Charged vacuum bubble stability

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A type of scenario is considered where electrically charged vacuum bubbles, formed from degenerate or nearly degenerate vacua separated by a thin domain wall, are cosmologically produced due to the breaking of a discrete symmetry, with the bubble charge arising from fermions residing within the domain wall. Stability issues associated with wall tension, fermion gas, and Coulombic effects for such configurations are examined. The stability of a bubble depends upon parameters such as the symmetry breaking scale and the fermion coupling. A dominance of either the Fermi gas or the Coulomb contribution may be realized under certain conditions, depending upon parameter values.
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I. INTRODUCTION

Domain wall formation [1–3] can result from the spontaneous breaking of a discrete symmetry, such as a Z_2 symmetry. If, however, the discrete symmetry is *biased* [4], where the formation of one protodomain is favored over that of the other (or if the discrete symmetry is *approximate* [5,6], rather than exact, so that the probabilities of forming domains of different vacua become unequal), then there can result a network of bounded domain wall surfaces. This network of ''*vacuum bubbles*'' will evolve in a way that is dictated by the interactions between the scalar field giving rise to the domain wall and other fields. A vacuum bubble formed from an ordinary domain wall not coupled to other particles or fields undergoes an unchecked collapse due to the tension in the wall. However, if there is a coupling between the domain wall scalar field and one or more fermions, it is possible for the fermions to help stabilize the bubble. For instance, an interaction term $G_F\phi\bar{\psi}\psi$, where ϕ represents the scalar field forming the domain wall, generates a mass m_F for the fermion ψ. If $\phi\rightarrow\pm\eta$, $m_F\rightarrow G_F\eta$ asymptotically outside the wall and $\phi\rightarrow 0$ in the core of the wall, then it becomes energetically favorable for the fermions to populate the core of the wall, with the fermions experiencing an attractive force $\vec{F}\sim -G_F\nabla\phi$. A thin walled vacuum bubble may then feel a force, due to the existence of an effective two-dimensional Fermi gas pressure, that tends to slow or halt the collapse of the bubble. This type of effect plays an essential role in the ''Fermi ball'' model [7], for example, where heavy neutral fermions acquire mass from the domain wall scalar field. The resulting bag configuration in the Fermi ball model can possess a finite equilibrium surface area, but, because the bag is unstable against flattening, the bag can flatten and subsequently fragment into many tiny, thick walled Fermi balls that are supported by the Fermi gas. Fermi balls can serve as candidates for cold dark matter, and such a model can arise quite naturally in supersymmetric theories in response to softly broken supersymmetry [8].

Here, a similar type of model is considered where the fermions populating the bubble wall have an electric U(1) gauge charge, and attention is focused upon the effects of the domain wall tension, the Fermi gas, and the Coulombic forces on the stability of an electrically charged vacuum bubble. In this type of model, the long range Coulombic force tends to stabilize a thin walled bubble against flattening and fragmenting, so that the final equilibrium configuration can consist of a larger, thin walled, charged vacuum bubble instead of the smaller Fermi ball. For a bubble populated by only one species of fermion, the relative importance of the Coulombic and the Fermi gas contributions depends upon the number of fermions in the bubble. Furthermore, as pointed out in Sec. II, the stability of the charged bubble against fermion emission and charge evaporation is found to depend on the strengths of model parameters such as the fermion coupling constant G_F and the symmetry breaking scale η. A bubble that is not stable against charge evaporation, due to the existence of a critical strength electric field, can cause electron-positron pairs to be produced near its surface. Either electrons or positrons are then attracted to the bubble surface to partially or completely neutralize it, with the result that the vacuum bubble ends up supporting two species of fermions, which increases the Fermi gas contribution while decreasing the Coulombic one. Such vacuum bubbles inhabited by two fermion species are examined in Sec. III. Specifically, we look at completely neutralized bubbles and near-critically charged bubbles (with near-critical surface electric fields). Bubble sizes and masses are estimated for limiting cases of special interest, and parameter constraints are estimated. A brief summary of the results is presented in Sec. IV.

II. CHARGED VACUUM BUBBLES WITH A SINGLE FERMION SPECIES

As stated in the introduction, if the mass of an electrically charged fermion is generated by a scalar field, as might be described by the interaction term $G_F\phi\bar{\psi}\psi$, and the scalar field forms domain walls where $\phi=0$ in the core of a wall, then it becomes energetically favorable for the fermion to reside inside the wall where it is massless. A domain wall then acquires electrical charge due to a population of charged fermions. Consider the case where the domain wall arises in response to a broken Z_2 symmetry, with the domain wall interpolating between two distinct, but energetically degenerate, vacuum states. If the discrete Z_2 symmetry is biased

[4], so that the probabilities of forming domains of different vacua become unequal, then there can result a network of bounded domain wall surfaces which may evolve to give rise to stable or metastable charged vacuum bubbles. (We could also consider the case where the Z_2 symmetry is approximate, with the difference in vacuum energy densities being sufficiently small that it can be safely ignored.) Let us focus on a single spherical domain wall bubble that encloses vacuum (say, $\phi = \mp \eta$) and is surrounded by vacuum (say, $\phi = \pm \eta$). In the thin wall approximation, the bubble can be considered as a two dimensional surface populated by massless, electrically charged fermions. The bubble is then considered to carry a uniform charge $q = Ne$ (but no spatial electric current). The configuration energy $\mathcal{E}$ of the bubble receives contributions from the surface energy Σ of the wall, the two dimensional Fermi gas energy $\mathcal{E}_F$, and the Coulomb self energy $\mathcal{E}_C$. The new feature in this model is the inclusion of a long ranged U(1) gauge field which can help to stabilize the bubble against collapse and fragmentation, as takes place in the electrically neutral Fermi ball model, for instance. A model of the type under consideration here might be described by a Lagrangian of the form

$$L = \frac{1}{2}(\partial\phi)^2 + \overline{\psi}(i\gamma\cdot D - G_F\phi)\psi - \frac{1}{4}(F^{\mu\nu})^2 - \frac{\lambda^2}{2}(\phi^2 - \eta^2)^2. \tag{1}$$

