Vacuum defects without a vacuum

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Topological defects can arise in symmetry breaking models where the scalar field potential $V(\phi)$ has no minima and is a monotonically decreasing function of $|\phi|$. The properties of such vacuumless defects are quite different from those of the "usual" strings and monopoles. In some models such defects can serve as seeds for structure formation, or produce an appreciable density of mini-black holes. [S0556-2821(99)50102-1]

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Symmetry-breaking phase transitions in the early universe can result in the formation of topological defects: domain walls, strings, monopoles, and their hybrids [1,2]. A prototypical symmetry breaking model is of the form

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \phi_a \partial^\mu \phi_a - V(f), \qquad (1)$$

where ϕ_a is a set of scalar fields, $a = 1,\ldots,N$, $f = (\phi_a \phi_a)^{1/2}$, and the potential $V(f)$ has a minimum at a non-zero value of f. Domain walls, for example, are obtained for a singlet scalar field, $N=1$. Domain wall solutions, $\phi = \phi(x)$, interpolate between $-\eta$ at $x \to -\infty$ and $+\eta$ at $x \to +\infty$, where $f = \eta$ is the minimum of $V(f)$.

In this paper, we are going to consider defects in models where $V(f)$ has a local maximum at $f=0$ but no minima; instead, it monotonically decreases to zero at $f \to \infty$. Potentials with this asymptotic behavior can arise, for example, due to non-perturbative effects in supersymmetric gauge theories [3]. In a cosmological context, they have been discussed in the so-called quintessence models [4]. Defects arising in such models could be called "quintessential," but we shall resist this temptation and use a more descriptive term "vacuumless." We shall see that the properties and evolution of vacuumless defects can be quite different from the usual ones.

We begin with the simplest case of a domain wall described by a real scalar field ϕ with a potential

$$V(f) = \lambda M^4 \cosh^{-2}(f/M). \qquad (2)$$

The field equation

$$f''(x) = V'(f) \qquad (3)$$

has a domain wall solution

$$\sinh(\phi/M) = (2\lambda)^{1/2}Mx \qquad (4)$$

with a finite energy per unit area

$$\sigma = \int T_0^0 dx = \pi(2\lambda)^{1/2}M^3. \qquad (5)$$

Similarly, for an inverse power-law potential

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$$V(f) = \lambda M^{4+n}(M^n + f^n)^{-1}, \qquad (6)$$

one finds $f \propto |x|^{2/(n+2)}$ and $T_0^0(x) \propto |x|^{-2n/(n+2)}$ at $|x| \to \infty$. For $n>2$, this energy distribution is integrable and the mass per unit area σ is finite. In this respect, vacuumless walls are quite similar to ordinary domain walls.

The situation with strings and monopoles is more interesting. Let us first consider global strings described by a scalar doublet (ϕ_1, ϕ_2) with a power-law potential (6). In the cylindrical coordinates (r, θ, z), the string ansatz is $\phi_1 = f(r)\cos\theta$, $\phi_2 = f(r)\sin\theta$, and $f(r)$ satisfies the equation

$$f'' + \frac{f'}{r} - \frac{f}{r^2} - \frac{dV}{df} = 0. \qquad (7)$$

The asymptotic solution of Eq. (7) at $r \gg \lambda^{-1/2}M^{-1}$ is

$$f(r) = AM(r/\delta)^{2/n+2}, \qquad (8)$$

where $A = (n+2)^{2/(n+2)}(n+4)^{-1/(n+2)} \sim 1$, and $\delta = \lambda^{-1/2}M^{-1}$ is the size of the string core. The energy density around the string is $T_0^0 \propto r^{-2n/(n+2)}$ and the energy per unit length of string is

$$\mu(R) = 2\pi \int_0^R T_0^0 r \, dr \sim M^2(R/\delta)^{4/n+2} \sim [f(R)]^2. \qquad (9)$$

The cutoff radius R has the meaning of a distance to the nearest string (or of the loop radius in the case of a closed loop). We see from Eq. (9) that vacuumless strings are very diffuse objects with most of the energy distributed at large distances from the string core. They are much more diffuse than ordinary global strings which have $\mu(R) \propto \ln R$, so that most of the energy is concentrated near the core.

