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Domain walls and the decay rate of the excited vacua in large *N* Yang-Mills theory

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In the (nonsupersymmetric) Yang-Mills theory in the large N limit, there exists an infinite set of nondegenerate ''vacua.'' The distinct vacua are separated by domain walls whose tension determines the decay rate of the false vacua. I discuss the phenomenon from a field-theoretic point of view, starting from supersymmetric gluodynamics and then breaking supersymmetry by introducing a gluino mass. By combining previously known results, the decay rate of the excited vacua is estimated, $\Gamma \sim \exp(-\text{const}\times N^4)$. The fourth power of N in the exponent is a consequence of the fact that the wall tension is proportional to N. [S0556-2821(98)50324-4]

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The θ dependence of the pure Yang-Mills theories in the strong coupling regime has been investigated for a long time. A qualitative picture gradually emerged explaining various observations regarding the θ dependence of the vacuum energy E. This picture (for a very clear summary see Sec. 1 in Ref. [1]) predicts the existence of a set of states—only one of them is the true vacuum, while all others are ''excited vacua''—intertwined together in the process of the θ evolution. Each time θ crosses π, 3π, and so on, the levels cross and change their relative roles: one of the excited vacua becomes the true one and *vice versa*. Then, $E(\theta)$ is a multi-branch function of θ of the type

$$E(\theta)=N^2\ \min_k\ F\left(\frac{\theta+2\pi k}{N}\right),$$

where N is the number of colors and $F(x)$ is some N independent function. This vacuum structure can be proven in softly broken supersymmetric (SUSY) theories [2,3]. A similar picture emerges from the M-theory five-brane approach [4]. Recently, it was derived [1] in the context of ideas connected with the correspondence between the conformal gauge field theory and quantum gravity on anti–de-Sitter space [5]. Maldacena's duality is believed to give rise to a large N gauge theory belonging to the same universality class as QCD, starting from a string theory. It was shown [1], that for every θ, there is a set of infinitely many vacua stable at $N=\infty$. The true vacuum is obtained by minimizing energy over this set. Cusps occur at $\theta=\pi(2k+1)$ where k is an integer. At these points, an additional twofold degeneracy emerges. The adjacent vacua from the above set are separated by domain walls which can be described in terms of wrapped six-branes [1].

Here I describe how these qualitative results regarding the structure of the QCD vacuum are naturally obtained in the field theory *per se*, and calculate the life time of the ''false'' vacua, which turns out to be proportional to $\exp(-CN^4)$. The constant C can be found up to a numerical factor of order unity; the uncertainty is due to extrapolation from the supersymmetric to non-SUSY limit. The method is based on this extrapolation and on the fact that in the SUSY limit both, the vacuum structure and the domain wall tension, are exactly known [6–8]. I start from SUSY gluodynamics and, in order to break supersymmetry, introduce a mass term m_g to the gluino fields. At $m_g \ll \Lambda$ (where Λ is the dynamical mass scale) calculations are exact. As m_g grows and, eventually, crosses Λ the gluinos decouple, and one recovers pure Yang-Mills theory. Extrapolating the small m_g results to $m_g \sim \Lambda$ yields the overall structure of the pure Yang-Mills theory we are interested in, allowing one to establish fully the N dependencies. In fact, this approach has been already applied previously [2,3]. A new element which I add is combining it with the N counting. A surprising finding is the existence of the stationary domain walls in the (nonsupersymmetric) Yang-Mills theory in the limit $N\to\infty$, whose tension can be evaluated. These domain walls occur as the boundaries separating the distinct stable vacua from the intertwined set.

The vacuum structure in SUSY gluodynamics is very simple [9,10,6]. We have N degenerate chirally asymmetric vacua, labeled by the value of the gluino condensate

$$\langle \text{Tr}\ \lambda\lambda\rangle = N\Lambda^3 \exp\left(\frac{i(2\pi k+\theta)}{N}\right),\quad k=0,1,\ldots,N-1, \tag{1}$$

plus a possible chirally symmetric vacuum at $\langle \text{Tr}\ \lambda\lambda\rangle=0$. The chirally asymmetric vacua form a family of N states intertwined in the process of the θ evolution. At $\theta=\pi,3\pi,\ldots$ the vacuum restructuring takes place, so that the physical 2π periodicity in θ is maintained. The chirally symmetric vacuum at $\langle \text{Tr}\ \lambda\lambda\rangle=0$ plays no role in this process, and will be disregarded hereafter.

