Stars and halos of degenerate relativistic heavy-neutrino and neutralino matter

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Heavy-neutrino (or neutralino) stars are studied using the general relativistic equations of hydrostatic equilibrium and the relativistic equation of state for degenerate fermionic matter. The Tolman-Oppenheimer-Volkoff equations are then generalized to include a system of degenerate neutrino and neutralino matter that is gravitationally coupled. The properties and implications of such an interacting astrophysical system are discussed in detail. [S0556-2821(98)05124-8]

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I. INTRODUCTION

One of the most tantalizing puzzles of the universe is the issue of dark matter, the presence of which is inferred from the observed flat rotation curves in spiral galaxies [1,2] and the diffuse emission of x rays in elliptical galaxies and clusters of galaxies, as well as from cluster dynamics. Primordial nucleosynthesis entails that most of baryonic matter in this universe is nonluminous, and such an amount of dark matter falls suspiciously close to that required by galactic rotation curves. However, although a significant component of dark matter in galactic halos is presumably baryonic [3], the bulk part of dark matter in this universe is believed to be nonbaryonic. Many candidates have been proposed [4], both baryonic as well as nonbaryonic, to explain the dark matter paradigm, but the issue of the nature of dark matter is still far from being resolved.

One of the most conservative candidates for nonbaryonic dark matter is, of course, the massive neutrino. In this paper we are particularly interested in neutrinos with masses between 10 and 25 keV/c^2, as these could form supermassive degenerate neutrino stars, which may explain, without invoking the black-hole hypothesis, some of the features observed around supermassive compact dark objects with masses ranging from $10^{6.5}M_\odot$ to $10^{9.5}M_\odot$. These have been reported to exist at the center of a number of galaxies [5–8] including our own [9–15] and quasistellar objects. It is interesting to note that neutrinos in this mass range can also cluster around ordinary stars, and thus these neutrinos could account for at least part of galactic dark matter. A further motivation for studying the collapsed structures of heavy neutrino matter is the recent increased interest in fermionic cold dark matter models [16] in which massive neutrinos play an important role in structure formation in the early universe.

A 10–25 keV/c^2 neutrino is in conflict neither with particle and nuclear physics experiments nor with astrophysical observations [17]. On the contrary, if the conclusion of the Liquid Scintillation Neutrino Detector (LSND) Collaboration which claims to have detected $\overline{\nu_\mu}\rightarrow\overline{\nu_e}$ flavor oscillations [18] is confirmed, and the quadratic seesaw mechanism involving up, charm, and top quarks [19,20] is the correct mechanism for neutrino mass generation, the ν_τ mass may be between 6 and 32 keV/c^2 [21], which is well within the cosmologically forbidden range. It is well known that such a quasistable neutrino would lead to an early neutrino-matter-dominated phase, which may have started as early as 3 weeks after the big bang. Thus, a critical universe that remained neutrino-matter dominated all the time would have reached the current microwave background temperature in less than 1 Gyr, i.e., much too early to accommodate the oldest stars in globular clusters, nuclear cosmochronometry, and the Hubble expansion age.

It is well accepted [22], however, that the cosmological bounds on neutrino mass can be bypassed by (i) reheating, (ii) decay of the neutrinos and/or (iii) annihilation of the neutrinos and antineutrinos. Reheating of the plasma would have to take place between nucleosynthesis and (re)combination. The temperature can only increase by a factor of 2 or 3 at most during reheating, because otherwise it would, through a baryon to photon ratio which differs from that after 3 min, also reduce the number of baryons below the number that is observed in the stars of our universe. Therefore, reheating alone is certainly not sufficient to bypass the cosmological bounds on neutrino mass. If the decay involves photons, it should happen at temperatures that are not too different from the energy of the decay photons, so that they have enough time to thermalize and do not distort the microwave background. Regardless of whether photons are involved in the decay or not, this would obviously involve nonstandard particle physics. Annihilation via the Z^0 can only be effective in gravitationally condensed objects. In fact, it has been shown [16] that a scenario with moderate reheating and annihilation of neutrinos in supermassive neutrino stars exists in which the cosmological bounds can be bypassed.

It is thus conceivable that, in the presence of such heavy neutrinos, the early universe might have evolved quite differently than described in the homogeneous standard model of cosmology. Neutrino stars may have emerged in local condensation processes during a gravitational phase transition, after the neutrino-matter-dominated epoch began. The latent heat produced in such a first-order phase transition, corresponding to an average binding energy of a few percent of the neutrino rest mass, might have reheated the radiation background apart from reheating the gaseous phase. Annihilation of heavy neutrinos into light neutrinos via the Z^0 would take place in the interior of neutrino stars [23,17]. Both these processes would decrease the number of heavy neutrinos, as perceived today, and also increase the time which photons need to cool down to the present microwave background temperature. Thus a quasistable neutrino in the mass range between 10 and 25 keV/c^2 is presumably not in contradiction with cosmological and astrophysical observations [17].

In fact, it has recently been shown [13,21,24] that degenerate neutrino stars [17,23,25,26] may indeed have been formed during a gravitational phase transition in the early universe. As the universe cools, heavy neutrino matter will start dominating the universe and, at a certain temperature, undergo a first-order phase transition in which supermassive neutrino stars are formed. This conclusion is robust and based on the well-accepted Thomas-Fermi model at finite temperature [24]. Of course, we cannot claim that *all* the neutrinos will end up in such neutrino stars. However, as the formation process is a genuine phase transition, most of the matter, if not all, will be in the condensed phase below the critical temperature. It is, therefore, reasonable to assume that less than 0.1–1% of the neutrinos will be left out in the form of nondegenerate gas, thus rendering the annihilation process effective. Moreover, the latent heat associated with this first-order phase transition will be released and the plasma will be moderately reheated.

Whereas the existence of this first-order phase transition is firmly established, the microscopic mechanism through which the latent heat is released during the phase transition and dissipated into observable and perhaps unobservable matter or radiation remains to be identified. At this stage, however, it is still not clear whether an efficient dissipation mechanism can be found within the minimal extension of the standard model of particle physics or whether new physics is required in the right-handed neutrino sector. We therefore have to assume in the following that such an efficient dissipation mechanism exists, in order to make sure that fermions can actually settle in the state of lowest energy in a time much shorter than the age of the universe.