For the exponential potential (2), numerical integration of Eq. (7) indicates that the derivative terms in that equation are negligible at large r, while the last two terms are nearly equal. The asymptotic behavior of $f(r)$ at large r is given by

$$f(r) \approx M \ln(r/\delta) \qquad (10)$$

and the dominant contribution to $\mu(R)$ is

$$\mu(R) \approx \int_\delta^R \frac{f^2}{2r^2} 2\pi r \, dr \approx \frac{\pi}{3}M^2[\ln(R/\delta)]^3. \qquad (11)$$

A global monopole is described by a triplet of scalar fields ϕ_a, $a=1,2,3$. The monopole ansatz is $\phi_a = f(r)x_a/r$, where r is the distance from the monopole center. For the power-law potential (6), it is easily verified that the field equation for $f(r)$ admits a solution of the same form (8) as for a global string (with a different coefficient $A \sim 1$). The total energy of a global monopole is

$$\mathcal{M} \sim M^2 R(R/\delta)^{4/n+2} \sim R[f(R)]^2, \tag{12}$$

where the cutoff radius R is set by the distance to the nearest antimonopole. For the exponential potential (2), $f(r)$ is given by Eq. (10) and $\mathcal{M}(R) \sim 4\pi M^2 R[\ln(R/\delta)]^2$.

The structure of gauge vacuumless strings and monopoles is more unusual. A gauge symmetry breaking model is obtained from Eq. (1) by adding the gauge field Lagrangian and replacing ∂_μ by gauge covariant derivatives. We shall first consider strings with a power-law potential (6). The string ansatz for the gauge field is $A_\theta(r) = -\alpha(r)/er$, where e is the gauge coupling. We found numerical solutions to the field equations for $\alpha(r)$ and $f(r)$

$$f'' + \frac{f'}{r} - (\alpha-1)^2 \frac{f}{r^2} - \frac{dV}{df} = 0 \tag{13}$$

$$\alpha'' - \frac{\alpha'}{r} - e^2 f^2(\alpha-1) = 0 \tag{14}$$

in a finite range $0 \leqslant r \leqslant R$, with boundary conditions $f(0) = \alpha(0) = 0$, $\alpha(R) = 1$. The value of $f(R)$ was chosen so that the total field energy in the volume under consideration is minimized. As $f(r)$ grows towards large r, the effective mass of the gauge field also grows, and $\alpha(r)$ approaches its asymptotic value, $\alpha=1$, very quickly [see Fig. 1(a)]. The gauge flux tube is therefore very well localized and the boundary condition $\alpha(R) = 1$ is a good approximation.

The solution for $f(r)$ in the case $n=2$ and $e^2/\lambda = 8$ is shown in Fig. 1(b) for several values of R. The most remarkable feature of these solutions is that, at a given r, they do not approach any limit as the integration range R is increased. This can be understood as follows. Outside the gauge flux tube, we can set $\alpha=1$ and drop the third term in Eq. (13). Then there are two possibilities. Either all three remaining terms in Eq. (13) are of comparable magnitude in the asymptotic region of large r, or two of the three are comparable, while the third term is negligible. Now, if the potential term is comparable to one or both of the derivative terms, then the solution is a power law, Eq. (8). It is easily seen that the coefficient A for this solution cannot be a positive real number, and thus the solution is unphysical. The only remaining possibility is that the two derivative terms are comparable, while the potential term is negligible. This gives the solution