With the exact supersymmetry, all N vacua from the above family have the vanishing energy density and are physically equivalent. If one considers two distinct vacua separated ''geographically,'' the border between them is a domain wall discussed in [7,8,11]. If the wall is BPS saturated, then the tension of the wall separating two adjacent vacua is

$$\varepsilon=\frac{N}{8\pi^2}\langle \text{Tr}\ \lambda\lambda\rangle_0\left|\exp\left(\frac{2\pi i k}{N}\right)-\exp\left(\frac{2\pi i(k+1)}{N}\right)\right|, \tag{2}$$

where θ is set equal to zero, and $\langle \mathrm{Tr}\,\lambda\lambda\rangle_0$ is the gluino condensate at $k=0$. Since $\langle \mathrm{Tr}\,\lambda\lambda\rangle_0$ scales as N, in the large N limit the tension of the saturated wall is

$$\varepsilon=\frac{N}{4\pi}\,\Lambda^3. \tag{3}$$

It is important to note that the asymptotic behavior is linear in N. Even in the unlikely case the wall is not saturated,[1] its tension must differ from Eq. (3) only by a numerical factor which does not alter the N dependence of ε.

Now, what happens if one adds to the Lagrangian of SUSY gluodynamics a soft SUSY-breaking term?

The gluino mass term has the form

$$\Delta\mathcal{L}_m=\frac{m_g}{g^2}\langle \mathrm{Tr}\,\lambda\lambda\rangle+\mathrm{H.c.} \tag{4}$$

To begin with, we assume that $m_g/g^2\ll\Lambda$. Now SUSY is broken, and with it is gone the degeneracy of N vacua of supersymmetric gluodynamics. To first order in m_g, the energy density of the kth vacuum becomes

$$E_k=-\mathrm{Re}\,\frac{m_g}{g^2}\,\langle \mathrm{Tr}\,\lambda\lambda\rangle_k=-\left(2\cos\frac{2\pi k}{N}\right)\left(\frac{m_g}{g^2}\right)N\Lambda^3. \tag{5}$$

I assume m_g/g^2 to be real and positive. (This can be always achieved by adjusting θ appropriately.) Note that the combination m_g/g^2 is renormalization-group invariant to leading order, and scales as N. The combination renormalization-group invariant to all orders can also be found [16],

$$\frac{m_g}{g^2}-m_g\frac{N}{8\pi^2}.$$

For our purposes, it is sufficient to limit ourselves to the leading order.

Generically, all vacua are shifted from zero by $\Delta E\sim N^2$, in full accordance with the general expectations regarding the vacuum energy in the nonsupersymmetric gauge theories. The true vacuum corresponds to $k=0$. The states at $k\neq 0$ have a higher energy density. The spectrum of the states corresponding to Eq. (5) consists of two distinct parts (call them the first and the second part, respectively). For k that does not scale with N, the argument of the cosine is small, and the level splitting between the neighboring vacua is

$$\Delta E\sim 8\pi^2\left(k+\frac{1}{2}\right)\frac{m_g}{Ng^2}\,\Lambda^3\sim N^0. \tag{6}$$

As is seen from this expression, for higher k the level splittings grow and become of order N when k becomes proportional to N. This is the maximal dependence of the level splittings on N. For $k\sim N$, Eq. (6) is not valid, since it was obtained by expanding Eq. (5). One can see directly from Eq. (5) that at $k\sim N$ the energy splittings $\Delta E\sim N$. Note that at $N=\infty$, the number of states belonging to the part of the vacuum family with the level splittings of order N^0 (i.e., the first part of the spectrum) is infinite by itself. The fate of the vacua from this part of the spectrum, on the one hand, and the higher-lying states (from the second part), on the other hand, is different. The height of the barrier in the functional space, separating the adjacent vacua is of order N [see Eq. (3)]. It is determined by the wall tension. One should keep in mind that the wall width $\sim\Lambda^{-1}\sim N^0$. Although so far the wall tension was obtained in SUSY gluodynamics, the gluino mass term does not affect it as long as $m_g\ll\Lambda$. Even at $m_g\gtrsim\Lambda$ the walls, interpolating between those vacua that belong to the first part of the spectrum, persist as static objects, and their tension changes only by order unity. The N dependence of ε remains intact. Therefore, the vacua from the first part of the spectrum are stable in spite of the fact that they are nondegenerate. Below we will evaluate their decay rate to be $\exp(-CN^4)$.

