Top-bottom splitting in technicolor with composite scalars

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We present a model of dynamical electroweak symmetry breaking in which the splitting between the top and bottom quark masses arises naturally. The W and Z masses are produced by a minimal technicolor sector, the top quark mass is given by the exchange of a weak-doublet technicolored scalar, and the other quark and lepton masses are induced by the exchange of a weak-doublet technicolor-singlet scalar. We show that, in the presence of the latter scalar, the vacuum alignment is correct even in the case of SU(2) technicolor. The fit of this model to the electroweak data gives an acceptable agreement ($\chi^2 = 28$, for 20 degrees of freedom). The mass hierarchy between the standard fermions other than top can also be explained in terms of the hierarchy of squared-masses of some additional scalars. We discuss various possibilities for the compositeness of the scalars introduced here. [S0556-2821(99)01501-5]

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I. INTRODUCTION

While the dynamics responsible for the generation of mass remains obscure, there are a few known theoretical possibilities that explain certain relationships between the masses of the observed particles. The success of the standard model in fitting the experimental results may appear to favor models that include a Higgs boson in the low energy effective theory, such as supersymmetric standard models or top quark condensation models [1,2]. Yet the current precision of the electroweak measurements does not actually distinguish between the standard model and certain models that do not have a decoupling limit. The latter theories use technicolor to give the W and Z masses, and additional fields to communicate electroweak symmetry breaking to the quarks and leptons. If these fields are heavy gauge bosons, as in extended technicolor [3], then one is led to consider complicated dynamics [4,5].

On the other hand, if the additional fields are scalars, one has the flexibility to generate the observed masses without immediate dynamical assumptions. For example, technicolor models with weak-doublet technicolor-singlet scalars [6,7,8,9] have been found to have phenomenology consistent with experiment. Alternatively, technicolor models that include weak-singlet technicolored scalars [10,11,12,13] give a natural explanation for the mass hierarchy between the fermion generations. The existence of scalars much lighter than the Planck scale does require some further explanation. Their masses can be protected by supersymmetry [14,15,10,12], or they can be bound states arising within a high energy theory [16]. It is also conceivable that the fundamental scale where quantum gravity becomes strong is not 10^{19} GeV, but rather some TeV scale [17].

In this paper we show that a technicolor model that includes weak-*doublet* technicolored scalars explains not only the intergenerational fermion mass hierarchy, but also the intragenerational mass hierarchies, in terms of relationships among the squared masses of different scalars. For example the top-bottom splitting arises naturally in such models because hypercharge prevents the techniscalar responsible for the top quark mass from inducing a bottom quark mass.

We start by constructing the low-energy effective theory that gives rise to the W, Z and t masses, without specifying a dynamical origin for the scalars. In Sec. III we explore the electroweak phenomenology of the low-energy effective theory. Next, we discuss possible mechanisms for generating the masses of the other quarks and leptons. Dynamics that could create the scalar bound states in our models are addressed in Sec. V. We present conclusions in Sec. VI. In Appendix A we show that SU(2) technicolor breaks the electroweak symmetry correctly in the presence of the scalar used to give mass to the light fermions. In Appendix B we present the fit to the electroweak data.

II. TECHNICOLOR AND THE TOP QUARK MASS

Our model includes the standard model gauge and fermion sectors together with a minimal technicolor sector intended to break the electroweak symmetry dynamically. The latter consists of an asymptotically free SU(N_{TC}) gauge group, which becomes strong at a scale of order 1 TeV, and one doublet of technifermions which transform under the SU(N_{TC})$\times$SU(3)$_C$$\times$SU(2)$_W$$\times$U(1)$_Y$ gauge group as

$$\Psi_L = \begin{pmatrix} P_L \\ N_L \end{pmatrix} : (N_{TC}, 1, 2)_0,$$

$$P_R : (N_{TC}, 1, 1)_{+1}, \qquad (2.1)$$

$$N_R : (N_{TC}, 1, 1)_{-1}.$$

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The dynamics of the technicolor interactions is taken from QCD: the $SU(2)_L \times SU(2)_R$ chiral symmetry of the technifermions is spontaneously broken by the condensates

$$\langle \bar{P}P \rangle \approx \langle \bar{N}N \rangle \approx 4\pi f^3 \left(\frac{3}{N_{TC}} \right)^{1/2}, \qquad (2.2)$$

where f, the technipion decay constant, is the analog of f_π in QCD. Since the $SU(2)_W \times U(1)_Y$ group is embedded in the chiral symmetry, the technifermion condensates break the electroweak symmetry. If minimal technicolor is the only source of electroweak symmetry breaking, then the observed W and Z masses require $f = v$, where $v \approx 246$ GeV is the electroweak scale.

The only constraints on this electroweak symmetry breaking mechanism come from the oblique radiative correction parameter S, which measures the momentum-dependent mixing of the neutral electroweak gauge bosons. The technifermion contribution to S can be estimated by using the QCD data [18,19]:

$$S \approx 0.1 N_{TC}. \qquad (2.3)$$

A fit to the electroweak data (using the standard model with a Higgs boson mass of 300 GeV as a reference) yields [13] a 1σ ellipse in the $S-T$ plane whose projection on the S-axis is $S = -0.09 \pm 0.34$. Thus, S in the minimal technicolor model is smaller than the 2σ upper bound provided $N_{TC} < 6$. The cancelation of the Witten anomaly for $SU(2)_W$ requires N_{TC} to be even. If the only interactions, in addition to technicolor, experienced by the technifermions were the electroweak interactions, then the value $N_{TC} = 2$ would be ruled out because the most attractive channel for condensation, $\langle P_L N^c + N_L P^c \rangle$, breaks the electroweak group completely [20]. However, the generation of quark and lepton masses requires additional interactions of the technifermions, which may easily tilt the condensate in the correct direction. For example, in Sec. IV A, we introduce a weak-doublet scalar to communicate electroweak symmetry breaking to the light fermions. As shown in Appendix A, the scalar's interactions with the technifermions would have a sufficiently large effect on the technifermion condensate to make the case $N_{TC} = 2$ viable. Therefore, in what follows we adopt the values