In this paper, we focus primarily on gravitationally clustered, degenerate nonbaryonic matter consisting of two species of weakly interacting stable or quasistable fermions: one with a mass around 15 keV/c^2, which we subsequently call ''neutrino,'' and the other with a mass around 1 GeV/c^2, which we henceforth call ''neutralino.'' As to the neutralino mass, the general consensus is that neutralinos should have masses of tens of GeV. Of course, there is nothing that prevents us from applying our formalism to a standard $m_{\tilde{\chi}_1^0} > 23\ \mathrm{GeV}/c^2$ (C.L.=95%) neutralino or to a $m_{\tilde{\gamma}} > 15\ \mathrm{GeV}/c^2$ (C.L.=90%) photino which are the experimental and observational limits [27]. This would only reduce the Oppenheimer-Volkoff limit by a factor of a few hundred without invalidating the main substance of our paper. However, in most models of low-energy supersymmetry in which dimension-3 supersymmetry breaking operators are highly suppressed [28], photinos and gluinos are very light. In this quite attractive supersymmetry breaking scenario, the lightest R-odd particle may be a color-singlet state containing a gluino, the R^0, with mass m_{R^0} in the 1–2 GeV range [29]. Moreover, it has been recently pointed out [30], within this framework, that a photino $\tilde{\gamma}$ slightly lighter than the R^0, in the mass range of 100 MeV to 1.4 GeV, would survive as the relic R-odd species and it might be an attractive dark matter candidate. Indeed, a light photino with a mass in the range $1.2 \leq m_{R^0}/m_{\tilde{\gamma}} \leq 2.2$ is cosmologically acceptable and in the range $1.6 \leq m_{R^0}/m_{\tilde{\gamma}} \leq 2.2$ even an excellent dark matter candidate.

Furthermore, the chosen neutralino mass offers the possibility of replacing the neutralino with a neutron, as the strong-interaction effects of the neutron in neutron star matter can be simulated by an effective mass. Of course, this substitution makes sense only as long as the binding energy of the neutron is larger than the Q value for the neutron decay, so that the neutron can be considered stable in neutron star matter.

It is interesting to note that a variety of similar scenarios can be treated within the same framework: Apart from a neutrino halo around a neutron star, one could also study a neutrino halo around a white dwarf or around an ordinary star [25], since all these baryonic stars can be approximated using similar polytropic equations of state which eventually result in the same nonlinear differential equations of the Lané-Emden type. Moreover, by varying the polytropic index of the equation of state, one can also investigate the properties of a cold neutrino star immersed in a hot radiation field, or in a hot baryonic background, or in a vacuum with nonzero energy density, which all may have played a role in the formation process of primordial neutrino stars. Thus the study of this simple interacting neutrino-neutralino system allows us to learn a great deal about the properties of gravitationally clustered baryonic and nonbaryonic matter.

This paper is organized as follows: In Sec. II in a general relativistic framework, we discuss the properties and implications of degenerate neutrino (and neutralino) stars and their Newtonian and ultrarelativistic limits. In Sec. III we generalize the Tolman-Oppenheimer-Volkoff (TOV) equations to include gravitationally clustered, degenerate nonbaryonic matter, consisting of neutrinos and neutralinos. Our results are summarized in Sec. IV.

II. DEGENERATE NEUTRINO STARS

A spherically symmetric cloud of degenerate neutrino matter can be characterized by its mass density $\rho_\nu(r)$, pressure $P_\nu(r)$, and the metric in the Schwarzschild form [31]

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2). \qquad (1)$$

The pressure and the density satisfy the general relativistic TOV equations of hydrostatic equilibrium [32,33]:

$$\frac{dP_\nu}{dr} = -\frac{1}{2}(\rho_\nu c^2 + P_\nu)\,\frac{d\nu}{dr}, \tag{2}$$

$$e^\lambda = \left(1 - \frac{2Gm}{c^2 r}\right)^{-1}, \tag{3}$$

$$\frac{dP_\nu}{dr} = -G\,\frac{(\rho_\nu + P_\nu/c^2)(m + 4\pi r^3 P_\nu/c^2)}{r(r - 2Gm/c^2)}, \tag{4}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho_\nu(r), \tag{5}$$

where $m(r)$ is the mass enclosed within a radius r. The relevant boundary conditions are $m(0)=0$, $P_\nu(R)=0$, and $\rho_\nu(R)=0$, as the pressure and the density vanish at the radius R of the star. Outside the star, the functions ν and λ are determined by the usual Schwarzschild solution

$$e^\nu = e^{-\lambda}, \quad e^\lambda = (1 - 2GM/c^2 r)^{-1}, \tag{6}$$

$$M = \int_0^R 4\pi \rho_\nu(r) r^2 dr. \tag{7}$$

We now introduce the equation of state, neglecting possible effects of the dissipation mechanism. Of course, in the formation process, which we are not discussing in this paper, such a dissipation mechanism is very important. However, as soon as the degenerate star is formed, the dissipation mechanism is irrelevant and does not affect the equation of state. Thus, the equation of state may be approximated by that of a degenerate relativistic Fermi gas [34], parametrized as

$$P_\nu = K\left[X(1+X^2)^{1/2}\left(\frac{2}{3}X^2 - 1\right) + \log[X + (1+X^2)^{1/2}]\right], \tag{8}$$

$$\rho_\nu = \frac{K}{c^2}\{X(1+X^2)^{1/2}(2X^2+1) - \log[X + (1+X^2)^{1/2}]\}, \tag{9}$$

$$n_\nu = \frac{8KX^3}{3m_\nu c^2}. \tag{10}$$

Here, n_ν denotes the neutrino-number density, and K and X are given by

$$K = \frac{g_\nu m_\nu^4 c^5}{16\pi^2 \hbar^3}, \quad X = \frac{p_\nu}{m_\nu c}, \tag{11}$$