$$f(r) = A \ln(r/\delta) + B. \tag{15}$$

The problem with this solution is that with $f(r)$ given by Eq. (15), the potential term in Eq. (13) decreases more slowly than the derivative terms. This indicates that Eq. (15) cannot
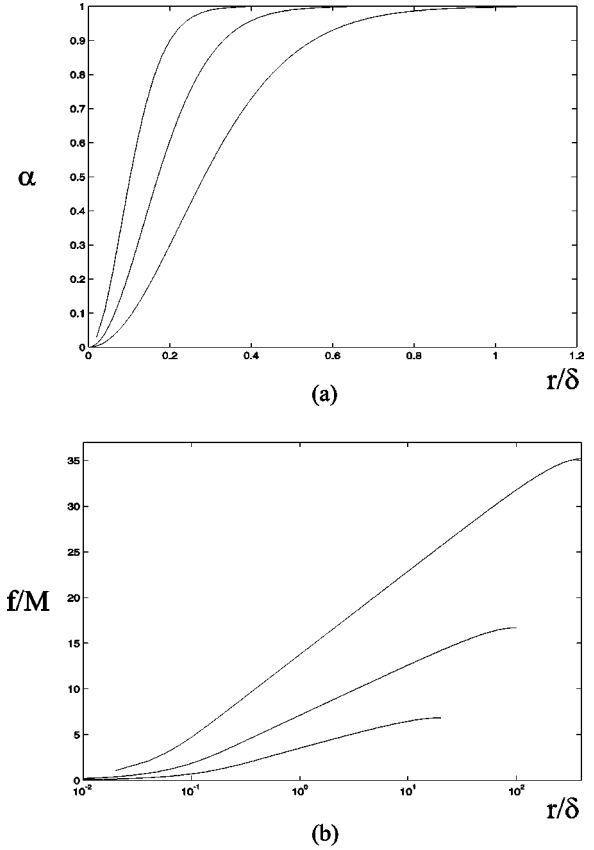

FIG. 1. (a) Gauge field $\alpha(r)$ and (b) scalar field $f(r)$ of a gauge vacuumless string for a potential (6) with $n=2$ and $e^2/\lambda = 8$. The top, middle, and bottom curves are for $R=400$, 100, and 20, respectively.

apply in the whole asymptotic range $r \gg \delta$. The potential term catches up with the derivative terms near the point $r = R$, where the boundary conditions are imposed. Assuming that $B \lesssim A \ln(R/\delta)$ and requiring that $f'' \sim dV/df$ at $r \sim R$, we have

$$A \sim M(R/\delta)^{2/n+2}[\ln(R/\delta)]^{-n+1/n+2}. \tag{16}$$

We have integrated Eqs. (13), (14) with $n=2$ numerically for several values of parameters R/δ and λ/e^2. The results are in a good agreement with Eqs. (15), (16).

The energy per unit length of a gauge vacuumless string is $\mu(R) \sim 2\pi A^2(R)\ln(R/\delta)$, with the main contribution coming from $f'^2/2$ and $V(f)$ terms in T_0^0. Apart from a logarithmic factor, this is the same as Eq. (9) for a global vacuumless string. In the case of an exponential potential (2), a similar analysis yields $A \approx M$ and $\mu(R) \sim \pi M^2 \ln(R/\delta)$.

The force per unit length due to inter-string interaction is $F_i \sim d\mu(R)/dR$, and the force due to the string tension is $F_t \sim \mu(R)/R$ (assuming that the typical curvature radius of strings in the network is comparable to the average separation R). For vacuumless strings with an exponential potential, $F_t/F_i \sim \ln(R/\delta)$, and we expect the Nambu action to give an adequate description of macroscopic strings [$\ln(R/\delta) \sim 100$]. For a power-law potential, $F_i \sim F_t$, and the Nambu action cannot be used.