A planar domain wall solution is given by $\phi(x) = \eta\,\tanh(x/w)$, where $w = 1/(\lambda\eta)$ is the thickness, or width, of the wall. The wall has a surface energy of $\Sigma = \frac{4}{3}\lambda\eta^3$. We consider a spherical bubble of domain wall inhabited by $N \gg 1$ fermions, each of charge e, which, in the thin wall approximation, has a radius $R \gg w$, i.e. $\lambda\eta R \gg 1$. (For simplicity, we shall normally take $\lambda \sim 1$.) The configuration energy of the bubble is

$$\mathcal{E} = \mathcal{E}_W + \mathcal{E}_F + \mathcal{E}_C, \tag{2}$$

where $\mathcal{E}_W = \Sigma S$ is the energy contribution from the domain wall, with $S = 4\pi R^2$ the surface area,

$$\mathcal{E}_F = \frac{4\sqrt{\pi}N^{3/2}}{3\sqrt{g}S^{1/2}} \tag{3}$$

is the energy contribution from the two dimensional Fermi gas [7], with $g = 2$ being the number of spin degrees of freedom for a spin $\frac{1}{2}$ fermion, and

$$\mathcal{E}_C = 2\pi\sigma^2 R^3 = \frac{q^2}{8\pi R} = \frac{N^2e^2}{8\pi R} = \sqrt{\pi}\alpha\frac{N^2}{S^{1/2}} \tag{4}$$

is the Coulomb energy for the bubble with surface charge density $\sigma = q/(4\pi R^2)$ and $\alpha = e^2/4\pi$. (As in the Fermi ball model, we also make the assumption that a fermion-antifermion asymmetry exists so that fermions within a bubble wall do not all annihilate away.) The bubble mass

$$\mathcal{E} = \Sigma S + \left[\frac{4\sqrt{\pi}N^{3/2}}{3\sqrt{g}} + \alpha\sqrt{\pi}N^2\right]S^{-1/2} \tag{5}$$

can be minimized with a surface area of

$$S = 4\pi R^2 = \left\{\frac{1}{2\Sigma}\left[\frac{4\sqrt{\pi}N^{3/2}}{3\sqrt{g}} + \alpha\sqrt{\pi}N^2\right]\right\}^{2/3}, \tag{6}$$

corresponding to an equilibrium radius of

$$R = \left(\frac{1}{4\pi}\right)^{1/2}\left\{\frac{1}{2\Sigma}\left[\frac{4\sqrt{\pi}N^{3/2}}{3\sqrt{g}} + \alpha\sqrt{\pi}N^2\right]\right\}^{1/3}. \tag{7}$$

Note that the surface area independent ratio of the Coulomb energy to Fermi gas energy is

$$\frac{\mathcal{E}_C}{\mathcal{E}_F} = \frac{3}{4}\sqrt{g}\,\alpha N^{1/2} \approx \alpha N^{1/2}. \tag{8}$$

From Eq. (6) we can write the surface area of the bubble as

$$S = \frac{1}{2\Sigma}\left(\frac{4\sqrt{\pi}N^{3/2}}{3\sqrt{g}S^{1/2}} + \frac{\alpha\sqrt{\pi}N^2}{S^{1/2}}\right) = \frac{1}{2\Sigma}(\mathcal{E}_F + \mathcal{E}_C). \tag{9}$$

The configuration energy is $\mathcal{E} = \Sigma S + \mathcal{E}_F + \mathcal{E}_C$, so that by Eqs. (5) and (9),

$$\mathcal{E} = \Sigma\left[\frac{1}{2\Sigma}(\mathcal{E}_F + \mathcal{E}_C)\right] + \mathcal{E}_F + \mathcal{E}_C = \frac{3}{2}(\mathcal{E}_F + \mathcal{E}_C). \tag{10}$$

[Also, by Eq. (9) $\mathcal{E}_F + \mathcal{E}_C = 2\Sigma S = 2\mathcal{E}_W$, so that $\mathcal{E} = \mathcal{E}_W + \mathcal{E}_F + \mathcal{E}_C = 3\mathcal{E}_W$.]

A. Limiting cases

We can consider the limiting cases where either the Fermi gas contribution dominates the Coulomb contribution, or vice versa, the Coulomb energy dominates the Fermi gas energy. Let these cases be referred to as ''Fermi gas dominance'' and ''Coulomb dominance,'' respectively. For Fermi gas dominance $\mathcal{E}_C/\mathcal{E}_F \ll 1$, which by Eq. (8) implies that $\alpha N^{1/2} \ll 1$, whereas for Coulomb dominance $\mathcal{E}_C/\mathcal{E}_F \gg 1$, which implies that $\alpha N^{1/2} \gg 1$. Therefore a stable bubble with a sufficiently *small* number of fermions will be Fermi gas dominated with a mass $\mathcal{E} \approx \frac{3}{2}\mathcal{E}_F$, while one with a sufficiently *large* number of fermions will be Coulomb dominated with $\mathcal{E} \approx \frac{3}{2}\mathcal{E}_C$.

