Bimaximal neutrino mixing in $\mathrm{SO}(10)_{\mathrm{GUT}}$

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We find a grand unified SO(10) model which accommodates the bimaximal neutrino mixing for vacuum-oscillation solutions to the atmospheric and solar neutrino problems. This model maintains the original SO(10) mass relation between neutrino and up-type quark masses $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$. [S0556-2821(98)07023-4]

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The recent data on the atmospheric neutrino from the SuperKamiokande (SuperK) Collaboration [1] have presented convincing evidence for neutrino oscillation with a mass-squared difference $\delta m^2_{\mathrm{atm}} \simeq 5\times 10^{-3}$ eV^2. It is now understood that the long-standing puzzle of the atmospheric muon neutrino (ν_μ) deficit in underground detectors [2] is indeed due to neutrino oscillations. As for the solar neutrino problem, there are still two allowed solutions: one is matter enhanced neutrino oscillation [i.e., the Mikheyev-Smirnov-Wolfenstein (MSW) solution[1] [3]] and the other is the long-distance vacuum neutrino oscillation called the ''just-so'' oscillation [5,6].

It is known [7] that the small angle MSW solution and the maximal mixing between the atmospheric ν_μ and ν_τ are quite naturally explained in a large class of seesaw models [8]. However, the electron energy spectrum recently reported by the SuperK Collaboration [9] seems to favor the ''just-so'' vacuum oscillation with $\delta m^2_{\mathrm{sun}} \simeq 10^{-10}$ eV^2 and the maximal mixing. If this vacuum oscillation of the solar neutrino is confirmed in future solar-neutrino experiments [9,10], we will be led to a quite surprising situation that two independent mixing angles in the lepton sector are very large in contrast with the quark sector in which all observed mixing angles among different families are small. This may point to a rule that governs the lepton mass matrices is significantly different from the one for the quark sector, which seems to be a contradiction to the idea of complete unification of quarks and leptons.[2]

On the other hand, as noted recently by Barger *et al.* [12] the required neutrino mass ratio $m_{\nu_2}/m_{\nu_3} \simeq 10^{-4}$ (provided $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$) is approximately equal to $(m_c/m_t)^2$ as predicted by a seesaw model in the SO(10) grand unified theory (GUT) [13].

The purpose of this paper is to construct a simple $\mathrm{SO}(10)_{\mathrm{GUT}}$ model which naturally accomodates the bimaximal neutrino mixing for the atmospheric and the solar neutrino vacuum oscillations, keeping the interesting $\mathrm{SO}(10)_{\mathrm{GUT}}$ mass relation $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$.[3] We assume supersymmetry throughout this paper.

[1]The MSW solution has two distinct regions: the small and the large angle ones [4].

[2]The so-called democratic mass matrices for quarks and leptons can generate large mixing in the neutrino sector [11].

[3]Recent analyses on phenomenological consequences of the bimaximal neutrino mixing are given in Refs. [12,14].

Let us first discuss the minimal $\mathrm{SO}(10)_{\mathrm{GUT}}$ model which contains three families of quarks and leptons $\psi_i(\mathbf{16})$ ($i=1,\cdots,3$) belonging to **16** of the $\mathrm{SO}(10)_{\mathrm{GUT}}$ and one Higgs field $H(\mathbf{10})$. We will consider Higgs multiplets responsible for the breaking of $\mathrm{SO}(10)_{\mathrm{GUT}}$ down to the standard-model gauge group later. This minimal model is known to yield a mass degeneracy of up-type and down-type quarks and vanishing Cabibbo-Kobayashi-Maskawa (CKM) mixing [15].

The simplest extension of the minimal model avoiding this unwanted mass degenercy is to introduce another Higgs field $H'(\mathbf{10})$. This two-Higgs field **10** model, in fact, gives less stringent relations among quark and lepton mass matrices as

$$M_{\nu D} = M_u, \quad M_l = M_d, \tag{1}$$

where $M_{\nu D}$ is 3×3 Dirac mass matrix for neutrinos. It is well known [16] that we need a large hierarchy in the Majorana mass matrix for right-handed neutrinos N_i($i=1,\ldots,3$) to obtain large neutrino mixing. However, if one assumes that the large hierarchy in the Majorana mass matrix for N_i one loses the original $\mathrm{SO}(10)_{\mathrm{GUT}}$ relation, $m_{\nu_2}/m_{\nu_3} \sim (m_c/m_t)^2$, discussed in the Introduction.

