Initial data and coordinates for multiple black hole systems

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We present here an alternative approach to data setting for spacetimes with multiple moving black holes generalizing the Kerr-Schild form for rotating or nonrotating single black holes to multiple moving holes. Because this scheme preserves the Kerr-Schild form near the holes, it selects out the behavior of null rays near the holes, may simplify horizon tracking, and may prove useful in computational applications. For computational evolution, a discussion of coordinates (lapse function and shift vector) is given which preserves some of the properties of the single-hole Kerr-Schild form. [S0556-2821(98)03918-6]

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I. INTRODUCTION

In the numerical simulation of gravitational spacetimes Einstein's equations are cast as a Cauchy problem in the Arnowitt-Deser-Misner (ADM) $3+1$ formalism. In this form the equations are split into a set of evolution and constraint equations. The latter set, the Hamiltonian and momentum constraints, are elliptic partial differential equations which must be imposed at the initial slice in the Cauchy evolution. Thereafter, analytically, the evolution equations preserve them. York, in a series of papers [1–4] gave a framework for solving the constraint equations known as the conformal formalism. We take $G=1$ (Newton's constant) and $c=1$ (the speed of light). The 3-metric, g_{ij}, is assumed to be of the form

$$g_{ij}=\phi^4\hat{g}_{ij}, \tag{1}$$

where $\hat{g}_{ij}$ is the base metric; g_{ij} is then termed the physical metric and ϕ is the conformal factor. The extrinsic curvature is decomposed into its trace and trace-free parts:

$$\hat{K}_{ij}=\hat{E}_{ij}+\frac{1}{3}\hat{g}_{ij}K, \tag{2}$$

where again the hatted quantities refer to the tensors in the base metric and $\hat{E}_{ij}$ is the traceless part of the extrinsic curvature. In this approach K is assumed to be a given scalar function. The physical extrinsic curvature is related to the base extrinsic curvature through the following conformal transformation:

$$E^{ij}=\phi^{-10}\hat{E}^{ij}. \tag{3}$$

The Hamiltonian and momentum constraints then can be written as

$$8\hat{\Delta}\phi-\hat{R}\phi-\frac{2}{3}K^2\phi^5+\hat{E}^{ij}\hat{E}_{ij}\phi^{-7}=0 \tag{4}$$

and

$$D_jE^{ij}-\frac{2}{3}D^iK=0, \tag{5}$$

where $\hat{\Delta}$ is the Laplacian with respect to $\hat{g}_{ij}$, $\hat{D}$ is the covariant derivative compatible with $\hat{g}_{ij}$ and since we have a vacuum spacetime we take the matter terms to be zero.

York's method, as it has been applied in the black hole case, assumes that the 3-space is conformally flat, with holes, and that the expansion of the 3-space (TrK) vanishes. For the case we consider, we assume the initial spatial domain contains holes. A method has been given [5,6] for specifying an essentially analytic solution to the momentum constraint with symmetric boundary conditions (an infinite series has to be summed) for a multiple black hole spacetime. The remaining difficulty is then to solve the Hamiltonian constraint (4), which is a nonlinear elliptic equation, for the conformal factor. A completely convergent method for doing this was exhibited [7], where a controllable convergent algorithm produces solutions of specifiable accuracy. The static single hole initial-data solution found in this manner is a $t=$const slice of the isotropic coordinate representation of Schwarzschild [8]. (One of the features is that the initial slice does not penetrate *inside* the horizon.) Multiple moving spinning black holes can be specified.

A new alternative to the conventional method based on throats and conformal imaging was proposed and implemented by Brandt and Brügmann [9] where the black holes are treated as punctures. The internal asymptotically flat regions are compactified to obtain a domain without inner boundaries leading to significant computational simplications.

Recent computational black hole work has turned to solutions of the Kerr-Schild [10,11] form, because these analytical models have no coordinate pathology at the horizon and allow slicings which penetrate within the horizon [12]. The present philosophy of handling the singularity hidden within a black hole is to compute only up to the horizon [13,14], though such computational techniques almost certainly require a consideration of points slightly within the horizon. Further, having an analytic, tractable example is a great asset in developing a computational scheme. Hence we propose a scheme where we utilize our freedom in setting data for the base 3-metric and extrinsic curvature and use this analytical specification based on the Kerr-Schild form.