where p_ν stands for the local Fermi momentum of the neutrinos of mass m_ν, and g_ν is the spin degeneracy factor of neutrinos and antineutrinos, i.e., $g_\nu = 2$ for Majorana and $g_\nu = 4$ for Dirac neutrinos and antineutrinos. Using Eqs. (8) and (9), and introducing dimensionless variables $x = r/a_\nu$ and $\mu = m/b_\nu$ with the scales

$$a_\nu = 2\sqrt{\frac{\pi}{g_\nu}}\left(\frac{M_{\rm Pl}}{m_\nu}\right)^2 L_{\rm Pl} = 2.88233\times 10^{10} g_\nu^{-1/2}\left(\frac{17.2\ \text{keV}}{m_\nu c^2}\right)^2\ \text{km}, \tag{12}$$

$$b_\nu = 2\sqrt{\frac{\pi}{g_\nu}}\left(\frac{M_{\rm Pl}}{m_\nu}\right)^2 M_{\rm Pl} = 1.95197\times 10^{10} M_\odot g_\nu^{-1/2}\left(\frac{17.2\ \text{keV}}{m_\nu c^2}\right)^2, \tag{13}$$

where $M_{\rm Pl} = (\hbar c/G)^{1/2}$ and $L_{\rm Pl} = (\hbar G/c^3)^{1/2}$ denote Planck's mass and length, respectively, the TOV equations (4) and (5) can be written as

$$\frac{dX}{dx} = -\frac{1+X^2}{X(x^2 - 2\mu x)} \times \left\{\mu + x^3\left[X(1+X^2)^{1/2}\left(\frac{2}{3}X^2 - 1\right) + \log[X + (1+X^2)^{1/2}]\right]\right\}, \tag{14}$$

$$\frac{d\mu}{dx} = x^2\{X(1+X^2)^{1/2}(2X^2+1) - \log[X + (1+X^2)^{1/2}]\}, \tag{15}$$

subject to the boundary conditions $X(0) = X_0$ and $\mu(0) = 0$. In addition to Eqs. (14) and (15), there is also an equation governing the number of neutrinos n within a radius $r = a_\nu x$:

$$\frac{d\tilde{n}}{dx} = x^2 X^3 (1 - 2\mu/x)^{-1/2}, \tag{16}$$

where $\tilde{n} = n/N_0$ is the rescaled neutrino-number density subject to the boundary condition $\tilde{n}(0) = 0$, with

$$N_0 = \frac{8b_\nu}{3m_\nu} = 3.3765\times 10^{72}\left(\frac{17.2\text{keV}}{m_\nu c^2}\right)^3 g_\nu^{-1/2}. \tag{17}$$

Equations (14)–(16) may be solved numerically. Picking up a value X_0 for the Fermi momentum at the center (in units of $m_\nu c$), one obtains the total mass of the star, M, the radius R, and the total number of particles, N, by integrating outward until X vanishes. The results are summarized in Figs. 1 and 2. In Fig. 1 the total mass is plotted against the radius of the neutrino star. The curve has a maximum, namely, the Oppenheimer-Volkov (OV) limit [33], at $\mu_{\rm OV} = 0.15329$, which corresponds to a neutrino star mass of

$$M_{\rm OV} = 0.15329 b_\nu = 0.54195 M_{\rm Pl}^3 m_\nu^{-2} g_\nu^{-1/2} = 2.9924\times 10^9 M_\odot\left(\frac{17.2\ \text{keV}}{m_\nu c^2}\right)^2 g_\nu^{-1/2}. \tag{18}$$

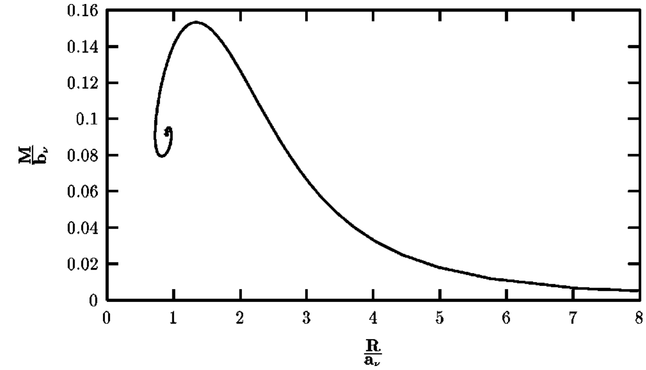

FIG. 1. The total mass M of a neutrino star in units of b_ν as a function of its radius R in units of a_ν. The maximum corresponds to the OV limit. The curve left from the maximum represents unstable configurations curling up around the point of infinite central density.

Owing to their large mass, neutrino stars could serve as candidates for supermassive compact dark objects observed in the mass range

$$2.5\times 10^6 M_\odot \lesssim M \lesssim 3\times 10^9 M_\odot \tag{19}$$

at the centers of a number of galaxies. Assuming that the most massive and violent objects are neutrino stars at the OV limit with $M_{\rm OV}=(3.2\pm 0.9)\times 10^9 M_\odot$, such as the supermassive compact dark object at the center of M87 [8], the neutrino mass required for this scenario is

$$\begin{aligned} 12.4 \ \ \mathrm{keV}/c^2 \leqslant m_\nu \leqslant 16.5 \ \ \mathrm{keV}/c^2 \quad \text{for } g_\nu=2, \\ 10.4 \ \ \mathrm{keV}/c^2 \leqslant m_\nu \leqslant 13.9 \ \ \mathrm{keV}/c^2 \quad \text{for } g_\nu=4. \end{aligned} \tag{20}$$

The radius of such a neutrino star is $R_{\rm OV}=4.4466 R^s_{\rm OV}$, where $R^s_{\rm OV}=2GM_{\rm OV}/c^2$ is the Schwarzschild radius of the mass $M_{\rm OV}$. Thus, at a distance of a few Schwarzschild radii away from the supermassive object, there is little difference between a neutrino star at the OV limit and a black hole, in particular since the last stable orbit around a black hole already has a radius of $3R^s_{\rm OV}$. A neutrino star of mass $M_{\rm OV} = 3\times 10^9 M_\odot$ would have a radius $R_{\rm OV}=3.9396\times 10^{10}$ km or 1.52 light-days.