We, therefore, consider a different extension of the minimal $\mathrm{SO}(10)_{\mathrm{GUT}}$ model in this paper. Instead of adding an extra Higgs field $H'(\mathbf{10})$, we introduce one extra matter multiplet $\psi(\mathbf{10})$ belonging to **10** of the $\mathrm{SO}(10)_{\mathrm{GUT}}$ [17]. Thus, the matter multiplets in our model are three families of $\psi_i(\mathbf{16})$ and one $\psi(\mathbf{10})$.[4]

We now assume that the $\mathrm{SO}(10)_{\mathrm{GUT}}$ is broken down to SU(5) by condensation of Higgs fields $\langle \chi(\mathbf{16})\rangle = \langle \overline{\chi}(\mathbf{16}^*)\rangle = V$ with V being $\sim 10^{16}$ GeV.[5] This GUT breaking also induces a mass term for the matter multiplets through the following superpotential:

[4]We introduce one extra matter multiplet $\psi(\mathbf{10})$ in this paper. We may, however, introduce three families of $\psi_i(\mathbf{10})$ ($i=1,\ldots,3$). In this case, the Froggatt-Nielsen mechanism [18] may be used to account for observed quark and lepton mass matrices, assuming different charges for $\psi_i(\mathbf{16})$ and $\psi_i(\mathbf{10})$ in each families.

[5]We need other Higgs multiplets such as **45** to complete the breaking of the $\mathrm{SO}(10)_{\mathrm{GUT}}$ down to the standard-model gauge group. We do not consider them in this paper, since they are irrelevant to our present analysis.

$$W=\sum_{i=1}^{3} f_i\psi_i(\mathbf{16})\psi(\mathbf{10})\langle\chi(\mathbf{16})\rangle. \tag{2}$$

Namely, a linear combination $\mathbf{5}_\psi^{*\prime}\equiv\Sigma_{i=1}^{3}f_i\mathbf{5}_i^*$ in $\psi_i(\mathbf{16})$ receives a GUT scale mass together with $\mathbf{5}_\psi$ in $\psi(\mathbf{10})$. We choose $f_2=f_3=0$, here. The reason for this will be clearly understood later on.

After the spontaneous breakdown of SO(10)$_{\rm GUT}$ to SU(5), massless matter multiplets are given by

$$\mathbf{10}_3+\mathbf{5}_3^*+N_3=\psi_3(\mathbf{16}),$$

$$\mathbf{10}_2+\mathbf{5}_2^*+N_2=\psi_2(\mathbf{16}),$$

$$\mathbf{10}_1+N_1\subset\psi_1(\mathbf{16}),$$

$$\mathbf{5}_\psi^*\subset\psi(\mathbf{10}). \tag{3}$$

It should be clear that $\mathbf{10}_i$ $(i=1,\dots,3)$ in Eq. (3) are not **10** of the SO(10)$_{\rm GUT}$, but **10** of the SU(5).

We take a basis where the original Yukawa coupling matrix of the Higgs field $H(\mathbf{10})$ to the matter $\psi_i(\mathbf{16})$ is diagonal:[6]

$$W=h_i\psi_i(\mathbf{16})\psi_i(\mathbf{16})H(\mathbf{10}). \tag{4}$$

This leads to a diagonal mass matrx for the up-type quarks such as

$$M_u=\begin{pmatrix} m_u & 0 & 0\\ 0 & m_c & 0\\ 0 & 0 & m_t\end{pmatrix}, \tag{5}$$

where M_u is defined as

$$(M_u)_{ii}=h_i\langle\mathbf{5}_H\rangle. \tag{6}$$

Here, $\mathbf{5}_H$ is a SU(5)-**5** component of $H(\mathbf{10})$.

The down-type quark mass matrix is, however, incomplete, since the SU(5)-$\mathbf{5}^*$ of $\psi(\mathbf{10})$ (i.e., $\mathbf{5}_\psi^*$), does not have any Yukawa coupling to $H(\mathbf{10})$. To solve this problem we introduce a pair of Higgs fields $H(\mathbf{16})$ and $\overline{H}(\mathbf{16}^*)$ and consider a superpotential

$$W=kH(\mathbf{10})\overline{H}(\mathbf{16}^*)\overline{\chi}(\mathbf{16}^*)+MH(\mathbf{16})\overline{H}(\mathbf{16}^*). \tag{7}$$

U(1) R symmetry may be useful to have this form of superpotential. The U$(1)_R$ charges are given in Table I. The GUT condensation $\langle\overline{\chi}(\mathbf{16}^*)\rangle\neq 0$ induces a mass mixing between $\mathbf{5}^*$'s of $H(\mathbf{10})$ and $H(\mathbf{16})$ (i.e., $\mathbf{5}^*_{H(\mathbf{10})}$ and $\mathbf{5}^*_{H(\mathbf{16})}$). Then, a linear combination

TABLE I. U$(1)_R$ charges.