The motivation for this work is to initialize computational evolutions being done by the Binary Black Hole Grand Challenge Alliance [15] which has shown some success at evolutions based on the Kerr-Schild form. The approach here focuses on the 3-dimensional initial value problem, and puts less emphasis on a representation which resembles the Kerr-Schild one in a 4-dimensional sense. A complementary approach [16] within the Alliance, which does emphasize the 4-dimensional aspects has recently appeared. The two approaches differ significantly, as we discuss more completely in Sec. VII below.

II. KERR-SCHILD FORMS FOR ISOLATED BLACK HOLES

The Kerr-Schild spacetime metric is given by

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + 2H(x^\alpha) l_\mu l_\nu dx^\mu dx^\nu, \tag{6}$$

where $\eta_{\mu\nu}$ is the usual flat space form, H is a scalar function of position and time and l_μ is an (ingoing) null vector (null both in the background, and in the full metric),

$$\eta^{\mu\nu} l_\mu l_\nu = g^{\mu\nu} l_\mu l_\nu = 0, \tag{7}$$

so that $l_t^2 = l_i l_i$. It is the fact that the Kerr-Schild form is based on and selects null surfaces near the black hole which makes it so appealing as a scheme for setting coordinates. The general Kerr-Schild black hole metric (written in Kerr's original rectangular coordinates) has

$$H = \frac{Mr}{r^2 + a^2 \cos^2\theta} \tag{8}$$

and

$$l_\mu = \left(1, \frac{rx+ay}{r^2+a^2}, \frac{ry-ax}{r^2+a^2}, \frac{z}{r}\right). \tag{9}$$

Here M is the mass of the Kerr black hole and $a = J/M$ is the specific angular momentum of the black hole, r, θ (and ϕ) are auxiliary spheroidal coordinates. $z = r\cos\theta$ and ϕ is the axial angle. $r(x,y,z)$ is obtained from the relation

$$\frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1 \tag{10}$$

as

$$r^2 = \frac{1}{2}(\rho^2 - a^2) + \sqrt{\frac{1}{4}(\rho^2-a^2)^2 + a^2 z^2}, \tag{11}$$

with $\rho = \sqrt{x^2+y^2+z^2}$. Notice that the function H given by Eq. (8) is harmonic:

$$\nabla^2 H = 0, \tag{12}$$

where this ∇^2 is the flat space background Laplacian. In the limit $r \to \infty$ or $a \to 0$ we recover the Schwarzschild metric in Kerr-Schild form $[H = M/r,\ l_\mu = (1, n_i)]$. This corresponds to the ingoing Eddington-Finkelstein [18] form of the Schwarzschild metric. Notice that for the stationary Kerr (and Schwarzschild) black holes we choose a particular normalization of $l_\mu : l_t \equiv l_\mu t^\mu = 1$. In the $3+1$ form used in many computational approaches, one splits the 4-metric into a 3-metric, a lapse and a shift. The 3-metric gives distances measured in a given spacelike 3-surface (at a particular instant of time). The inverse of the lapse function gives the ratio of the coordinate time (measured normally to the $t = \text{const}$ 3-space) to the evolution of proper time in moving along that timelike direction. The shift vector (multiplied by the interval of coordinate time dt) gives the amount that coordinate labels shift in going from one $t = \text{const}$ 3-space to the next.