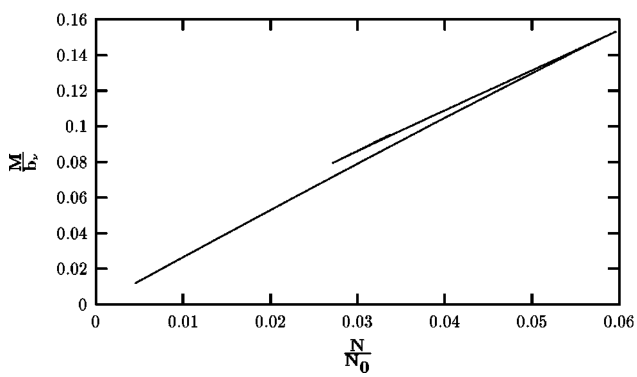

FIG. 2. The total mass M of a neutrino star in units of b_ν as a function of its total number of particles, N, in units of N_0. The configurations represented by the upper part of the curve are energetically less favorable and hence unstable.

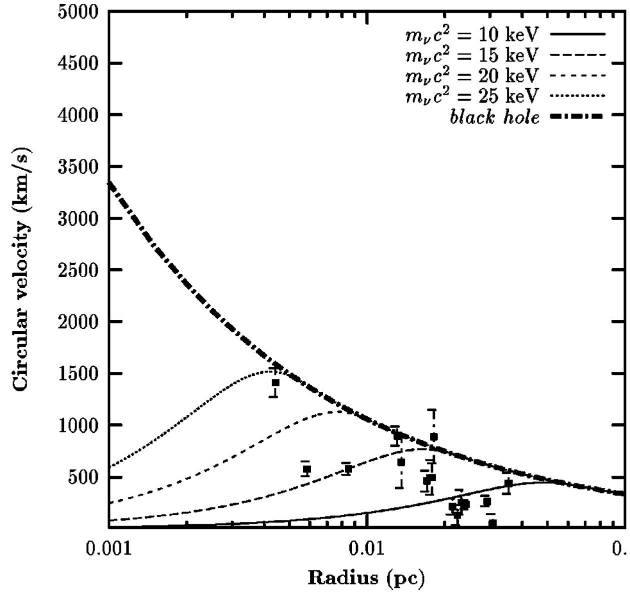

FIG. 3. The circular velocity as a function of the distance from Sgr A* for the black hole and neutrino-star scenarios. The data are taken from Ref. [11] assuming that the projected velocity and distance from Sgr A* are equal to the true velocity and distance, respectively.

Of course, neutrino stars that are well below the OV limit will have a size much larger than black holes of the same mass, although they will still be dark and much more compact than any known baryonic object of the same mass. As the gravitational potential of such an extended neutrino star is much shallower, significantly less energy will be dissipated through accreting matter than in the case of a black hole of the same mass. In fact, there is compact dark matter at the center of our galaxy with $(2.45\pm 0.40)\times 10^6 M_\odot$ concentrated within a radius smaller than 0.0254 pc or 30.3 light-days [9–11], determined from the motion of stars in the vicinity of Sgr A*. Interpreting this supermassive compact dark object in terms of a degenerate neutrino star of $2.5\times 10^6 M_\odot$, the upper limit for the size of the object provides us with a lower limit for the neutrino mass: i.e.,

$$\begin{aligned} m_\nu \geqslant 14.3 \ \ \mathrm{keV}/c^2 \quad \text{for } g_\nu=2, \\ m_\nu \geqslant 12.0 \ \ \mathrm{keV}/c^2 \quad \text{for } g_\nu=4. \end{aligned} \tag{21}$$

In Figs. 3, 4 we show the escape and circular velocities as functions of the distance from Sgr A*, for both black-hole and neutrino-star scenarios. In these graphs, we have also included the data of Ghez [11] with error bars, assuming that the velocity component and distance from Sgr A* in the line of sight are both zero, i.e., $v_z=0$ and $z=0$. We therefore must allow for a shift of the data upwards and to the right by an unknown amount, because v_z has not been and z cannot be measured. Taking a reasonable shift of the data into account, we can say with some confidence that the nearest and

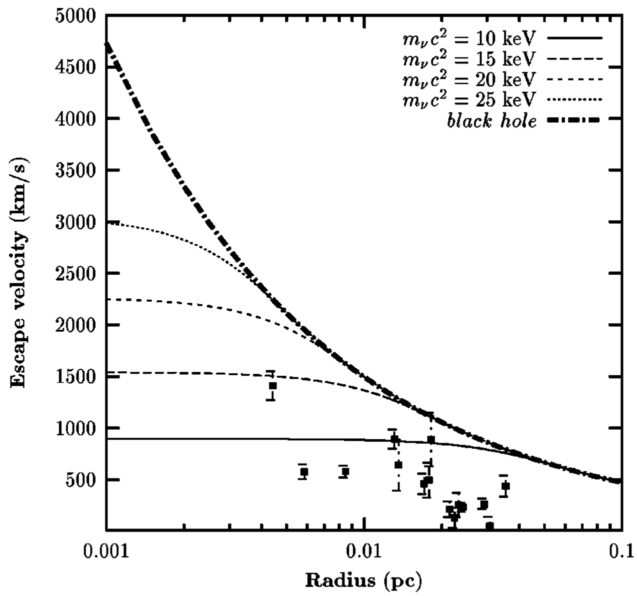

FIG. 4. The escape velocity as a function of the distance from Sgr A* for the black-hole and neutrino-star scenarios. The data are taken from Ref. [11] assuming that the projected velocity and distance from Sgr A* are equal to the true velocity and distance, respectively.

fast moving star S1 [10] or S0-1 [11], moving at the projected velocity $\sqrt{v_x^2+v_y^2}=(1400\pm100)$ km/s, is consistent with the local escape velocity at the projected distance $\sqrt{x^2+y^2}$ of S0-1 from Sgr A*. The data of Ghez imply a lower bound for the neutrino mass of $m_\nu \geqslant 15.9$ keV/c^2 for $g_\nu = 2$, yielding an upper bound for the radius of the neutrino star of $R \leqslant 22.4$ light-days for a mass of $M = 2.6\times10^6 M_\odot$. Using $m_\nu = 15.9$ keV/c^2 and v_x and v_y without error bars and assuming $v_z = z = 0$, the star S0-1 is bound on an elongated orbit with maximal and minimal distances from Sgr A* of 42 and 4 light-days, respectively. Thus there seems to be no contradiction between the data of Ghez and the neutrino star scenario with $m_\nu = 15.9$ keV/c^2.