	$H(\mathbf{10})$	$H(\mathbf{16})$	$\overline{H}(\mathbf{16}^*)$	$\chi(\mathbf{16})$	$\overline{\chi}(\mathbf{16}^*)$	$\psi_i(\mathbf{16})$	$\psi(\mathbf{10})$
R	0	0	2	0	0	1	1

$$\tilde{\mathbf{5}}_H^*=\cos\theta\mathbf{5}^*_{H(\mathbf{10})}+\sin\theta\mathbf{5}^*_{H(\mathbf{16})}, \tag{8}$$

$$\tan\theta=-\frac{k\langle\overline{\chi}(\mathbf{16}^*)\rangle}{M}, \tag{9}$$

remains as a massless Higgs field $H(\mathbf{5}^*)$ in the standard SU$(5)_{\rm GUT}$ and contributes to the quark and lepton mass matrix. Then, $\tilde{\mathbf{5}}_H^*$ can couple to $\mathbf{5}_\psi^*$ as

$$W_{\rm eff}=\sin\theta\sum_{i=1}^{3} g_i\mathbf{10}_i\mathbf{5}_\psi^*\tilde{\mathbf{5}}_H^*, \tag{10}$$

where the coupling constants g_i are defined as

$$W=\sum_{i=1}^{3} g_i\psi_i(\mathbf{16})\psi(\mathbf{10})H(\mathbf{16}). \tag{11}$$

Now, the Yukawa coupling of $\tilde{\mathbf{5}}_H^*$ is given by

$$W_{\rm eff}=\cos\theta(\mathbf{10}_1,\ \mathbf{10}_2,\ \mathbf{10}_3)\times\begin{pmatrix}0 & g_1\tan\theta & 0\\ h_2 & g_2\tan\theta & 0\\ 0 & g_3\tan\theta & h_3\end{pmatrix}\begin{pmatrix}\mathbf{5}_2^*\\ \mathbf{5}_\psi^*\\ \mathbf{5}_3^*\end{pmatrix}\tilde{\mathbf{5}}_H^*, \tag{12}$$

which yields the down-type quark and the charged lepton mass matrix

$$M_{d/l}=m_t\begin{pmatrix}0 & x & 0\\ m_c/m_t & y & 0\\ 0 & z & 1\end{pmatrix}\times\frac{\cos\theta}{\tan\beta}. \tag{13}$$

Here, $\tan\beta\equiv\langle\mathbf{5}_H\rangle/\langle\tilde{\mathbf{5}}_H^*\rangle$ and

$$x=\frac{g_1}{h_3}\tan\theta,\quad y=\frac{g_2}{h_3}\tan\theta,\quad z=\frac{g_3}{h_3}\tan\theta. \tag{14}$$

We see that a choice of $x\sim m_c/m_t$, $y\sim\sqrt{m_c/m_t}$, and $z\sim 1$ produces a nice fit of the observed quark and lepton mass ratios and the CKM matrix.[7] Thus, we take $x\simeq m_c/m_t$, $y\simeq\sqrt{m_c/m_t}$, and $z\simeq 1$. The $\tan\beta$ may be very large unless $\cos\theta$ is very small $[\tan\beta\simeq\sqrt{2}(m_t/m_b)\cos\theta]$. It is now clear that in contrast with the CKM mixing we have a large mix-

[6]Precisely speaking, we assume $f_2=f_3=0$ in Eq. (2) in this basis. We discard small deviations from our assumption in the present analysis.

[7]To explain quark and lepton masses more precisely one must introduce SU(5) breaking effects, otherwise we have wrong SU$(5)_{\rm GUT}$ relations, $m_\mu=m_s$ and $m_e=m_d$. A detailed analysis including these effects will be given in Ref. [19].

ing closed to the maximal between $\mathbf{5}_3^*$ and $\mathbf{5}_\psi^*$ which corresponds to a mixing between charged leptons of the third and second families.

Let us turn to the Dirac mass term for neutrinos which is given by the superpotential (4). This mass matrix is also incomplete, since $\mathbf{5}_\psi^*$ never couples to N_i's in $\psi_i(\mathbf{16})$. However, the following nonrenormalizable interaction gives a desired coupling:

$$W=\sum_{i=1}^{3} k_i \psi_i(\mathbf{16})\psi(\mathbf{10})H(\mathbf{10})\frac{\overline{\chi}(\mathbf{16}^*)}{M_G}, \tag{15}$$

where M_G is the gravitational scale $M_G \simeq 2\times 10^{18}$ GeV. Together with the original coupling in Eq. (4), the nonrenormalizable interaction (15) yields

$$M_{\nu D}=m_t\begin{pmatrix} 0 & \delta_1 & 0 \\ m_c/m_t & \delta_2 & 0 \\ 0 & \delta_3 & 1 \end{pmatrix}. \tag{16}$$

Here, $M_{\nu D}$ is defined as

$$W_{\mathrm{eff}}=(N_1,N_2,N_3)M_{\nu D}\begin{pmatrix} \mathbf{5}_2^* \\ \mathbf{5}_\psi^* \\ \mathbf{5}_3^* \end{pmatrix}, \tag{17}$$

and $\delta_i=(k_i/h_3)(\langle\chi(\mathbf{16})\rangle/M_G)$. Notice that $\delta_i\simeq O(10^{-2})$ as long as $k_i/h_3\sim O(1)$.