The general Kerr-Schild metric can be cast in a $3+1$ form:

$$(\text{lapse}) \quad \alpha = \frac{1}{\sqrt{1 + 2H l_{t^2}}}, \tag{13}$$

$$(\text{shift}) \quad \beta_i = 2H l_t l_i, \tag{14}$$

$$(\text{3-metric}) \quad g_{ij} = \delta_{ij} + 2H l_i l_j. \tag{15}$$

For black hole spacetimes the relation between the lapse and the shift obtained from this splitting guarantees that the horizon stays at a constant location, even though the lapse is nonzero at the horizon. This is in contrast to the isotropic coordinate representation of the Schwarzschild solution, in which the shift vanishes everywhere and the lapse vanishes at the horizon. Hence, for example, the static Schwarzschild black hole (the ''Eddington-Finkelstein [18] form''):

$$H = M/r \tag{16}$$

$$\alpha^2 = \frac{1}{1+2M/r}, \quad \beta_i = \frac{2M}{r}\frac{x_i}{r}, \tag{17}$$

$$g_{ij} = \eta_{ij} + \frac{2M}{r}\frac{x_i}{r}\frac{x_j}{r}, \tag{18}$$

$$K_{ij} = \frac{2M}{r^4}\frac{1}{\sqrt{1+2M/r}}\left[r^2\eta_{ij} - \left(2 + \frac{M}{r}\right)x_i x_j\right], \tag{19}$$

is smooth at the horizon $r = 2M$. Notice that the nonmoving Eddington-Finkelstein metric uses

$$l_i = \partial_i (M/H), \tag{20}$$

where ∂_i is the partial derivative.

III. BOOSTED BLACK HOLES

The Kerr-Schild metric is form-invariant under a boost due to its structure. This makes it ideal as a metric for constructing a model problem for moving black holes [17]. One simply applies a constant Lorentz (boost velocity $\mathbf{v}$ as specified in the background Minkowski spacetime) transformation Λ^α_β to the 4-metric. The resulting metric retains the Kerr-Schild form, but with straightforwardly transformed H and l_μ:

$$x'^\beta = \Lambda^\beta_\alpha x^\alpha,$$

$$H(x^\alpha)\rightarrow H(\Lambda^{-1\alpha}{}_\beta x'^\beta),$$

$$l'_\delta=\Lambda^\gamma{}_\delta l_\gamma(\Lambda^{-1\alpha}{}_\beta x'^\beta),$$

$$g'_{\mu\nu}=\eta_{\mu\nu}+2Hl'_\mu l'_\nu. \tag{21}$$

Under this boost, l_t is no longer unity. Note that because we start with a stationary solution the only time dependence is in the motion of the center. In this sense the solution has no ''extra'' radiation loaded into the initial data for the boosted hole (since we see none escape to infinity).

Under such a boost, H no longer satisfies Eq. (12), but every solution of Eq. (12) in the nonmoving frame satisfies

$$\nabla^2 H-(\mathbf{v}\cdot\nabla)^2\mathbf{H}=\mathbf{0} \tag{22}$$

in the boosted frame, where ∇ is the flat background spatial derivative operator.

IV. BOOSTED SCHWARZSCHILD

For example, for the Eddington-Finkelstein (EF) system boosted in the z direction ($v\equiv v_z$), with new coordinates (x,y,z,t) we have

$$r^2=x^2+y^2+\gamma^2(z-vt)^2,$$

$$l_t=\gamma(1-v\gamma(z-vt)/r),$$

$$l_x=x/r,$$

$$l_y=y/r,$$

$$l_z=\gamma(\gamma(z-vt)/r-v),$$

$$H=M/r, \tag{23}$$

where $\gamma=1/\sqrt{(1-v^2)}$

Under a boost, the metric becomes explicitly time dependent (because r is time dependent). Because the boost of the Schwarzschild solution merely ''tilts the time axis,'' we can consider all of the boosted $3+1$ properties at an instant $t=0$, in the frame which sees the hole moving. Subsequent times t simply offset the solution by an amount vt.

After the boost, l_i no longer solves Eq. (20), but

$$l_i=\partial_i(M/H)-\gamma v_i(\text{boosted E F}). \tag{24}$$

With Eq. (22) and Eq. (24) α, β^i are defined via Eq. (13) and Eq. (14). Figure 1 shows a slice through an Eddington-Finkelstein solution boosted to 0.5c (moving upward in the figure). The heavy inner contour is the horizon, the cardioid contours are lines of constant α, and the line segments indicate the direction and magnitude of β_i.

