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Electromagnetic form factor corrections to collisional energy loss of pions and protons, and spin correction for muons

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At very high energies, the collisional energy loss of hadrons passing through matter is reduced relative to that of a point particle because of their extended electromagnetic structures. For fermions, spin and magnetic moment effects work in the opposite direction. Analytic and numerical results are given for pions, muons, and protons. The reductions are small, but increase with energy. At 1 TeV, for example, the reduction is 2.7–3.4 % for pions and 1.4–1.9 % for protons, depending on the material. The influence of form factors on the so-called z^3 effect at high energies is also addressed. [S0556-2821(98)06623-5]

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I. INTRODUCTION

Collisional energy loss divides naturally into two contributions: one from ''soft'' collisions involving energy transfers from zero to an intermediate energy T_0 and another for ''hard'' collisions with energy transfers from T_0 to the maximum $T_{\max}$. For the soft collisions, atomic binding effects are important; for the hard collisions, the electrons in the struck atom or molecule can be regarded as free. The conventional (Bethe-Bloch) expressions are almost totally insensitive to the exact choice of T_0 because the dominant contribution to the ''Bethe logarithm'' from one is $\ln(T_0/\varepsilon_{\rm eff})$ and the other, $\ln(T_{\max}/T_0)$.

The standard results (see, e.g., Ref. [1]) treat the incident heavy particle as a spinless point charge in its interaction with the atomic electrons. At high energies the maximum momentum transfer in a collision with a free electron at rest can be large enough that the extended size of the hadron's charge distribution begins to come into play. Table I gives some representative values of both $[Q_{\max}]^2$ and $T_{\max}$ for pions and protons. The mean square charge radius of the pion is $\langle r^2\rangle_\pi \approx 11\ (\mathrm{GeV})^{-2}$ [2], while that of the proton is $\langle r^2\rangle_p \approx 17\ (\mathrm{GeV})^{-2}$ [3]. From the expansion of a form factor, $F(Q^2)\approx 1-\langle r^2\rangle Q^2/6+\cdots$, we see that at high energies the extended charge distribution will have a softening effect on the hardest collisions: the energy loss is reduced relative to that of point particles, although spin effects can work in the opposite direction.

In this paper I present for the record the results for the corrections (reductions) in collisional energy loss of pions with a simple propagator form factor [2] and of protons whose electron scattering is described by the Rosenbluth formula [4] with the standard dipole form factor [3]. These form factors describe the elastic electron scattering data (or equivalent) adequately well beyond the range of Q^2 considered here. A preliminary version of the results was archived with the Particle Data Group (PDG) [5]. I also include the correction to the muon's energy loss from its spin and the very small high-energy z^3 effect [6] with form factors, but only in the approximation of their expansion to first order in Q^2.

TABLE I. Representative values of $[Q_{\max}]^2$ and $T_{\max}$ for pions and protons versus energy.

E (GeV)	Pions		Protons	
	$[Q_{\max}]^2$ (GeV)^2	$T_{\max}$ (GeV)	$[Q_{\max}]^2$ (GeV)^2	$T_{\max}$ (GeV)
100	0.086	84	0.011	10
500	0.492	482	0.188	184
1000	1.003	981	0.549	537

II. FORMULAS

For a heavy particle of charge ze and mass M passing through a medium of atomic number Z, the standard expression for energy loss in MeV cm^2/g is

$$\frac{dE}{dx}=C\,\frac{z^2}{\beta^2}\,L(\beta,Z),\quad C=0.307\,\frac{Z}{A}, \tag{1}$$

where $L(\beta,Z)$ is the ''Bethe logarithm,'' given as the square bracketed expression in Eq. (22.1) of the Particle Data Group [1]. Note that my constant C is related to Rossi's [7] constant C_R by $C=4m_ec^2C_R$, and to the PDG's K by $C=KZ/A$.