$$N_{TC} = 2, 4. \qquad (2.4)$$

In order to generate the large top quark mass, we have to specify some new physics at a scale of order 1 TeV that allows the minimal technicolor sector discussed so far to couple to the top quark. A particularly attractive alternative is to introduce a scalar multiplet, χ^t, which transforms under the $SU(N)_{TC} \times SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge group as

$$(\overline{N_{TC}}, 3, 2)_{4/3}. \qquad (2.5)$$

The most general Yukawa interactions are contained in

$$\mathcal{L}_t = C_q^j \overline{q_L'^j} N_R \chi^t + C_u^j \bar{\Psi}_L u_R'^j i\sigma_2 \chi^{t\dagger} + \text{H.c.}, \qquad (2.6)$$

where $j \in 1,2,3$ counts the generations, σ_i are the Pauli matrices; C_q^j, C_u^j are Yukawa couplings, $u_R'^j$ are the right-handed up-type quarks, and $q_L'^j \equiv (u_L'^j, d_L'^j)^\mathsf{T}$ are the left-handed quarks, defined in an arbitrary eigenstate. At first glance, it appears that all three generations of right-handed up-type and left-handed quarks couple to the χ^t scalar. However, these couplings are linear in the quark fields (unlike the bilinear quark couplings to the Higgs doublet in the standard model), and therefore only one linear combination of the three generations couples to χ^t. Because this is the combination that becomes heavy, it is identified by convention with the third generation in the weak eigenstate. Therefore, the Lagrangian in Eq. (2.6) is equivalent to

$$\mathcal{L}_t = C_q \overline{q_L^3} N_R \chi^t + C_t \bar{\Psi}_L t_R i\sigma_2 \chi^{t\dagger} + \text{H.c.}, \qquad (2.7)$$

where $q_L^3 \equiv (t_L, b_L)^\mathsf{T}$ is the left-handed weak eigenstate $t - b$ quark doublet, and the Yukawa coupling constants, C_q and C_t, are defined to be positive.

If the mass of the χ^t scalar is much larger than f, then at energies of the order of the electroweak scale the effects of χ^t exchange are well described by the following four-fermion operators:

$$\mathcal{L}_{4F} = -\frac{1}{2M_{\chi^t}^2} \left\{ C_q^2 (\bar{N}_R \gamma^\mu N_R)(\bar{q}_L^3 \gamma_\mu q_L^3) \right.$$

$$+ C_t^2 (\bar{\Psi}_L \gamma^\mu \Psi_L)(\bar{t}_R \gamma_\mu t_R) + \left[C_q C_t (\bar{\Psi}_L N_R)(\bar{t}_R q_L^3) \right.$$

$$\left. \left. + \frac{C_q C_t}{4} (\bar{\Psi}_L \sigma^{\mu\nu} N_R)(\bar{t}_R \sigma_{\mu\nu} q_L^3) \right] + \text{H.c.} \right\}. \qquad (2.8)$$

Upon technifermion condensation, the third operator in Eq. (2.8) induces a top quark mass

$$m_t \approx \frac{C_q C_t}{M_{\chi^t}^2} \pi f^3 \left(\frac{3}{N_{TC}} \right)^{1/2}. \qquad (2.9)$$

Using $m_t \approx 175$ GeV and $f \approx 246$ GeV we get

$$M_{\chi^t} \approx 570 \text{ GeV } \sqrt{C_q C_t} \left(\frac{2}{N_{TC}} \right)^{1/4}, \qquad (2.10)$$

which shows that the assumption $M_{\chi^t} \gg f$ is valid only if the Yukawa coupling constants are rather large. This situation seems plausible if χ^t is a bound state, but in this case loop corrections to the operators in Eq. (2.8) might need to be included in the low energy theory. Alternately, for C_q and C_t of order one or smaller, M_{χ^t} is not much larger than f and the technicolor dynamics might be modified by the existence of χ^t. With these limitations in mind, we will assume that the effects of χ^t are described sufficiently well by the operators in Eq. (2.8).

It is remarkable that the hypercharge of χ^t allows it to couple to t_R but not to b_R. As a consequence, with the field content discussed so far, the only standard fermion that becomes massive is the top quark. Provided that the other quark and lepton masses are produced by physics above the

technicolor scale, it will be natural for the top quark to be the heaviest fermion. This situation is in contrast with the case of a weak-singlet technicolored scalar, which can couple to both t_R and b_R [11,12,13] and needs the top-bottom mass ratio to be provided by a ratio of Yukawa couplings. Note that the models with weak-singlet techniscalars naturally explain the small Cabibbo-Kobayashi-Maskawa (CKM) elements associated with the third-generation quarks, because the t_L and b_L mass eigenstates are automatically aligned.

III. ELECTROWEAK OBSERVABLES AND χ^t

As mentioned earlier, if the mass of the χ^t scalar is much larger than f, at energies below the weak scale the effects of χ^t exchange are captured by the four-fermion operators in Eq. (2.8). One consequence of those four-fermion operators is the generation of a large mass for the top quark. Another, as we shall now discuss, is a significant contribution to the couplings of the $SU(2)_W \times U(1)_Y$ gauge bosons, W_i^μ ($i=1,2,3$) and B^μ, to the t and b quarks. This causes both direct and oblique corrections to electroweak observables.