In order for the thin wall approximation to be respected, we require that $R/w \gg 1$, where $w = 1/(\lambda\eta)$, and we assume for simplicity that λ is of order unity. Therefore, in the thin wall approximation, the equilibrium radius of the bubble must satisfy $R\eta \gg 1$. Let us take $R\eta \gtrsim \alpha^{-1}$, so that from Eq. (7) we have (dropping factors of order unity),

$$R\eta \sim \left(\frac{1}{4\pi}\right)^{1/2} N^{1/2}[1 + \alpha N^{1/2}]^{1/3} \gtrsim \alpha^{-1}, \tag{11}$$

from which we conclude that, roughly, $\alpha N^{1/2} \gtrsim O(1)$. Therefore, Fermi gas dominance is *not* realized for a stable, spherical, thin walled bubble with a single species of electrically charged fermion trapped within the wall, and Coulombic effects are therefore necessarily nonnegligible. However, in what follows we can consider two limits: (1) $\alpha N^{1/2} \sim 1$, in which case the Fermi gas and Coulomb energy contributions are of comparable magnitude, and (2) $\alpha N^{1/2} \gg 1$, in which case the bubble is Coulomb dominated.

Although we have assumed that a mechanical equilibrium has been established for the bubble, we must check for electrodynamic stability against charge evaporation and also for stability against fermion emission from the wall, i.e., the attractive force pulling a fermion into the wall must be larger than the forces that would otherwise squeeze the fermion out of the wall (e.g., the tension in the domain wall, the Coulomb force, and the Fermi gas pressure) allowing the bubble to contract to a smaller radius.

B. Stability against fermion emission

Consider a static bubble with fermion number $N \gg 1$ and mass $\mathcal{E}^{(N)}$. For it to be stable against releasing a fermion with mass m_F in the vacuum, we require that $\mathcal{E}^{(N+1)} - \mathcal{E}^{(N)} = \delta\mathcal{E} < m_F$. In the case of Coulomb dominance, the N-dependent energy contributions in $\mathcal{E}$ are power functions of N, and we have, approximately, for $N \gg 1$, $\delta\mathcal{E} \sim (\partial\mathcal{E}/\partial N)\delta N$. Therefore, for the Coulomb dominated bubble, from Eqs. (9) and (10) we have $\mathcal{E} \sim \mathcal{E}_C \sim \alpha N^2/S^{1/2} \sim \alpha N^2(\Sigma^{1/2}/\mathcal{E}_C^{1/2})$ which implies that $\mathcal{E}_C \sim \alpha^{2/3} N^{4/3} \Sigma^{1/3}$, so that $\partial\mathcal{E}_C/\partial N \sim \mathcal{E}_C/N$. Setting $\delta N = 1$ we then have that $\delta\mathcal{E} < m_F$ implies that, roughly,

$$\mathcal{E} < N m_F \tag{12}$$

for stability against fermion emission. The fermion mass (in vacuum) is $m_F = G_F \eta$, the domain wall surface energy is $\Sigma \sim \eta^3$, and for the Coulomb dominated bubble $\mathcal{E} \sim \alpha^{2/3} N^{4/3} \eta$, so that Eq. (12) implies that

$$(\alpha^2 N)^{1/3} < G_F = \left(\frac{m_F}{\eta}\right). \tag{13}$$

Therefore, for the Coulomb dominance condition $\alpha^2 N \gg 1$ and Eq. (13) to be simultaneously satisfied, we require

$$1 \ll \alpha^2 N < G_F^3. \tag{14}$$

The condition given by Eq. (14) can be met for a sufficiently large fermion-scalar coupling G_F, but for a weaker coupling with G_F of order unity or smaller, a Coulomb dominated bubble apparently will not stabilize when the wall is inhabited by a single species of charged fermion.

For the Fermi gas-Coulomb balance case $\alpha^2 N \sim 1$, we have $\mathcal{E}_C/\mathcal{E}_F \sim 1$ and by Eq. (6) $S \sim (\alpha\eta)^{-2}$, so that by Eqs. (4) and (10) $\mathcal{E} = \frac{3}{2}(\mathcal{E}_F + \mathcal{E}_C) \sim 3\mathcal{E}_C \sim \alpha^{-2}\eta$. Then, since $\alpha^{-2} \sim N$, we get $\mathcal{E}^{(N)} \sim N\eta$ and $\delta\mathcal{E} \sim \eta$. In this case the bubble is stable against fermion emission if

$$\eta < m_F \Rightarrow G_F > 1. \tag{15}$$

Thus, the requirement imposed upon G_F by stability against fermion emission seems to be a little more relaxed for the Fermi gas-Coulomb balance case $\alpha N^{1/2} \gtrsim 1$ than for the Coulomb dominance case $\alpha N^{1/2} \gg 1$.

