Zumino interaction in this model may be understood as generated by quantum effects due to coupling to fermions. For instance, one can start from the above-mentioned anomalous gauge model with the Lagrangian density which describes the coupling of chiral fermions to an Abelian gauge field A_μ:

$$\mathcal{L}_0[A_\mu,\psi,\overline{\psi}]=\overline{\psi}\gamma^\mu\{i\partial_\mu+eA_\mu P_L\}\psi-m\overline{\psi}\psi, \qquad (1.1)$$

where $P_L\equiv(1/2)(\mathbf{1}-\gamma_5)$. After fermion quantization it leads to the chiral anomaly, thereby breaking the classical invariance under local gauge transformations of the left chiral sector and making a serious obstruction to derive a unitary and renormalizable gauge theory. This obstruction is essentially induced by coupling to the longitudinal part of the gauge potential, which is in turn described by the Wess-Zumino interaction in the following sense. If we rewrite the gauge potential in Eq. (1.1) as

$$A_\mu(x)=A_\mu^\perp(x)+A_\mu^\parallel(x)=\left(\delta_\mu^\nu-\frac{\partial_\mu\partial^\nu}{\partial^2}\right)A_\nu(x)+\partial_\mu\chi(x), \qquad (1.2)$$

it is well known that integration over fermion fields drives from the classical Lagrangian density (1.1), in the limit when the mass m can be disregarded, to the quantum effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}}=\frac{e^3}{48\pi^2}\,\chi\tilde{F}_{\mu\nu}F^{\mu\nu}+\mathcal{L}_{\mathrm{eff}}^\perp[A_\mu^\perp], \qquad (1.3)$$

where the last term indicates the gauge-invariant nonanomalous part. Furthermore, it is also evident that the gauge-invariant part of the effective Lagrangian (1.3) is subleading within the low-momentum regime we are dealing with here, as its quadratic part can always be reabsorbed into the renormalization of the photon wave function (see below), whereas the quartic term is of order $\alpha^2[(k/\mu_{\mathrm{IR}})\ln(k/\mu_{\mathrm{IR}})]^4$, k and μ_{IR} being the low-energy photon momentum and normalization scale, respectively.

When the gauge fields are massless, i.e., photons, a more realistic model providing at low energies the Wess-Zumino interaction of (1.3) type is QED with the additional Yukawa coupling to a scalar chiral field:

$$\mathcal{L}_0[A_\mu,\psi,\overline{\psi}]=\overline{\psi}\gamma^\mu\{i\partial_\mu+eA_\mu\}\psi-m\overline{\psi}\exp(2i\gamma_5Y\tilde{\chi})\psi, \qquad (1.4)$$

where Y stands for the hyperchange of (charged) fermions. In turn, this Lagrangian may arise as a low-energy part of the Higgs field model or of a theory with dynamically generated fermion masses. After fermion quantization the corresponding effective action at low momenta or for heavy fermions yields the pertinent Wess-Zumino vertex as a main contribution.

It can be shown as well—see the Appendix—that, in the limit of small gauge particle momenta, quadratic kinetic terms for the fields χ and $\tilde{\chi}$ are also generated by quantum effects. To sum up, the low-momentum regime of the Abelian chiral gauge theory (1.1) or QED with the ‘‘chiral mass’’ term (1.4) is faithfully described by the following nonrenormalizable effective Lagrangian density: namely,

$$\mathcal{L}_{\mathrm{eff}}[\theta,F_{\mu\nu}]=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa}{2M}\,\theta\tilde{F}_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\,\partial_\mu\theta\partial^\mu\theta, \qquad (1.5)$$

in which we have set $\theta\equiv M\chi$, standing for a pseudoscalar-axion-like field, and M is some reference mass scale, while κ is the dimensionless WZ coupling parameter of order $\alpha\sqrt{\alpha}$ in the case of the chiral gauge model (1.1) or αY for QED with the chiral mass term, α being the fine structure constant.

This latter model may also have a different origin, the pseudoscalar field being a scalar gravitational [14] or quintessence field [15] or even associated with the torsion field of a particular [16], divergenceless type: $T_{\mu\nu\sigma}=\epsilon_{\mu\nu\sigma\rho}\partial^\rho\chi(x)$, $\partial^\mu T_{\mu\nu\sigma}=0$.

Phenomenologically, the overall inverse coupling of pseudoscalar particles to photons is actually constrained from laboratory experiments, as well as from astrophysical and cosmological observations [17] to be more than 10^{12} GeV: namely, we can reasonably suppose our reference mass M to be of the same very large order of magnitude when we remain within the perturbation approach.

On the other hand, one of the aims of the present paper is to show that the effective Lagrangian (1.5), which describes quantum effects of the Abelian anomalous gauge theory or QED with chiral mass interaction, at small momenta p such that $(p/M)\ll 1$, can lead to the dynamical breaking of Lorentz symmetry, the nonperturbative phenomenon which changes drastically the photon spectrum and induces the birefringence of photons with opposite helicities. In this regime the pseudoscalar field loses time derivatives in the kinetic term (when treated in the static frame) and therefore cannot describe a propagating particle, thereby making the bounds from [17] inapplicable.

The paper is organized as follows. In Sec. II the renormalizable version of the Abelian AWZ model is implemented with the help of higher derivatives in the kinetic term for photons. The remaining divergences are analyzed; the β function and anomalous dimensions are exactly calculated. It is proved within perturbation theory that the ghostlike vector-field degrees of freedom decouple at small momenta and the model is infrared stable and nontrivial at low energies.

In Sec. III the one-photon-loop effective potential for the axion field is derived in the renormalizable AWZ model by employing the ζ-function technique [18,19]. This effective potential is shown to possess a minimum at nonzero values of $\partial_\mu\theta$ for large values of normalization scale, i.e., in the strong coupling regime.

This phenomenon of axion-field condensation leads to Lorentz symmetry breaking, whose consequences for the photon spectrum are examined in Sec. IV. In particular, it is elucidated that the tachyon modes appear in the photon spectrum [2] and photons of different helicities propagate with different phase velocities, which leads to the birefringence of arbitrarily polarized photon waves. The possible instability of the Fock vacuum arises if the vacuum expectation value (VEV) of $\partial_\mu\theta$ is a timelike vector, whereas for the spacelike one consistent LSB may be induced by infrared radiative effects.

In our Conclusion perturbation theory in the symmetry-broken phase is shortly outlined and the propagators for distorted photons are obtained.

II. RENORMALIZABLE AXION-WESS-ZUMINO MODEL

The renormalizable Abelian vector-field model (in Euclidean space) we consider is described by the Lagrangian density which contains the Wess-Zumino coupling of pseudoscalar axion and Abelian gauge field, as well as a higher-derivative kinetic term for the Abelian gauge field:

$$\mathcal{L}_{\mathrm{AWZ}}=\frac{1}{4M^2}\,\partial_\rho F_{\mu\nu}\partial_\rho F_{\mu\nu}+\frac{1}{4}\,F_{\mu\nu}F_{\mu\nu}+\frac{1}{2\xi}\,(\partial_\mu A_\mu)^2+\frac{1}{2}\,\partial_\mu\theta\partial_\mu\theta-i\,\frac{\kappa}{2M}\,\theta F_{\mu\nu}\tilde{F}_{\mu\nu}, \tag{2.1}$$

where $\tilde{F}_{\mu\nu}\equiv(1/2)\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$, and some suitable dimensional scale M is introduced, κ and ξ being the dimensionless coupling and gauge-fixing parameters, respectively.

The Wess-Zumino interaction can be equivalently represented in the following form:

$$\int d^4x\,\frac{\kappa}{2M}\,\theta F_{\mu\nu}\tilde{F}_{\mu\nu}=-\int d^4x\,\frac{\kappa}{M}\,\partial_\mu\theta A_\nu\tilde{F}_{\mu\nu}, \tag{2.2}$$

at the level of the classical action. Therefore the pseudoscalar field is involved in the dynamics only through its gradient $\partial_\mu\theta(x)$ due to topological triviality of Abelian vector fields.