Below the technicolor scale, only technipion dynamics has an impact on the electroweak observables, and these effects can be evaluated using an effective Lagrangian approach. Recalling that $f \approx v$ in our minimal one-doublet technicolor sector we find

$$\bar\Psi_L \gamma^\mu \Psi_L = i \frac{v^2}{2} \, \mathrm{Tr}(\Sigma^\dagger D^\mu \Sigma),$$

$$\bar N_R \gamma^\mu N_R = -i \frac{v^2}{2} \, \mathrm{Tr}\left(D^\mu \Sigma \, \frac{-\sigma_3 + 1}{2} \, \Sigma^\dagger\right),$$

$$\bar P_R \gamma^\mu P_R = -i \frac{v^2}{2} \, \mathrm{Tr}\left(D^\mu \Sigma \, \frac{\sigma_3 + 1}{2} \, \Sigma^\dagger\right), \quad (3.1)$$

where Σ transforms as $W\Sigma R^\dagger$ under $SU(2)_W \times SU(2)_R$ [where $SU(2)_R$ is the global symmetry which has $U(1)_Y$ as a subgroup]. Note that the last two terms in Eq. (2.8) do not affect the technipions to leading order. The covariant derivative is

$$D^\mu \Sigma = \partial^\mu \Sigma - ig \frac{\sigma^k}{2} W_k^\mu \Sigma + ig' \Sigma \frac{\sigma^3}{2} B^\mu, \quad (3.2)$$

which gives

$$\bar\Psi_L \gamma^\mu \Psi_L = 0,$$

$$\bar N_R \gamma^\mu N_R = \frac{v^2}{4} (g W_3^\mu - g' B^\mu) = \frac{v^2}{4} \frac{g}{c_W} Z^\mu$$

$$= -\bar P_R \gamma^\mu P_R. \quad (3.3)$$

The result is that, in addition to the standard model couplings of the $SU(2)_W \times U(1)_Y$ gauge bosons to third generation quarks, the following coupling is induced by the exchange of the χ^t scalar:

$$\mathcal{L}_{\mathrm{eff}} = \delta g \, \frac{g}{c_W} Z^\mu (\bar q_L \gamma_\mu q_L), \quad (3.4)$$

where

$$\delta g = -\frac{C_q}{C_t} \frac{m_t}{8\pi v} \left(\frac{N_{TC}}{3}\right)^{1/2} \approx -2.3 \times 10^{-2} \frac{C_q}{C_t} \left(\frac{N_{TC}}{2}\right)^{1/2}. \quad (3.5)$$

We will now explore the consequences of this additional coupling.

The first noticeable effect is on the oblique radiative parameter S [19]:

$$S \equiv -16\pi \left[\frac{d}{dq^2} \Pi_{3B}\right]_{q^2=0}. \quad (3.6)$$

In this model, S is given by

$$S = S^0 + S^{(t,b)}, \quad (3.7)$$

where S^0 is the technifermion contribution noted earlier in Eq. (2.3), and $S^{(t,b)}$ is an additional contribution from the effective coupling in Eq. (3.4) (see Ref. [13]),

$$S^{(t,b)} \approx \frac{4}{3\pi} \delta g \, \ln\left(\frac{\Lambda}{M_Z}\right) < 0, \quad (3.8)$$

and Λ is a scale of order 1 TeV. This negative contribution to S is certainly welcome.

Similarly, χ^t contributes to weak isospin violation, as measured by the parameter

$$T \equiv \frac{4}{\alpha v^2} [\Pi_{11}(0) - \Pi_{33}(0)], \quad (3.9)$$

where $\Pi_{ii}(q^2)$ are the vacuum polarizations of the W_i^μ gauge fields due to nonstandard model physics, with the gauge couplings factored out. The operator in Eq. (2.8) contributes directly to the T parameter:

$$T^{(t,b)} \approx -\delta g \, \frac{3m_t^2}{\pi^2 \alpha v^2} \ln\left(\frac{\Lambda}{m_t}\right) \approx -34.0 \delta g. \quad (3.10)$$

In addition to the direct isospin violation $T^{(t,b)}$, there are "indirect" contributions to T from the technifermion mass spectrum which can be only roughly estimated:

$$T^0 \sim \frac{N_{TC}}{16\pi^2 \alpha v^2} (\Sigma_P(0) - \Sigma_N(0))^2, \quad (3.11)$$

where $\Sigma_P(q^2)$ and $\Sigma_N(q^2)$ are the technifermion self-energies. In this model, the indirect isospin violation is due to the last two terms in Eq. (2.8). These four-fermion interactions induced by χ^t exchange give a one-loop correction to Σ_N which is quadratically divergent:

$$\Sigma_P(0) - \Sigma_N(0) = -\frac{3}{16\pi^3} \frac{m_t^2}{v^3} \Lambda'^2, \quad (3.12)$$

with Λ' a scale of order 1 TeV, potentially different than Λ. Putting these contributions together and taking $\Lambda = 1$ TeV in Eq. (3.8), we obtain

$$T \approx 4.2 \times 10^{-3} \frac{N_{TC}}{2} \left(\frac{\Lambda'}{1 \text{ TeV}} \right)^4 + 0.79 \frac{C_q}{C_t} \left(\frac{N_{TC}}{2} \right)^{1/2}, \tag{3.13}$$

$$S \approx 0.1 N_{TC} - 2.4 \times 10^{-2} \frac{C_q}{C_t} \left(\frac{N_{TC}}{2} \right)^{1/2}. \tag{3.14}$$

The 2σ upper bound $T < 0.71$ from Ref. [13] then suggests that the Yukawa coupling C_t must be at least as large as C_q; if Λ' is not significantly larger than 1 TeV, one finds $C_t \gtrsim C_q$.

In addition to the oblique corrections, the shift in the coupling of the Z boson to third-generation left-handed quarks also makes direct corrections to several observables measured at the Z-pole: the total Z decay width Γ_Z, the peak hadronic cross section σ_h, the rate of Z decays to b-quarks relative to other hadrons R_b, the front-back asymmetry in Z decays to b-quarks $A_{FB}(b)$, and the rates of Z decays to leptons relative to hadrons R_e, R_μ, R_τ. The oblique and direct corrections that χ^t causes in the full set of electroweak observables are summarized in Appendix B. We derived the expressions by adapting the analysis of [21] to our model and using Eqs. (3.5), (3.13) and (3.14) to write the results in terms of C_q and C_t.