It is extremely interesting that when $\delta_2\sim m_c/m_t$ we have a large mixing between $\mathbf{5}_\psi^*$ and $\mathbf{5}_2^*$ which produces a large mixing between left-handed neutrinos of the second and first families. This observation is crucial for our purpose, since this tells us that when we maintain the $\mathrm{SO}(10)_{\mathrm{GUT}}$ relation, $m_{\nu_2}/m_{\nu_3}\sim(m_c/m_t)^2$ (i.e., $\delta_2\simeq m_c/m_t$), we necessarily obtain a large mixing closed to the maximal between ν_e and ν_μ (provided that the Majorana mass matrix for N_i does not have hierarchy). We take, for simplicity, $\delta_1\simeq\delta_3\simeq 0$ and $\delta_2 \simeq m_c/m_t$.[8]

The Majorana masses for the right-handed neutrino N_i are given by the following nonrenormalizable superpotential:

$$W=j_{ij}\frac{1}{M_G}\psi_i(\mathbf{16})\psi_j(\mathbf{16})\overline{\chi}(\mathbf{16}^*)\overline{\chi}(\mathbf{16}^*). \tag{18}$$

After the $\mathrm{SO}(10)_{\mathrm{GUT}}$ breaking we obtain the Majorana mass matrix

$$(M_N)_{ij}=\frac{\langle\overline{\chi}(\mathbf{16}^*)\rangle^2}{M_G}(j_{ij}). \tag{19}$$

Simply assuming $\langle\overline{\chi}(\mathbf{16}^*)\rangle=V\sim 10^{16}$ GeV and $j_{ij}\sim\delta_{ij}$ we get

$$M_N\sim(10^{14}\ \mathrm{GeV})\times\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

From the see-saw mechanism the light neutrino masses are given by

$$M_\nu\sim\frac{(m_t)^2}{M_N}\begin{pmatrix} 0 & 0 & 0 \\ 0 & (m_c/m_t)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{21}$$

with the Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix [20] defined in the basis where the charged lepton mass matrix is diagonal:

$$U_{\mathrm{MNS}}\sim\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \epsilon \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}, \tag{22}$$

$$\epsilon=O(\sqrt{m_c/m_t}). \tag{23}$$

This neutrino mass matrix given by Eqs. (21), (22) is nothing but the one used for explaining the atmospheric neutrino oscillation[9] and the ''just-so'' oscillation of the solar neutrino [12,14].

In this paper we have found a simple $\mathrm{SO}(10)_{\mathrm{GUT}}$ model which naturally generates the bimaximal neutrino mixing suggested from the atmospheric ν_μ deficit and the ''just-so'' oscillation solution to the solar neutrino problem. This model maintains the original $\mathrm{SO}(10)_{\mathrm{GUT}}$ mass relation $m_{\nu_2}/m_{\nu_3} \sim(m_c/m_t)^2$ which is required for the ''just-so'' scenario [12]. However, one may think that the present model is already too complicated, and in this sense the ''just-so'' oscillation seems very unlikely as stressed by Ramond and one of the authors (T.Y.) [7].

Nevertheless, if it turns out to be the case, we will be forced to consider drastic changes of the underlying physics governing the Yukawa couplings for quarks and leptons. We think that our modified $\mathrm{SO}(10)_{\mathrm{GUT}}$ model presented in this paper will be a rather mild change among them.

Note added in proof. In the text we have restricted our discussion to a specific case of $f_2=f_3=0$ in Eq. (2). For a general case $(f_1,f_2,f_3\neq 0)$ we have the following mass matrix for down-type quarks and charged leptons in the limit $m_u=0$:

[8] If $\delta_2\simeq 1/5$, the small angle MSW solution can be accommodated instead of the ''just-so'' solution. In this case some SO(10)-singlet fields are required at the GUT scale.

[9] From Eq. (21), we obtain $m_{\nu_3}\simeq\sqrt{\delta m^2_{\mathrm{atm}}}\simeq O(0.1)$ eV.

$$M_{d/l}=m_t\begin{pmatrix} 0 & x & 0 \\ (\cos\,\alpha_1)m_c/m_t & y & (\sin\,\alpha_1\ \ \sin\,\alpha_2)m_c/m_t \\ 0 & z & \cos\,\alpha_2 \end{pmatrix}\times\frac{\cos\phi}{\tan\beta}, \tag{24}$$

which may yield a better fit to the observations.

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