C. Stability against charge evaporation

Although the bubble may be stable against fermion emission, it may not be stable against charge evaporation. Consider, for example, a region where the electric field strength is large enough to separate a virtual electron-positron pair by a distance $r \sim 2m^{-1}$, where m is the electron mass, and bring the particles onto the mass shell. The work done by the external electric field eEr will be $\gtrsim 2m$ for a field strength $E \gtrsim m^2/e$, allowing the field to create $e^{\pm}$ pairs from the vacuum. Therefore, if the electric field just outside the charged bubble is above a critical value of $E_c \sim m^2/e$, where m is the mass of a charged particle, then the pair creation of charged particles becomes probable. For a subcritical field strength $E < E_c$, pair creation from the vacuum is suppressed. Taking m to be the electron mass, we see that if the electric field at the bubble surface becomes supercritical, i.e., $E > E_c$, then electron-positron pairs are created from the vacuum, and a positively charged bubble attracts electrons and repels positrons. The electrons partially neutralize the bubble charge, thereby reducing the field until it reaches a subcritical value $E < E_c$, so that the initial bubble charge effectively evaporates through positron emission.

The electric field at the bubble's surface is $E = \sigma = Ne/S$, which is to be compared to the critical field strength $E_c \sim m^2/e$. Now, for the case of a Coulomb dominated bubble ($\alpha N^{1/2} \gg 1$) at equilibrium, the surface area, from Eq. (6), is $S \sim \alpha^{2/3} N^{4/3}/\Sigma^{2/3} \sim \alpha^{2/3} N^{4/3}/\eta^2$, so that

$$\frac{E}{E_c} \sim \frac{Ne/S}{m^2/e} = \frac{\alpha N}{4\pi m^2 S} \sim \frac{\alpha}{4\pi}\left(\frac{1}{\alpha^2 N}\right)^{1/3}\left(\frac{\eta}{m}\right)^2. \tag{16}$$

The field of a Coulomb dominated bubble will be subcritical for $(\alpha^2 N)^{1/3} > (\alpha/4\pi)(\eta/m)^2$, so that, in the case of Coulomb dominance,

$$E < E_c \Rightarrow \left(\frac{\eta}{m}\right)^2 < \left(\frac{4\pi}{\alpha}\right)(\alpha^2 N)^{1/3}. \tag{17}$$

Combining this condition with that of Eq. (14) then implies that

$$\left(\frac{\eta}{m}\right)^2 < \left(\frac{4\pi}{\alpha}\right)(\alpha^2 N)^{1/3} < \left(\frac{4\pi}{\alpha}\right) G_F, \tag{18}$$

which, for a given value of G_F, places an upper limit on the symmetry breaking energy scale η.

On the other hand, for the Fermi gas-Coulomb balance case $\alpha^2 N \sim 1$,

$$\frac{E}{E_c} \sim \frac{e\eta^2}{m^2/e} = 4\pi\alpha\left(\frac{\eta}{m}\right)^2, \tag{19}$$

and for the field to be subcritical,

$$E<E_c\Rightarrow\left(\frac{\eta}{m}\right)^2<\frac{1}{4\pi\alpha}. \tag{20}$$

In either case, unless the Z_2 symmetry breaking energy scale is sufficiently close to the electron mass so that either Eq. (18) or Eq. (20) can be satisfied, a bubble containing only one species of positively (negatively) charged fermion will undergo charge evaporation by positron (electron) emission, reducing the net bubble charge to a value that renders the external electric field subcritical. During such a process the bubble will also change its lepton number. The stable bubble will therefore be inhabited by *two* species of charged fermions, each contributing a Fermi gas energy term, but the Coulomb energy term will be lowered.

In summary, a charged, thin walled, stable vacuum bubble populated with a single charged fermion species will not be Fermi gas dominated, but a thin walled bubble may stabilize if it is either Coulomb dominated or if there is a Fermi gas–Coulomb balance. However, stability against fermion emission and charge evaporation requires that (i) there be a sufficiently large fermion coupling G_F and (ii) that the symmetry breaking energy scale η be sufficiently small. This last condition may be easily violated for a value of η on the order of the electroweak scale or higher, in which case we expect stable charged vacuum bubbles to be populated with at least two species of fermions, if stable bubbles exist at all.

III. VACUUM BUBBLES WITH TWO FERMION SPECIES—NEUTRAL AND NEAR-CRITICAL CHARGED BUBBLES

When the conditions for a subcritical electric field at a bubble's surface cannot be reached at equilibrium, so that electric field at the surface of a charged bubble reaches a critical value $\sigma_{crit}\sim m^2/e$, where m is the electron mass, then electron-positron pairs are produced resulting in charge evaporation through electron absorption and positron emission, until the electric field at the bubble surface drops slightly below its critical value. (For definiteness, we take the charge of the heavy fermion attached to the domain wall to be $+e$.) Since the pair production is strongly suppressed for $\sigma<\sigma_{crit}$, we expect the bubble to equilibrate by adjusting its radius, keeping its surface charge density slightly subcritical. Therefore, for a bubble that has not been completely neutralized, we take the surface charge density to be approximately constant during the equilibration process, $\sigma\sim\sigma_{crit}\sim m^2/e$. Of course, if the bubble is completely neutralized, $\sigma=0$, in which case there are as many electrons in the bubble wall as there are heavy positively charged fermions, i.e. $N_e=N_F$, where N_e is the number of electrons in the bubble and N_F is the number of heavy fermions. For $N_e<N_F$, we have $\sigma=(N_F-N_e)e/4\pi R^2\sim\sigma_{crit}$ which implies that the electron number

$$N_e\sim N_F-\frac{4\pi\sigma_{crit}}{e}R^2\sim N_F-\frac{m^2}{\alpha}R^2 \tag{21}$$

varies with the bubble radius R.