Finally, we consider vacuumless gauge monopoles. Once again, we find numerically that the potential term in the equation for $f(r)$ is negligible for $r \ll R$. An analytic solution for both scalar and gauge fields can be given in this range. It is the Prasad-Sommerfield solution [5]

$$f(r) = A \coth(eAr) - 1/er,$$

$$\alpha(r) = eAr/\sinh(eAr),$$

where $\alpha(r)$ is defined in terms of the gauge field A_μ^a as $A_i^a = -[1 - \alpha(r)]\epsilon^{aij}x^j/er^2$. As before, A can be estimated by requiring that the potential term catches up with the gradient terms at $r \sim R$. For the power-law potential, this gives

$$A(R) \sim M(\lambda e M^3 R^3)^{1/(n+1)}. \tag{17}$$

The size of the monopole core is $r_A \sim 1/eA$. Much of the monopole energy is concentrated near the core; the total energy in that region is

$$\mathcal{M} = \frac{4\pi}{e}A. \tag{18}$$

In addition, there is a nearly constant energy density $T_0^0 \approx V(A)$ outside the core. The corresponding total energy is $\mathcal{E} \sim V(A)R^3 \sim \mathcal{M}$. We note that the monopole core becomes a black hole for $f(R) \approx A \gtrsim m_p$, where m_p is the Plank mass. Quite similarly, for the exponential potential we find $A \sim M \ln(R/\delta)$, while $\mathcal{M}$ is still given by Eq. (18).

Let us now briefly discuss the formation and cosmological evolution of vacuumless defects. At high temperatures in the early universe, the potential $V(f)$ acquires the usual temperature-dependent term,

$$V_T(f) = aT^2f^2, \tag{19}$$

where a depends on the coupling of ϕ to other fields. When T is sufficiently large, the minimum of the potential is at $f = 0$. As T decreases, this minimum turns into a maximum at some critical temperature T_c. For the exponential potential (2) and for the power-law potential (6) with $n = 2$, $T_c = (\lambda/a)^{1/2}M$ [6]. When the universe cools down to $T = T_c$, the symmetry-breaking phase transition occurs and the defects are formed in the usual way.

It should be noted that Eq. (19) for the thermal correction to the potential is valid only at small f. As f gets large, particles coupled to ϕ acquire large masses and disappear from the thermal bath. As a result, $V_T(f)$ decreases exponentially at large f. The thermal defect production mechanism will work only if f is initially localized near $f = 0$.

An alternative way of triggering the symmetry-breaking phase transition is to use the coupling of ϕ to the scalar curvature, $V_\mathcal{R}(f) = \xi\mathcal{R}f^2$. The curvature $\mathcal{R}$ drops from a large value to near zero at the end of inflation, and defect formation can occur at that point. The phase transition can also be triggered by a coupling of f to the inflation field.

By analogy with ordinary defects, one can expect that vacuumless strings and monopoles will eventually reach a scaling regime in which the typical distance between the de-

fects is comparable to the horizon, $R \sim t$. The main dissipation mechanism of vacuumless defects is similar to that of ordinary global strings and monopoles which lose most of their energy by radiating massless Goldstone bosons. Global vacuumless defects will also radiate Goldstone bosons, but in addition they will radiate massless f-particles (which correspond to radial excitations of the field ϕ). For gauge vacuumless defects, there are no Goldstone bosons, but f-particles will still be emitted. Thus in all cases the dominant dissipation mechanism is the radiation of massless particles.

For global strings with a power-law potential, the mass per unit length of string is given by Eq. (9) with R replaced by t. The relative contribution of strings to the energy density of the universe is given by

$$\rho_s/\rho \sim \mu(t)/m_p^2 \sim [f(t)/m_p]^2, \tag{20}$$

where $f(t) \sim M(t/\delta)^{2/(n+2)}$ is the characteristic magnitude of ϕ in the space between the strings. We see that the fraction of energy in strings monotonically grows with time, and the universe becomes dominated by the strings when $f(t) \sim m_p$.

The observed isotropy of the cosmic microwave background (CMB) implies $\mu(t_0)/m_p^2 \lesssim 10^{-5}$, where t_0 is the present time. The corresponding constraint on M is $M \lesssim 1$ MeV for $n = 2$ and $M \lesssim 100$ GeV for $n = 4$ (assuming $\lambda \sim 1$). For values of M saturating this constant, the strings would make a non-negligible contribution to the observed CMB anisotropy on the largest scales. Such strings, however, cannot be used to explain structure formation. The characteristic scale of the observed large-scale structure crossed the horizon at $t \sim t_{eq} \sim 10^{-6}t_0$. The density fluctuations due to strings on that scale are of the order $\delta\rho/\rho \sim \mu(t_{eq})/m_p^2 \lesssim 10^{-5}$. Very similar conclusions are reached for gauge strings and global monopoles.