The extrinsic curvature for the boosted Kerr-Schild metric is given by computing $\dot{g}_{ij}$ in the boosted frame [given by combining the boosted terms from Eq. (21) into Eqs. (13)–(15)].

Now, one of the Einstein equations is

$$\dot{g}_{ij}=\beta_{i,j}+\beta_{j,i}-2\Gamma^k_{ij}\beta_k-2\alpha K_{ij}, \tag{25}$$

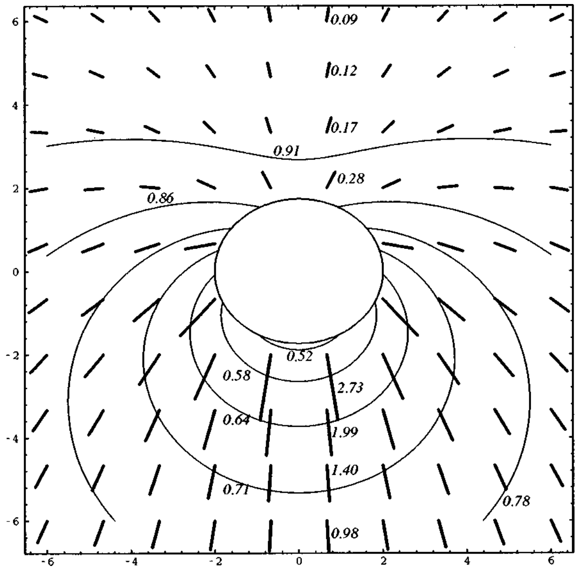

FIG. 1. Contours of lapse and shift vector β_i for a boosted Schwarzschild (Eddington-Finkelstein) black hole. This is a cut through the equator; the hole is boosted at 0.5 upward in the figure. The shift is smaller ahead of the hole than behind, which allows the black hole to move through the computational grid. The ovoid figure is the horizon (distorted in these coordinates). The cardioid curves are contours of constant lapse α, with values from the topmost contour down of 0.91, 0.86, 0.78, 0.71, 0.64, 0.58, 0.52. Along the axis of motion the lapse for this $v=0.5$ case is $\sqrt{3}/2$ at the leading point and 1/2 at the trailing point.

where the dot indicates partial time derivatives. Equation (25) enables the computation of K_{ij}.

We find that

$$K_{ij}\sim O\left(\frac{M}{r^2}\right)(1+O(v)). \tag{26}$$

V. SETTING MULTIPLE BLACK HOLE DATA

We will work by setting data for *two* Kerr-Schild black holes of comparable mass $M_1\sim M_2$. For purposes of development here (e.g., to estimate the size of terms), we also assume $M_1/r_{12}\ll 1$ where r_{12} is the coordinate separation between holes. Each hole has a velocity $\mathbf{v}_1$ or $\mathbf{v}_2$, as appropriate, assigned to it.

Define

$${r_1}^2=(x-x_1)^i(x-x_1)^j\delta_{ij},$$

$${r_2}^2=(x-x_2)^i(x-x_2)^j\delta_{ij}, \tag{27}$$

where x_1^i and x_2^i are the coordinate positions of the holes on the initial slice.

Here we use our freedom to set data and fix the background 3-metric as follows:

$$ {}^{(3)}\hat{d}s^2=\hat{g}_{ij}dx^i dx^j=\delta_{ij}dx^i dx^j + {}_1H(r_1)_1l_{i1}l_j dx^i dx^j + {}_2H(r_2)_2l_{i2}l_j dx^i dx^j. \tag{28}$$

Here ${}_aH$, and ${}_al_i(a=1,2)$ are the functions defined from each single (perhaps boosted) black hole. This background metric has two vectors corresponding to the null vector of the Kerr-Schild form (although at this point we see only their spatial components).

The ˆ symbol indicates that this is a conformally related metric, while the physical metric is

$$g_{ij}=\phi^4\hat{g}_{ij}. \tag{29}$$

Here ϕ is a strictly positive conformal factor which will be determined in the process of solving the constraints.