From the above Lagrangian it is easy to derive the Feynman rules: namely, the free vector-field propagator reads

$$D_{\mu\nu}(p)=-M^2\,\frac{d_{\mu\nu}(p)}{p^2(p^2+M^2)}+\frac{\xi}{p^2}\,\frac{p_\mu p_\nu}{p^2}, \tag{2.3}$$

with $d_{\mu\nu}(p)\equiv-\delta_{\mu\nu}+(p_\mu p_\nu/p^2)$ being the transversal projector; the free axion propagator is the usual $D(p)=(p^2)^{-1}$ and the axion-vector-vector WZ vertex turns out to be given by

$$V_{\mu\nu}(p,q,r)=-i(\kappa/M)\epsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma\quad(p+q+r=0), \tag{2.4}$$

all momenta being incoming, r being the axion momentum. It is worthwhile to recall that the Fock space of asymptotic states, in the Minkowskian formulation of the present model, exhibits an indefinite metric structure. Actually, from the algebraic identity

$$\frac{M^2}{p^2(p^2+M^2)}\equiv\frac{1}{p^2}-\frac{1}{p^2+M^2},$$

it appears that negative norm states are generated by the asymptotic vector-field transversal component with ghost mass M; in addition, the longitudinal component of the asymptotic vector field also gives rise to negative norm states.

Now let us develop the power-counting analysis of the superficial degree of divergence within the model. The number of loops is as usual $L=I_v+I_s-V+1$, $I_{v(s)}$ being the number of vector (scalar) internal lines and V the number of vertices. Next we have $2V=2I_v+E_v$ and $V=2I_s+E_s$, where $E_{v(s)}$ is the number of vector (scalar) external lines. As a consequence, the overall UV behavior of a graph G is provided by

$$\omega(G)=4L-4I_v-2I_s+2V-E_s-E_v=4-2E_v-E_s-2I_v+2I_s, \tag{2.5}$$

and therefrom we see that the *only* divergent graph corresponds to $I_s=1$, $I_v=1$, $E_s=0$, $E_v=2$, and it turns out to be the one-loop vector self-energy.[1] Thus we conclude that the model is superrenormalizable. We notice that the number of external vector lines has to be even. The computation of the divergent self-energy can be done using dimensional regularization (in 2ω-dimensional Euclidean space) and gives

$$\Pi^{(1)}_{\mu\nu}(p)=\frac{g}{16\pi\epsilon}\,p^2 d_{\mu\nu}(p)+\hat{\Pi}^{(1)}_{\mu\nu}(p), \tag{2.6}$$

with $\epsilon\equiv2-\omega$, $g\equiv(\kappa^2/4\pi)$, while the finite part reads

[1] Actually, the tadpole $E_s=I_v=1$, $I_s=0$ indeed vanishes owing to the tensorial structure of the AWZ vertex.

$$\hat{\Pi}^{(1)}_{\lambda\nu}(p) = -\frac{g}{16\pi} p^2 d_{\lambda\nu}(p) \left\{ \ln \frac{M^2}{4\pi\mu^2} - \psi(2) + \frac{2}{3} \left[1 + \frac{p^2+M^2}{p^2} \ln\left(1+\frac{p^2}{M^2}\right) \right] - \frac{M^2}{3p^2} \left[1 - \frac{p^2+M^2}{p^2} \ln\left(1+\frac{p^2}{M^2}\right) \right] - \frac{p^2}{3M^2} \left[1 - \frac{p^2+M^2}{p^2} \ln\left(1+\frac{p^2}{M^2}\right) + \ln \frac{p^2}{M^2} \right] \right\}, \tag{2.7}$$

where μ denotes as usual the mass parameter in the dimensional regularization. It follows therefore that the single countergraph to be added, in order to make finite the whole set of proper vertices, is provided by the two-point one-particle-irreducible (1PI) structure

$$\Gamma^{(\mathrm{c.t.})}_{\lambda\nu}(p) \equiv -\Pi^{(1)}_{\lambda\nu}(p)\big|_{\mathrm{div}} = -\frac{1}{16}\frac{g}{\pi} p^2 d_{\lambda\nu}(p) \left[\frac{1}{\epsilon} + F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right) \right], \tag{2.8}$$

in which F_1 denotes the scheme-dependent finite part (when $\epsilon \to 0$) of the countergraph.

As a result, it is clear that we can write the renormalized Lagrangian in the forms

$$\mathcal{L}^{(\mathrm{ren})}_{\mathrm{AWZ}} = \frac{1}{4M_0^2} \partial_\rho F^{(0)}_{\mu\nu} \partial_\rho F^{(0)}_{\mu\nu} + \frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)}_{\mu\nu} + \frac{1}{2\xi_0} (\partial_\mu A^{(0)}_\mu)^2 + \frac{1}{2} \partial_\mu \theta \partial_\mu \theta - i\mu^\epsilon \frac{\kappa_0}{2M_0} \theta F^{(0)}_{\mu\nu} \tilde{F}^{(0)}_{\mu\nu}$$
$$= \frac{1}{4M^2} \partial_\rho F_{\mu\nu} \partial_\rho F_{\mu\nu} + \frac{Z}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2} \partial_\mu \theta \partial_\mu \theta - i\mu^\epsilon \frac{\kappa}{2M} \theta F_{\mu\nu} \tilde{F}_{\mu\nu}, \tag{2.9}$$

where, as a result of superrenormalizability, the exact wave function renormalization constant Z is provided by

$$Z = c_0\left(g, \frac{M}{\mu}; \epsilon\right) + \frac{1}{\epsilon} c_1(g); \tag{2.10}$$

here, we can write, up to the one-loop approximation,

$$c_0\left(g, \frac{M}{\mu}; \epsilon\right) = 1 - \frac{g}{16\pi} F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(g^2),$$
$$c_1(g) = -\frac{g}{16\pi}. \tag{2.11}$$

Moreover, the relationships between bare and renormalized quantities turn out to be the following: namely,

$$A^{(0)}_\mu = \sqrt{Z} A_\mu, \tag{2.12a}$$

$$M_0 = \sqrt{Z} M, \tag{2.12b}$$

$$\xi_0 = Z\xi, \tag{2.12c}$$

$$\kappa_0 = \frac{\kappa}{\sqrt{Z}}, \quad g_0 = \frac{g}{Z}. \tag{2.12d}$$

In particular, from the Laurent expansion of Eq. (2.12d), we can write

$$\kappa_0 = a_0\left(\kappa, \frac{M}{\mu}; \epsilon\right) + \frac{1}{\epsilon} a_1(\kappa), \tag{2.13}$$

with

$$a_0\left(\kappa, \frac{M}{\mu}; \epsilon\right) = \kappa + \frac{\kappa^3}{128\pi^2} F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(\kappa^5),$$
$$a_1(\kappa) = \frac{\kappa^3}{128\pi^2}. \tag{2.14}$$

This entails that, within this model, we can solve the renormalization group equations (RGEs) in the minimal subtraction (MS) scheme $F_1 \equiv 0$: namely,

$$\mu \frac{\partial \kappa}{\partial \mu} = -\epsilon\kappa - a_1(\kappa) + \kappa \frac{d}{d\kappa} a_1(\kappa), \tag{2.15}$$

to get the exact MS prescription β function

$$\beta(\kappa) = \frac{\kappa^3}{64\pi^2}, \tag{2.16}$$

telling us, as expected, that $g=0$ is an IR-stable fixed point. It follows that we can integrate Eq. (2.15) and determine the running coupling exact behavior

$$g(\mu) = \frac{g(\mu_0)}{1 - [g(\mu_0)/8\pi] \ln(\mu/\mu_0)}. \tag{2.17}$$

Furthermore, always from Eqs. (2.12a)–(2.12d) and within the MS prescription, it is straightforward to recognize the remaining RG coefficients to be

$$\gamma_M \equiv \frac{1}{2} \mu \frac{\partial \ln M^2}{\partial \mu} = -\frac{g}{16\pi}, \tag{2.18a}$$

$$\gamma_d \equiv \frac{1}{2} \mu \frac{\partial \ln Z}{\partial \mu} = \frac{g}{8\pi}, \tag{2.18b}$$

$$\gamma_\xi \equiv \mu \frac{\partial \ln \xi}{\partial \mu} = -\frac{g}{4\pi}. \tag{2.18c}$$