In the case of an exponential potential, the evolution of global and gauge vacuumless strings is similar to that of ordinary global strings, and the evolution of global monopoles is similar to that of ordinary global monopoles. The strings serve as seeds for structure formation with $M \sim 10^{13}$ GeV and $M \sim 10^{15}$ GeV in the global and gauge cases, respectively. Global monopoles can seed structure formation for $M \sim 10^{14}$ GeV. The difference in the required energy scale M for different defects is due to the different power of the large logarithm in the expression for their mass.

Let us finally discuss the evolution of vacuumless gauge monopoles. A monopole and an antimonopole separated by a distance R are attracted with a force $F \sim d\mathcal{M}/dR \sim \mathcal{M}/R$ (for the power-law potential) and develop an acceleration $a \sim R^{-1}$. They reach relativistic speeds and oscillate with a characteristic period $\sim R$, rapidly losing their energy by radiation of massless ϕ-particles. Eventually, they annihilate into ϕ- and gauge bosons. If R is large enough so that $f(R) > m_p$, then the monopole cores are black holes, and a Schwarzschild black hole of mass $\sim \mathcal{M}$ (without a magnetic charge) is produced as a result of annihilation.

Apart from close monopole-antimonopole pairs which are about to annihilate, the typical distance between the mono-

poles at time t is $R \sim t$, and their relative contribution to the energy density of the universe is

$$\rho_m / \rho \sim \mathcal{M}(t)/m_p^2 t \propto t^{2-n/n+1}. \qquad (21)$$

For $n=2$, $\rho_m / \rho \sim (\lambda/e^2)^{1/3}(M/m_p)^2 = \mathrm{const}$, while for $n > 2$ it decreases with time.

Monopoles turn into black holes at time t_* such that $f(t_*) \sim m_p$. The mass of black holes formed at time t_f is $m \sim \mathcal{M}(t_f)$. For t_f in the radiation era, $t_f < t_{eq}$, the mass distribution of black holes at the present time is

$$m \frac{d\Omega_{BH}}{dm} \sim \frac{m t_{eq}^{1/2}}{m_p^2 t_f^{3/2}} \sim 10^{27} \left(\frac{\lambda}{e} \right)^{1/2} \left(\frac{M}{m_p} \right)^{5/2} \left(\frac{M}{em} \right)^{(n-1)/2}, \qquad (22)$$

and for $t_f > t_{eq}$,

$$m \frac{d\Omega_{BH}}{dm} \sim \frac{m}{m_p^2 t_f} \sim \left(\frac{\lambda}{e^2} \right)^{1/3} \left(\frac{M}{m_p} \right)^2 \left(\frac{M}{em} \right)^{(n-2)/3}. \qquad (23)$$

The usual bounds [7] on the density of primordial black holes can be used to constrain the parameters M, λ, and n.

[1] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).

[2] For a review of topological defects, see, e.g., A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).

[3] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B241**, 493 (1984); P. Binetruy, M. Gaillard, and Y. Y. Wu, Phys. Lett. B **412**, 288 (1997).

[4] P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); I. Zlatev, L. Wang, and P. J. Steinhardt, astro-ph/9807002.

[5] M. K. Prasad and C. H. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975).

[6] For $n > 2$, the finite temperature potential always has a local minimum at $f = 0$. However, this minimum is protected by a potential barrier whose width and height rapidly decrease with the temperature. This barrier can be easily overcome by thermal fluctuations.

[7] See, e.g., B. J. Carr, J. H. Gilbert, and J. E. Lidsey, Phys. Rev. D **50**, 4853 (1994).