In this context, it is important to note that if Sgr A* is a matter-accreting neutrino star [13–15], one can, in a natural way, explain the so-called ''blackness problem'' of Sgr A*, i.e., the fact that Sgr A* does not seem to emit detectable x rays of a few tens of keV, which would be emitted by baryonic matter falling towards a black hole. As this unmistakable black-hole signature is missing, the concept of a ''black hole on starvation'' has been created in order to save the black-hole idea. However, the neutrino-star model also fits the infrared and part of the enigmatic radio emission spectrum of Sgr A* much better than the ''black hole on starvation'' model [15].

The total mass of the neutrino star M is plotted against the total number of particles N in Fig. 2. For masses much smaller than the OV limit, the relation between M and N is unique. However, as M approaches the OV limit, M becomes a multivalued function of N. The part of the curve on the left side of the maximum in Fig. 1, which corresponds to the upper part of the curve in Fig. 2, represents unstable configurations [31,35] for which the relative mass defect

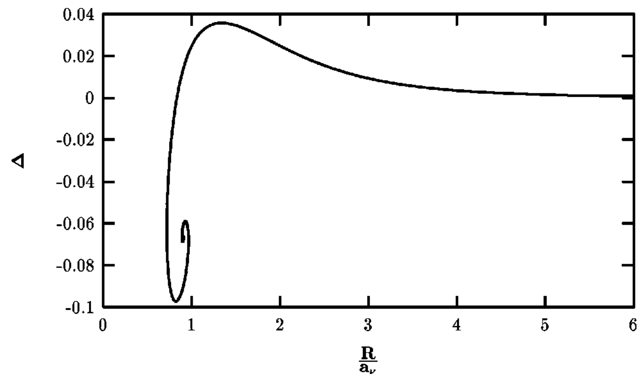

FIG. 5. The relative mass defect Δ as a function of the radius R of the neutrino star. The configurations with $\Delta<0$ are absolutely unstable since the system can gain energy by disintegrating.

$$\Delta = \frac{Nm_\nu - M}{Nm_\nu} \tag{22}$$

eventually becomes negative, as seen in Fig. 5. Thus, for $\Delta < 0$, the system can gain energy by disintegrating. The maximal relative mass defect, or the strongest binding, is obtained at the OV limit with $\Delta_{OV} = 3.5807\times10^{-2}$.

For completeness, we note that in the Newtonian limit $X_0 \ll 1$, the TOV equations (14) and (15) reduce to

$$\frac{dX}{dx} = -\frac{\mu}{x^2 X}, \tag{23}$$

$$\frac{d\mu}{dx} = \frac{8}{3} x^2 X^3, \tag{24}$$

which, using the substitution $\Theta = X^2$ and $\xi = 4x/\sqrt{3}$, can be cast into the nonlinear Lané-Emden differential equation with the polytropic index 3/2 [36]:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\Theta}{d\xi}\right) = -\Theta^{3/2}. \tag{25}$$

Owing to the scaling property of the Lané-Emden equation, the mass and radius of a nonrelativistic neutrino star scale as [17]

$$MR^3 = \frac{91.869\hbar^6}{G^3 m_\nu^8}\left(\frac{2}{g_\nu}\right)^2. \tag{26}$$

In the limit $X_0 \ll 1$, Eqs. (8) and (9) yield the equation of state of a nonrelativistic degenerate Fermi gas, i.e.,

$$P_\nu = \left(\frac{6}{g_\nu}\right)^{2/3} \rho_\nu^{5/3} \frac{\pi^{4/3}\hbar^2}{5m_\nu^{8/3}}, \tag{27}$$

as expected.

For large central densities $X_0 \gg 1$, μ oscillates around $\mu_\infty = 0.09196$, which corresponds to a neutrino star mass $M_\infty = 1.795\times10^9 M_\odot g_\nu^{-1/2}$ for a neutrino mass $m_\nu = 17.2$ keV/c^2. For a gas of neutralinos of a mass precisely equal to the neutron mass $m_n = 0.93955$ GeV/c^2 and a degeneracy factor $g_n=2$, the infinite density limit is $M_\infty = 0.4164 M_\odot$, whereas the OV limit is $M_{\rm OV}=0.7091 M_\odot$ and $R_{\rm OV}=9.1816$ km [35]. Thus, owing to their compactness, neutralino stars could mimic the properties of massive compact halo objects (MACHOs) which have been detected in the line of sight towards the Large Magellanic Cloud (LMC) [37,38]. If these objects are located in our galactic halo (rather than in the LMC), their masses seem to be about $0.4 M_\odot$. Thus we are faced with the dilemma that, on the one hand, they are 5 times too heavy for brown dwarfs and, on the other hand, they cannot be luminous stars, because they would have been easily detected. Therefore, the baryonic matter interpretation of these ''MACHOs'' is disfavored [38]. If one wants to interpret these objects as neutralino stars, the mass of the neutralino m_n is restricted to $4.22\ {\rm MeV}/c^2 \leqslant m_n < 1.251\ {\rm GeV}/rmc^2$, for a degeneracy factor of $g_n=2$. The lower limit is obtained restricting the size of the dark object; i.e., we have somewhat arbitrarily constrained the radius of the ''MACHO'' to $R \leqslant 0.25$ AU which is much smaller than the average Einstein radius of about 3 AU. As 0.25 AU is the distance a star would travel at a speed of $v_\odot = 220$ km/s in approximately 2 days, this would not affect the light curve too much, since the average time scale of the lensing events is about 88 days. The upper limit is obtained assuming that $M_{\rm OV} \approx 0.4 M_\odot$ is at the Oppenheimer-Volkoff limit; i.e., it is almost a black hole.