In conclusion, we are able to summarize the asymptotic behavior of the ghost-mass parameter M and of the gauge-fixing parameter ξ at large distances, where perturbation theory is reliable in the model we are considering and within the MS renormalization scheme. Actually, if we set $s \equiv (\mu/\mu_0)$, we can easily derive

$$\overline{g}(s;g) = \frac{g}{1-(g/8\pi)\ln s} \overset{s\to 0}{\sim} -\frac{8\pi}{\ln s}, \tag{2.19a}$$

$$\overline{M}(s;M,g) = M\sqrt{1-\frac{g}{8\pi}\ln s} \overset{s\to 0}{\sim} M\sqrt{\frac{g|\ln s|}{8\pi}}, \tag{2.19b}$$

$$\overline{\xi}(s;\xi,g) = \xi + \ln\left(1-\frac{g \ln s}{8\pi}\right) \overset{s\to 0}{\sim} \xi + 2\ln\left(\frac{g}{4\pi}|\ln s|\right), \tag{2.19c}$$

showing that longitudinal as well as ghostlike transverse vector-field degrees of freedom decouple at small momenta where perturbation theory has to be trusted. Owing to this asymptotic decoupling of negative norm states, within the domain of validity of perturbation theory, the present super-renormalizable model might be referred to as *asymptotically unitary*.

Now, since Eq. (2.17) holds exactly within the MS renormalization prescription, it is important to analyze the matter of triviality in the present model. First of all, it is worthwhile to notice, taking Eqs. (2.12b), (2.12d) into account, that the quantity $\kappa_0 M_0 = \kappa M \equiv 4\pi M_{\rm inv}$ is a RG-invariant mass parameter. Furthermore, it is useful to rewrite the renormalized Lagrangian in the form

$$\begin{aligned}\mathcal{L}^{(\rm ren)}_{\rm AWZ} = &\frac{1}{4M^2}\partial_\rho F_{\mu\nu}\partial_\rho F_{\mu\nu} + \frac{Z}{4}F_{\mu\nu}F_{\mu\nu}\\ &+\frac{1}{2\xi}(\partial_\mu A_\mu)^2 + \frac{1}{2}\partial_\mu\theta\partial_\mu\theta\\ &- i\mu^\epsilon \frac{g_0}{2M_{\rm inv}}\theta F^{(0)}_{\mu\nu}\tilde{F}^{(0)}_{\mu\nu}.\end{aligned} \tag{2.20}$$

Remembering that in the MS scheme we have the following relationships: namely,

$$g_0(\epsilon) = Z^{-1}_{\rm MS} g_{\rm MS}(\mu) = \frac{g_{\rm MS}(\mu)}{1-[g_{\rm MS}(\mu)/16\pi\epsilon]}, \tag{2.21}$$

where $g_{\rm MS}(\mu)$ is given by Eq. (2.17), we are indeed allowed to specify arbitrarily the mass $M_{\rm inv}(\epsilon)$, which turns out to be some *free* mass parameter, analytic when $\epsilon\to 0$, in the present model.

Now, let us suppose $\epsilon>0$, $g_0(\epsilon)\ll 1$ ($\sim 10^{-2}$, e.g.); when $g_{\rm MS}(\mu) = 16\pi\epsilon>0$, then Eq. (2.21) can no longer be satisfied unless $g_{\rm MS}(\mu) = g_0(\epsilon) = 0$, $\forall \mu>0$. On the other hand, this situation does not entail triviality of the model as we can always choose $M_{\rm inv}\to 0$ in such a way that the ratio $[g_0(\epsilon)/M_{\rm inv}(\epsilon)] = (1/M_*)\neq 0$. As a consequence, the nontrivial renormalized model is most suitably parametrized as follows: namely,

$$\begin{aligned}\mathcal{L}^{(\rm ren)}_{\rm AWZ} = &\frac{\rho}{4M_*^2}\partial_\rho F_{\mu\nu}\partial_\rho F_{\mu\nu} + \frac{Z(\epsilon)}{4}F_{\mu\nu}F_{\mu\nu}\\ &+\frac{1}{2\xi}(\partial_\mu A_\mu)^2 + \frac{1}{2}\partial_\mu\theta\partial_\mu\theta\\ &-\mu^\epsilon\frac{i}{2M_*}\theta F_{\mu\nu}\tilde{F}_{\mu\nu},\end{aligned} \tag{2.22}$$

in terms of the free RG-invariant mass M_* and of the unitarity violation running parameter $\rho \equiv (M_*^2/M^2)$, which asymptotically vanishes at large distances as already stressed. We notice that it is just the above RG-invariant free mass that has to be eventually identified with the "physical value" $M_* \geqslant 10^{12}$ GeV, as discussed in Ref. [17]. However, it is important to note that what has been discussed in the present section is actually pertinent to the unbroken Lorentz-covariant phase. As a matter of fact, we shall see in the next section that quantum radiative effects may lead, in the present model, to the onset of another phase in which Lorentz symmetry appears to be dynamically broken and a nontrivial VEV for the quantity $\partial_\mu\theta$ arises.

III. EFFECTIVE POTENTIAL

We are ready now to investigate a further interesting feature of this simple but nontrivial model: the occurrence of spontaneous breaking at the quantum level of the SO(4) symmetry in the Euclidean version or the $O(3,1)^{++}$ space-time symmetry in the Minkowskian case. As a matter of fact, we shall see in the following that the effective potential for the pseudoscalar axion field θ may exhibit nontrivial minima and, consequently, some privileged direction has to be fixed by boundary conditions, in order to specify the true vacuum of the model. More interesting, those nontrivial minima lie within the perturbative domain. Since we are looking for the effective potential of the pseudoscalar field, we are allowed to ignore the renormalization constant $Z(\epsilon)$ in Eq. (2.22) and restart from the classical action in four dimensions.

The axion background-field-generating functional is defined as

$$\mathcal{Z}[\theta] \equiv \mathcal{N}^{-1}\int [\mathcal{D}A_\mu]\exp\{-\mathcal{A}_{\rm AWZ}[A_\mu,\theta]\},$$

$$\mathcal{A}_{\rm AWZ}[A_\mu,\theta] \equiv \int d^4x(\mathcal{L}_{\rm AWZ} - A_\mu J_\mu), \tag{3.1}$$

where we have included the photon coupling to (external) matter sources J_μ. The classical field configurations $\overline{A_\mu}(x)$ are solutions of the Euler-Lagrange equations

$$\frac{\delta\mathcal{A}_{\rm AWZ}[A_\mu,\theta]}{\delta A_\mu(x)}=K_{\mu\nu}[\theta]\bar{A}_\nu(x)=J_\mu, \tag{3.2}$$

with $(\Delta\equiv\partial_\mu\partial_\mu)$

$$K_{\mu\nu}[\theta]\equiv\left(\rho\,\frac{\Delta}{M_*^2}-1\right)(\delta_{\mu\nu}\Delta-\partial_\mu\partial_\nu)-\frac{1}{\xi}\,\partial_\mu\partial_\nu-\frac{2}{M_*}\,\epsilon_{\lambda\mu\sigma\nu}\partial_\lambda\theta(x)(-i\partial_\sigma) \tag{3.3}$$

being an elliptic invertible local differential operator. After integrating over photon fluctuations $A_\mu(x)-\bar{A}_\mu(x)$, we eventually obtain