We start the constraint-solution process with a trial $\hat{K}_a{}^b$:

$${}_0\hat{K}_a{}^b=\hat{K}_a{}^b(1)+\hat{K}_a{}^b(2). \tag{30}$$

These $\hat{K}_a{}^b(1)$, $\hat{K}_a{}^b(2)$ are the individual extrinsic curvatures, computed and indices raised using the single-hole boosted Kerr-Schild metric appropriate to either M_1 or M_2. (Henceforth we use the 2-hole physical or conformal metric.) The leading subscript on the left indicates that this is a zeroth order approximation (in the sense of ''zeroth'' guess; this is *not* an iteration method).

Following York, we separate the trace: $K=\hat{K}(1)+\hat{K}(2)$ from the traceless part of ${}_0\hat{K}_{ab}$:

$${}_0\hat{E}^a_b={}_0\hat{K}^a_b-\frac{1}{3}\,\delta^a_b K. \tag{31}$$

K is considered a given scalar function and is not conformally scaled. The conformal scaling for ${}_0\hat{E}^{ab}$ is chosen as

$${}_0E^{ab}=\phi^{-10}{}_0\;{}_0\hat{E}^{ab}. \tag{32}$$

Here ${}_0E^{ab}$ is the traceless part of the extrinsic curvature in the physical space associated with the zeroth guess.

We attempt to write the momentum constraint:

$$D_{b0}E_c{}^b-\frac{2}{3}\,D_cK\neq 0, \tag{33}$$

where D_b is the covariant derivative compatible with the 3-metric. The momentum constraint is violated because of the appearance of connections from hole (1) multiplying an extrinsic curvature computed from hole (2), and vice versa.

We can solve the momentum constraint equation by adding a term which contributes to the longitudinal part of the solution:

$$A^{cb}\equiv{}_0E^{cb}+(lw)^{cb}, \tag{34}$$

where w^a is a vector to be solved for [4], and

$$(lw)^{cb}=D^c w^b+D^b w^c-\frac{1}{3}\,g^{bc}D_d w^d. \tag{35}$$

We then demand

$$D_bA^{cb}-\frac{2}{3}\,D^cK=0, \tag{36a}$$

or

$$D_b(lw)^{cb}=\frac{2}{3}\,D^cK-D_{b0}E^{cb}. \tag{36b}$$

This is an elliptic equation for w^a, which gives an addition to the extrinsic curvature guaranteeing the solution of the momentum constraint. As it stands, however, we cannot directly solve Eq. (29) because it involves the full physical metric, which we have not yet specified.

Using the result due to York [4],

$$D_bA^{cb}=\phi^{-10}\hat{D}_b\hat{A}^{cb}, \tag{37}$$

the momentum constraint can be written as

$$\hat{D}_b(\hat{l}w)^{cb}=\frac{2}{3}\,\hat{g}^{cb}\phi^6\hat{D}_bK-\hat{D}_b({}_0\hat{E}^{cb}). \tag{38}$$

Here, note $\hat{D}_bK$ is just $\partial K/\partial x^b$.

Form (38) allows solution for w^a in the conformal frame, except that the value of ϕ is not known; in fact this appearance of ϕ leads to a coupling between this momentum constraint, and the Hamiltonian constraint to which we now turn.

The Hamiltonian constraint solution follows York's development exactly. Since $R=\hat{R}\phi^{-4}-8\phi^{-5}\hat{\Delta}\phi$ we have

$$0=8\Delta\phi-\hat{R}\phi-\frac{2}{3}\,K^2\phi^5 +\phi^{-7}({}_0\hat{E}^{ij}+(\hat{l}w)^{ij})({}_0\hat{E}_{ij}+(\hat{l}w)_{ij}). \tag{39}$$

Solution of the coupled set Eq. (38) and Eq. (39) for ϕ and w constitutes a solution of the constraint equations. For boundary conditions on ϕ we impose $\phi=1$ at infinity, perhaps through a Robin condition [19,6], and at the surface horizons of the black holes (located in the background metric) we set $\phi=1$. We impose the condition $w^a\to 0$ at the surfaces of the black holes and at $r\to\infty$.