For a bubble that is microscopic in size, we expect the electron Fermi gas to be relativistic. For this type of bubble containing two species of charged fermion, the Fermi gas energy increases and the Coulomb energy decreases as N_e increases for a given value of R. In the Fermi gas energy term of Eq. (3) we have $N^{3/2}\to N_F^{3/2}+N_e^{3/2}$, while in the Coulomb energy term of Eq. (4) we have $N\to N_F-N_e$.

A. Neutral bubbles

For the case that the stabilized bubble has been completely neutralized, i.e. $N_e=N_F$, the Coulomb energy term vanishes, so that the configuration energy of a spherical bubble is

$$\mathcal{E}=\mathcal{E}_W+\mathcal{E}_F=4\pi\Sigma R^2+\frac{4N_F^{3/2}}{3\sqrt{g}R} \tag{22}$$

giving an equilibrium radius of

$$R=\left(\frac{1}{6\pi\sqrt{g}\Sigma}\right)^{1/3}N_F^{1/2}\sim\left(\frac{1}{6\pi\sqrt{g}}\right)^{1/3}\frac{N_F^{1/2}}{\eta} \tag{23}$$

and a bubble mass

$$\mathcal{E}\sim 4\pi N_F\eta. \tag{24}$$

For a thin-walled bubble ($\eta R\gg 1$) Eq. (23) implies that $N_F\gg 1$.

As in the case of a single fermion bubble, we can examine the conditions under which the neutral bubble will be stable against emission of heavy fermions. We again require (assuming that $N_F\gg 1$) that $\mathcal{E}^{(N+1)}-\mathcal{E}^{(N)}=\delta\mathcal{E}\sim\partial\mathcal{E}/\partial N_F<m_F=G_F\eta$. For the neutral bubble, $\mathcal{E}\sim 4\pi N_F\eta$, so that for the bubble to be stable against heavy fermion emission we must have $G_F\gtrsim 4\pi$. For a fermion coupling much smaller than this, it becomes energetically favorable for the heavy fermions to be expelled from the bubble as it collapses, thus preventing stabilization.

However, although the neutral bubble can stabilize with a finite surface area, as in the case with uncharged false vacuum bags the bubble is not stable against flattening, so that as in the Fermi ball scenario, the bubble can ultimately fragment into many small Fermi balls, each with a radius of roughly $R_0\sim\eta^{-1}$ at which point the fragmentation process stops. (If there were a false vacuum volume energy term $\mathcal{E}_V\sim\Lambda V$ due to a slight breaking of the vacuum degeneracy, as is the case in the original Fermi ball model, the tendency to fragment would be enhanced.) These massive, neutral Fermi balls could serve as candidates for cold dark matter.

Finally, let us note that a Fermi ball sized bubble with radius $R_0\sim\eta^{-1}$ cannot be electrically charged if (i) $\eta/m\gg 1$ and (ii) $\alpha(N_F-N_e)\sim O(1)$ (as in the original Fermi ball model), since the surface electric field would be supercritical in that case. This can be seen by writing $R^2=(N_F-N_e)e/(4\pi\sigma)$ and noting that for $\sigma\lesssim\sigma_{crit}\sim m^2/e$, the minimum radius the bubble with a subcritical electric field can have is $R_{\min}\sim[(N_F-N_e)\alpha]^{1/2}/m$. Thus, $R_{\min}/R_0\sim[(N_F-N_e)\alpha]^{1/2}(\eta/m)$ is not near unity and, consequently, we have that $R_{\min} \gg R_0$ if we allow $\eta/m \gg 1$ while keeping $\alpha(N_F - N_e)$ roughly to an order of unity. On the other hand, for $\alpha(N_F - N_e) \sim (m/\eta)^2$, we could have $R_{\min} \sim R_0$, but such a bubble would have a charge number $(N_F - N_e) \sim (1/\alpha)(m/\eta)^2 < 1$ for $(\eta/m) \gtrsim 1/\sqrt{\alpha}$, i.e., the bubble would have to be effectively neutral for $(\eta/m) \gg 1$.

B. Near-critical charged bubbles

For the case $N_F - N_e > 0$, let us assume that the surface charge density is near-critical, i.e., $\sigma \approx \sigma_{crit} \sim m^2/e$. Since $0 < N_e < N_F$, we take the Fermi gas energy to be

$$\mathcal{E}_F = \frac{2}{3\sqrt{g}R}(N_F^{3/2} + N_e^{3/2}) \sim \frac{N_F^{3/2}}{R}. \tag{25}$$

The Coulomb energy is

$$\mathcal{E} = 2\pi\sigma^2 R^3 \sim \frac{m^4 R^3}{\alpha}, \tag{26}$$

where $\alpha = e^2/4\pi$ and m is the electron mass. For a spherical bubble stabilized at a radius R, we have

$$\frac{\mathcal{E}_C}{\mathcal{E}_F} \sim \frac{(mR)^4}{\alpha N_F^{3/2}}, \tag{27}$$

which can be rewritten as

$$R^2 \sim \frac{\sqrt{\alpha} N_F^{3/4}}{m^2}\left(\frac{\mathcal{E}_C}{\mathcal{E}_F}\right)^{1/2}. \tag{28}$$

Several limiting cases can be considered.