$$\mathcal{Z}[\theta]\equiv\mathcal{N}^{-1}\,\exp\{-\mathcal{A}_{\rm AWZ}[\bar{A}_\mu,\theta]\}(\det\|\mathcal{K}_{\mu\nu}[\theta]\|)^{-1/2}, \tag{3.4}$$

with $\mathcal{N}=\mathcal{Z}[\theta=0]$ and where the dimensionless operator has been introduced: namely,

$$\mathcal{K}_{\kappa\nu}[\theta]\equiv\mu^{-2}K_{\kappa\nu}[\theta]=\mathsf{T}_{\kappa\nu}\,\frac{\Delta}{\mu^2}\left(\rho\,\frac{\Delta}{M_*^2}-1\right)-\frac{1}{\xi}\,\frac{\Delta}{\mu^2}\,l_{\kappa\nu}-\frac{2}{\mu^2}\,\epsilon_{\kappa\nu\lambda\sigma}\eta_\lambda(x)(-i\partial_\sigma), \tag{3.5}$$

where we have set

$$\mathsf{T}_{\mu\nu}\equiv\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Delta}, \tag{3.6a}$$

$$l_{\mu\nu}\equiv\frac{\partial_\mu\partial_\nu}{\Delta}, \tag{3.6b}$$

$$\eta_\mu(x)\equiv\frac{\partial_\mu\theta(x)}{M_*}, \tag{3.7}$$

in which μ represents the subtraction point, i.e., the momentum scale at which the effective action is defined, whose actual value is constrained by physical requirements as we shall see below.

We want to evaluate the determinant in Eq. (3.4) for constant vector η_μ; to this aim, we can rewrite the relevant operator in the form

$$\mathcal{K}_{\kappa\nu}(\eta)\equiv\frac{\Delta}{\mu^2}\left\{-\mathsf{T}_{\kappa\nu}\left(1-\rho\,\frac{\Delta}{M_*^2}\right)-\frac{1}{\xi}\,l_{\kappa\nu}\right\}+\frac{\mathcal{E}_{\kappa\nu}(\eta)}{\mu^2}, \tag{3.8}$$

with

$$\mathcal{E}_{\mu\nu}(\eta)\equiv-2\,\epsilon_{\mu\nu\lambda\sigma}\eta_\lambda(-i\partial_\sigma). \tag{3.9}$$

From the conjugation property

$$(\mathcal{E}^\dagger)_{\mu\nu}=-\mathcal{E}_{\mu\nu}, \tag{3.10}$$

it follows that

$$(\mathcal{K}^\dagger[\eta])_{\mu\nu}=(\mathcal{K}[-\eta])_{\mu\nu}, \tag{3.11}$$

which shows that the the relevant operator is *normal*. As a consequence, after compactification of the Euclidean space, we can safely define its complex power [18] and its determinant [19] by means of the ζ-function technique: namely,

$$\det\|\mathcal{K}[\eta]\|=(\det\|\mathcal{K}[\eta]\mathcal{K}^\dagger[\eta]\|)^{1/2}\equiv\exp\left\{-\frac{1}{2}\,\frac{d}{ds}\,\zeta_H(s;\eta)\right\}\Bigg|_{s=0}, \tag{3.12}$$

where we have set[2]

$$(H[\eta])_{\mu\nu}\equiv(\mathcal{K}[\eta])_{\mu\lambda}(\mathcal{K}^\dagger[\eta])_{\lambda\nu}, \tag{3.13}$$

$$\zeta_H(s;\eta)\equiv{\rm Tr}(H[\eta])^{-s}. \tag{3.14}$$

Going into the momentum representation, it is easy to obtain from Eq. (3.13) the Fourier transform of our relevant operator: namely,

$$(\tilde{H}[p;\eta])_{\mu\nu}=\left(\frac{p^2}{\mu^2}\right)^2\left\{-\left(1+\rho\,\frac{p^2}{M_*^2}\right)^2t_{\mu\nu}+\frac{1}{\xi^2}\,l_{\mu\nu}\right\}-4\,\frac{(\eta\cdot p)^2-\eta^2p^2}{\mu^4}\,e_{\mu\nu}, \tag{3.15}$$

in terms of the projectors

$$t_{\mu\nu}=\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}, \tag{3.16a}$$

$$l_{\mu\nu}=\frac{p_\mu p_\nu}{p^2}, \tag{3.16b}$$

$$e_{\mu\nu}\equiv\{\mathbf{e}_2(p;\eta)\}_{\mu\nu}=\frac{p^2}{(\eta\cdot p)^2-\eta^2p^2}\left\{-\eta^2t_{\mu\nu}+\eta_\mu\eta_\nu+\frac{(\eta\cdot p)^2}{p^2}\,\delta_{\mu\nu}-\frac{\eta\cdot p}{p^2}\,(\eta_\mu p_\nu+\eta_\nu p_\mu)\right\}; \tag{3.16c}$$

notice that the following properties hold:

$$e_{\mu\nu}p_\nu=0,\quad e_{\mu\nu}\eta_\nu=0,\quad e_{\mu\nu}t_{\nu\lambda}=e_{\mu\lambda}. \tag{3.17}$$

Taking all those definitions and properties carefully into account, it is straightforward to rewrite the relevant operator according to the orthogonal decomposition as follows:

$$\tilde{H}[p;\eta]=\tilde{H}_0(p)\{\mathbf{Id}_4-\mathbf{e}_2(p;\eta)+\mathbf{e}_2(p;\eta)\tilde{R}[p;\eta]\}, \tag{3.18}$$

[2]The same regularized determinant is obtained by considering $H'[\eta]\equiv\mathcal{K}^\dagger[\eta]\mathcal{K}[\eta]$.

in which

$$(\tilde{H}_0(p))_{\mu\nu}=\left(\frac{p^2}{\mu^2}\right)^2\left\{-\left(1+\rho\,\frac{p^2}{M_*^2}\right)^2 t_{\mu\nu}+\frac{1}{\xi^2}\,l_{\mu\nu}\right\}, \quad (3.19)$$

$$\tilde{R}[p;\eta]=\left(1+\frac{4[(\eta\cdot p)^2-\eta^2p^2]}{(p^2)^2[1+\rho(p^2/M_*^2)]^2}\right), \quad (3.20)$$

while the projector $\mathbf{e}_2(p;\eta)$ onto a two-dimensional subspace satisfies

$$\mathrm{tr}\mathbf{e}_2(p;\eta)=2, \quad \mathbf{e}_2(p;\eta=0)=0, \quad (3.21)$$

where ''tr'' means contraction over four-vector indices.

As a consequence, from Eqs. (3.4) and (3.12), we can eventually write

$$\mathcal{Z}[\eta_\mu]=\exp\{-\mathcal{A}_{\mathrm{AWZ}}[\bar{A},\eta]+\mathcal{A}_{\mathrm{AWZ}}[\bar{A},\eta=0]\}\left\{\frac{\det\|H_0[\mathbf{Id}_4-\mathbf{e}_2+\mathbf{e}_2R(\eta)]\|}{\det\|H_0\|}\right\}^{-1/4}; \quad (3.22)$$

here, H_0 and $R(\eta)$ stand, obviously, for the integro-differential operators whose Fourier transforms are given by Eqs. (3.19), (3.19), respectively.