We used a least-squares fit to evaluate the models' agreement with the electroweak data [22,13] for different values of N_{TC}; the resulting values of the observables are given in Table I. For $N_{TC} = 2$, a fit setting $\Lambda' = 1$ TeV yields a ratio of Yukawa couplings

$$\frac{C_q}{C_t} = 0.025 \pm 0.013. \tag{3.15}$$

The central value has $\chi^2/N_{\text{DOF}} \approx 30.5/21$, which corresponds to a goodness of fit $P(N_{\text{DOF}}, \chi^2) = 8.3\%$. Leaving both C_q/C_t and Λ' free yields best-fit values

$$\frac{C_q}{C_t} = 0.017 \pm 0.033, \quad \Lambda' = 2.4 \pm 0.36 \text{ TeV} \tag{3.16}$$

with $\chi^2/N_{\text{DOF}} \approx 28/20$ and $P(N_{\text{DOF}}, \chi^2) = 11\%$. The goodness-of-fit is comparable to, or slightly better than, that of the standard model $[P(N_{\text{DOF}}, \chi^2) = 6\%$ for a Higgs boson mass of 300 GeV] as evaluated in [13]. For the best-fit values of the model parameters, the predicted value of each observable is within 3σ of the experimental value (except for A_{LR}, which is slightly further away). Moreover, the error ellipses for the model parameters overlap the region of parameter space ($\Lambda' \sim 2.3$ TeV, $C_q/C_t < 0.07$) in which all of the observables are within 3σ of their experimental values. In contrast, the fits for $N_{TC} = 4$ are much poorer, with goodness-of-fit less than 1%, and it is never possible to have all observables within 3σ of their experimental values. The larger value of N_{TC} increases S^0 enough to prevent Γ_Z and A_{LR} from simultaneously agreeing with experiment.

TABLE I. Experimental and standard model values [22,13] and predicted values of electroweak observables for $N_{TC} = 2$ and 4. Both C_q/C_t and Λ' were set equal to their best-fit values: for $N_{TC} = 2(4)$, these are $C_q/C_t = 0.017$ (0.0027) and $\Lambda' = 2.4$ TeV (3.0 TeV).

Quantity	Experiment	SM	$N_{TC} = 2$	$N_{TC} = 4$
Γ_Z	2.4948 ± 0.0025	2.4966	2.4977	2.4986
R_e	20.757 ± 0.056	20.756	20.756	20.756
R_μ	20.783 ± 0.037	20.756	20.756	20.756
R_τ	20.823 ± 0.050	20.756	20.756	20.756
σ_h	41.486 ± 0.053	41.467	41.467	41.467
R_b	0.2170 ± 0.0009	0.2158	0.2173	0.2162
R_c	0.1734 ± 0.0008	0.1723	0.1718	0.1720
A_{FB}^e	0.0160 ± 0.0024	0.0162	0.0156	0.0151
A_{FB}^μ	0.0163 ± 0.0014	0.0162	0.0156	0.0151
A_{FB}^τ	0.0192 ± 0.0018	0.0162	0.0156	0.0151
$A_\tau(P_\tau)$	0.1411 ± 0.0064	0.1470	0.1443	0.1424
$A_e(P_\tau)$	0.1399 ± 0.0073	0.1470	0.1443	0.1424
A_{FB}^b	0.0984 ± 0.0024	0.1031	0.1014	0.1000
A_{FB}^c	0.0741 ± 0.0048	0.0736	0.0722	0.0712
A_{LR}	0.1550 ± 0.0034	0.1470	0.1443	0.1424
M_W	80.41 ± 0.09	80.375	80.375	80.375
$g_L^2(\nu N \to \nu X)$	0.3003 ± 0.0039	0.3030	0.3035	0.3041
$g_R^2(\nu N \to \nu X)$	0.0323 ± 0.0033	0.0300	0.0302	0.0303
$g_{eA}(\nu e \to \nu e)$	-0.503 ± 0.018	-0.507	-0.5076	-0.5083
$g_{eV}(\nu e \to \nu e)$	-0.025 ± 0.019	-0.037	-0.036	-0.036
$Q_W(Cs)$	-7.211 ± 0.93	-72.88	-73.04	-73.20
$R_{\mu\tau}$	0.9970 ± 0.0073	1.0	1.0	1.0

Overall, a one-doublet technicolor model with an extra technicolored χ^t scalar gives reasonable agreement with electroweak data only for $N_{TC} = 2$. The model presented thus far is incomplete, as it does not provide masses for the light quarks and leptons. We will see that the additional physics required to address this goal also causes the vacuum alignment of $SU(2)_{TC}$ to occur in the pattern that breaks the electroweak symmetry appropriately.

IV. MASSES FOR THE OTHER QUARKS AND LEPTONS

We turn, now, to addressing the origin of the masses of the other quarks and leptons. These much smaller masses can be generated by physics well above the electroweak scale. Note that an additional small contribution to the top quark's mass may also result from this physics.

One question that will naturally arise when the origins of the other quarks' masses and mixing angles are considered is the extent to which flavor-changing neutral currents constrain the model. Such considerations are unlikely to place significant limits on the properties of χ^t. The strength of the χ^t state's interactions with the d and s quarks is not particularly large, being given roughly by the size of the intergenerational mixing (i.e., of order 0.1–0.01). Furthermore, χ^t couples only to left-handed down-type quarks. So any constraints arising from χ^t exchange in the box diagrams for $K^0 \bar{K}^0$ and $B^0 \bar{B}^0$ mixing or the loop diagrams for $b \to s\gamma$

would limit only the mixing angles of the left-handed quarks. If the flavor-symmetry-breaking mixings for down-type quarks are largely in the right-handed sector, extra flavor changing neutral current (FCNC) contributions from χ^t will be suppressed. Since the χ^t is the only new physics that couples to the large mass of the top quark, this line of reasoning suggests that FCNC need not be a problem in the class of models we are considering.