(1) *Fermi gas dominance*

$$\frac{\mathcal{E}_C}{\mathcal{E}_F} \ll 1 \Rightarrow R^2 \ll \frac{\sqrt{\alpha} N_F^{3/4}}{m^2}. \tag{29}$$

(2) *Coulomb dominance*

$$\frac{\mathcal{E}_C}{\mathcal{E}_F} \gg 1 \Rightarrow R^2 \gg \frac{\sqrt{\alpha} N_F^{3/4}}{m^2}. \tag{30}$$

(3) *Fermi gas–Coulomb balance*

$$\frac{\mathcal{E}_C}{\mathcal{E}_F} \sim 1 \Rightarrow R^2 \sim \frac{\sqrt{\alpha} N_F^{3/4}}{m^2}. \tag{31}$$

We write the total configuration energy of the bubble *at equilibrium* (dropping factors of order unity) as

$$\mathcal{E} = \mathcal{E}_W + \mathcal{E}_F + \mathcal{E}_C \sim 4\pi\Sigma R^2 + \frac{N_F^{3/2}}{R} + \frac{m^4 R^3}{\alpha}. \tag{32}$$

The equilibrium radius of the bubble is determined by a balance of the radial forces acting on the bubble wall. Caution must be taken here to not simply minimize $\mathcal{E}$ in Eq. (32), which would give a radially inward Coulombic force, but rather to determine the force by considering a *virtual* displacement of the bubble wall, holding the charge Q on the wall fixed. (The charge of the bubble will vary with R, and hence time, in general, as the bubble changes its radius during the physical equilibration process. We can view $Q \approx 4\pi R^2 \sigma_{crit}$ as a constraint on the charge Q. We find the radial force at an instant by holding Q fixed, and considering how the energy of the configuration varies with R at this instant. The final result for the force at this instant is then obtained by using the constraint $Q \approx 4\pi R^2 \sigma_{crit}$.) For a *virtual* change in the bubble's radius, holding the bubble charge Q fixed at any instant of time, we have $\mathcal{E}_C = Q^2/(8\pi R)$, and a virtual change in energy due to a change in the radius alone is $\delta\mathcal{E}_C = -Q^2/(8\pi R^2)\delta R$, allowing us to identify the radial electrostatic force by $F_R = -\delta\mathcal{E}/\delta R = Q^2/(8\pi R^2)$. Inserting the charge $Q \approx 4\pi R^2 \sigma_{crit}$ into the expression for F_R gives

$$F_R \sim \frac{m^4 R^2}{2\alpha}, \tag{33}$$

which is a radially outward force tending to stabilize the bubble against contraction. The total radial force on the bubble at equilibrium is given by

$$-F_R^{Tot} \sim 8\pi\Sigma R - \frac{N_F^{3/2}}{R^2} - \frac{m^4 R^2}{2\alpha} \approx 0. \tag{34}$$

Taking $\Sigma \sim \eta^3$, we have

$$8\pi\eta^3 R^3 - N_F^{3/2} - \frac{m^4 R^4}{2\alpha} \approx 0. \tag{35}$$

Each of the limiting cases can be examined separately.

(1) *Fermi gas dominance*

In this case the Fermi gas contribution to the force and configuration energy is assumed to be much larger in magnitude than the Coulomb contribution. The equilibrium radius is determined by $8\pi\eta^3 R^3 \sim N_F^{3/2}$, giving an equilibrium bubble radius

$$R \sim \frac{N_F^{1/2}}{\eta}. \tag{36}$$

This is compatible with the condition given by Eq. (29) provided that $N_F^{1/4} \ll \eta^2/m^2$. For a thin walled bubble $(\eta R \gg 1)$, we therefore require

$$1 \ll N_F \ll \left(\frac{\eta}{m}\right)^8. \tag{37}$$

From Eq. (21) we find the electron number for the bubble to be $N_e \sim N_F[1 - \alpha(m^2/\eta^2)]$, which for $\alpha(m/\eta) \ll 1$ gives $N_e \sim N_F$. The bubble mass is given by $\mathcal{E}/\eta \sim N_F$. For the bubble to be stable against heavy fermion emission, we require that $G_F \gtrsim 1$.

(2) *Coulomb dominance*

The bubble radius obtained from $8\pi\eta^3R^3 - m^4R^4/(2\alpha) \approx 0$ is

$$R \sim \frac{\eta^3}{m^4}. \tag{38}$$

This is compatible with Eq. (30) if $N_F^{1/4} \ll \eta^2/m^2$, and the bubble respects the thin wall approximation if $\eta R \sim (\eta/m)^4 \gg 1$. The bubble mass $\mathcal{E} \sim \mathcal{E}_W + \mathcal{E}_C$ is roughly given by $\mathcal{E}/\eta \sim (\eta/m)^8$. This mass expression is independent of N_F, since the Fermi gas term has been neglected, but from Eq. (32) we have that $\delta\mathcal{E} \sim \partial\mathcal{E}/\partial N_F \sim N_F^{1/2}/R \sim (m^4/\eta^3)N_F^{1/2}$, so that $\delta\mathcal{E}/\eta < G_F$, i.e., for the bubble to be stable against heavy fermion emission, we have $G_F \gtrsim N_F^{1/2}(m/\eta)^4$. Since $N_F^{1/2} \ll (\eta/m)^4$, the constraint on G_F can be satisfied for $G_F \gtrsim 1$.