We can definitely obtain

$$\mathcal{W}[\eta_\mu,\rho]=-\ln\,\mathcal{Z}[\eta_\mu,\rho]\equiv\mathcal{A}_{\mathrm{AWZ}}[\bar{A},\eta,\rho]-\mathcal{A}_{\mathrm{AWZ}}[\bar{A},\eta=\rho=0]-\frac{1}{4}\frac{d}{ds}\,\zeta_h(s=0;\eta,\rho)+\frac{1}{4}\frac{d}{ds}\,\zeta_{h_0}(s=0), \quad (3.23)$$

in which

$$\zeta_h(s;\eta,\rho)=2(\mathrm{vol})_4\mu^{4s}\int\frac{d^4p}{(2\pi)^4}\left\{(p^2)^2\left(1+\rho\,\frac{p^2}{M_*^2}\right)^2+4[(\eta\cdot p)^2-\eta^2p^2]\right\}^{-s}, \quad (3.24)$$

while, obviously, $\zeta_{h_0}(s)=\zeta_h(s;\eta=\rho=0)$. The effective potential for constant η_μ appears eventually to be expressed as

$$\mathcal{V}_{\mathrm{eff}}(\eta,\rho)\equiv\frac{1}{2}\,M_*^2\eta^2-\frac{1}{(\mathrm{vol})_4}\left\{\frac{1}{4}\frac{d}{ds}\,\zeta_h(s=0;\eta,\rho)-\frac{1}{4}\frac{d}{ds}\,\zeta_{h_0}(s=0)\right\}, \quad (3.25)$$

and therefore we have to carefully compute the integral in Eq. (3.24). To this aim, it is convenient to select a coordinate system in which

$$p_\mu=(\mathbf{p},p_4), \quad p_4=\frac{\eta\cdot p}{\sqrt{\eta^2}}, \quad (3.26)$$

in such a way that, after rescaling variables as $\mathbf{x}=(\mathbf{p}/\mu)$, $y=(p_4/\mu)$, we obtain

$$\zeta_h(s;\eta,\rho)=\frac{4\mu^4(\mathrm{vol})_4}{(2\pi)^4\Gamma(s)}\int_0^\infty d\tau\,\tau^{s-1}\int_0^\infty dy\int d^3x\,\exp\{-\tau(\mathbf{x}^2+y^2)^2[1+\varrho(\mathbf{x}^2+y^2)]^2+\tau v^2\mathbf{x}^2\}, \quad (3.27)$$

where $\varrho\equiv\rho(\mu/M_*)^2$ and $v_\nu\equiv(2/\mu)\eta_\nu$. A straightforward calculation leads eventually to the following integral representation[3] [20]: namely,

$$[\mu^4(\mathrm{vol})_4]^{-1}\zeta_h(s;\eta,\rho)=\frac{(v^2)^{2-2s}}{8\pi^2}\int_0^\infty dt\,\frac{t^{1-2s}}{(1-\varrho v^2t)^{2s}}\,{}_2F_1\left(\frac{3}{2},s;2;\,\frac{-1}{t(1-\varrho v^2t)^2}\right). \quad (3.28)$$

Let us first analyze the case $\rho=0$, which corresponds to the low-energy unitary regime; in this limit, the integration in the previous formula can be performed explicitly $[1<\mathrm{Re}\,s<(7/4)]$ to yield

$$[\mu^4(\mathrm{vol})_4]^{-1}\zeta_h(s;\eta,\rho=0)=\frac{(v^2)^{2-2s}}{16\pi^2\sqrt{\pi}}\,\frac{2^{4s-4}}{(s-1)}\,\frac{\Gamma[s-(1/2)]\Gamma[(7/2)-2s]}{\Gamma[(5/2)-s]}. \quad (3.29)$$

In the present case $\rho\to0$, the effective potential for constant η_μ within the ζ-function regularization is given by

[3]We notice that, from the integral representation (3.28) for Re $s<1$, it turns out that $\zeta_{h_0}(s)$ is regularized to zero.

$$\mathcal{V}_{\text{eff}}(\eta,\rho=0)=\frac{1}{2}M_*^2\eta^2-\frac{1}{(\text{vol})_4}\frac{1}{4}\frac{d}{ds}\zeta_h(s=0;\eta,\rho=0)=\frac{5\mu^4}{32\pi^2}\left\{az+z^2\left(\ln z+\frac{7}{30}\right)\right\}, \tag{3.30}$$

where $a\equiv(16\pi^2M_*^2/5\mu^2)$ and $z\equiv(v_\nu v_\nu/4)=\eta_\nu\eta_\nu/\mu^2$. We can easily check that stable O(4)-degenerate nontrivial minima appear for $a\leqslant a_{\text{cr}}=\exp\{-(37/30)\}\simeq0.2913$. Notice that the latter interval of values of a just corresponds to $\mu\geqslant10.4M_*$.

It follows therefrom that, for $0\leqslant a\leqslant a_{\text{cr}}$, the corresponding symmetry-breaking values satisfy

$$z_0\geqslant z_{\text{SB}}\geqslant z_{\text{cr}},\quad z_0=\exp\left\{-\frac{11}{15}\right\}\simeq0.480,\quad z_{\text{cr}}=a. \tag{3.31}$$

We remark that the above result, within the ζ-function regularization, actually reproduces our previous calculation [4] using large momenta cutoff regularization. To be more precise, Eq. (3.30) indeed corresponds to a specific choice of the subtraction terms in the large momenta cutoff method, something we could call *minimal subtraction for the effective potential*.[4]

It is eventually very interesting to study the dependence of the symmetry-breaking value z_{SB} upon the parameter ρ, which measures the departure of the model from unitarity. To this aim, it is necessary to come back to the general expression of Eq. (3.28) and to make use of the Mellin-Barnes transform for the hypergeometric function [20]. The result eventually reads

$$[\mu^4(\text{vol}_4)]^{-1}\zeta_h(s;\eta,\rho)=\frac{1}{4\pi^2\sqrt{\pi}}\frac{(v^2)^{2-2s}}{\Gamma(s)}\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(\varrho v^2)^{n+2s-2}\frac{\Gamma(s+n)\Gamma(n+\frac{3}{2})\Gamma(2-2s-n)\Gamma(4s-2+3n)}{\Gamma(2+n)\Gamma(2s+2n)}\right.$$
$$\left.+\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(\varrho v^2)^n\frac{\Gamma(2-s+n)\Gamma(n-2s+\frac{7}{2})\Gamma(2s-2-n)\Gamma(4-2s+3n)}{\Gamma(4-2s+n)\Gamma(4-2s+2n)}\right\}, \tag{3.32}$$

namely, a convergent power series for $\varrho v^2<(4/27)$, with $(1/2)<\text{Re}\,s<(7/4)$. As a check, we notice that, when $1<\text{Re}\,s<(7/4)$, it is possible to set $\rho\to0$ in the previous formula: in so doing, Eq. (3.29) is indeed recovered.

It would be possible, now, to study the behavior of $\mathcal{V}_{\text{eff}}(\eta,\rho)$ up to any order in ρ. Nonetheless, a first indication on the shift of the true minima, in the renormalizable nonunitary model, is clearly given already at first order. It reads

$$\mathcal{V}_{\text{eff}}(\eta,\rho)=\frac{5\mu^4}{32\pi^2}\left\{az+z^2\left(\ln z+\frac{7}{30}+14\varrho z\ln z+\frac{74}{15}\varrho z\right)+\mathcal{O}(\varrho^2)\right\}. \tag{3.33}$$

In the present case nontrivial minima appear for

$$a\leqslant a_{\text{cr}}(\varrho)=a_{\text{cr}}(0)+\frac{37}{3}\varrho a_{\text{cr}}^2(0)+\cdots\simeq0.2913+1.046\varrho, \tag{3.34}$$

whose corresponding values are between

$$z_0(\rho)\geqslant z_{\text{SB}}(\varrho)\geqslant z_{\text{cr}}(\varrho),$$
$$z_0(\varrho)=\exp\left\{-\frac{11}{15}\right\}+\varrho\exp\left\{-\frac{22}{15}\right\}+\mathcal{O}(\varrho^2)\simeq0.480+0.231\varrho,$$
$$z_{\text{cr}}(\varrho)=a_{\text{cr}}(0)+\frac{32}{3}\varrho a_{\text{cr}}^2(0)+\cdots\simeq0.2913+0.905\varrho. \tag{3.35}$$

It appears therefore that, within the renormalizable but nonunitary regime, the dynamical breaking of the O(4) symmetry is enhanced with respect to the unitary limit $\rho\to0$. The persistence of a nonvanishing VEV of the operator $\partial_\mu\theta$ for any ρ is a quite unexpected result and, thereby, indeed remarkable. As a matter of fact, the renormalizable and/or unitary formulations have, in general, radically different behaviors [6,10]. The possible occurrence of dynamical symmetry breaking for any nonvanishing ρ (renormalizable model), which remains there in the limit $\rho\to0$ (unitary model), actually shows that this feature has a deep meaning closely connected to infrared properties of the Wess-Zumino interaction to massless photons, i.e., to the presence of the chiral local U(1) anomaly.