For large X, the solutions of the TOV equations (14) and (15) tend to

$$\mu = \frac{3}{14} x \quad \text{and} \quad X = \left(\frac{3}{28}\right)^{1/4} x^{-1/2}. \tag{28}$$

The pressure and the density thus become

$$P_\nu = \frac{c^4}{56\pi} \frac{1}{r^2} \quad \text{and} \quad \rho_\nu = \frac{3c^2}{56\pi} \frac{1}{r^2}, \tag{29}$$

yielding the equation of state of radiation

$$P_\nu = \frac{1}{3} c^2 \rho_\nu, \tag{30}$$

as expected.

III. DEGENERATE NEUTRINO AND NEUTRALINO MATTER

We now turn to the discussion of an astrophysical system consisting of degenerate heavy-neutrino and neutralino matter that is gravitationally coupled. As each component satisfies the equation of hydrostatic equilibrium separately, i.e., Eq. (2) and

$$\frac{dP_n}{dr} = -\frac{1}{2}(\rho_n c^2 + P_n) \frac{d\nu}{dr}, \tag{31}$$

the total pressure $P=P_n+P_\nu$ and the total mass density $\rho = \rho_n + \rho_\nu$ will also obey the same equation

$$\frac{dP}{dr} = -\frac{1}{2}(\rho c^2 + P) \frac{d\nu}{dr}. \tag{32}$$

In addition to the equation of state for neutrino matter, Eqs. (8) and (9), we now have the equation of state for neutralino matter:

$$P_n = K \frac{g_n}{g_\nu} \left(\frac{m_n}{m_\nu}\right)^4 \left[Y(1+Y^2)^{1/2} \left(\frac{2}{3} Y^2 - 1\right) + \log[Y + (1+Y^2)^{1/2}] \right], \tag{33}$$

$$\rho_n = \frac{K}{c^2} \frac{g_n}{g_\nu} \left(\frac{m_n}{m_\nu}\right)^4 \{ Y(1+Y^2)^{1/2}(2Y^2+1) - \log[Y + (1+Y^2)^{1/2}] \}, \tag{34}$$

where g_n is the spin-degeneracy factor for neutralinos and antineutralinos, and Y is the local Fermi momentum of neutralino matter (in units of $m_n c$). Inserting Eqs. (33) and (34) into Eq. (31), after integration we arrive at

$$Y = [(1+Y_0^2) e^{\nu(0)-\nu(r)} - 1]^{1/2}, \tag{35}$$

with $Y_0 = Y(0)$. Using Eqs. (8), (9), and the equation of hydrostatic equilibrium Eq. (2), a similar relation for the Fermi momentum of neutrinos (in units of $m_\nu c$) is obtained:

$$X = [(1+X_0^2) e^{\nu(0)-\nu(r)} - 1]^{1/2}. \tag{36}$$

Combining Eqs. (35) and (36), the two local Fermi momenta are related by

$$X^2 = \frac{(X_0^2+1)Y^2 + X_0^2 - Y_0^2}{1+Y_0^2}. \tag{37}$$

The condition $X^2 \geqslant 0$ restricts the range of allowed values of Y to

$$Y^2 \geqslant \frac{Y_0^2 - X_0^2}{1+X_0^2}. \tag{38}$$

The total pressure and mass densities are given by

$$P(Y) = P_n(Y) + P_\nu(X(Y)) \tag{39}$$

and

$$\rho(Y) = \rho_n(Y) + \rho_\nu(X(Y)), \tag{40}$$

respectively.

We now formulate the coupled differential equations describing a gravitationally interacting system of degenerate heavy-neutrino and neutralino matter. We first keep the mass of the neutrino halo constant while varying the mass of the neutralino star. Introducing the dimensionless variables $x = r/a_n$ and $\mu = m/b_n$ with the scales

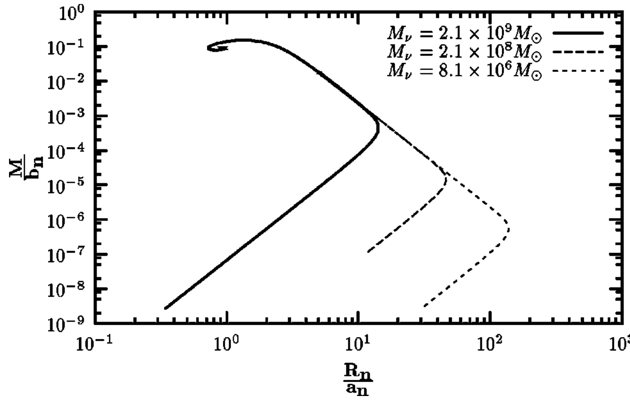

FIG. 6. The total mass (including neutralinos and neutrinos) M enclosed within the radius R_n of the neutralino star for various masses M_ν of the neutrino halo. The maximal radius of the neutralino star decreases with increasing M_ν.

$$a_n = \frac{2}{m_n^2}\sqrt{\frac{\pi\hbar^3}{g_n c G}} \quad \text{and} \quad b_n = \frac{2}{m_n^2}\sqrt{\frac{\pi\hbar^3 c^3}{g_n G^3}}, \tag{41}$$

the relevant TOV equations can be written in the form

$$\frac{dY}{dx} = -\frac{1+Y^2}{Y(x^2-2\mu x)}\Bigg(\mu + x^3\Bigg\{Y(1+Y^2)^{1/2}\left(\frac{2}{3}Y^2-1\right) + \log[Y+(1+Y^2)^{1/2}] + \left(\frac{m_\nu}{m_n}\right)^4 \frac{g_\nu}{g_n}\left[X(1+X^2)^{1/2}\left(\frac{2}{3}X^2-1\right) + \log[X+(1+X^2)^{1/2}]\right]\Bigg\}\Bigg), \tag{42}$$

$$\frac{d\mu}{dx} = x^2\Bigg\{Y(1+Y^2)^{1/2}(2Y^2+1) - \log[Y+(1+Y^2)^{1/2}] + \left(\frac{m_\nu}{m_n}\right)^4 \frac{g_\nu}{g_n}\{X(1+X^2)^{1/2}(2X^2+1) - \log[X+(1+X^2)^{1/2}]\}\Bigg\}, \tag{43}$$

where X is related to Y through Eq. (37). If the condition (38) is not satisfied, i.e., if the neutrino pressure and density have already vanished, the system is solved with the Y terms describing the neutralinos only.