As for the vacua from the second part of the spectrum, at $k\propto N$ they may disappear at all as local minima in the functional space. Or, else, some of them may survive as shallow minima. In any case, they disappear as stationary states, and the walls interpolating between these former vacua, even if they survive as shallow minima, are not static objects, they tend to ''decay.'' Needless to say that for such walls the estimate of their tension from Eq. (3) would be wrong.

If the decay rates of the vacua from the first part of the family tend to zero at $N\to\infty$ as $\exp(-CN^4)$, the decay rates of the states from the second part are either of order unity at $k\sim N$ or vanish slower than $\exp(-CN^4)$ if k scales as N^σ with $\sigma<1$.

Now we estimate the decay rate of the stable vacua. The false ones decay into the true vacuum through the bubble formation. The quasiclassical theory of these decays is well-developed [17], it is applicable if the radius of the critical bubble is large, much larger than the wall width. In our case, the radius of the critical bubble is proportional to N (this is the radius corresponding to a balance between the volume energy gained and the surface energy lost), while the wall width is N independent. Therefore, at large N the quasiclassical theory is valid. The general result of this theory is

$$\Gamma\propto\exp\left(-\frac{27}{2}\pi^2\frac{\varepsilon^4}{(\Delta E)^3}\right), \tag{7}$$

where ΔE is the difference of the vacuum energy densities in

[1]The issue whether or not the walls interpolating between the adjacent vacua (the so called complex walls) are Bogomol'nyi-Prasad-Sommerfield (BPS) saturated is being debated [12]. For $N=2$ and 3, the amended [6] Veneziano-Yankielowicz Lagrangian [13] exhibits no complex walls at all. For large N, the walls separating the adjacent vacua must exist. Arguments were given [15] that the Veneziano-Yankielowicz Lagrangian is inappropriate for the explorations of the complex walls. To see whether or not the BPS saturated walls are present, cusps inherent to this Lagrangian must be smoothed out, see, e.g., [14].

the false and true vacua, and ε is the surface energy density of the domain wall. With our values of ε and ΔE, we get[2]

$$\Gamma \sim \exp\left(-\left| \frac{\mathrm{Tr}\langle \lambda\lambda \rangle_0}{[m/(Ng^2)]^3} \right| \frac{N^3}{(k+1/2)^3} \frac{3^3}{2^{18}\pi^8} \right). \qquad (8)$$

The result for the exponent is rigorously valid for $m_g \ll \Lambda$ and $k \ll N$. I will now extrapolate it to the point of the gaugino decoupling, i.e., $m_g/(Ng^2) \sim \Lambda$ (still assuming that $k \ll N$). Then

$$\Gamma \sim \exp\left(-\eta \frac{3^3}{2^{18}\pi^8} \frac{N^4}{(k+1/2)^3} \right), \qquad (9)$$

where a dimensionless coefficient η is introduced to take account of the uncertainty of the extrapolation, $\eta \sim 10^0$. This coefficient is purely numerical, it is N independent.

Equation (9) presents an estimate of the false vacuum decay rate to its neighbor in the large N (nonsupersymmetric) Yang-Mills theory. Even though we know the exponent only by an order of magnitude, the presence of a very strong numerical suppression of the exponent seems evident. If so, our derivations are practically applicable only to very large $N \gtrsim 100$ even at $k \sim 1$.

It would be very interesting to check how both conclusions—the N^4 functional dependence of $\ln \Gamma$ and a numerical suppression of the coefficient in front of N^4—appear directly within the Maldacena-Witten approach. Narrow quasistable excited vacua were detected recently within an effective Lagrangian approach in Ref. [18]. Although some aspects in this consideration remain questionable and require further clarification, it seems worth trying to apply the method to check whether some of the excited vacua survive at low N, and, if so, to estimate their decay rate and possible phenomenological manifestations.

One of potentially important points is the lattice calculations. Since they are always done in finite volume, which reduces the field-theoretic system to quantum-mechanical, all vacua contribute to the correlation functions calculated in the lattice Yang-Mills theory, generally speaking. This might lead to a contamination of the lattice results by false vacua. Certainly, practically all calculations are done at $N=2$ or 3. At such low values of N, the false vacua may not exist as local minima in the functional space, or may be so shallow, that there is no barrier separating them from the true one. In this case they do not affect determination of the physically measurable quantities (such as the particle masses and coupling constants) from the finite-volume lattice results.

In summary, starting from supersymmetric gluodynamics and extrapolating in the gluino mass, one can argue that an infinite set of the stable vacua exist in the large N nonsupersymmetric Yang-Mills theory. Static domain walls interpolate between these vacua. The decay rate of the false vacua is given by the formula (9).

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