The effect of a form factor is felt only in the hard collisions. Here the struck electrons can be considered as free particles. Furthermore, the form factor effects only occur at extremely high energies. We therefore put $\beta\to 1$ and $p_{\rm lab}=E_{\rm lab}$ throughout. The difference in energy loss with and without a form factor $F(Q^2)$ in the Born approximation calculation is then

$$\Delta\,\frac{dE}{dx}=\frac{C}{2}\,z^2\int_{T_0}^{T_{\max}}\frac{dT}{T}\left(1-\frac{T}{T_{\max}}\right)[|F(Q^2)|^2-1], \tag{2}$$

where $T=Q^2/2m_e$ is the energy transfer, T_0 is the intermediate energy transfer, assumed very small compared to $T_{\max}$, and

$$T_{\max}=\frac{2m_ep_{\rm lab}^2}{s}\approx\frac{2m_eE_{\rm lab}^2}{s},$$

$$s=m_e^2+M^2+2m_eE_{\rm lab}. \tag{3}$$

The maximum momentum transfer squared is $Q^2_{\max} = 2m_eT_{\max}$. Equation (2) is valid for a spinless incident particle with charge form factor $F(Q^2)$. Note that, because $|F(Q^2)|\leqslant 1$, the right-hand side of Eq. (2) is negative.

For protons, the spin and magnetic moment complicate the expression beyond that given by Eq. (2). In terms of the so-called charge and magnetic form factors G_E and G_M in the Rosenbluth formula, the change in collisional energy loss for protons can be written conveniently as the sum of two terms:

$$\left[\Delta\,\frac{dE}{dx}\right]_P=\left[\Delta\,\frac{dE}{dx}\right]_1+\left[\Delta\,\frac{dE}{dx}\right]_2, \tag{4}$$

where

$$\left[\Delta\,\frac{dE}{dx}\right]_1=\frac{C}{2}\,z^2\int_{T_0}^{T_{\max}}\frac{dT}{T}\left(1-\frac{T}{T_{\max}}\right)[|G_E(Q^2)|^2-1], \tag{5}$$

$$\left[\Delta\,\frac{dE}{dx}\right]_2=\frac{m_eC}{4M^2}\,z^2\int_{T_0}^{T_{\max}}dT\left\{\left(1-\frac{T}{T_{\max}}\right)\times\frac{[|G_M|^2-|G_E|^2]}{(1+m_eT/2M^2)}+\frac{M^2T}{m_eE_{\text{lab}}^2}\,|G_M|^2\right\}. \tag{6}$$

The first contribution has the same form as Eq. (2); it corresponds to a spinless ‘‘proton’’ and is negative. The second contribution results from the proton’s spin and magnetic moment and its associated form factor. Since $|G_M|>|G_E|$, the second contribution is positive, a consequence of the magnetic moment scattering increasing relative to the charge scattering at large angles.

The form factors for the pion and the proton are taken to be

$$F_\pi(Q^2)=\frac{\mu_\pi^2}{\mu_\pi^2+Q^2}, \tag{7}$$

$$G_E(Q^2)=\frac{G_M(Q^2)}{\xi}=F_p(Q^2)=\left(\frac{\mu_p^2}{\mu_p^2+Q^2}\right)^2, \tag{8}$$

where $\xi=2.79285$ is the proton’s magnetic moment in nuclear magnetons. The mass parameter for the pions is chosen to be $\mu_\pi=0.736$ GeV [2] and that for the proton, $\mu_p=0.8426$ GeV or $(\mu_p)^2=0.71$ GeV^2 [3].