(3) *Fermi gas-Coulomb balance*

We consider the Fermi gas force to be comparable to the Coulomb force in this case, and therefore require $8\pi\eta^3R^3 \sim N_F^{3/2}$ and $N_F^{3/2} \sim m^4R^4/(2\alpha)$. The bubble radius is roughly

$$R \sim \frac{\eta^3}{m^4}, \tag{39}$$

and $N_F \sim (\eta/m)^8$. The bubble mass is roughly given by $\mathcal{E}/\eta \sim N_F \sim (\eta/m)^8$. For stability against heavy fermion emission, $G_F \gtrsim 1$.

C. Black hole formation

For a vacuum bubble to stabilize before forming a black hole, we require that the stabilization radius R be larger than the radius R_H of the outer horizon of the corresponding black hole state. For a neutral, nonrotating black hole $R_H = 2G\mathcal{E}$, where $\mathcal{E}$ is the black hole mass, and for an extreme Reissner-Nordstrom black hole (with charge $Q = \sqrt{G}\mathcal{E}$) the outer horizon is located by $R_H = G\mathcal{E}$, so that for the case of nonextemal or extremal nonrotating black holes we have, roughly, $R_H \sim G\mathcal{E} = \mathcal{E}/M_P^2$, where $M_P = (G)^{-1/2}$ is the Planck mass. For the bubble to stabilize with a radius R and avoid the formation of a gravitationally collapsed black hole state we therefore require that the equilibrium radius R be larger than $R_H \sim \mathcal{E}/M_P^2$.

For the case of a Fermi gas dominated bubble,

$$\frac{R}{R_H} \sim \left(\frac{M_P}{\eta}\right)^2 \frac{1}{N_F^{1/2}}, \tag{40}$$

so that for this bubble to avoid black hole formation, the number of heavy fermions must be smaller than $N_{F,\max} \sim (M_P/\eta)^4$. The maximum size and mass of such a bubble would then be, respectively,

$$R_{\max} \sim \frac{N_{F,\max}^{1/2}}{\eta} \sim \left(\frac{M_P}{\eta}\right)^2 \frac{1}{\eta}, \tag{41}$$

$$\mathcal{E}_{\max} \sim N_{F,\max}\eta \sim \left(\frac{M_P}{\eta}\right)^4 \eta. \tag{42}$$

[For a bubble that stabilizes at a grand unified theory (GUT) scale value $\eta \sim 10^{16}$ GeV, for example, we have a maximum bubble radius $R_{\max} \sim 10^{-10}$ GeV^{-1} and a maximum mass $\mathcal{E}_{\max} \sim 10^{28}$ GeV~ 10 kg. On the other hand, for $\eta \sim 10^4$ GeV, for example, $R_{\max} \sim 10^{26}$ GeV$^{-1} \sim 10^{10}$ m and $\mathcal{E}_{\max} \sim 10^{64}$ GeV, describing a very massive compact astrophysical object with a mass of roughly 10 million solar masses and a radius of roughly 100 solar radii.] Bubbles larger than that allowed by Eq. (41) would evidently form black hole states, since the stabilization radius would lie inside the horizon. Similar results hold for the neutral vacuum bubble. Also notice that for the Fermi gas dominated (charged) bubble, by Eq. (37) we have $N_{F,\max} \ll (\eta/m)^8$, which implies that

$$\eta > (M_P m^2)^{1/3} \sim 10^4 \text{ GeV}. \tag{43}$$

Therefore, a Fermi gas dominated charged bubble can evidently reach a stable equilibrium only for a value of the symmetry breaking scale in excess of roughly 10^4 GeV. We conclude that such a bubble formed at the electroweak scale $\eta \sim 10^2$ GeV would necessarily collapse to a black hole.

Here, it is interesting to note that for a sufficiently small value of η (e.g., $\eta \sim 10^4$ GeV), the Fermi gas dominated bubbles described above can have sizes and masses that become comparable to those of *neutrino balls* [9], which are particular examples of *cosmic balloons* [10]. (The spherical domain wall of a cosmic balloon entraps fermions within its volume that become heavy outside the balloon.) However, a fundamental difference between a bubble and a neutrino ball (NB) is that the domain wall of the neutrino ball is essentially transparent to all matter and radiation, except for neutrinos, whereas the domain wall of the bubble is not transparent, as it is inhabited by charged fermions. Stars, gravitationally attracted to the neutrino ball, can simply drift through the NB domain wall [11]. Inside the NB, there will be a frictional force exerted on a star by the ambient neutrinos, so that eventually the star will tend to reside at or near the center of the NB. The ambient neutrino gas can speed the star's evolution, enhancing its probability rate to undergo a supernova type of explosion. Holdom and Malaney [11] have proposed a mechanism wherein the neutrino emissions from such explosions within NBs can be converted into intense gamma ray bursts. This mechanism, however, would not apply to a vacuum bubble, since the domain wall of the bubble is not invisible to matter and radiation, and hence stars can not simply drift through the domain wall of the bubble. Furthermore, we have considered the case where vacuum, rather than fermionic gas, occupies the bubble's interior.