A. Weak-doublet technicolor-singlet scalar, ϕ

As a simple realization of the higher-scale physics responsible for the light fermions' masses, we consider the existence of a scalar, ϕ, which transforms as $(1,1,2)_{+1}$ under the (technicolor×standard model) gauge group. Although its quantum numbers are the same as those of the standard model Higgs doublet, the behavior of ϕ is considerably different because we assume that its mass-squared is *positive*, as in Ref. [6]. Furthermore, ϕ need not couple predominantly to the top quark. Following Ref. [6], we allow the most general Yukawa couplings of ϕ to technifermions,

$$\lambda_+ \bar{\Psi}_L P_R i \sigma_2 \phi^\dagger + \lambda_- \bar{\Psi}_L N_R \phi + \text{H.c.,} \qquad (4.1)$$

and also to the quarks and leptons. If M_ϕ is larger than the technicolor scale, or if $\lambda_\pm$ is smaller than order one, then the effect of ϕ on the technicolor dynamics is small, as we make explicit below. When the technifermions condense, the interactions in expression (4.1) give rise to a tadpole term for ϕ,

$$\pi f^3 \left(\frac{3}{N_{TC}} \right)^{1/2} (\lambda_+ + \lambda_-)(1 - \sigma_3)\phi + \text{H.c.,} \qquad (4.2)$$

which leads to a vacuum expectation value (VEV):

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ f' \end{pmatrix}. \qquad (4.3)$$

If the quartic scalar operators have small coefficients, then

$$f' \approx 2\sqrt{2}\pi f^3 \frac{\lambda_+ + \lambda_-}{M_\phi^2} \left(\frac{3}{N_{TC}} \right)^{1/2}. \qquad (4.4)$$

The electroweak symmetry is broken not only by the technifermion condensates, but also by the VEV of ϕ, so that the electroweak scale is given by

$$v = \sqrt{f^2 + f'^2}. \qquad (4.5)$$

The four real scalar components of the ϕ doublet form an isotriplet and an isosinglet. In general, the isotriplet mixes with the three technipions, forming the longitudinal W and Z, as well as a triplet of physical pseudoscalars [7]. Finally, as discussed in Appendix A, the interaction between ϕ and the technifermions affects the vacuum alignment enough to make even SU(2)$_{TC}$ viable

We assume that the bulk of the top mass is given by the technicolor sector, as discussed in Sec. II. In this case, f cannot be much smaller than v, suggesting

$$f' \ll f \approx v, \qquad (4.6)$$

so that the longitudinal W and Z are predominantly composed of technipions.

The inclusion of the ϕ doublet in the technicolor model with a weak-doublet techniscalar provides masses for all the standard fermions while evading the usual constraints on the standard model Higgs boson, which arise from the requirement that the Higgs doublet be responsible for electroweak symmetry breaking and for the top quark's mass. The contributions from ϕ to the elements of the quark and lepton mass matrices are given by $f'/\sqrt{2}$ times the corresponding Yukawa coupling constants. The top quark's mass separately receives the contribution estimated in Eq. (2.9).

A lower bound for f' comes from requiring the b-quark's Yukawa coupling constant not to be larger than order one:

$$f' \gtrsim \sqrt{2}m_b. \qquad (4.7)$$

Due to the relation (4.4) between the ϕ mass and VEV, the bounds on f' from Eqs. (4.6) and (4.7) impose constraints on M_ϕ:

$$1 \text{ TeV} \gtrsim \frac{M_\phi}{\sqrt{\lambda_+ + \lambda_-}} \gtrsim 5 \text{ TeV.} \qquad (4.8)$$

The major advantage of such a model, as far as predicting the fermion spectrum is concerned, lies in the fact that the top quark is naturally the heaviest. Note also that it appears that a theory of a composite ϕ, in which the Yukawa couplings are determined, is in principle easier to construct than in the case of the standard model Higgs boson, because one does not have to worry about the W, Z and t masses. However, the low-energy Yukawa couplings of ϕ are no more constrained by theory than those of the standard model Higgs boson.

The presence of heavy weak-doublet scalars ϕ need not greatly alter the low-energy electroweak or flavor-changing neutral current phenomenology of the model. First, consider the electroweak effects. Recall [cf. Eq. (4.5)] that the technifermion condensate will generate a small VEV (f') for ϕ, and it is the combination of decay constants $f^2 + f'^2$ which now equals $v \approx 246$ GeV. The factor v^2 in Eq. (3.3) therefore becomes an f^2 and the expression (3.5) for the coupling shift will be multiplied by the ratio f^2/v^2. Yet the net effect must be small in order for our analysis to remain self-consistent: according to Eq. (2.9), keeping the top quark mass fixed while lowering f requires either raising the values of the Yukawa couplings C_q and C_t or reducing the mass of χ_t—both of which are problematic. A further effect of the presence of ϕ is to make an additional contribution to the S parameter (3.7); as long as $f' \ll v$, however, this contribution will be negligible [6]. There are also contributions from ϕ to the T parameter to the extent that the coupling of technifermions to ϕ violates weak isospin; again, these can be made small [6]. Finally, we come to flavor-changing neutral currents. The size of the contributions ϕ makes to $K^0 \bar{K}^0$ mixing, $B^0 \bar{B}^0$ mixing, or $b \to s\gamma$ is proportional to powers of the ϕ state's Yukawa couplings to quarks. Such contributions have

been found [6,11,8] to be significant in the case where the Yukawa coupling of ϕ to t is large enough to generate the full top quark mass. In our model, however, ϕ need contribute no more to m_t than to m_b; this suppresses the extra FCNC contributions by several powers of m_b/m_t, making them far less restrictive.

B. Weak-doublet technicolored scalars, χ_f

The situation is different in models with additional weak-doublet technicolored scalars. In addition to χ^t, there are only three scalar representations of the (technicolor×SM) gauge group, χ_b, χ_τ and χ_{ν_τ}, that can have Yukawa couplings involving a technifermion and a standard fermion:

$$\chi_b:(\overline{N_{TC}},3,2)_{-2/3}, \quad \chi_\tau:(\overline{N_{TC}},1,2)_{-2}, \quad \chi_{\nu_\tau}:(\overline{N_{TC}},1,2)_0. \tag{4.9}$$

The most general Yukawa couplings of χ_b, for conveniently chosen quark eigenstates, are given by

$$\mathcal{L}_b = C_q^b \overline{q_L^3} P_R \chi_b + C_q^{b'} \overline{q_L^2} P_R \chi_b + C_b \overline{\Psi}_L b_R i\sigma_2 \chi_b^\dagger + \text{H.c.} \tag{4.10}$$

Note that after a U(3) flavor redefinition as in the case of Eq. (2.7), only one down-type right-handed quark, namely b_R, couples to χ_b. For left-handed quarks, however, only a U(2) flavor transformation is available, because q_L^3 is already defined by Eq. (2.7). Therefore, χ_b couples to both q_L^2 and q_L^3. As a result, both a b quark mass,

$$m_b \approx \frac{C_q^b C_b}{M_{\chi_b}^2} \pi f^3 \left(\frac{3}{N_{TC}}\right)^{1/2}, \tag{4.11}$$

and a $b-s$ quark mixing are induced (it is, thus, necessary that $C_q^b \gg C_q^{b'}$). Comparing Eqs. (2.9) and (4.11) one can see that the ratio $m_t/m_b \approx 40$ can have its origin in a scalar mass ratio

$$\frac{M_{\chi_b}}{M_{\chi^t}} \approx 6.5, \tag{4.12}$$

instead of a large ratio of Yukawa coupling constants.