First note that if we apply these conditions for a single black hole, since those exact solutions already satisfy the constraints we obtain a solution $\phi=1$ and $w^a=0$ immediately. Then note that if the initial configuration has holes widely separated, we expect $\phi\sim 1$ everywhere. Then the momentum equation is an elliptic operation for w^a with source $\sim r^{-4}$ at large distances (since it arises from the Christoffel symbol×extrinsic curvature cross terms). Hence the ''total charge'' of the source is well localized, and $w^a\sim r^{-1}$, leading to corrections to K_{ab}, which are the same order in r as the background $K_{ab}\sim r^{-2}$. Further, for well-separated holes these corrections are small. In the Hamiltonian constraint (39), the extrinsic curvature terms are all squared, i.e., $\sim r^{-4}$. Hence (again assuming $\phi\sim 1$) the elliptic equation for ϕ has finite inhomogeneous source $\sim K_{ij}{}^2\sim r^{-4}$. The remaining question is the behavior of the $\hat{R}\phi$ piece. However, in the single-hole Eddington-Finkelstein metric, and hence in our conformal space, the behavior of $\hat{R}$ is such: $\hat{R}>0$, $R \sim r^{-3}$ at infinity, with deviations at infinity from the Eddington-Finkelstein value that go as r^{-4}. Hence there is a finite contribution to the ''charge'' for the conformal factor equation arising from R, and ϕ differs only finitely from from the Eddington-Finkelstein value $\phi=1$.

VI. COORDINATE CONDITIONS FOR MULTIPLE BLACK HOLES

This section deals only with the case of boosted Eddington-Finkelstein (nonspinning) holes. This development is prepared for immediate computational implementation of a multiple hole spacetime. As such, these ideas are tentative and need experimental (computational) verification. We present two formulations of coordinate setting for multiple black hole systems. Both are based on the idea that near the holes the spacetime should look as closely as possible like a single hole Eddington-Finkelstein solution. In both cases, we set boundary conditions or lapse and shift near the holes based on the single hole analysis above.

A defect of this current presentation is that some features of these coordinate specifications may be inappropriate for situations with net angular momentum (e.g., spiraling merger that settles to a Kerr black hole), but these coordinate specifications will suffice for evolution a finite time into the future, because they are *most* accurate near the black holes, where the time scales associated with curvatures are shortest.

We first present a scheme which is very closely linked to the single hole factor H. First consider the single boosted hole case Eq. (22), with appropriate boundary conditions: $H=0.5$ at the horizon. H satisfies a mixed boundary condition at infinity. Specifying H [solving Eq. (22) for H] provides a complete solution for the gauge conditions, and guarantees that the algebraic relation between lapse and shift are consistent at the horizon with the Eddington-Finkelstein form. Hence the horizon will be locked at a finite (moving, but not expanding or contracting) coordinate location.

Implementing this idea even in the single black hole case requires a kind of horizon-locking algorithm. The logic is locate the coordinate position of the apparent horizon, solve Eq. (22) for H, and use H to determine β_a and α, and adjust β_a to move the horizon normally back to its desired position. Iteration over these steps may be required to correctly generate H and β_a.

The multiple-hole case is complicated by the fact that $\mathbf{v}$ appears in Eq. (22), and there is no uniquely defined $\mathbf{v}$ in a multiple-hole scenario. We take the point of view that (a) the velocity of a black hole against its surroundings is available by examining its recent history, (b) for multiple hole systems the total momentum in the computational frame will be zero; where we compute the total momentum in a completely naive way as $\Sigma_{i=1,2}M_i\mathbf{v}_{(i)}$, where M_i and $\mathbf{v}_{(i)}$ are the parameters used in the initial data setting.