For the case of either a Coulomb dominated bubble or a Fermi gas–Coulomb balanced bubble, we have $R \sim (\eta/m)^3(1/m)$ and $\mathcal{E} \sim (\eta/m)^8\eta$, so that

$$\frac{R}{R_H} \sim \left(\frac{m}{\eta}\right)^4 \left(\frac{M_P}{\eta}\right)^2. \tag{44}$$

Therefore, for one of these bubbles to avoid black hole formation we must have $R > R_H$ which implies that

$$\eta \lesssim (M_P m^2)^{1/3} \sim 10^4 \text{ GeV}. \tag{45}$$

For sufficiently small symmetry breaking scales (e.g., the electroweak scale, $\eta \sim 10^2$ GeV), black hole formation is avoided, but for values of η much greater than that of Eq. (45) (e.g. the GUT scale, $\eta \sim 10^{16}$ GeV), black hole formation evidently cannot be avoided. (For a bubble that stabilizes at an electroweak scale value $\eta \sim 10^2$ GeV, for example, we have a bubble radius and mass of $R \sim 10^{18}\,\text{GeV}^{-1} \sim 10^2$ m and $\mathcal{E} \sim 10^{42}\,\text{GeV} \sim 10^{15}$ kg, respectively.) Therefore, whether or not a particular type of charged bubble can eventually stabilize depends upon whether η is above or below a value of roughly 10 TeV.

IV. SUMMARY AND CONCLUSIONS

If either a *biased*, exact discrete symmetry or an *approximate* discrete symmetry is spontaneously broken, a network of bounded domain wall surfaces giving rise to ''vacuum bubbles'' may result. The dynamical evolution of a bubble will depend upon what other fields couple to the scalar field forming the domain wall. If fermions couple to the scalar field in such a way that it becomes favorable for the fermions to reside within the bubble wall, the resulting degenerate Fermi gas can help to stabilize the bubble against an unchecked collapse. A scenario of this type incorporating electrically neutral fermions plays an essential role in the Fermi ball model [7], for example, where the domain wall forms a thin skin enclosing a false vacuum. For a sufficiently strong fermion coupling to the scalar field, the fermions remain within the wall and allows the resulting bag-like configuration to equilibrate with a finite nonzero surface area. In turn, this vacuum bag can flatten and fragment, resulting in the production of many smaller ''Fermi balls.'' Thus, the cosmological domain wall problem can be evaded through the formation of a bubble network, and ultimately, Fermi balls. Here, attention has been focused upon an extension of this type of scenario, where the fermions are assumed to have an electric U(1) gauge charge, introducing nontrivial electromagnetic effects that must be taken into consideration when examining the stability of a *charged* vacuum bubble. It has been demonstrated that the Coulombic effects *cannot* be considered negligible in comparison to the Fermi gas effects for the case of a thin walled bubble inhabited by a single species of charged fermion.

The physical realization of stable, static, charged vacuum bubbles also depends upon two particular stability issues: (i) stability of the bubble against an emission of the fermion coupled to the scalar field, and (ii) stability against charge evaporation, which can occur when the surface electric field of the bubble becomes too large, or supercritical. The stability against fermion emission can, in many cases, be satisfied if the fermion-scalar coupling occupies a range, given roughly by $G_F \gtrsim 1-10$, which may be regarded as fairly natural. However, for stability against charge evaporation, a relatively severe constraint is placed upon the symmetry breaking energy scale η. More specifically, unless the Z_2 symmetry breaking energy scale is sufficiently close to the electron mass, so that either Eq. (18) or Eq. (20) can be satisfied, a bubble initially containing only one species of positively (negatively) charged fermion will undergo charge evaporation by positron (electron) emission, reducing the net bubble charge to a value that renders the external electric field subcritical. This may easily be the case for a value of η on the order of the electroweak scale or higher. During such a process the bubble will also change its lepton number. The stable bubble will then be inhabited by *two* species of charged fermions, each contributing a Fermi gas energy term, but the Coulomb energy term will be lowered.

Therefore, consideration has subsequently been given to the case of a bubble populated by *two* species of fermions—the heavy fermions coupling directly to the domain wall scalar field, and electrons that have been absorbed by the bubble in order to partially or completely neutralize the bubble, rendering the surface electric field subcritical. The charge evaporation allows the Coulombic effects to be diminished, while the Fermi gas effects are enhanced. These two-fermion species bubbles may stabilize through Fermi gas dominance, through Coulomb dominance, or through a Fermi gas–Coulomb balance, depending upon the value of η and final configuration parameters, such as the bubble mass and the numbers of fermions populating the bubble (see Sec. III). Finally, constraints have been estimated for stable bubbles that do not form black holes. These constraints take the form of limits for either the number of heavy fermions that can populate a particular type of bubble (for the cases of neutral or charged, Fermi gas dominated bubbles), and/or for the symmetry breaking scale η (for the cases of near-critically charged bubbles). For instance, if $\eta < 10^4$ GeV, then Fermi gas dominated bubbles do not stabilize before forming black holes, whereas if $\eta > 10^4$ GeV, then Coulomb dominated or Fermi gas–Coulomb balanced bubbles necessarily collapse into black holes.

In summary, it has been argued that if there existed a biased, or an approximate, discrete symmetry which was broken in the early universe, and if heavy charged fermions coupled to the domain wall-forming scalar field, then it is possible for stable, charged vacuum bubbles to be produced, provided that certain parameters, such as the symmetry breaking energy scale, the fermion-scalar coupling constant, and fermion numbers, occupy appropriate ranges. If the parameters do not lie within such ranges, the bubbles are expected to undergo an unchecked collapse. Stable bubbles may indeed form, but it is not known what fraction of bubbles will actually stabilize, since this presumably depends upon how the data of initial conditions are distributed over the collection of evolving bubbles. At any rate, it is possible that such bubble configurations, even if rare, could be physically realized, and the physical existence of such vacuum bubbles could have interesting consequences for particle physics and cosmology.

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