Since the integrands in Eqs. (2), (5), and (6) are not singular as T^{-1} at small T, the intermediate energy transfer T_0 can be set to zero with negligible error. The results for the change in energy loss are then

$$\left[\Delta\,\frac{dE}{dx}\right]_\pi=-\frac{C}{2}\,z^2\ln(1+x^\pi_{\max}), \tag{9}$$

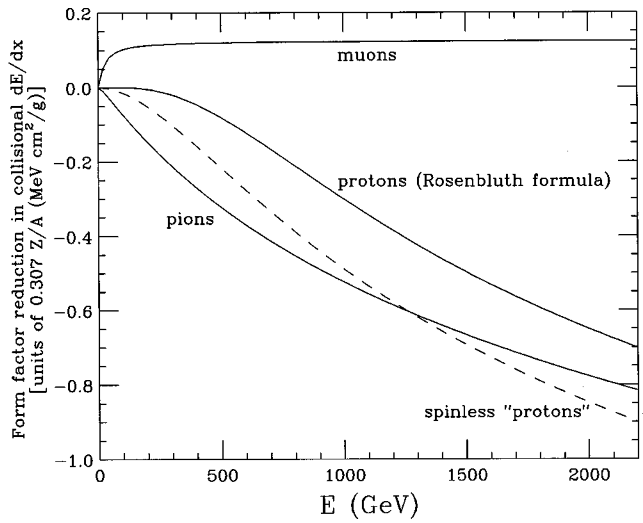

FIG. 1. Reduction in collisional energy loss for pions and protons from the hadronic electromagnetic form factors and spin correction to muon energy loss versus incident particle energy in GeV. Units of energy loss are $C=0.307\ Z/A$ MeV cm^2/g. The dashed curve represents Eq. (10), the reduction for a spinless ‘‘proton.’’ The solid proton curve is the sum of Eqs. (10) and (11).

where $x^\pi_{\max}=2m_eT_{\max}/\mu_\pi^2$. For protons the dipole form factor yields

$$\left[\Delta\,\frac{dE}{dx}\right]_1=-\frac{C}{2}\,z^2\left[\ln(1+x^p_{\max})+\frac{5}{6}-\frac{(5+4x^p_{\max})}{6(1+x^p_{\max})^2}\right], \tag{10}$$

where $x^p_{\max}=2m_eT_{\max}/\mu_p^2$, and

$$\left[\Delta\,\frac{dE}{dx}\right]_2=\frac{C}{2}\,z^2\lambda[(\xi^2-1)I_a(x^p_{\max})+\nu\xi^2I_b(x^p_{\max})], \tag{11}$$

where $\lambda=\mu_p^2/4M^2$, $\nu=M^2\mu_p^2/2m_e^2E_{\text{lab}}^2$, and

$$I_a(x)=\frac{\lambda^2}{(1-\lambda)^4}\,\frac{(1+\lambda x)}{x}\ln\left(\frac{1+\lambda x}{1+x}\right)+\frac{\lambda^2}{(1-\lambda)^3}-\frac{\lambda x}{2(1-\lambda)^2(1+x)}+\frac{x(3+2x)}{6(1-\lambda)(1+x)^2}, \tag{12}$$

$$I_b(x)=\frac{x^2(3+x)}{6(1+x)^3}. \tag{13}$$

For muons, a small correction to the standard Bethe formula results from the Dirac current of the muon. A simple Feynman diagram computation, appeal to the result given by Rossi [8], or specialization of Eq. (11) to $\xi=1$ and $\mu_p\to\infty$ gives the correction

$$\left[\Delta\,\frac{dE}{dx}\right]_{\text{muon}}=\frac{C}{2}\left(\frac{T_{\max}}{E_{\text{lab}}}\right)^2=\frac{C}{8}\left(1+\frac{m_\mu^2}{2m_eE_{\text{lab}}}\right)^{-2}\to\frac{C}{8}. \tag{14}$$

III. RESULTS

Figure 1 displays the reduction in energy loss for pions and protons and the spin correction for muons in units of C as functions of the energy of the incident particle. Since the main ''Bethe logarithm'' in Eq. (1) is in the range of 13–22 for $E_{\mathrm{lab}}=100-1000$ GeV, the reduction is less than 1% for pions of energies less than 200 GeV and protons of less than 700 GeV. At 2 TeV, the reduction is 4–5 % for pions and 3–4 % for protons. The difference between the dashed curve and the solid curve for protons shows the magnitude of the positive magnetic moment contribution, $[\Delta dE/dx]_2$.