In order to solve Eqs. (42) and (43) numerically, we fix the Fermi momentum of neutrinos (in units of $m_\nu c$) at the center and vary the central values of the corresponding quantity Y_0 for neutralinos. The total mass (including neutrinos and neutralinos) enclosed within the radius R_n of the neutralino star is shown in Fig. 6. Here, the neutrino mass and the degeneracy factor are taken to be $m_\nu = 17.2$ keV/c^2 and $g_\nu = 2$, respectively, while for the neutralino mass we have chosen $m_n = 939.55$ MeV/c^2 and $g_n = 2$, with the scales $a_n = 6.8304$ km and $b_n = 4.6257 M_\odot$. For small neutralino-star masses, the total mass enclosed in R_n scales as R_n^3, corresponding to a constant density governed by the gravitational potential of the surrounding supermassive neutrino halo. However, as the radius of the neutralino star approaches that of a ''free'' neutralino star, the gravitational potential of the neutralino star becomes dominant and the mass now scales as R_n^{-3} up to the OV limit. Thus there is always a maximal radius of a neutralino star within a neutrino halo of a given mass. Substituting neutralinos by neutrons, we must take care of the fact that (i) the neutron interacts strongly in the nuclear medium (simulated, e.g., by an effective mass) and (ii) the neutron decays through weak interactions. Thus, stable neutron stars can exist only in the range from $0.2 M_\odot$ to $2 M_\odot$ [39], where the average binding energy is larger than the Q value for the neutron decay.

It is instructive to study the properties of a degenerate gas of neutralinos and neutrinos in the nonrelativistic approximation. In the limits $X \ll 1$ and $Y \ll 1$, Eqs. (42) and (43) simplify to

$$\frac{dY}{dx} = -\frac{\mu}{x^2 Y}, \tag{44}$$

$$\frac{d\mu}{dx} = \frac{8}{3}x^2\left[Y^3 + \frac{g_\nu}{g_n}\left(\frac{m_\nu}{m_n}\right)^4 (Y^2 + X_0^2 - Y_0^2)^{3/2}\right], \tag{45}$$

with the boundary conditions

$$\mu(0) = 0, \quad Y^2 \geqslant Y_0^2 - X_0^2, \quad Y(0) = Y_0. \tag{46}$$

This system of equations can be rewritten in the form of a Lané-Emden-type equation by introducing $\Theta_n = Y^2$, $\Theta_\nu = X^2$, and a new dimensionless radial variable $\xi = 4x/\sqrt{3}$:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\Theta_n}{d\xi}\right) = -\left[\Theta_n^{3/2} + \frac{g_\nu}{g_n}\left(\frac{m_\nu}{m_n}\right)^4 \times (\Theta_n + \Theta_{\nu 0} - \Theta_{n0})^{3/2}\right], \tag{47}$$

where Θ_{n0} and $\Theta_{\nu 0}$ are the central values of the neutralino and neutrino densities, respectively. For very small neutralino densities, i.e., $Y \ll 1$ and $Y_0 \ll 1$, the mass equation (45) can be integrated to give

$$\mu(x) = \frac{8}{9}\left(\frac{m_\nu}{m_n}\right)^4 X_0^3 x^3, \tag{48}$$

which confirms the conclusion drawn in the context of Fig. 6.

We now turn to the case of a neutralino star of constant mass surrounded by a neutrino halo of variable mass. The TOV equations written in terms of the functions X and μ may be obtained from Eqs. (42) and (43), in which we make the replacements $X \leftrightarrow Y$, $g_\nu \leftrightarrow g_n$, and $m_\nu \leftrightarrow m_n$. Thus, we find

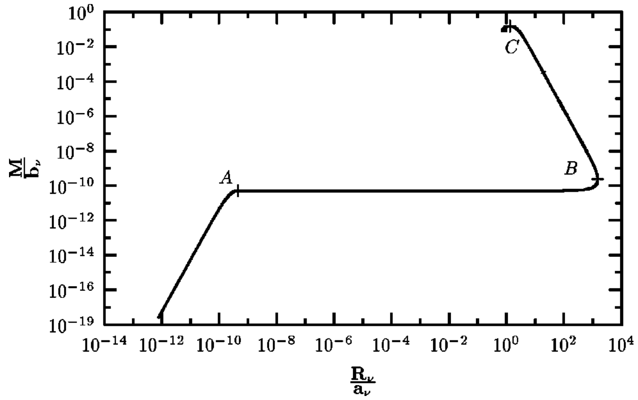

FIG. 7. The total mass of neutralinos and neutrinos M contained within the radius R_ν of the neutrino halo around a neutralino star. The plateau between points A and B reflects the fact that the neutralino part, which is saturated at point A, dominates the total mass up to the turning point B. Here C represents the OV limit.

$$\frac{dX}{dx}=-\frac{1+X^2}{X(x^2-2\mu x)}\left(\mu+x^3\left\{X(1+X^2)^{1/2}\left(\frac{2}{3}X^2-1\right)+\log[X+(1+X^2)^{1/2}]+\left(\frac{m_n}{m_\nu}\right)^4\frac{g_n}{g_\nu}\left[Y(1+Y^2)^{1/2}\left(\frac{2}{3}Y^2-1\right)+\log[Y+(1+Y^2)^{1/2}]\right]\right\}\right), \quad (49)$$

$$\frac{d\mu}{dx}=x^2\left\{X(1+X^2)^{1/2}(2X^2+1)-\log[X+(1+X^2)^{1/2}]+\left(\frac{m_n}{m_\nu}\right)^4\frac{g_n}{g_\nu}\{Y(1+Y^2)^{1/2}(2Y^2+1)-\log[Y+(1+Y^2)^{1/2}]\}\right\}, \quad (50)$$