Likewise, the most general Yukawa couplings of χ_τ are given by

$$\mathcal{L}_\tau = C_l \overline{l_L^3} P_R \chi_\tau + C_\tau \overline{\Psi}_L \tau_R i\sigma_2 \chi_\tau^\dagger + \text{H.c.}, \tag{4.13}$$

where $l_L^3 = (\nu_\tau, \tau)^\mathsf{T}$ is the left-handed third generation lepton. The τ mass is produced by the exchange of χ_τ, with the condition

$$M_{\chi_\tau} \approx 5.7 \text{ TeV} \sqrt{C_l C_\tau} \left(\frac{2}{N_{TC}}\right)^{1/4}. \tag{4.14}$$

Including the scalar χ_{ν_τ} would be useful only for producing a Dirac mass for ν_τ.

A second generation of weak-doublet techniscalars would give masses to the second generation of quarks and leptons

(this scenario is discussed briefly for the case of weak-singlet techniscalars in [11]). It is possible then to trade all the large ratios of Yukawa couplings required in the standard model for smaller ratios of scalar masses, with the hope that the scalar spectrum is correctly produced by the high energy theory responsible for scalar compositeness or supersymmetry breaking.

Exchange of the numerous technicolored scalars χ^f would make contributions to electroweak radiative corrections analogous to those from χ^t. However, such effects are suppressed relative to the effects of χ^t by the ratio of the lighter fermion mass to m_t. Since the minimum suppression is by a factor of 40, these corrections are small enough to ignore.

V. COMPOSITE SCALARS

In the previous sections we showed that the inclusion of scalar fields in a minimal technicolor model may be useful in explaining certain features of the quark and lepton spectrum, such as the heaviness of the top quark, or the mass hierarchy between the third generation and the others. However, in the absence of supersymmetry, the existence of scalar fields much lighter than the Planck scale is natural only if these scalars are composite. In this section we discuss various possibilities for the existence of fermion-antifermion states bound by new dynamics at a scale of order a TeV, or higher.

A. Quark-technifermion bound states

The simplest possibility that leads to compositness for the scalars discussed in Secs. II and IV, is the existence of a new nonconfining gauge interaction that binds together standard fermions and technifermions. In this case, the χ^t techniscalar that is responsible for the top quark mass can be a $\overline{t}_R \Psi_L$ or a $\overline{N}_R q_L^3$ state. More generally, both these states are present, and because they have the same transformation properties under the (technicolor×SM) gauge group, a large mixing between them is induced by technicolor interactions. The $\overline{t}_R \Psi_L$ composite scalar (labeled χ_R^t) has a large Yukawa coupling to the t_R and $\overline{\Psi}_L$ fields, while the $\overline{N}_R q_L^3$ composite scalar (χ_L^t) couples to $\overline{q}_L^3$ and N_R. This situation is reminiscent of the supersymmetric technicolor models of Refs. [10,12], where a combination of superpotential holomorphy and gauge anomaly cancellation requires the existence of *two* techniscalars which mix. Because of the scalar mixing, the exchange of the two physical scalar states gives rise to four-fermion operators as in Eq. (2.8), but with modified coefficients: $M_{\chi^t}^2$ is replaced by

$$M_{\chi_L^t}^2 - \frac{M_{\chi_{LR}^t}^4}{M_{\chi_R^t}^2}, \quad M_{\chi_R^t}^2 - \frac{M_{\chi_{LR}^t}^4}{M_{\chi_L^t}^2}, \quad \frac{M_{\chi_L^t}^2 M_{\chi_R^t}^2}{M_{\chi_{LR}^t}^2} - M_{\chi_{LR}^t}^2,$$

respectively in the first, second, and last two terms of $\mathcal{L}_{4F}$ [also in Eqs. (2.9) and (2.10)], where $M_{\chi_L^t}$ and $M_{\chi_R^t}$ are the χ_L^t and χ_R^t masses, and $M_{\chi_{LR}^t}$ is the mass mixing. Conse-

quently, the results of Sec. IV survive, modified only by having the ratio C_q/C_t multiplied by $M^2_{\chi^t_R}/M^2_{\chi^t_{LR}}$.

An example of the nonconfining interaction that can bind together the top quark fields and technifermions is a $U(1)_{new}$ gauge symmetry, attractive in the $\bar{t}_R\Psi_L$ and $\bar{N}_R q^3_L$ channels, and broken at a scale in the TeV range. In order to form sufficiently narrow bound states, this interaction has to be rather strong, though it need not be strong enough to produce fermion/antifermion condensates. Avoiding a Landau pole for the $U(1)_{new}$ gauge coupling requires the embedding of $U(1)_{new}$ in a non-Abelian gauge group at a scale just slightly higher than the composite scalar masses. There are also several constraints on the $U(1)_{new}$ charges. First, exchange of the heavy $U(1)_{new}$ gauge boson gives rise to four-technifermion operators; to prevent these operators from making a large contribution to the isospin breaking parameter T, the $U(1)_{new}$ charges of P_R and N_R must be equal. Second, requiring anomaly cancellation imposes relations between the various fermions' $U(1)_{new}$ charges. These relations constrain the co-efficients of the four-fermion operators induced by the strong $U(1)_{new}$. For example, in the simplest scenario the only fields charged under $U(1)_{new}$ are the third generation fermions (including the right-handed neutrino[1]) and the technifermions (with P_R and N_R having equal charges). In this case, the $\bar{\tau}_R l^3_L$ and $\bar{\nu}_{\tau_R} l^3_L$ channels turn out to be much more attractive than $\bar{t}_R\Psi_L$ and $\bar{N}_R q^3_L$. As a result, a couple of composite ϕ scalars will form and will even be narrower than the $\chi^t_{L,R}$ scalars. Other attractive channels lead to the formation of lepto-quarks and color-octet scalars. The current limited knowledge of strongly coupled field theory does not allow us to decide whether the $U(1)_{new}$ gauge group gives rise to the precise scalar spectrum we need.