We are not claiming that this will give a final configuration which is on average completely nonmoving, but it seems likely to remove the majority of the residual average motion. Furthermore, computational science is experimental science, and the results of a run showing a final drift are a signal to adjust the initial total momentum-setting. To apply coordinate ideas as in the preceding section, we would then expect that $\mathbf{v}$ in the coordinate specification Eq. (22) is a function of position having a local value appropriate to black hole 1 when at the horizon of black hole 1; and smoothly changing to the value of $\mathbf{v}$ associated with hole 2. A simple scheme would be to take the coordinate specification

$$\mathbf{v}=\mathbf{v}_1\left(\frac{r_{h1}}{r_1}\right)^p+\mathbf{v}_2\left(\frac{r_{h2}}{r_2}\right)^p. \tag{40}$$

Thus velocities approach the appropriate velocity as the surface of the black hole is approached. The quantities r_1, r_2 are computed as in Eq. (27). The quantities r_{h1}, r_{h2} are the horizon radii along a ray to the point we wish to evaluate $\mathbf{v}$. The exponent p will be chosen by experiment. $p=1$ gives the situation simplest to analyze. Then if we assume slowly moving black holes:

$$\mathbf{v}=\frac{2M_1\mathbf{v}_1}{r_1}+\frac{2M_2\mathbf{v}_2}{r_2}. \tag{41}$$

By our ''zero momentum'' assumption, this is

$$\mathbf{v}=2M_1\mathbf{v}_1\left[\frac{1}{r_1}-\frac{1}{r_2}\right]. \tag{42}$$

For $r_1=r_2$, the point equidistant from the holes, one obtains $\mathbf{v}=0$. At infinity one has $v\to 0$ at least as r^{-2}. At the surface of hole 1:

$$\mathbf{v}=\mathbf{v}_1\left[1-\frac{2M_1}{r_2}\right], \tag{43}$$

which gives a first order fractional correction to $\mathbf{v}$. This may require the use of a higher positive power p in practical applications.

At intermediate locations we would expect H to recognize the orbital angular momentum, and take on a Kerr-like form, but our present development supports only the Schwarzschild-like case. Note, however, that the Kerr-like effects are ''mild'' until a final black hole forms, at which point the problem will become essential.

We mention here a second approach which can be considered: maximal slicing [20] (which determines the lapse α) with minimal shear specification of the shift β_i [21]. These elliptic equations require boundary conditions on lapse and shift. The boundary conditions specified in this section determine the horizon values of the lapse and shift, even when we solve them via methods other than solving for H as suggested above. Hence those boundary conditions can be applied directly to the lapse and shift to be solved in the elliptic maximal slicing-minimal shear system.

VII. DISCUSSION

The description in Sec. V of an algorithm to set binary black hole data stars proceeds by generalizing the spatial metric to two centers, and erecting spatial ''unit'' vectors from those centers. With this form, a variant of standard 3 +1 data setting is used to set up the data. The exposition in this paper concentrated on understanding the initial data structure of a single black hole, and describing how it can be used for a computational evolution. This method is to a large extent complementary to the other Alliance paper on setting data via the Kerr-Schild approach [16] and assumes a general metric (even for two-hole data) of the form of Eq. (6), up to terms $O(t^2)$ where $t=0$ is the time of the initial spatial slice. Furthermore, the initial value problem is solved in terms of conditions on the null vector l_v, and on the multiplying scalar H, rather than expressing it directly in terms of the usual $3+1$ objects, g_{ij} and K_{ij}. While the usual Kerr-Schild l_α is shear-free, that condition is not imposed in this more general situation. Because of the close connection of the approach in [16] to the structure of the known Kerr-Schild solutions, a large number of cross checks and analytical simplifications apply even in this more general situation. In part because if those simplifications, [16] is able to present data for perturbed Schwarzschild black holes. An example set of data for ''close'' black holes is given in a perturbative limit where the deviations from sphericity are small. In this case then analysis can be carried through completely.

Attacking the binary black hole problem computationally has sharpened our perspective that all aspects of the relativistic problem must be understood, for the computation to proceed. These two approaches, or some subsequent combination, will be invaluable in setting and evolving binary black hole data.

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