IV. LORENTZ SYMMETRY BREAKING IN QED DUE TO *CPT*-ODD INTERACTIONS

In electrodynamics, when one retains its fundamental character provided by the renormalizability, it is conceivable to have LSB in the $(3+1)$-dimensional Minkowski space-time by the $(C)PT$-odd Chern-Simons (CS) coupling of photons to the vacuum [2] mediated by a constant CS vector η_μ

[4]We recall that, in general, the ζ-regularized functional determinant of elliptic invertible normal operators is defined up to local polynomials of the background fields.

(Carroll-Field-Jackiw model):[5]

$$\mathcal{L}_{\rm LSB}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}A_\mu F_{\nu\lambda}\eta_\sigma. \tag{4.1}$$

One can guess that the CS vector η_μ originates from the VEV of the gradient of the axion field θ in the AWZ model (2.22): $\langle\partial_\mu\theta(x)\rangle_0=M_*\eta_\mu$, up to Wick rotation back to Minkowski space-time.

This supplement to electrodynamics does not break the gauge symmetry of the action, but splits the dispersion relations for different photon helicities [2]. As a consequence, the *linearly* polarized photons exhibit birefringence when they propagate in the vacuum, i.e., the rotation of the polarization direction depending on the distance.

If the vector η_μ is timelike, $\eta^2>0$, then this observable effect is isotropic in the preferred frame (presumably, the rest frame of the Universe where the cosmic microwave background radiation is maximally isotropic), since $\eta_\mu=(\eta_0,0,0,0)$. However, it is essentially anisotropic for spacelike $\eta^2<0$. The first possibility was thoroughly examined [2,21], resulting in the bound $|\eta_0|<10^{-33}\,\text{eV}\simeq10^{-28}\,\text{cm}^{-1}$. Last year, a new compilation of data on the polarization rotation of photons from remote radio galaxies was presented [22], and it was argued that the space anisotropy with $\eta_\mu\simeq(0,\vec{\eta})$ of order $|\vec{\eta}|\sim10^{-32}\,\text{eV}\simeq10^{-27}\,\text{cm}^{-1}$ exists. However, the subsequent analysis [23] about the confidence level of the above-mentioned compilation has made it clear that the existence of such an effect cannot presently be inferred.

We can use now the effective potential derived in the previous section and conclude that the timelike pattern for the CS interaction is *naively* inconsistent as it is accompanied by the creation of tachyonic photon modes[6] from the vacuum; i.e., such a vacuum would be unstable under QED radiative effects [24]—although it seems more likely that the back reaction of the electromagnetic field on the pseudoscalar will occur until some equilibrium is reached.

On the contrary, the spacelike anisotropy carrying CS vector does not generate any ‘‘at first glance’’ vacuum instability and may be naturally induced [4] by a Coleman-Weinberg mechanism [5] in any scale-invariant scenario where the CS vector is related to the VEV of the gradient of a pseudoscalar field.

Indeed, let us analyze the photon energy spectrum which can be derived from wave equations on the gauge potential $A_\mu(p)$ in the momentum representation:

$$\left[p^2g^{\mu\nu}+\left(\frac{1}{\xi}-1\right)p^\mu p^\nu+2i\epsilon^{\mu\nu\lambda\sigma}\eta_\lambda p_\sigma\right]A_\nu =-K^{\mu\nu}[\eta]A_\nu=0, \tag{4.2}$$

where $K^{\mu\nu}[\eta]$ is given (in Euclidean notation) by Eq. (3.3) and we put $\rho=0$, focusing on the infrared part of photon spectrum.

It is evident that the CS interaction changes the spectrum only in the polarization hyperplane orthogonal to the momentum p_μ and the CS vector η_ν. The relevant projector on this plane is $e_{\mu\nu}\equiv\{\mathbf{e}_2(p;\eta)\}_{\mu\nu}$ described in Eq. (3.16c). After employing the notation (3.9), $\mathcal{E}^{\mu\nu}\equiv2i\epsilon^{\mu\nu\lambda\sigma}\eta_\lambda p_\sigma$, one can prove that[7]

$$\mathbf{e}_2=\frac{\mathcal{E}\cdot\mathcal{E}}{N},\quad N\equiv4[(\eta\cdot p)^2-\eta^2p^2], \tag{4.3}$$

and $\mathcal{E}\cdot\mathbf{e}_2=\mathcal{E}$. Respectively, one can unravel the energy spectrum of the wave equation (4.2) in terms of two polarizations of different helicity:

$$\mathbf{e}_{L,R}=\frac{1}{2}\left(\mathbf{e}_2\pm\frac{\mathcal{E}}{\sqrt{N}}\right),\quad \mathcal{E}\cdot\mathbf{e}_{L,R}=\pm\sqrt{N}\mathbf{P}_{L,R}. \tag{4.4}$$

Then the dispersion relation can be read out of the equation

$$(p^2)^2+4\eta^2p^2-4(\eta\cdot p)^2=0. \tag{4.5}$$

From Eq. (4.5) one obtains the different physical properties depending on whether η^2 is timelike, lightlike, or spacelike.

If $\eta^2>0$, one can examine photon properties in the rest frame for the CS vector $\eta_\mu=(\eta_0,0,0,0)$. Then the dispersion relation

$$(p_0)^2_\pm=\vec{p}^2\pm2|\eta_0||\vec{p}| \tag{4.6}$$

shows that the upper type of solutions can be interpreted as describing massless states because their energies vanish for $\vec{p}=0$. Meanwhile, the lower type of distorted photons behave as tachyons [2] with a real energy for $|\vec{p}|>2|\eta_0|$ (when their phase velocity is taken into account). There are also static solutions with $p_0=0\Leftrightarrow|\vec{p}|=2|\eta_0|$ and unstable solutions (tachyons) with a negative imaginary energy for $|\vec{p}|<2|\eta_0|$.

For lightlike CS vectors $\eta^2=0$, one deals with conventional photons of shifted energy-momentum spectra for different polarizations:

$$(p_0\pm\eta_0)^2=(\vec{p}\pm\vec{\eta})^2. \tag{4.7}$$

If the CS vector is spacelike, $\eta^2<0$, the photon spectrum is more transparent in the static frame where $\eta_\mu=(0,\vec{\eta})$. The corresponding dispersion relation reads

$$(p_0)^2_\pm=\vec{p}^2+2\vec{\eta}^2\pm2\sqrt{|\vec{\eta}|^4+(\vec{\eta}\cdot\vec{p})^2}. \tag{4.8}$$

It can be checked that in this case $p_0^2\geqslant0$ for all $\vec{p}$ and neither static nor unstable tachyonic modes do actually arise. The upper type of solution describes the massive particle with a mass $m_+=2|\vec{\eta}|$ for small space momenta $|\vec{p}|\ll|\vec{\eta}|$. The lower type of solutions represents a massless state as p_0

[5]We notice that our constant vector η_μ is denoted as s_μ in Ref. [2].

[6]The presence of tachyonic modes in the photon spectrum was mentioned in [2].

[7]In what follows the matrix product is provided by contraction with $g_{\mu\nu}$.

$\rightarrow 0$ for $|\vec{p}|\rightarrow 0$. It might also exhibit the acausal behavior when $p_\mu p^\mu < 0$, but even in this case $p_0^2 \geqslant 0$ for all $\vec{p}$, so that unstable tachyonic modes never arise.