An alternative method for producing top-technifermion bound states from nonconfining interactions might arise in composite models. If the top quark and technifermion fields were composites created by some underlying high-energy theory, additional top-technifermion interactions able to produce the requisite scalar states could be present.

B. New fermions as constituents

An alternative possibility is that composite χ^t scalar could be produced by the action of a new strong gauge group $SU(n)_{NJL}$ on a set of additional fermions charged under that group.

For example, consider two Dirac fermions, A and B, which transform under the gauge groups as shown in Table II. Note that these new fermions are vectorlike, so that they do not have large contributions to the S and T parameters. If the $SU(n)_{NJL}$ gauge group is spontaneously broken and undercritical, a $\bar{B}A$ scalar with positive mass-squared will be formed, as can be proven in the large n limit (as has been shown [1] in the Nambu–Jona-Lasinio model [23]). This

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[1]Otherwise the anomaly-cancellation conditions would make the $\bar{t}_R\Psi_L$ or $\bar{N}_R q^3_L$ channels repulsive.

TABLE II. Fermion constituents of χ^t.

	$SU(n)_{NJL}$	$SU(4)_{TC}$	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$
A	□	□	**1**	**1**	y
B	□	**1**	□	□	$4/3+y$

bound state has the transformation properties of χ^t. Inducing the Yukawa interactions of χ^t requires additional 4-fermion operators:

$$\mathcal{L}_{4f}=\frac{1}{M^2}\,\bar{q}^3_L N_R \bar{B}A + \frac{1}{M'^2}\,\epsilon_{ij}\bar{\Psi}_{L_i} t_R \bar{A}_j B + \text{H.c.}, \quad (5.1)$$

which must be provided by physics at higher scales. The χ^f and ϕ scalars may have a similar origin.

C. Strongly coupled ETC

Finally, we note that a composite technicolor-singlet scalar ϕ state and its couplings to fermions could result from strongly-coupled extended technicolor (ETC) interactions between standard fermions and technifermions [16]. The fact that ϕ need not provide the entire large mass of the top quark would provide useful flexibility in keeping the model's phenomenology consistent with experiment. Given that the quarks and technifermions need to belong to the same ETC multiplets, it is even possible that strongly-coupled ETC could give rise to a composite χ^t techniscalar.

VI. CONCLUSIONS

We have introduced a class of technicolor models in which the top quark mass is produced by exchange of a weak-doublet technicolored scalar multiplet. Such models can explain the large top quark mass while remaining consistent with precision electroweak data.

A single scalar multiplet χ^t with $SU(N)_{TC}\times SU(3)_C \times SU(2)_W \times U(1)$ charges $(\bar{N}_{TC},3,2)_{4/3}$ gives mass only to a single up-type quark. The hypercharge quantum number prevents this scalar from coupling to the right-handed bottom quark and generating m_b. Thus, a model containing a χ^t scalar naturally produces both the large top quark mass and the large splitting between m_t and m_b. We have identified several promising dynamical mechanisms through which χ^t could be created as a fermion/anti-fermion bound state. These include models with no new fermions and a strong $U(1)$ gauge interaction, models with new fermions as subconstituents of χ_t, and models with strongly-coupled extended technicolor interactions.

A minimal model including one-doublet $SU(2)_{TC}$ technicolor and a χ^t scalar is in agreement with the precision electroweak data. For larger N_{TC} the agreement is poor; additional physics would be required to ammend the corrections to the electroweak observables.

The simplest way of generating the masses of the lighter fermions is to include a weak-doublet technicolor-singlet scalar ϕ. Including ϕ has several virtues. The vacuum alignment of $SU(2)_{TC}$ produces the correct electroweak symme-

try breaking pattern in the presence of a ϕ scalar. The presence of ϕ need not modify the successful match between the minimal model and the electroweak data. The ϕ scalar can be readily generated by the same dynamics that produces a χ^t bound state. So we have a complete and appealing package.

Another alternative is for the masses of some of the lighter fermions to be created by additional weak-doublet technicolored scalars χ_f. This could neatly explain fermion mass hierarchies in terms of relationships among the χ_f masses while leaving intact the agreement between the predicted and measured electroweak observables.

APPENDIX A: VACUUM ALIGNMENT IN SU(2) TECHNICOLOR

In this appendix we show that $SU(2)_{TC}$ technicolor correctly breaks the electroweak symmetry in the presence of the weak-doublet ϕ scalar discussed in Sec. IV A.

We start by reviewing the argument against minimal $SU(2)_{TC}$ technicolor [20,24]. The $SU(2)_{TC}$ group has only real representations, so that, in the absence of the electroweak interactions, there is an $SU(4)_F$ chiral symmetry acting on the P_L, N_L, P^c, N^c technifermions (P^c and N^c are the charge conjugate fermions corresponding to P_R and N_R). At the scale where $SU(2)_{TC}$ becomes strong, the technifermions condense and break the chiral symmetry down to Sp(4). Therefore, the vacuum manifold is Sp(4) symmetric, which implies that the two condensation channels, $\langle P_L P^c \rangle = \langle N_L N^c \rangle$ and $\langle P_L N_L \rangle = \langle P^c N^c \rangle$, are equally attractive.