At extremely high energies, of course, radiative losses from bremsstrahlung and pair production dominate. Extensive computations have been made of energy loss by bremsstrahlung and pair production for muons [9]. The dominant scaling with incident particle mass M is as $(m_\mu/M)^2$ for bremsstrahlung and as m_μ/M for pair production, each times an insensitive logarithm. With such scaling, I find that in iron, for example, the combined radiative energy loss for pions (protons) equals the collisional loss at roughly 500 GeV (4.5 TeV). Even for protons, as the energy increases radiative processes obviously dominate the energy losses before the form factor corrections to the collisional losses become appreciable.

IV. z^3 CORRECTION WITH FORM FACTOR

The so-called z^3 correction to energy loss is given at high energies by an expression similar to Eq. (2), but with the difference between the second-order McKinley-Feshbach-Dalitz cross section [10,11] and the lowest order result for the elastic scattering of an electron by a point Coulomb field. The second-order cross section is the relativistic Rutherford expression times

$$X=1-\beta^2\sin^2\theta/2+\pi z\alpha\beta\,\sin\,\theta/2(1-\sin\,\theta/2).$$

The first two terms are the well-known lowest order result for a spin-$\frac{1}{2}$ incident electron. The remainder gives the z^3 correction to the energy loss for a point charge incident on a free electron [6]:

$$\left[\frac{dE}{dx}\right]^0_{z^3}=\frac{\pi\alpha z^3C}{2\beta}\int_0^{T_{\max}}\frac{dT}{T}\left[\left(\frac{T}{T_{\max}}\right)^{1/2}-\frac{T}{T_{\max}}\right]=\frac{\pi\alpha z^3C}{2\beta}. \tag{15}$$

With $\pi\alpha/2\approx 1.15\times 10^{-2}$, this correction is tiny for particles with $|z|=1$. What happens if the incident particle has extended electromagnetic distributions? It seems clear that, since the second-order diagram has *two* softened external vertices, the z^3 contribution to the scattering cross section will be reduced more than the z^2 contribution. The new integrand for Eq. (15) will surely be smaller than for a point particle.

Since the z^3 effect is very small already, we content ourselves with an expansion of the form factors to first order in Q^2 in an extension of Dalitz's calculation. This approximation make tractable an otherwise fairly horrendous computation. The calculation follows closely that of Dalitz (in some ways simpler); we present only the result. The extension of Eq. (2) to include the z^3 terms, but with the form factor expanded only to first order in Q^2, is

$$\left[\Delta\,\frac{dE}{dx}\right]_{z^2+z^3}=-\frac{z^2Cm_eT_{\max}\langle r^2\rangle}{3}\int_0^1 dx\left\{(1-x)+\pi\alpha z\times\left[(1-x)+\frac{1}{2}\,x(\sqrt{x}-x)\right]\right\}, \tag{16}$$

where $x=T/T_{\max}$. The z^2 part of Eq. (16) is just what Eq. (2) gives if $|F(Q^2)|^2$ is expanded to first order in Q^2. The full high-energy z^3 correction, to first order in $\langle r^2\rangle$ [the sum of Eq. (15) and the z^3 part of Eq. (16)] is

$$\left[\frac{dE}{dx}\right]^1_{z^3}=\frac{\pi\alpha z^3C}{2}\left[1-\frac{16}{45}\langle r^2\rangle m_eT_{\max}\right]=\frac{\pi\alpha z^3C}{2}\left[1-\frac{8}{45}\langle r^2\rangle Q^2_{\max}\right]. \tag{17}$$

For pions (spinless ''protons'') at 100 GeV, the form factor reduction is 17% (3%). At 500 GeV (beyond the range of validity), the reduction is 96% (57%). The only reliable message at very high energies is that the z^3 contribution to the energy loss is well below the point-charge result (15).

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