with X and Y subject to the condition

$$X^2 \geqslant \frac{X_0^2-Y_0^2}{1+Y_0^2}. \quad (51)$$

If this condition is not satisfied, i.e., if the pressure and density of neutralinos have already vanished, Eqs. (49) and (50) are solved without the Y terms, i.e., for neutrinos only. Choosing the OV limit as the mass of the neutralino star, i.e., $M_{\rm OV}^n=0.7091M_\odot$ for $m_n=0.93955$ GeV/c^2 and $g_n=2$, and varying the central Fermi momentum X_0, one can find the total mass (including neutralinos and neutrinos) as a function of the radius R_ν of the neutrino halo. This scenario is reflected in Fig. 7, where the length and mass scales are $a_\nu=2.0381\times10^{10}$ km and $b_\nu=1.3803\times10^{10}M_\odot$, respectively. Here the neutrino mass and the degeneracy factor have been chosen as $m_\nu=17.2$ keV/c^2 and $g_\nu=2$, respectively. At the turning point A, the total mass enclosed within the radius $R_A=R_{\rm OV}^n=9.1816$ km of the neutrino halo is $M_A=M_{\rm OV}^n=0.7091M_\odot$. At the turning point B, the total mass enclosed within the radius $R_B=0.9912$ pc of the neutrino halo is $M_B=3.3453M_\odot$. It is interesting to note that, also in this case, there is a maximal radius R_B of the neutrino halo, for a given mass of the neutralino star.

Replacing the neutralino star by a baryonic star, such as a neutron star, a white dwarf, or an ordinary star, the only thing that will change in Fig. 7 is the point A at which the enclosed mass starts deviating from the constant value, which depends, of course, on the mass M_n of the central object. Thus, for $M_n \gtrsim M_\odot$, the halo will have a size of a few light-years and a mass of a few times that of the central baryonic or nonbaryonic star.

To investigate the consequences of this idea in more detail, let us assume that the Sun is surrounded by a degenerate neutrino halo. In spite of the non-negligible probability that, during its lifetime, the solar system has been visited by an intruder star passing as close as 5000 AU from the Sun, the planetary system has and a possible neutrino halo within 40 AU would have survived such a disruption unharmed, because the Fermi momentum p_F of the degenerate neutrino matter is given by $p_F=m_\nu v_\infty$, where v_∞ is the escape velocity. In the vicinity of the Sun, in the region of the size of the planetary system, the neutrino density is governed by the gravitational potential of the Sun. In fact, the mass due to neutrinos contained within a radius r is, in the vicinity of a baryonic or a nonbaryonic star of mass M_n, given in the nonrelativistic approximation [17,25] by

$$\frac{M_\nu}{M_\odot}=1.34\times10^8 g_\nu\left(\frac{M_n}{M_\odot}\right)^{3/2}\left(\frac{m_\nu c^2}{17.2\ \ \text{keV}}\right)^4\left(\frac{r}{\text{AU}}\right)^{3/2}, \quad (52)$$

where AU$=1.496\times10^8$ km is the astronomical unit. This means that for $m_\nu c^2=17.2$ keV, $g_\nu=2$, and $M_n=M_\odot$, the mass of the neutrino (and antineutrino) halo contained within Earth's orbit would be $M_\nu=2.68\times10^{-8}M_\odot$.

From the Pioneer 10 and 11 and the Voyager 1 and 2 ranging data [40] we know that the dark mass contained within Jupiter's orbit is $M_d=(0.12\pm0.027)\times10^{-6}M_\odot$ and within Neptune's orbit $M_d\leqslant3\times10^{-6}M_\odot$. Of course, the Jupiter data should be taken only as a lower limit, as Jupiter tends to eject almost any matter within its orbit [40]. Nevertheless, taking the Jupiter data at face value, and interpreting dark matter as degenerate neutrino matter, the neutrino mass limits are

$$12.6\ \ \text{keV}/c^2\leqslant m_\nu\leqslant14.2\ \ \text{keV}/c^2 \quad \text{for}\ \ g_\nu=2,$$
$$10.6\ \ \text{keV}/c^2\leqslant m_\nu\leqslant12.0\ \ \text{keV}/c^2 \quad \text{for}\ \ g_\nu=4. \quad (53)$$

For dark matter within Neptune's orbit, the neutrino mass limits are

$$m_\nu\leqslant15.6\ \ \text{keV}/c^2 \quad \text{for}\ \ g_\nu=2,$$
$$m_\nu\leqslant13.1\ \ \text{keV}/c^2 \quad \text{for}\ \ g_\nu=4. \quad (54)$$

In summary, considering Eqs. (20), (21), and (54), a neutrino mass range

$$14.3\ \text{keV}/c^2 \leq m_\nu \leq 15.6\ \text{keV}/c^2 \quad \text{for}\ g_\nu = 2,$$
$$12.0\ \text{keV}/c^2 \leq m_\nu \leq 13.1\ \text{keV}/c^2 \quad \text{for}\ g_\nu = 4, \qquad (55)$$

seems to be consistent with all reliable data.

IV. CONCLUSIONS

We have studied degenerate fermion stars, consisting of massive neutrinos or neutralinos, or both. We have shown that the existence of such objects may have important astrophysical implications.

For neutrino masses in the range of several keV, neutrino stars are natural candidates for the supermassive dark objects at the centers of galaxies. Assuming that the most massive object, such as the compact dark object at the center of M87, is a neutrino star at the OV limit, the neutrino mass required for this scenario should be between 10 keV/c^2 and 16 keV/c^2, depending on the degeneracy factor g_ν.

Furthermore, interpreting the supermassive dark object at the center of our galaxy as a neutrino star, we obtain, from the upper limit of the size of this object, a lower bound on the neutrino mass which overlaps with the range mentioned above. In addition, our interpretation explains the so-called ''blackness problem'' of Sgr A* in a natural way.

By studying a two-component system consisting of neutralinos in the GeV mass range and neutrinos in the keV mass range, we have found that there is always a maximal mass and radius of a neutralino star within a neutrino halo of a given mass. Owing to their compactness, neutralino stars could mimic the properties of ''MACHOs'' for neutralino masses between 4.22 MeV and 1.25 GeV.

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