In the presence of the $SU(2)_W \times U(1)_Y$ interactions, the degeneracy of the vacuum manifold is lifted, Exchange of the $SU(2)_W \times U(1)_Y$ gauge bosons, W^a_μ and B_μ, contributes at one loop to the $P_L N_L$ and $P^c N^c$ dynamical masses, respectively, but makes no contribution to the $P_L P^c$ or $N_L N^c$ dynamical masses. The net effect is

$$\delta(M_{P_L N_L} + M_{P^c N^c}) \approx \frac{3g^2 + g'^2}{8\pi^2} M_{P_L N_L} \ln\left(\frac{M_{TC}}{M_{P_L N_L}}\right) > 0,$$
(A1)

where $M_{P_L N_L} \sim 1$ TeV (given by scaling the constituent quark mass from QCD), M_{TC} is a physical cutoff of order $4\pi M_{P_L N_L}$, while g and g' are the $SU(2)_W \times U(1)_Y$ gauge couplings. The increase in the dynamical masses implies that

the $SU(2)_W \times U(1)_Y$ interactions make the $\langle P_L N_L \rangle \approx \langle P^c N^c \rangle$ channel more attractive than $\langle P_L P^c \rangle = \langle N_L N^c \rangle$, leading to a complete breaking of the electroweak symmetry. Thus, minimal $SU(2)_{TC}$ technicolor is not viable on its own.

However, the vacuum is tilted in the wrong direction only by the electroweak interactions, which are a small perturbation on the technicolor dynamics. Additional interactions of the technifermions may easily change the vacuum alignment. Consider the effect of the ϕ scalar that communicates electroweak symmetry breaking to the lighter quarks and leptons in the scenarios discussed in Sec. IV A. In the vacuum where $\langle P_L P^c \rangle = \langle N_L N^c \rangle$, ϕ acquires a VEV which enhances the $P_L P^c$ and $N_L N^c$ masses by an amount [see Eqs. (4.1) and (4.4)]

$$\delta(M_{P_L P^c} + M_{N_L N^c}) \approx \frac{(\lambda_+ + \lambda_-)^2}{M_\phi^2} 2\pi f^3 \left(\frac{3}{N_{TC}}\right)^{1/2} > 0.$$
(A2)

The correct symmetry breaking pattern, $SU(2)_W \times U(1)_Y \rightarrow U(1)_Q$, requires $\langle P_L P^c \rangle = \langle N_L N^c \rangle$ to be the most attractive channel, which is satisfied provided the contribution in Eq. (A2) is larger than the one in Eq. (A1). This is equivalent to the requirement

$$\frac{M_\phi}{\lambda_+ + \lambda_-} \lesssim 1.6 \text{ TeV},$$
(A3)

which is consistent with Eq. (4.8) and the assumption that the $\lambda_\pm$ Yukawa couplings are of order one.

APPENDIX B: ELECTROWEAK OBSERVABLES

In this appendix, adapting the analysis of [21], we present the expressions for the electroweak observables in our models relative to those in the standard model. The contributions from a minimal technicolor sector and χ^t are included explicitly; as argued in the text, the effects of additional ϕ or χ^f scalars would be negligible by comparison.

Using the convenient notation

$$r \equiv 10^{-2} \sqrt{\frac{N_{TC}}{2}},$$
(B1)

the values of the electroweak observables may be expressed as follows:

$$\Gamma_Z = (\Gamma_Z)_{SM}\left[1 - 7.7r^2 - 0.44r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 2.5r\frac{C_q}{C_t}\right],$$

$$R_{e,\mu,\tau} = (R_{e,\mu,\tau})_{SM}$$
$$\times\left[1 - 5.8r^2 - 8.2 \times 10^{-2}r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 2.5r\frac{C_q}{C_t}\right],$$

$$\sigma_h = (\sigma_h)_{SM}$$
$$\times\left[1 + 0.44r^2 + 6.7 \times 10^{-3}r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 - 0.95r\frac{C_q}{C_t}\right],$$

$$R_b = (R_b)_{SM}\left[1 + 1.3r^2 - 1.7 \times 10^{-2}r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 8.3r \frac{C_q}{C_t}\right],$$

$$R_c = (R_c)_{SM}\left[1 + 2.6r^2 - 4.2 \times 10^{-2}r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 - 2.3r \frac{C_q}{C_t}\right],$$

$$A_{FB}^{e;\mu,\tau} = (A_{FB}^{e;\mu,\tau})_{SM} - 14r^2 - 0.2r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 0.39r \frac{C_q}{C_t},$$

$$A_{e,\tau}(P_\tau) = (A_{e,\tau}(P_\tau))_{SM} - 57r^2$$
$$- 0.84r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 1.6r \frac{C_q}{C_t},$$

$$A_{FB}^b = (A_{FB}^b)_{SM} - 380r^2 - 0.56r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 1.2r \frac{C_q}{C_t},$$

$$A_{FB}^c = (A_{FB}^c)_{SM} - 290r^2 - 0.43r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 0.85r \frac{C_q}{C_t},$$

$$A_{LR} = (A_{LR})_{SM} - 57r^2 - 0.84r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 5.6r \frac{C_q}{C_t},$$

$$M_W = (M_W)_{SM}\left[1 - 7.2r^2 - 0.23r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 0.44r \frac{C_q}{C_t}\right],$$

$$g_L^2(\nu N \to \nu X) = (g_L^2(\nu N \to \nu X))_{SM} - 5.4r^2$$
$$- 0.28r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 + 0.53r \frac{C_q}{C_t},$$

$$g_R^2(\nu N \to \nu X) = (g_R^2(\nu N \to \nu X))_{SM} + 1.9r^2$$
$$- 8.0 \times 10^{-3}r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 - 0.017r \frac{C_q}{C_t},$$

$$g_{eA}(\nu e \to \nu e) = (g_{eA}(\nu e \to \nu e))_{SM} - 0.17r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4$$
$$- 0.31r \frac{C_q}{C_t},$$

$$g_{eV}(\nu e \to \nu e) = (g_{eV}(\nu e \to \nu e))_{SM} + 15r^2$$
$$- 0.23r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4 - 0.44r \frac{C_q}{C_t},$$

$$Q_W(C_s) = (Q_W(C_s))_{SM} - 1.6 \times 10^3 r^2 + 0.47r^2\left(\frac{\Lambda'}{1 \text{ TeV}}\right)^4$$
$$+ 1.0r \frac{C_q}{C_t},$$

$$R_{\mu\tau} \equiv \frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})} = R_{\mu\tau}^{SM}. \qquad \text{(B2)}$$

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