In a general frame, for high momenta $|\vec{p}| \gg |\vec{\eta}|$, $|p_0| \gg \eta_0$, one obtains the relation

$$|p_0| - |\vec{p}| \simeq \pm(\eta_0 - |\vec{\eta}|\cos\varphi), \tag{4.9}$$

where φ is an angle between $\vec{\eta}$ and $\vec{p}$. Hence, for a given photon frequency p_0, the phase shift induced by the difference between wave vectors of opposite helicities does not depend upon this frequency. Moreover, the linearly polarized waves—a combination of left- and right-handed ones—reveal the birefringence phenomenon of the rotation of the polarization axis with the distance [2].

Let us now examine the radiative effects induced by the emission of distorted photons. In principle, the energy and momentum conservation allows for pairs of tachyons to be created from the vacuum due to the CS interaction. Thereby, in any model where the CS vector plays a dynamical role, being related to the condensate of a matter field, one may expect that, owing to tachyon pair creation, the asymptotic Fock vacuum state becomes unstable and transforming towards a true nonperturbative state without tachyonic photon modes. But if we inherit the causal prescription for propagating physical waves, then the physical states are assigned to possess a non-negative energy sign. As a consequence, tachyon pairs can be created out of the vacuum only if $\eta^2 > 0$. In particular, the static waves with $p_0 = 0 \Leftrightarrow |\vec{p}| = 2|\eta_0|$ are well produced to destroy the vacuum state. On the contrary, for $\eta^2 < 0$ the causal prescription for the energy sign together with the energy-momentum conservation prevents the vacuum state from photon pair emission.

Thus the decay process holds when static and unstable tachyonic modes exist. Let us clarify this point with the help of the radiatively induced effective potential (3.25), (3.30) for the variable η_μ treated as an average value[8] of the gradient of a pseudoscalar field, $\eta^2 = -z\mu^2$. In this case the infrared normalization scale $\mu = \sqrt{-\eta^2/z}$ has to be of the order of 10^{-32}–10^{-33} eV in such a way to fit the Carrol-Field-Jackiw (or would-be Nodland-Ralston-like) effect [2,22].

One can see from Eq. (3.30) the following.

(a) If $\eta^2 > 0$, there appears an imaginary part for the vacuum energy,

$$\mathrm{Im}\, \mathcal{V}_{\mathrm{eff}} = -\frac{5}{32\pi}(\eta_\mu \eta^\mu)^2, \tag{4.10}$$

which characterizes the rate per unit volume of tachyon pairs production out of the vacuum state.

(b) For $\eta^2 \leqslant 0$, the effective potential is real and has a maximum at $\eta^2 = 0$, whereas the true minima arise at the nonzero spacelike value $\eta^2 = -\mu^2 z_{\mathrm{SB}}$ from Eq. (3.31).

We conclude that it is unlikely to have Lorentz symmetry breaking by the CPT-odd interaction (4.1) by means of a timelike CS vector preserving rotational invariance in the η_μ rest frame. Rather intrinsically, the pseudoscalar matter interacting with photons has a tendency to condensate along a spacelike direction. In turn, as we have seen, it leads to photon mass formation. Of course, this effect of a Coleman-Weinberg type does not yield any explanation for the magnitude of the scale μ, which, however, is implied to be a physical infrared cutoff of a cosmological origin. Therefore its magnitude can be thought to be the inverse of the maximal photon wavelength in the Universe: namely, $\lambda_{\max} = 1/\mu \simeq 10^{27}$ cm.

V. CONCLUSIONS: SKETCH OF PERTURBATION THEORY IN THE LSB PHASE

In the previous section we used the quasiclassical, one-photon-loop approach to argue for the existence of a phase with dynamical LSB. We remark that this phenomenon can be well realized in the perturbative low-energy domain provided that the values of the free parameters involved, μ, M_*, and ρ, are appropriately tuned according to Eqs. (3.31) and (3.35). Thus in this feature the AWZ model is closely analogous to the second one—the Abelian Higgs model—in the original Coleman-Weinberg paper [5].

A natural question arises about quantum fluctuations with respect to the LSB vacuum as well as about higher-loop corrections. In order to reply to it, one should develop perturbation theory in the LSB phase. It can be built with the help of three basic ingredients: the photon propagator in the background of constant η_μ, the AWZ vertex (2.2), (2.4), which remains unchanged, and the effective propagator for the θ field, which should be derived from the second variation of the one-loop effective action $\mathcal{W} = -\ln \mathcal{Z}[\theta]$ given by Eq. (3.23), in the vicinity of its LSB minimum. The latter definition implies that the calculation of photon-loop self-energy diagram is to be supplemented with a particular subtraction of that part which is borrowed by the effective θ-field propagator.

Let us display the structure of distorted photon and θ propagators. The photon propagator can be obtained (in the limit $\rho = 0$) by setting

$$\partial_\mu \theta = M_* \eta_\mu + \partial_\mu \vartheta \tag{5.1}$$

in the Lagrangian (2.22) and subsequent inversion of the photon kinetic operator: namely,

$$\tilde{K}_{\mu\nu} = -g_{\mu\nu}p^2 + p_\mu p_\nu\left(1 - \frac{1}{\xi}\right) + i\epsilon_{\mu\nu\rho\sigma}(\eta^\rho p^\sigma - p^\rho \eta^\sigma) \tag{5.2}$$

in Minkowski space-time. The inversion can be easily performed by means of a decomposition in terms of a suitable complete set of tensors: namely,

[8] It may be a mean value over a large volume for a slowly varying classical background field or, eventually, the vacuum expectation value (VEV) for an axion-type field.

$$\tilde{D}_{\mu\nu}(p)=i(1-\xi)\,\frac{p_\mu p_\nu}{(p^2+i\epsilon)^2}+\frac{i}{\Delta(p,\eta)}\times\Bigg\{-g_{\mu\nu}p^2+4\,\eta^2\,\frac{p_\mu p_\nu}{p^2+i\epsilon}+4\,\eta_\mu\eta_\nu-4\,\frac{\eta\cdot p}{p^2+i\epsilon}\times(\eta_\mu p_\nu+p_\mu\eta_\nu)-2i\epsilon_{\mu\nu\rho\sigma}\eta^\rho p^\sigma\Bigg\}, \tag{5.3}$$

where

$$\Delta(p,\eta)\equiv\Delta_+(p,\eta)\Delta_-(p,\eta),\qquad \Delta_\pm=p_0^2-\vec{p}^{\,2}-2\,\vec{\eta}^{\,2}\pm\sqrt{|\vec{\eta}|^4+(\vec{\eta}\cdot\vec{p})^2}+i\epsilon, \tag{5.4}$$

and the causal prescription for two poles is indicated. Herein, in order to make the pole structure of the above propagator more transparent, we have referred to the static frame where $\eta_\mu=(0,\vec{\eta})$, according to Eq. (4.8).

In turn, the modified kinetic term for pseudoscalar field at low momenta is derived from the second variation of the effective potential (3.30) in terms of Eq. (5.1):

$$\mathcal{W}^{(2)}\simeq\frac{1}{2}\int d^4x\;\partial_\mu\vartheta(x)\,\frac{1}{M_*^2}\,\frac{\delta^2\mathcal{V}_{\rm eff}}{\delta\eta_\mu\delta\eta_\nu}\,\partial_\nu\vartheta(x)=\frac{1}{2}\int d^4x\,\frac{5}{4\pi^2M_*^2}\,[\eta^\mu\partial_\mu\vartheta(x)]^2. \tag{5.5}$$

This kinetic term does not correspond to a relativistic propagating particle as it does not contain time derivatives. This, of course, is a consequence of spontaneous LSB in accordance with the Goldstone theorem. The related ''propagator'' takes the following form:

$$\tilde{D}(p)=\frac{4i\pi^2M_*^2}{5}\,\frac{1}{(\eta\cdot p)^2}\equiv-\frac{4i\pi^2M_*^2}{5}\,\frac{\partial}{\partial(\eta\cdot p)}\,\mathrm{CPV}\left(\frac{1}{\eta\cdot p}\right), \tag{5.6}$$

where we adopted (as it customary [25]) the Cauchy principal value prescription for this spacelike singularity. With this prescription, the emission of the ϑ field will never take place and thereby astrophysical bounds [17] are no longer applicable. One could guess that in space-time directions orthogonal to η_μ radiatively induced higher derivative terms play an essential role to restore a particle like or ghost like dynamics.

Formally, with these propagators we do not change the power counting of Sec. II for UV divergences and the UV renormalizability is still available. But with the (infrared) ϑ ''propagator'' (5.6), one anticipates drastical changes in the β function and anomalous dimensions as both the divergent and finite parts of the photon polarization function are no longer presented by Eqs. (2.6) and (2.7).

We conclude that, in contrast to the spontaneous breaking of internal symmetries, LSB leads to a substantial modification of the particle dynamics at low momenta up to the disappearance of those particles which implement the Goldstone theorem. We postpone a more detailed development of perturbation theory in the LSB phase and a discussion of higher-order loop effects to the next paper.

Note added in proof. Soon after the completion of the present work, we became aware of the paper by Colladay and Kostelecký [26]. We would like to acknowledge this paper together with references therein, in which some LSB modifications of the standard model are thoroughly discussed.

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APPENDIX

In this appendix we compute the fermion chiral determinant in the case of a constant homogeneous gauge potential. In so doing, we shall be able to show that the low-momentum effective action for the pseudoscalar axion (the longitudinal component of the gauge potential) exhibits a purely quadratic kinetic term.

The classical action for a Dirac fermion, in Minkowski space-time, coupled with vector and axial-vector gauge potentials reads

$$\mathcal{A}_M=\int dx^0d^3\mathbf{x}\;\bar{\psi}\{i\gamma^\mu\partial_\mu-m+e\gamma^\mu(V_\mu+\gamma_5A_\mu)\}\psi. \tag{A1}$$

For our purposes, it is convenient to take the Weyl representation for the Dirac's matrices: namely,

$$\gamma^0=\begin{pmatrix}\mathbf{0} & \mathbf{Id}_2\\ \mathbf{Id}_2 & \mathbf{0}\end{pmatrix},\qquad \gamma^j=\begin{pmatrix}\mathbf{0} & \sigma^j\\ -\sigma^j & \mathbf{0}\end{pmatrix}, \tag{A2}$$

where σ^j, $j=1,2,3$, are the Pauli matrices, in such a way that

$$\gamma_5\equiv i\gamma^0\gamma^1\gamma^2\gamma^3=\begin{pmatrix}-\mathbf{Id}_2 & \mathbf{0}\\ \mathbf{0} & \mathbf{Id}_2\end{pmatrix}. \tag{A3}$$

The effective action is nothing but—up to the factor $(-i)$—the logarithm of the determinant of the vector–axial-vector (VAV) Dirac's operator. Now, in order to have a well-defined expression for such a quantity, it is necessary to make a transition to Euclidean space, i.e., to perform the usual Wick's rotation, which leads to the following Euclidean VAV Dirac operator: namely,

$$(i\mathcal{D}_E)=i\gamma_\mu\partial_\mu+im+e\gamma_\mu(v_\mu+\gamma_5a_\mu), \tag{A4}$$

where $\gamma_j=-i\gamma^j$, $\gamma_4=\gamma^0$, (v_μ,a_μ) being the Euclidean VAV potentials. If we perform the analytic continuation $a_\mu\mapsto i\hat{a}_\mu$, then the continued Euclidean VAV Dirac operator (A4) turns out to be elliptic [18], normal, and, if zero modes are absent as we now suppose, invertible. As a consequence, its determinant is safely defined to be [18,19]

$$\det[i\mathcal{D}_E]\equiv\exp\left\{-\frac{1}{2}\frac{d}{ds}\zeta_{\hat{h}_E}(s)\Big|_{s=0}\right\}\Bigg|_{\hat{a}_\mu=-ia_\mu}, \quad \text{(A5)}$$

where

$$\hat{h}_E\equiv(i\hat{\mathcal{D}}_E)^\dagger(i\hat{\mathcal{D}}_E), \quad \text{(A6)}$$

with

$$(i\hat{\mathcal{D}}_E)=i\gamma_\mu\partial_\mu+im+e\gamma_\mu(v_\mu+i\gamma_5\hat{a}_\mu). \quad \text{(A7)}$$

Let us compute the above quantity in the case of homogeneous VAV potentials. We have, in momentum space,

$$\hat{h}_E=\{p^2+m^2+e^2(v^2+\hat{a}^2)-2ep_\mu v_\mu\}\mathbf{Id}_4 \\ -iep_\mu\hat{a}_\nu\{\gamma_\mu,\gamma_5\gamma_\nu\}, \quad \text{(A8)}$$

and if we choose $\hat{a}_\mu=(0,0,0,\hat{a})$, we come to the result

$$\hat{h}_E=\{p^2+m^2+e^2(v^2+\hat{a}^2)-2ep_\mu v_\mu\}\mathbf{Id}_4 \\ -2ie\hat{a}\begin{pmatrix} p_j\sigma^j & \mathbf{0} \\ \mathbf{0} & p_j\sigma^j\end{pmatrix}. \quad \text{(A9)}$$

It is now easy to obtain

$$\zeta_{\hat{h}_E}(s)=\frac{(\mathrm{vol})_4\mu^4}{\pi^{5/2}\Gamma(s)}\int_0^\infty dt\ t^{s-3/2}\exp\left\{-t\,\frac{m^2+e^2\hat{a}^2}{\mu^2}\right\} \\ \times\int_0^\infty dp\ p^2\exp\{-tp^2\}\cosh\left\{2t\,\frac{e\hat{a}p}{\mu^2}\right\}. \quad \text{(A10)}$$

If we now come back to the original Euclidean axial-vector potential—i.e., $\hat{a}\to-ia$—we easily find

$$\zeta_{h_E}(s)=\frac{m^4(\mathrm{vol})_4}{4\pi^2}\exp\left\{-s\ \ln\left(\frac{m}{\mu}\right)^2\right\}\frac{1}{(s-1)(s-2)} \\ \times\left\{1-2(s-2)\left(\frac{ea}{m}\right)^2\right\}. \quad \text{(A11)}$$

Let us consider the chiral limit $v=\pm a\equiv(m\eta/2e)$; we can rewrite the previous formula as

$$\zeta_{h_E}(s)=\frac{m^4(\mathrm{vol})_4}{8\pi^2}\exp\left\{-s\ \ln\left(\frac{m}{\mu}\right)^2\right\}\frac{1}{(s-1)(s-2)} \\ \times\{2-(s-2)\eta^2\}, \quad \text{(A12)}$$

from which it is easy to read the chiral effective action we were looking for: namely,

$$W_\chi=-\ln\det(i\mathcal{D}_\chi)\equiv\frac{1}{2}\frac{d}{ds}\zeta(s=0)=\frac{m^4(\mathrm{vol})_4}{(4\pi)^2}\chi(\eta), \quad \text{(A13)}$$

with

$$\chi(x)=\frac{3}{2}-\eta^2-(1+\eta^2)\ln\left(\frac{m}{\mu}\right)^2. \quad \text{(A14)}$$

First, we notice that the first two terms on the right-hand side of the last formula may be ignored, as the effective action is always defined up to polynomials of momenta and masses. Second, the effective action—in the case of a homogeneous chiral potential—turns out to contain only quadratic terms in the chiral potential. Therefrom, we can see that functional integration over massive left-coupled spinors leads, in the low-momentum regime, to the effective Euclidean kinetic Lagrangian

$$\mathcal{L}_{\mathrm{kin}}(\partial_\nu\theta)=\frac{1}{2}\ \partial_\nu\theta\partial_\nu\theta, \quad \text{(A15)}$$

as we claimed in Sec. I, whose constant LSB value is

$$\mathcal{L}_{\mathrm{kin}}(\eta_\nu)=\frac{m^4}{2\pi^2}\ln\left(\frac{m}{\mu}\right)z_{\mathrm{SB}}. \quad \text{(A16)}$$

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